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Online Appendix for "Multinational Banks and Financial Stability."

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A Proofs

Proofs from the baseline model, the macroprudential regulation model, and the general model are contained in this appendix. It is worthwhile to note that the baseline model (and the macropru section) is an application of the model of Section 6. However, for transparency we present direct proofs for these results.

A.1 Competitive Equilibrium FOCs (Section 2.3)

The bank Lagrangian is (without loss of generality, multiplying utility by a weight $\omega_i > 0$, in anticipation of the planning problem)

$$
\begin{aligned}
\mathcal{L}_i = & \omega_i \int_s c_i(s) f(s) ds + \lambda_i^0 \left[A_i + D_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right] \\
& + \int_s \lambda_i^1(s) \Big[\gamma_i(s) L_{ii}(s) + (1+r_{ii})(R_i(s) I_{ii} - L_{ii}(s)) \\
& \qquad + \int_j \Big[\gamma_j(s) L_{ij}(s) + (1+r_{ij})(R_j(s) I_{ij} - L_{ij}(s)) \Big] dj - c_i(s) - D_i \Big] f(s) ds \\
& + \int_s \Lambda_i^1(s) \left[-D_i + \gamma_i(s) L_{ii}(s) + \int_j \gamma_j(s) L_{ij}(s) dj + (1-h_i(s)) \mathcal{C}_{ii}(s) + \int_j (1-h_j(s)) \mathcal{C}_{ij}(s) dj \right] f(s) ds \\
& + \int_s \left[\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ii}(s)(R_i I_{ii} - L_{ii}(s)) + \int_j \left(\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ij}(s)(R_j I_{ij} - L_{ij}(s)) \right) \right] f(s) ds
\end{aligned}
$$

where we recall that $\mathcal{C}_{ij}(s) = \gamma_j(s) \left[R_j(s) I_{ij} - L_{ij}(s) \right]$.

FOC for I_{ij}. Taking the first order condition in I_{ij}, we obtain

$$
0 \geq -\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial I_{ij}} + E\left[\lambda_i^1 (1+r_{ij}) R_j \right] + E\left[\Lambda_i^1 (1-h_j) \gamma_j R_j \right] + E\left[\overline{\xi}_{ij} R_j \right].
$$

Expanding the first expectation, we obtain the result.

FOC for $L_{ij}(s)$. Taking the first order condition in $L_{ij}(s)$, we obtain

$$0 = \lambda_i^1(s)(\gamma_j(s) - (1 + r_{ij}))f(s) + \Lambda_i^1(s)(\gamma_j(s) - (1 - h_j(s))\gamma_j(s))f(s) + \underline{\xi}_{ij}(s)f(s) - \overline{\xi}_{ij}(s)f(s)$$

which simplifies to the result.

A.2 Proof of Proposition 1

Give the definition of the private bank Lagrangian in Section A.1, we can define the Lagrangian of the global planner as (recall that we have already incorporated the welfare weights ω_i into the private Lagrangians)

$$\mathcal{L}^G = \int_{i'} \mathcal{L}_{i'} di - \lambda^0 \int_{i'} \mathcal{T}_{i'} di',$$

where $\int_{i'} \mathcal{T}_{i'} di' = 0$ are inter-country transfers (so that $\mathcal{T}_i$ increases A_i). First, optimal transfers satisfy

$$0 = \lambda_i^0 - \lambda^0$$

so that $\lambda_i^0 = \lambda^0$ for all i (date-0 weighted marginal value of wealth is equalized across countries). Now, consider the social optimality for liquidations $L_{ji}(s)$ for either $i = j$ or $i \neq j$. The social optimality condition is

$$0 = \frac{\partial \mathcal{L}_j}{\partial L_{ji}(s)} + \frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di'$$

Next, consider the decision rule of private banks, who are subject to wedges τ. Their decision rule for liquidations is

$$0 = \frac{\partial \mathcal{L}_j}{\partial L_{ji}(s)} - \lambda_i^0 \tau_{ji}^L(s) f(s),$$

accounting for the effect of the wedge. Combining these equations, we obtain

$$\tau_{ji}^L(s) = -\frac{1}{\lambda_j^0} \frac{1}{f(s)} \frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di'.$$

It remains now to evaluate the derivative. From the private Lagrangian definition, we have for $i' \neq i$

$$\frac{\partial \mathcal{L}_{i'}}{\partial L_i^A(s)} = \lambda_{i'}^1(s)\frac{\partial \gamma_i(s)}{\partial L_i^A(s)} L_{i'i}(s) f(s) + \Lambda_{i'}^1(s)\left[\frac{\partial \gamma_i(s)}{\partial L_i^A(s)} L_{i'i}(s) + (1-h_i(s))\frac{\partial \mathcal{C}_{i'i}(s)}{\partial L_i^A(s)}\right] f(s)$$

while for $i' = i$, we have $\frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} di$ equal to the same expression (due to home bias). From the definition, we have $\frac{\partial \mathcal{C}_{i'i}(s)}{\partial L_i^A(s)} = \frac{\partial \gamma_i(s)}{\partial L_i^A(s)}\left[R_i(s)I_{i'i} - L_{i'i}(s)\right]$, so that we have

$$\frac{\partial \mathcal{L}_{i'}}{\partial L_i^A(s)} = \lambda_{i'}^0 \frac{\partial \gamma_i(s)}{\partial L_i^A(s)}\left[\frac{\lambda_{i'}^1(s)}{\lambda_{i'}^0} L_{i'i}(s) + \frac{\Lambda_{i'}^1(s)}{\lambda_{i'}^0}\left[L_{i'i}(s) + (1-h_i(s))\left[R_i(s)I_{i'i} - L_{i'i}(s)\right]\right]\right] f(s) = \lambda_{i'}^0 \Omega_{i'i}(s) f(s)$$

under the definition of Ω given in the statement of the proposition. Finally, substituting into the integral and using that $\lambda_{i'}^0 = \lambda^0$ for all i',

$$\frac{\partial}{\partial L_i^A(s)}\int_{i'} \mathcal{L}_{i'} di' = \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} + \int_{i' \neq i} \frac{\partial \mathcal{L}_{i'}}{\partial L_i^A(s)} di' = \lambda^0 \left[\Omega_{ii}(s) + \int_{i'} \Omega_{i'i}(s) di'\right] f(s).$$

Finally, substituting into the wedge formula and using $\lambda_j^0 = \lambda^0$, we obtain

$$\tau_{ji}^L(s) = -\Omega_{ii}(s) - \int_{i'} \Omega_{i'i}(s) di',$$

giving the result. This derivation did not rely on the identity of country j, and so is valid for all j.

Finally, for all other choice variables (I, c, D), these choice variables do not directly impact the liquidation price, so that the private and social optimality conditions coincide. As a result, $\tau_i^c = 0$, $\tau_i^I = 0$, and $\tau_i^D = 0$ for all i.

A.3 Proof of Lemma 2

The first part of implementability, that the country i planner can directly choose domestic bank allocations (c_i, D_i, I_i, L_i) subject to constraints, is standard given complete wedges (a standard constrained efficient planning problem). Consider the domestic allocations I_{ji} and L_{ji} of foreign bank j. At interior solutions,[68] given taxes τ_i and τ_j on foreign bank j we have the first order

[68] Appendix E.2 shows that this argument generalizes under corner solutions, because it is optimal for country planner i to ensure that even at a corner solution, the foreign bank is on its FOC even at the corner solution.

conditions for liquidations given by

$$0 = -\lambda_j^0 \left[\tau_{i,ji}^L(s) + \tau_{j,ji}^L(s) \right] + \frac{\partial \mathcal{L}_j(s)}{\partial L_{ji}(s)},$$

and substituting in the competitive FOC at an interior solution ($\xi_{ji} = 0$), we have

$$0 = -\lambda_j^0 \left[\tau_{i,ji}^L(s) + \tau_{j,ji}^L(s) \right] + \lambda_j^1(s) \Big(\gamma_i(s) - (1 + r_{ji}) \Big) f(s) + \Lambda_j^1(s) \Big(\gamma_j(s) - (1 - h_i(s)) \gamma_i(s) \Big) f(s)$$

which rearranges to the result for the liquidation wedges. The same steps give us

$$0 = -\lambda_j^0 \left[\tau_{i,ji}^I + \tau_{j,ji}^I \right] - \lambda_j^0 \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E\left[\lambda_j^1 (1 + r_{ji}) R_i \right] + E\left[\Lambda_j^1 (1 - h_i) \gamma_i R_i \right]$$

which rearranges to the result for the investment wedges. Observe that because country $j \neq i$ only maintains a marginal (density) activities presence in country i, the country planner i takes as given the marginal values of wealth λ_j^0, λ_j^1 and the collateral constraint Lagrange multiplier Λ_j^1. As a result, the FOC for liqudiations is linear, and the country i planner can choose any interior value of liquidations $L_{ji}(s)$ by setting the wedge $\tau_{ji}^L(s)$ to clear this equation. Finally, the investment equation gives a demand function relating $\frac{\partial \Phi_{ji}}{\partial I_{ji}}$ to $\tau_{i,ji}^I$, so that again the social planner can enforce any demand I_{ji} by setting $\tau_{i,ji}^I$ to clear the first order condition. As such, the social planner can solve the problem by directly choosing foreign allocations (I_{ji}, L_{ji}) by using the wedges described above.

A.4 Proof of Proposition 3

The objective of country planner i under quantity regulation is to maximize domestic welfare by choosing feasible allocations $(c_i, I_i, L_i, D_i, \{I_{ji}, L_{ji}\})$ and wedges $\{\tau_{i,ji}^I, \tau_{i,ji}^L\}$ on foreign banks, taking as given foreign revenue remissions and that the implementing wedges for foreign banks must be given as in Lemma 2. However, given that revenue remissions are taken as given, the social planner's problem in country i can be represented from the Lagrangian $\mathcal{L}_i$, internalizing the liquidation function γ_i, combined with the implementability conditions of Lemma 2. However, because the implementing wedges $\{\tau_{i,ji}^I, \tau_{i,ji}^L\}$ do not appear in the Lagrangian $\mathscr{L}_i$, the social planner's Lagrange multipliers on the implementability conditions of Lemma 2 are 0. As such, we can represent country planner i's Lagrangian as $\mathcal{L}_i$ (internalizing γ_i), with the choice variables being allocations $(c_i, I_i, L_i, D_i, \{I_{ji}, L_{ji}\})$. Implementability simply gives the method of implementing the foreign allocations.

Now that we have the domestic planner's problem, we have the social optimality condition for domestic liquidations by domestic banks, $L_{ii}(s)$, given by

$$0 = \frac{\partial \mathcal{L}_i}{\partial L_{ii}(s)} + \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)}.$$

Using the same steps and definitions as in the proof of Proposition 1, we obtain

$$\tau_{i,ii}^L(s) = -\Omega_{ii}(s),$$

giving the first result.

Next, the social optimality condition for domestic liquidations by foreign banks, $L_{ji}(s)$, is given by

$$0 \geq \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)},$$

so that we have an allocation rule $0 = L_{ji}(s)\Omega_{ii}(s)$, with $L_{ji}(s) = 0$ if $\Omega_{ii}(s) < 0$.

Finally, for all other domestic choices (c_i, I_i, D_i), the private and social FOCs align, and we have $\tau_i^c = 0$, $\tau_i^I = 0$, and $\tau_i^D = 0$. Lastly, domestic investment by foreign banks has no welfare impact whatsoever, so by convention we set $\tau_{ji}^I = 0$.

A.5 Proof of Proposition 4

The objective of country planner i is the same as in Proposition 3, except for the internalized revenue remission. Total revenues collected by country planner i are given by

$$\Pi_i = \int_{i'} \left[\tau_{i,i'i}^I I_{i'i} + \tau_{i,i'i}^L L_{i'i} \right] di'.$$

This revenue is remitted lump sum to domestic banks into their budget constraint at date 0, and so is valued by the Lagrange multiplier λ_i^0. In other words, we can represent the Lagrangian of social planner i, internalizing both the liquidation function γ_i and the wedge formulas $\tau_{i,i'i}$, by

$$\mathcal{L}_i^{\Pi} = \mathcal{L}_i + \lambda_i^0 \Pi_i,$$

where as before the choice variables are allocations. Now, consider the impact of a change in price γ_i on revenues collected. Here, we have

$$\frac{\partial \Pi_i}{\partial \gamma_i(s)} = \int_{i'} \left[\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i} L_{i'i} \right] di'.$$

From Lemma 2, we have

$$\frac{\partial \tau^L_{i,ji}(s)}{\partial \gamma_i(s)} = \frac{\lambda_j^1(s)}{\lambda_j^0} + \frac{1}{\lambda_j^0} \Lambda_j^1(s) h_i(s)$$

$$\frac{\partial \tau^I_{i,ji}}{\partial \gamma_i(s)} = \frac{1}{\lambda_j^0} \Lambda_j^1 (1-h_i) R_i f(s) ds$$

From here, we have

$$\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i} L_{i'i} = \left[\frac{\lambda_{i'}^1(s)}{\lambda_{i'}^0} + \frac{1}{\lambda_{i'}^0} \Lambda_{i'}^1(s) h_i(s) \right] L_{i'i}(s) f(s) + \frac{1}{\lambda_{i'}^0} \Lambda_{i'}^1 (1-h_i) R_i I_{i'i} f(s)$$

And so adding and subtracting $\frac{1}{\lambda_{i'}^0} \Lambda_{i'}^1(s) h_i(s)$, we obtain

$$\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i} L_{i'i} = \left[\frac{\lambda_{i'}^1(s)}{\lambda_{i'}^0} L_{i'i}(s) + \frac{\Lambda_{i'}^1(s)}{\lambda_{i'}^0} \left[L_{i'i}(s) + (1-h_i(s)) \left[R_i I_{i'i} - L_{i'i}(s) \right] \right] \right] f(s) = \frac{\Omega_{i'i}(s)}{\partial \gamma_i(s) / \partial L_i^A(s)} f(s)$$

where we have substituted in the definition of $\Omega_{i'i}(s)$ from Proposition 1. As a result, we have

$$\frac{\partial \Pi_i}{\partial \gamma_i(s)} = \frac{f(s)}{\partial \gamma_i(s) / \partial L_i^A(s)} \int_{i'} \Omega_{i'i}(s) di'.$$

From here, we can find the social first order conditions. For domestic liquidations $L_{ii}(s)$, we have

$$0 = \frac{\partial \mathcal{L}_i}{\partial L_{ii}(s)} + \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} + \lambda_i^0 \frac{\partial \Pi_i}{\partial L_i^A(s)},$$

which using the results from the proof of Proposition 3 yields

$$0 = \lambda_i^0 \tau_{i,ii}^L(s) f(s) + \lambda_i^0 \Omega_{ii}^L(s) f(s) + \lambda_i^0 \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \frac{\partial \Pi_i}{\partial \gamma_i(s)}.$$

Substituting in the deriative from above and rearranging, we obtain

$$\tau_{i,ii}^L(s) = -\Omega_{ii}^L(s) - \int_{i'} \Omega_{i'i}(s) di',$$

yielding the tax formula for $\tau_{i,ii}^L(s)$.

Consider next the first order condition for $L_{ji}(s)$ for $j \neq i$. Now, we have

$$0 = \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} + \lambda_i^0 \left[\frac{\partial \Pi_i}{\partial L_i^A(s)} + \frac{\partial \Pi_i}{\partial L_{ji}(s)} \right],$$

where $\frac{\partial \Pi_i}{\partial L_{ji}(s)}$ is the direct effect of the change in liquidations on tax revenue (holding prices fixed). Using the same steps, this rearranges to

$$\frac{\partial \Pi_i}{\partial L_{ji}(s)} = -\Omega_{ii}^L(s) f(s) - \int_{i'} \Omega_{i'i}(s) di' f(s).$$

Finally, from the tax formula, we have

$$\frac{\partial \Pi_i}{\partial L_{ji}(s)} = \tau_{i,ji}^L(s) f(s) + \frac{\partial \tau_{i,ji}^L(s)}{\partial L_{ji}(s)} L_{ji}(s) f(s) + \frac{\partial \tau_{i,ji}^I}{\partial L_{ji}(s)} I_{ji} = \tau_{i,ji}^L(s) f(s),$$

since $L_{ji}(s)$ does not appear directly in the tax formulas of Lemma 2. Substituting back in, we obtain

$$\tau_{i,ji}^L(s) = -\Omega_{ii}^L(s) - \int_{i'} \Omega_{i'i}(s) di',$$

yielding the tax formula for $\tau_{i,ji}^L(s)$.

Consider next the first order condition for I_{ji} for $j \neq i$. We have

$$0 = \lambda_i^0 \frac{\partial \Pi_i}{\partial I_{ji}},$$

given that I_{ji} has no other welfare impact. From here, we have (since $\tau_{i,ji}^L$ does not depend directly of I_{ji})

$$\frac{\partial \Pi_i}{\partial I_{ji}} = \tau^I_{i,ji} + \frac{\partial \tau^I_{i,ji}}{\partial I_{ji}} I_{ji} = \tau^I_{i,ji} - \frac{\partial^2 \Phi_{ji}}{\partial I_{ji}^2}.$$

Substituting back in, we obtain

$$\tau^I_{i,ji} = \frac{\partial^2 \Phi_{ji}}{\partial I_{ji}^2} I_{ji},$$

giving the result for $\tau^I_{i,ji}$.

Finally, for all other domestic allocations (c_i, D_i, I_i), the private and social first order conditions align, and so we have $\tau^c_i = 0$, $\tau^D_i = 0$, and $\tau^I_{i,ii} = 0$.

A.6 Proof of Proposition 5

The proof follows immediately from Proposition 4: when $\frac{\partial^2 \Phi_{ji}}{\partial I_{ji}^2} = 0$ for all i and $j \neq i$, each country planner implements the same wedges as the global planner, that is to say $\tau^L_{i,ji}(s) = \tau^L_{ji}(s)$ for all i and j, while all other wedges are zero.

A.7 Proof of Proposition 6

The proof follows the same general steps as the proof of Proposition 1. The wedge on debt D_{ji} is thus given by[69]

$$\tau^D_{ji} = -\frac{1}{\lambda^0_j} \int_{s \in S^D_{ji}} \left[\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)} \frac{\partial}{\partial L^A_i(s)} \int_{i'} \mathcal{L}_{i'} di' \right] f(s) ds.$$

Note that we have $\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)} = \frac{1}{h_i(s)\gamma_i(s)}$ when $s \in S^D_{ji}$. It remains to characterize the spillover. Given the welfare function of country i' banks, we have

$$\frac{\partial \mathcal{L}_{i'}}{\partial L^A_i(s)} = \frac{d\gamma_i(s)}{\partial L^A_i(s)} \left[(\gamma_i(s) - 1) L_{i'i}(s) f(s) \right] = \frac{d\gamma_i(s)}{dL^A_i(s)} \left[L_{i'i}(s) + (\gamma_i(s) - 1) \frac{\partial L_{i'i}(s)}{\partial \gamma_i(s)} \right] f(s).$$

From here, the result follows given that

$$\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-\frac{\partial d^*_i(s)}{\partial \gamma_i(s)} h_i(s)\gamma_j(s) - (D_{i'i} - d^*_i(s) I_{i'i}) h_i(s)}{(h_i(s)\gamma_i(s))^2} = \frac{-h_i(s) d^*_i(s) I_{i'i} - (D_{i'i} - d^*_i(s) I_{i'i}) h_i(s)}{h_i(s)\gamma_i(s)} = \frac{-D_{i'i}}{h_i(s)\gamma_i(s)}.$$

[69] Note that the boundary term from Leibniz rule is zero since liquidations are zero at the boundary.

All that remains is to characterize the total derivative of the equilibrium price in liquidations, which we had denoted $\frac{d\gamma_i(s)}{\partial\varepsilon}$. This total derivative is non-trivial because liquidations are now endogenous to the price via the collateral constraint. In particular, we can characterize this impact by defining $L_i^{A,\varepsilon}(s) = L_{ii}(s) + \int_j L_{ij}(s)ds + \varepsilon$ and by totally differentiating the equilibrium price relationship $\gamma_i(s) = \frac{\partial\mathcal{F}(L_i^A(s),s)}{\partial L_i^A(s)}$ in ε around $\varepsilon = 0$. Evaluating this total derivative, we obtain

$$\frac{d\gamma_i(s)}{d\varepsilon} = \frac{\frac{\partial^2\mathcal{F}_i(s)}{\partial L_i(s)^2}}{1 - \frac{\partial^2\mathcal{F}_i(s)}{\partial L_i(s)^2}\left[\frac{\partial L_{ii}(s)}{\partial\gamma_i(s)} + \int_j \frac{\partial L_{ij}(s)}{\partial\gamma_i(s)}dj\right]} = \frac{\frac{\partial^2\mathcal{F}_i(s)}{\partial L_i(s)^2}}{1 - \frac{\partial^2\mathcal{F}_i(s)}{\partial L_i(s)^2}D_i^A(s)}.$$

where the last line follows from recalling that $\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-D_{i'i}}{h_i(s)\gamma_i(s)}$.

The result for τ^I_{ji} then follows in the same manner, with the only difference being the liquidation impact is instead $\frac{\partial L_{ji}(s)}{\partial I_{ji}} = \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}$ across states $s \in S^D_{ji}$.

A.8 Proof of Proposition 7

To begin with, starting from the Lagrangian of bank j, we have wedges given by

$$\tau^D_{i,ji} = -\tau^D_{j,ji} + \frac{1}{\lambda_j}\left[-1 + \mathbb{E}\left[(\gamma_i(s)-1)\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)}\right]\right]$$

$$\tau^I_{i,ji} = -\tau^I_{j,ji} + \frac{1}{\lambda_j}\left[E[R_i] + \mathbb{E}\left[(\gamma_i(s)-1)\frac{\partial L_{ji}(s)}{\partial I_{ji}}\right] - \lambda_j\frac{\partial\Phi_{ji}}{\partial I_{ji}}\right]$$

Once again, starting from a desired allocation (D_{ji}, I_{ji}), the planner of country i sets wedges according to the above in order to implement this allocation. Note that planner i takes as given the Lagrange multipliers. Now that we know the implementing wedges, we can solve the problem of country planner i as before. As in the proof of Proposition 4, we can internalize both the liquidation price and wedges to obtain a Lagrangian $\mathcal{L}_i + \lambda_i^0\Pi_i$, where

$$\Pi_i = \int_{i'}\left[\tau^D_{i,i'i}D_{i'i} + \tau^I_{i,i'i}I_{i'i}\right].$$

The steps are the same as in the proof of Proposition 4. Starting by characterizing the revenue derivative from a change in the equilibrium price, we have

$$\frac{\partial \Pi_i}{\partial \gamma_i(s)} = \int_{i'} \frac{\partial \left[\tau^D_{i,i'i} D_{i'i} + \tau^I_{i,i'i} I_{i'i}\right]}{\partial \gamma_i(s)} di'.$$

Taking the derivative, we have

$$\begin{aligned}
\frac{\partial [\tau^D_{i,ji} D_{ji} + \tau^I_{i,ji} I_{ji}]}{\partial \gamma_i(s)} &= \frac{\partial}{\partial \gamma_i(s)} \frac{1}{\lambda_{i'}} \mathbb{E}\left[(\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial D_{i'i}(s)} D_{i'i} + (\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial I_{i'i}(s)} I_{i'i}\right] \\
&= \frac{\partial}{\partial \gamma_i(s)} \frac{1}{\lambda_{i'}} \mathbb{E}\left[(\gamma_i(s)-1)\left(\frac{1}{h_i(s)\gamma_i(s)} D_{i'i} - \frac{d^*_i(s)}{h_i(s)\gamma_i(s)} I_{i'i}\right)\mathbf{1}_{s\in S^D_{i'i}}\right] \\
&= \frac{\partial}{\partial \gamma_i(s)} \frac{1}{\lambda_{i'}} \mathbb{E}\left[(\gamma_i(s)-1) L_{i'i}(s)\right]
\end{aligned}$$

where the last line follows from the collateral constraint and from noting that $L_{i'i} = 0$ for $s \notin S^D_{i'i}$. As such, again noting that the Leibniz boundary term drops out because $L_{i'i} = 0$ at the boundary, we have

$$\frac{\partial [\tau^D_{i,ji} D_{ji} + \tau^I_{i,ji} I_{ji}]}{\partial \gamma_i(s)} = \frac{1}{\lambda_{i'}} \mathbb{E}\left[\frac{\partial}{\partial \gamma_i(s)} [(\gamma_i(s)-1) L_{i'i}(s)]\right]$$

which is the spillover term from above.

From here, efficient setting of domestic wedges on domestic banks follows as in the baseline model. What remains is wedge setting on foreign banks. The first order condition for choice $D_{i'i}$ of debt by foreign banks is given by

$$0 = \mathbb{E}\left[\frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{d\gamma_i(s)}{dD_{i'i}} + \lambda^0_i \frac{d\Pi_i}{d\gamma_i(s)} \frac{d\gamma_i(s)}{dD_{i'i}}\right] + \lambda^0_i \frac{\partial [\tau^D_{i,i'i} D_{i'i} + \tau^I_{i,i'i} I_{i'i}]}{\partial D_{i'i}}.$$

As just shown above, the revenue derivative captured foreign spillovers. Similar to the steps above, the direct derivative of revenue from bank i' is

$$\frac{\partial [\tau^D_{i,i'i} D_{i'i} + \tau^I_{i,i'i} I_{i'i}]}{\partial D_{i'i}} = \frac{1}{\lambda_{i'}} \frac{\partial}{D_{i'i}} \left[-D_{i'i} + \mathbb{E}[(\gamma_i(s)-1) L_{i'i}(s)]\right] = \frac{1}{\lambda_{i'}} \left[-1 + \mathbb{E}[(\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial D_{i'i}}]\right] = \tau^D_{i,i'i},$$

giving that the wedge on debt is set efficiently.

Finally, we need to characterize the wedge on investment. From the same steps,

$$0 = \mathbb{E}\left[\frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)}\frac{d\gamma_i(s)}{dI_{i'i}} + \lambda_i^0 \frac{d\Pi_i}{d\gamma_i(s)}\frac{d\gamma_i(s)}{dI_{i'i}}\right] + \lambda_i^0 \frac{\partial[\tau^D_{i,i'i}D_{i'i} + \tau^I_{i,i'i}I_{i'i}]}{\partial I_{i'i}},$$

and from here we have

$$\begin{aligned}
\frac{\partial[\tau^D_{i,i'i}D_{i'i} + \tau^I_{i,i'i}I_{i'i}]}{\partial I_{i'i}} &= \frac{1}{\lambda_{i'}}\frac{\partial}{I_{i'i}}\left[E[R_i]I_{i'i} + \mathbb{E}[(\gamma_i(s)-1)L_{i'i}(s)] - \lambda_{i'}\frac{\partial \Phi_{i'i}}{\partial I_{i'i}}I_{i'i}\right] \\
&= \frac{1}{\lambda_{i'}}\left[E[R_i] + \mathbb{E}[(\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial I_{i'i}}] - \lambda_{i'}\frac{\partial \Phi_{i'i}}{\partial I_{i'i}}\right] - \frac{\partial^2 \Phi_{i'i}}{\partial I_{i'i}^2}I_{i'i} \\
&= \tau^I_{i,i'i} - \frac{\partial^2 \Phi_{i'i}}{\partial I_{i'i}^2}I_{i'i}
\end{aligned}$$

so that once again, efficiency requires $\frac{\partial^2 \Phi_{i'i}}{\partial I_{i'i}^2} = 0$.

A.9 Proof of Proposition 8

The Lagrangian of the global planner is

$$\mathcal{L}^G = \int_i \left[\omega_i U_i\left(u_i(a_i), u_i^A(a_i, a^A))\right) + \Lambda_i \Gamma_i\left(A_i + \mathcal{T}_i, \phi_i(a_i), \phi_i^A(a_i, a^A)\right)\right]di - \lambda^0 \int_i \mathcal{T}_i di.$$

From here, we have

$$\frac{d\mathcal{L}^G}{da_{ij}(m)} = \frac{\partial \mathcal{L}_i}{\partial a_{ij}(m)} + \frac{\partial \mathcal{L}_j}{\partial a_j^A(m)} + \int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_j^A(m)}di'$$

so that we obtain the required wedge

$$\tau_{ij}(m) = -\frac{1}{\lambda_i^0}\left[\frac{\partial \mathcal{L}_j}{\partial a_j^A(m)} + \int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_j^A(m)}di'\right]$$

where we define $\lambda_i^0 \equiv \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}$. Next, we can characterize the derivative

$$\frac{\partial \mathcal{L}_i}{\partial a_j^A(m)} = \omega_i \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_j^A(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_{ij}^A}{\partial a_j^A(m)}.$$

Finally, defining $\Omega_{i,j}(m) = \frac{1}{\lambda_i^0}\frac{\partial \mathcal{L}_i}{\partial a_j^A(m)}$ and using that $\lambda^0 = \lambda_i^0$ (from the FOC for $\mathcal{T}_i$), we obtain

$$\tau_{ij}(m) = -\Omega_{j,j}(m) - \int_{i'} \Omega_{i',j}(m) di'$$

giving the result.

B Section 5 Appendix

This appendix presents formal results underlying results and discussion in Section 5 that were not presented in the main text.

B.1 Baseline Model

In Section 5, we presented the global optimum (Proposition 6) as well as the Pigouvian efficiency result (Proposition 7). The following result characterizes the non-cooperative equilibrium under quantity regulation.

Proposition 11. *The non-cooperative equilibrium under quantity regulation has the following features.*

1. *Domestic regulation of domestic banks is given by*

$$\tau_{ii}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{1}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^D\right]$$

$$\tau_{ii}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^D\right]$$

where the domestic cost of liquidations $\Omega_{ii}(s)$ is

$$\Omega_{ii}(s) = \left|\frac{\partial \gamma_i(s)}{\partial \varepsilon}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)}\left[\frac{1}{\gamma_i(s)}D_{ii} - d_i^*(s)I_{ii}\right] \cdot \mathbf{1}_{s \in S_{ii}^D}$$

2. *Domestic regulation of foreign banks achieves an allocation rule $D_{ji} \leq d_i^* I_{ij}$, that is domestic regulation of foreign banks prevents foreign banks from liquidating the domestic asset.*

B.1.1 Proof of Proposition 11

The proof follows the same steps as the proof of Proposition 7, except that optimal allocations no longer include terms related to revenue derivatives. Hence, optimal rules are simply those without revenue derivatives. Domestic regulation of domestic banks only accounts for domestic spillovers (omitting foreign spillovers, which arise from the revenue derivative). Similarly, the allocation rule for foreign banks only features the domestic spillover that arises from foreign subsidiary distress, and hence sets $D_{ji} \leq d_i^* I_{ji}$.

B.2 Liquidity Regulation

We now present the formal results for cooperative and non-cooperative policies in Section 5.3, where we have incorporated liquid assets and liquidity regulation. Recall the modified definition of the distress region.

Proposition 12. *In the model with liquidity regulation,*

1. *The globally efficient allocation can be decentralized using wedges*

$$\tau_{ji}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s) \frac{1}{h_i(s)\gamma_i(s)} \middle| s \in S_{ji}^D\right]$$

$$\tau_{ji}^T = -\tau_{ji}^D$$

$$\tau_{ji}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s) \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)} \middle| s \in S_{ji}^D\right]$$

where the total social cost $\tau_i^L(s) \geq 0$ of liquidations in country i in state s is

$$\tau_i^L(s) = \left|\frac{d\gamma_i(s)}{dL_i^A(s)}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)} \left[\frac{1}{\gamma_i(s)}\left(D_i^A(s) - T_i^A(s)\right) - d_i^*(s) I_i^A(s)\right],$$

where the total price impact $\frac{d\gamma_i(s)}{dL_i^A(s)}$ is defined in the proof.

2. *Under non-cooperative quantity regulation,*

(a) *Domestic regulation of domestic banks is given by*

$$\tau_{ii}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{1}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^D\right]$$

$$\tau_{ii}^T = -\tau_{ii}^D$$

$$\tau_{ii}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^D\right]$$

where the domestic cost of liquidations $\Omega_{ii}(s)$ is

$$\Omega_{ii}(s) = \left|\frac{\partial \gamma_i(s)}{\partial \varepsilon}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)}\left[\frac{1}{\gamma_i(s)}[D_{ii} - T_{ii}] - d_i^*(s)I_{ii}\right] \cdot \mathbf{1}_{s \in S_{ii}^D}$$

(b) *Domestic regulation of foreign banks achieves an allocation rule $D_{ji} \leq d_i^* I_{ij} + T_{ij}$, that is domestic regulation of foreign banks prevents foreign banks from liquidating the domestic asset.*

3. *Suppose that for all i and $j \neq i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = \frac{\partial^2 \Phi_{ij}}{\partial T_{ij}^2} = \frac{\partial^2 \Phi_{ij}}{\partial T_{ij}\partial I_{ij}} = 0$. Then, the non-cooperative equilibrium under Pigouvian taxation is globally efficient. There is no scope for cooperation.*

B.2.1 Proof of Propositions 12

The proofs follow the same steps as the proofs of Propositions 6, 7, and 11, and we highlight here only the differences. First, in evaluating the spillover effect (Proposition 6, we now have instead

$$\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-\frac{\partial d_i^*(s)}{\partial \gamma_i(s)} h_i(s)\gamma_j(s) - (D_{i'i} - T_{i'i} - d_i^*(s)I_{i'i})h_i(s)}{(h_i(s)\gamma_i(s))^2} = \frac{-D_{i'i} - T_{i'i}}{h_i(s)\gamma_i(s)},$$

from which the social cost $\tau_i^L(s)$ follows. From here, the globally optimal wedges τ_{ji}^D and τ_{ji}^I follow exactly as before, which the liquidation wedge

$$\tau_{ji}^I = -\frac{1}{\lambda_j^0}\int_{s \in S_{ji}^D}\left[\frac{\partial L_{ji}(s)}{\partial T_{ji}(s)}\frac{\partial}{\partial L_i^A(s)}\int_{i'}\mathcal{L}_{i'}di'\right] f(s)ds = \frac{1}{\lambda_j^0}\int_{s \in S_{ji}^D}\left[\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)}\frac{\partial}{\partial L_i^A(s)}\int_{i'}\mathcal{L}_{i'}di'\right] f(s)ds = -\tau_{ji}^D.$$

Next, turning to the non-cooperative problem, we have the same implementability conditions for debt and illiquid investment, as well as the additional implementability condition for liquid investment

$$\tau^T_{i,ji} = -\tau^T_{j,ji} + \frac{1}{\lambda_j}\left[1 + \mathbb{E}\left[(\gamma_i(s)-1)\frac{\partial L_{ji}(s)}{\partial T_{ji}}\right] - \lambda_j \frac{\partial \Phi_{ji}}{\partial T_{ji}}\right]$$

Thus, taking the derivative we have

$$\begin{aligned}
\frac{\partial[\tau^D_{i,ji}D_{ji} + \tau^I_{i,ji}I_{ji} + \tau^T_{i,ji}T_{ji}]}{\partial \gamma_i(s)} &= \frac{\partial}{\partial \gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)\left(\frac{\partial L_{i'i}(s)}{\partial D_{i'i}(s)}D_{i'i} + \frac{\partial L_{i'i}(s)}{\partial I_{i'i}(s)}I_{i'i} + \frac{\partial L_{i'i}(s)}{\partial T_{i'i}(s)}T_{i'i}\right)\right] \\
&= \frac{\partial}{\partial \gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)\left(\frac{1}{h_i(s)\gamma_i(s)}[D_{i'i} - T_{i'i}] - \frac{d^*_i(s)}{h_i(s)\gamma_i(s)}I_{i'i}\right)\mathbf{1}_{s\in S^D_{i'i}}\right] \\
&= \frac{\partial}{\partial \gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)L_{i'i}(s)\right]
\end{aligned}$$

so that the revenue derivative results in internalizing foreign spillovers. From here, the remainder of the proof proceeds as before.

B.3 Cross-Border Support and Resolution

We now present the formal results for cooperative and non-cooperative policies in Section 5.4, where we have incorporated cross-border subsidiary support.

Proposition 13. *In the model with cross-border support,*

1. *The globally efficient allocation can be decentralized using wedges*

$$\tau^D_{ji} = \Pr(s \in S^D_{ji}) \cdot \mathbb{E}\left[\tau^L_i(s)\frac{1}{h_i(s)\gamma_i(s)}\middle| s \in S^D_{ji}\right]$$

$$\tau^G_{ji}(s) = -\tau^L_i(s)\frac{1}{h_i(s)\gamma_i(s)}\mathbf{1}_{s\in S^D_{ji}}$$

$$\tau^I_{ji} = \Pr(s \in S^D_{ji}) \cdot \mathbb{E}\left[\tau^L_i(s)\frac{-d^*_i(s)}{h_i(s)\gamma_i(s)}\middle| s \in S^D_{ji}\right]$$

where the total social cost $\tau^L_i(s) \geq 0$ of liquidations in country i in state s is

$$\tau^L_i(s) = \left|\frac{d\gamma_i(s)}{dL^A_i(s)}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)}\left[\frac{1}{\gamma_i(s)}\left(D^A_i(s) - G^A_i(s)\right) - d^*_i(s)I^A_i(s)\right],$$

where the total price impact $\frac{d\gamma_i(s)}{dL^A_i(s)}$ is defined in the proof.

2. *Under non-cooperative quantity regulation,*

   (a) *Domestic regulation of domestic banks is given by*

$$\tau_{ii}^{D} = \Pr(s \in S_{ji}^{D}) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{1}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^{D}\right]$$

$$\tau_{ii}^{G}(s) = -\Omega_{ii}(s)\frac{1}{h_i(s)\gamma_i(s)}\mathbf{1}_{s\in S_{ji}^{D}}$$

$$\tau_{ii}^{I} = \Pr(s \in S_{ji}^{D}) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^{D}\right]$$

   where the domestic cost of liquidations $\Omega_{ii}(s)$ is

$$\Omega_{ii}(s) = \left|\frac{\partial \gamma_i(s)}{\partial \varepsilon}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)}\left[\frac{1}{\gamma_i(s)}\Big(D_{ii} - G_{ii}(s)\Big) - d_i^*(s)I_{ii}\right] \cdot \mathbf{1}_{s\in S_{ii}^{D}}$$

   (b) *Domestic regulation of foreign banks achieves an allocation rule $D_{ji} \leq \inf_{s\in S}\{d_i^*(s)I_{ji} + G_{ji}(s)\}$, that is domestic regulation of foreign banks prevents foreign banks from liquidating the domestic asset.*

3. *Suppose that for all i and $j \neq i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$. Then, the non-cooperative equilibrium under Pigouvian taxation is globally efficient. There is no scope for cooperation.*

B.3.1 Proof of Propositions 13

The proofs follow the same steps as the proofs of Propositions 6, 7, and 11. We again highlight the key differences. Evaluating the spillover effect (Proposition 6, we now have instead

$$\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-D_{i'i} - G_{i'i}(s)}{h_i(s)\gamma_i(s)},$$

from which the social cost $\tau_i^L(s)$ follows. From here, the globally optimal wedges τ_{ji}^D and τ_{ji}^I follow exactly as before, which the liquidation wedge

$$\tau_{ji}^{I} = -\frac{1}{\lambda_j^0}\frac{\partial L_{ji}(s)}{\partial G_{ji}(s)}\frac{\partial}{\partial L_i^A(s)}\int_{i'}\mathcal{L}_{i'}di' = -\tau_i^L(s)\frac{1}{h_i(s)\gamma_i(s)}\mathbf{1}_{s\in S_{ji}^{D}}.$$

Next, turning to the non-cooperative problem, we have the same implementability conditions for debt and illiquid investment, as well as the additional implementability condition for cross-border support

$$\tau^G_{i,ji}(s) = -\tau^G_{j,ji}(s) + \frac{1}{\lambda_j}\left[1 + (\gamma_i(s)-1)\frac{\partial L_{ji}(s)}{\partial G_{ji}(s)}\right] - \mu_j(s)\Bigg]$$

where $\mu_j(s)$ is the Lagrange multiplier on the subsidiary support budget constraint. Thus, taking the derivative we have

$$\begin{aligned}
\frac{\partial[\tau^D_{i,ji}D_{ji} + \tau^I_{i,ji}I_{ji} + \tau^T_{i,ji}T_{ji}]}{\partial\gamma_i(s)} &= \frac{\partial}{\partial\gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)\left(\frac{\partial L_{i'i}(s)}{\partial D_{i'i}(s)}D_{i'i} + \frac{\partial L_{i'i}(s)}{\partial I_{i'i}(s)}I_{i'i} + \frac{\partial L_{i'i}(s)}{\partial G_{i'i}(s)}G_{i'i(s)}\right)\right] \\
&= \frac{\partial}{\partial\gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)\left(\frac{1}{h_i(s)\gamma_i(s)}[D_{i'i} - G_{i'i}(s)] - \frac{d^*_i(s)}{h_i(s)\gamma_i(s)}I_{i'i}\right)\mathbf{1}_{s\in S^D_{i'i}}\right] \\
&= \frac{\partial}{\partial\gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\Big[(\gamma_i(s)-1)L_{i'i}(s)\Big]
\end{aligned}$$

so that the revenue derivative results in internalizing foreign spillovers. From here, the remainder of the proof proceeds as before.

B.4 Bailouts and Fiscal Backstops

We incorporate bailouts into the baseline macroprudential regulatory model of Section 5. We model bailouts as ex ante lump sum transfer commitments $T^1_{ij}(s) \geq 0$, which provides a tractable way of representing the various possible bailout instruments.[70] At date 1, the country-level liquid net worth of a bank is therefore $A^1_{ij}(s) = -D_{ij} + T^1_{ij}(s)$, which may be negative. Notice that because bailouts are state contingent whereas debt is non-contingent, macroprudential regulation that reduces debt issuance is not a perfect substitute for bailouts. The country level collateral constraint now generates a liquidation rule $L_{ij}(s) = \frac{1}{h_j(s)\gamma_j(s)}\max\left\{D_{ij} - T^1_{ij}(s) - d^*_j(s)I_{ij}, 0\right\}$. Bailouts are financed by domestic taxpayers, with a utility cost $V^T_i(\mathcal{T}_i)$ of tax revenue collections.[71] Country planners

[70] Although in theory fiscal backstops such as deposit insurance and LOLR rule out bad equilibria without being used on the equilibrium path, in practice these measures often are associated with undesirable transfers and moral hazard.

[71] See Appendix B.5.1 for a foundation.

trade state-contingent claims on taxpayer revenue, yielding a tax-bailout budget constraint

$$\int_s \left[T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s) + \int_j T^1_{i,ij}(s) \right] f(s) ds \leq G_i + \mathfrak{T}_i \tag{19}$$

where $T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s) + \int_j T^1_{i,ij}(s)$ is required revenue for bailouts in state s. G_i is an existing inter-country tax revenue claim, with $\int_i G_i di = 0$, which we use in decentralizing the cooperative outcome. The ability for country planners to trade contingent bailout claims means that they could in principle implement a "common fiscal backstop" via trading of claims in a decentralized manner, that is they have the same set of tools that a common fiscal authority would have.

B.4.1 Globally Efficient Policies

We characterize the globally efficient bailout policies, and discuss the non-cooperative bailout rules. The formal characterization of globally efficient regulation and non-cooperative policies are contained in Appendix B.5.

Globally Efficient Bailouts. The global planning problem places welfare weights ω_i on countries and relative welfare weights ω_i^T on taxpayers. The following proposition characterizes the globally efficient bailout rule.[72]

Proposition 14. *The globally efficient bailout rule for $T^1_{ij}(s)$ is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathfrak{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ij}(s)}_{\text{Bank Benefit}} + \underbrace{\Omega^B_{j,j}(s) \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}}_{\text{Domestic Spillovers}} + \underbrace{\int_{i'} \Omega^B_{i',j}(s) di' \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}}_{\text{Foreign Spillovers}} \tag{20}$$

where the terms B^1_{ij}, $\Omega^B_{j,j}$, and $\Omega_{i',j}$ are defined in the proof.

The globally efficient bailout rules trade off the marginal cost of the bailout to taxpayers against both the direct benefit to the bank receiving the bailout, and the spillover benefits from reduced liquidations and fire sales. As in the baseline regulatory problem, globally efficient policy considers

[72] The Appendix also characterizes cooperative regulation, optimal tax collection, and an irrelevance result for bailout sharing rules.

the complete set of spillovers when designing bailouts. There is equal bailout treatment in the sense that domestic and foreign banks that have the same benefit $B^1_{ij}(s)$ and the liquidation responsiveness $\frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}$ from the bailout face the same marginal bailout rule.

Discussion.

Bailout Home Bias. Our model predicts that country planners provide stronger backstops for domestic banks and domestic operations. There are several examples of home bias in deposit insurance, including: US deposit insurance not applying to foreign branches of US banks; Iceland's decision not to honor deposit guarantee obligations to UK depositors after its despoit guarantee scheme was breached; and EU policies against deposit insurance discrimination by nationality.[73]

Common LOLR and Common Deposit Insurance in the EU. Our model predicts overly weak fiscal backstops. This coincides with the EU motivation for Common Deposit Insurance, whose purpose is to "increase the resilience of the Banking Union against future crises" (European Commision (2015)). It further coincides the ECB acting as a common LOLR to the European Union.

Asymmetric Contributions to Backstops. Concerns may arise about sharing a fiscal backstop if countries benefit asymmetrically from it, for example if some countries are net contributors while others are net recipients.[74] Proposition 14 implies that asymmetric bailouts can be optimal if it mitigates domestic fire sales and so promotes cross-border financial integration.

B.4.2 Non-Cooperative Quantity Regulation

We now characterize the optimal bailout rules that arise in under non-cooperative quantity regulation. Regulatory policy under non-cooperative quantity regulation in fact takes the same form as Proposition 11 (up to the modified definition of the distress region), which is shown formally in the proof.

[73] See 78 FR 56583; "Iceland Triumphs in Icesave court battle," *Financial Times*, January 28, 2013; and European Commision (2015)

[74] For example, Iceland had difficulty servicing its backstop because its banking system was large relative to taxpayer basis.

Proposition 15. *The bailout rules in the non-cooperative equilibrium under quantity regulation are as follows.*

1. *The optimal bailout rule for the domestic operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B_{ii}^1(s)}_{\text{Bank Benefit}} + \underbrace{\Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}}_{\text{Domestic Spillovers}} \tag{21}$$

2. *The optimal bailout rule for the foreign operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B_{ij}^1(s)}_{\text{Bank Benefit}} \tag{22}$$

3. *The optimal bailout rule for the domestic operations of a foreign bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{\Omega_{i,i}^B(s) \frac{\partial L_{ji}(s)}{\partial A_{ji}^1(s)}}_{\text{Domestic Spillovers}} \tag{23}$$

Three factors govern the non-cooperative bailout rules: the social cost of taxes, the direct benefit of bailouts to banks, and the domestic fire sale spillover. When choosing bailouts of domestic activities of domestic banks, the domestic planner considers all three factors, but neglects spillover costs to foreign banks. Moreover, the domestic planner neglects the benefits of alleviating foreign fire sales when choosing bailouts of foreign activities of domestic banks, and neglects the benefits of the bailout transfer when choosing bailouts of domestic activities of foreign banks. Country planners are *home biased* in their bailout decisions, generally preferring to bail out domestic activities of domestic banks.

Relative to the globally efficient bailout rule, non-cooperative planners under-value all bailout activities, including bailouts of domestic activities of domestic banks, not accounting for either benefits or spillovers to foreign banks. The cooperative agreement increases bailouts of both domestic and foreign banks. Multilateral fire sale spillovers imply the need for multilateral bailout cooperation.

B.4.3 Non-Cooperative Pigouvian Taxation

We next consider non-cooperative taxation in the model. We first show how efficiency breaks down even under Pigouvian taxation, and then we show how efficiency can be restored by taxing banks for the bailouts they expect to receive.

Proposition 16. *Suppose that the monopolist distortion is* 0*. Then, non-cooperative optimal taxation is as follows.*

1. *Domestic taxes on domestic banks' domestic activities are*

$$\tau^D_{i,ii} = E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta^T_{i'i}(s) T^1_{i'i}(s)di'\right] \frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}\right] \tag{24}$$

$$\tau^I_{i,ii} = -E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta_{i'i}(s) T^1_{i'i}(s)di'\right] \frac{\partial L_{ii}(s)}{\partial I_{ii}}\right] \tag{25}$$

   where $\Delta^T_{ij}(s)$ is defined in the proof.

2. *Domestic taxes on foreign banks' domestic activities are*

$$\tau^D_{i,ji} = E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta^T_{i'i}(s) T^1_{i'i}(s)di'\right] \frac{\partial L_{ji}(s)}{\partial A^1_{ji}(s)}\right] \tag{26}$$

$$\tau^I_{i,ji} = -E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta^T_{i'i}(s) T^1_{i'i}(s)di'\right] \frac{\partial L_{ji}(s)}{\partial I_{ji}}\right] \tag{27}$$

Moreover, the optimal bailout rules for banks are the same as in Proposition 19, but with the spillover effects defined above.

Although the result here appears largely as in the baseline model, there is one substantive difference: the additional terms $\Delta_{i'i}(s)T^1_{i'i}(s)$ that arise in the revenue derivatives. These terms arise whenever there are bailouts by *some country* (not necessarily i) of domestic activities of foreign banks. This effect arises because bailout revenue is not a choice variable of private agents, but rather is an untaxed and unpriced action of governments. In absence of bailouts ($T^1_{ji}(s) = 0$), this term disappears and we revert to the effective characterizations in the first half of this paper. In other words, bailouts lead to a violation of Assumption 9.[75]

Finally, we could consider the bailout rule for banks. The bailout rule for domestic banks is of the same form as in Proposition 19, except for the change in the spillover. Importantly, however, it is immediate to observe that the bailout rule for foreign banks does not consider the direct revenue benefit to banks from bailout revenue, because there is no tax on bailouts (i.e. it is not a private choice variable). As a result, cooperation is likely to be required over bailouts of cross-border banks even if non-cooperative Pigouvian taxation is able to achieve close-to-optimal internalization of spillovers. However, it is worthwhile to note that if the terms Δ_{ij}^T are close to zero, then Pigouvian taxation transforms the bailout problem to a *bilateral* problem, where the domestic planner simply neglects the benefit to foreign banks of receiving a bailout. Transforming the problem into a bilateral surplus problem, rather than a multilateral problem, may simplify cooperation over bailouts. For example, it may allow for simple agreements such as reciprocity on provision of deposit insurance and access to LOLR facilities.

B.4.4 Restoring Non-Cooperative Optimality With Bailout Levies

The above results imply that the existence of bailouts limits the ability for non-cooperative Pigouvian taxation to generate efficient policies. This failure arises because bailouts are not priced or otherwise optimally chosen by private banks. This implies that if bailouts *were* chosen by private banks, either explicitly or implicitly, we could restore efficiency.

Suppose that banks can in fact purchase bailout claims from the government, or alternatively that banks are charged ex ante for the bailout claims they will receive. In partciular, banks can purchase claims $T_{ij}^1(s) \geq 0$ at date 0, at a cost $\overline{q} > 1$ (i.e. the marginal cost of taxpayer funds). The first-order condition for bailout claim purchases in state s is

$$\tau_{j,ij}^T(s) = -\tau_{i,ij}^T(s) - \overline{q} + \frac{\lambda_i^1(s)}{\lambda_i^0}\left(1 + \left(\gamma_j(s) - 1\right)\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}\right). \tag{28}$$

Following the logic of previous sections, we have $\tau_{i,ij}^T = 0$, since banks now purchase bail out claims and since country i does not internalize impacts on foreign fire sales. As a result, domestic

[75] Note that the bailouts model features the nonlinear aggregates property of Appendix E.4, but that Assumption 9 is still the relevant assumption in that section.

planners never force banks to increase their backstop for foreign activities. On the other hand, the revenue that country planner j raises at date 0 from taxing the bailout purchases is of country i banks is $\tau^{T}_{j,ij}(s)T^{1}_{ij}(s)$. From here, it is easy to see that the efficiency results of the baseline model are restored. Planner j accounts for the direct benefits of bailouts, and also for the spillover costs.

The results of this section imply that *bailout* cooperation is also not necessary if it can be given a "market mechanism" and taxed. In practice, we could think about these taxes as corresponding to levies on banks for deposit insurance or access to lender of last resort, with the levies calibrated based on how much the bank expects to receive from them. Such levies are consistent with the fact that the Single Resolution Fund in the EU is funded by bank levies, and the Orderly Liquidity Fund in the US is designed to recoup expenditures from either the resolved bank or from other large financial institutions.[76]

The framework suggests that bailout policies are most naturally delegated to the *host* country, who can internalize the benefits and spillovers to foreign banks when using Pigouvian regulation combined with a market mechanism for bailouts. For example, this could correspond to a host country insuring the deposits of the local subsidiary of a foreign bank. This synergizes with other possible considerations, such as benefits to domestic depositors of deposit insurance, that might help to ensure that bailout policies are time consistent.

Time Consistency and Bailout Sharing. The results of this section assume that bailouts and bailout sharing rules are chosen ex ante with commitment. In practice, a key concern may be time consistency problems, where countries that ex post are obliged to send bailout funds to foreign countries renege on their international claims. If there are time consistency problems that prevent non-cooperative sharing of taxpayer funds, there may be a role for cooperation to enforce risk sharing agreements. However, the results of this section imply that the role of cooperation would be limited to enforcement of risk sharing, and would not need to specify the level of risk sharing.

Under Pigouvian taxation, bailout rules are still not efficient. The reason is that bailouts are chosen by governments, not by banks, so that there is not an equilibrium tax rate associated with them. This problem can be fixed if there is a mechanism in place to charge banks for the bailouts they expect to receive. Such a mechanism, which is effectively a Pigouvian tax on bailouts, restores

[76] See https://srb.europa.eu/en/content/single-resolution-fund for the former, and US Department of Treasury (2018) for the latter.

efficiency, including over bailout rules. One example of such a mechanism would be a deposit insurance levy.

B.5 Bailouts Model: Additional Results

In this appendix, we provide the additional results from the bailouts section, including the characterization of taxpayers, the relevant implementability conditions, characterization of regulatory policy, and characterization of non-cooperative taxation.

B.5.1 Taxpayers

We provide a foundation for the reduced-form indirect utility function $V_i(\mathfrak{T}_i)$ of tax revenue collections from taxpayers, and show a tax smoothing result.

A unit continuum of domestic taxpayers are born at date 1 with an endowment $\overline{T}_i^1(s)$ of the consumption good. Given tax collections $T_i^1(s) \leq \overline{T}_i^1(s)$, taxpayers enjoy consumption utility $u_i^T\left(\overline{T}_i^1(s) - T_i^1(s), s\right)$.[77] These tax collections generate total bailout revenue $\mathfrak{T}_i = \int_s q(s) T_i^1(s) f(s) ds$.[78] We characterize the optimal tax collection problem of country planner i, who has decided to collect a total $\mathfrak{T}_i$ in tax revenue for use in bailouts.

Lemma 17. *Taxpayer utility can be represented by the indirect utility function*

$$V_i^T(\mathfrak{T}_i) = \int_s u_i\left(\overline{T}_i^1(s) - T_i^1(\mathfrak{T}_i, s), s\right) f(s) ds \tag{29}$$

where $T_i^1(\mathfrak{T}_i, s)$ is given by the tax smoothing condition

$$\frac{1}{q(s)} u_i'\left(\overline{T}_i^1(s) - T_i^1(s), s\right) = \frac{1}{q(s')} u_i'\left(\overline{T}_i^1(s') - T_i^1(s'), s'\right) \qquad \forall s, s'.$$

Lemma 17 allows us to directly incorporate the indirect utility function $V_i^T(\mathfrak{T}_i)$ into planner i preferences, and to use total revenue collected $\mathfrak{T}_i$ as the choice variable. It implies that countries

[77] We impose $u_i^{T\prime}(0, s) = +\infty$. We think of u_i^T as incorporating both consumption preferences and distortionary effects of taxation.

[78] We could assume that the government pays a different price vector $\overline{q}$ for bailout claims, with derivations largely unchanged.

engage in tax smoothing without cooperation, but does not guarantee that they engage in the globally efficient level of bailouts.

Proof of Lemma 17. The optimization problem is

$$\max \omega_i^T \int_s u_i\left(\overline{T}_i^1(s) - T_i^1(s), s\right) f(s)ds \quad \text{s.t.} \quad \int_s q(s) T_i^1(s) f(s) ds \geq \mathfrak{T}_i$$

The FOCs are

$$\omega_i^T \frac{\partial u_i\left(\overline{T}_i^1(s) - T_i^1(s), s\right)}{\partial c_i^T(s)} f(s) - \mu q(s) f(s) = 0$$

Combining the FOCs across states, we obtain the result.

B.5.2 Implementability Conditions

We characterize the implementability conditions for domestic allocations of foreign banks, in a manner analogous to the characterization in Lemma 2. Note that now, the domestic choice variables of foreign banks are (D_{ij}, I_{ij}).

Lemma 18. *Country planner j can directly choose all domestic allocations of foreign banks, with implementing wedges*

$$\tau_{j,ij}^I = -\tau_{i,ij}^I - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda_i^0} E\left[\lambda_i^1(s)\left(\left(\gamma_j(s) - 1\right)\frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s)\right)\right] \tag{30}$$

$$\tau_{j,ij}^D = -\tau_{i,ij}^D - E\left[\frac{\lambda_i^1(s)}{\lambda_i^0}\left(1 + \left(\gamma_j(s) - 1\right)\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}\right)\right] \tag{31}$$

Using Lemma 18, we can characterize the non-cooperative equilibrium in the same manner as the baseline model. In particular, we isolate the decision problem of the country i planner, who optimizes domestic bank welfare choosing domestic and foreign allocations, subject to domestic bank constraints and to the implementability conditions of Lemma 18, taking as given foreign planner wedges and foreign bailouts.

B.5.3 Non-Cooperative and Cooperative Regulation

We now characterize optimal non-cooperative regulation.

In the non-cooperative equilibrium, country planners choose both the wedges and the bailouts $T^1_{i,ij}(s)$, taking as given the wedges and bailouts of other countries, to maximize domestic social welfare

$$V_i^P = \omega_i \left[\int_s c_i(s) f(s) ds + \omega_i^T V_i^T(\mathcal{T}_i) \right].$$

The following proposition describes optimal bailout policy under regulation.

Proposition 19. *The bailout rules in the non-cooperative equilibrium under quantity regulation are as follows.*

1. *The optimal bailout rule for the domestic operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ii}(s)}_{\text{Bank Benefit}} + \underbrace{\Omega^B_{i,i}(s) \frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}}_{\text{Domestic Spillovers}} \tag{32}$$

2. *The optimal bailout rule for the foreign operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ij}(s)}_{\text{Bank Benefit}} \tag{33}$$

3. *The optimal bailout rule for the domestic operations of a foreign bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{\Omega^B_{i,i}(s) \frac{\partial L_{ji}(s)}{\partial A^1_{ji}(s)}}_{\text{Domestic Spillovers}} \tag{34}$$

Three factors govern the non-cooperative bailout rules: the social cost of taxes, the direct benefit of bailouts to banks, and the domestic fire sale spillover. When choosing bailouts of domestic activities of domestic banks, the domestic planner considers all three factors, but neglects spillover costs to foreign banks. Moreover, the domestic planner neglects the benefits of alleviating foreign fire sales when choosing bailouts of foreign activities of domestic banks, and neglects the benefits of the bailout transfer when choosing bailouts of domestic activities of foreign banks. Country planners are *home biased* in their bailout decisions, generally preferring to bail out domestic activities of domestic banks.

Relative to the globally efficient bailout rule, non-cooperative planners under-value all bailout activities, including bailouts of domestic activities of domestic banks, not accounting for either benefits or spillovers to foreign banks. The cooperative agreement increases bailouts of both domestic and foreign banks. Multilateral fire sale spillovers imply the need for multilateral bailout cooperation.

Proposition 20. *Optimal non-cooperative regulation is given as follows.*

1. *Domestic taxes on domestic banks' domestic activities are given by*

$$\tau^D_{i,ii} = E\left[\Omega^B_{i,i}(s)\frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}\right] \tag{35}$$

$$\tau^I_{i,ii} = -E\left[\Omega^B_{i,i}(s)\frac{\partial L_{ii}(s)}{\partial I_{ii}}\right] \tag{36}$$

   while other domestic taxes on domestic banks are zero. $\Omega^B_{i,i}(s)$ *is defined in the proof.*

2. *If there is an adverse price spillover* $-\Omega^B_{i,i}(s) > 0$*, then regulation of foreign banks is equivalent to a ban on foreign liquidations.*

To understand Proposition 20, the fact that liquidations are now determined indirectly, rather than directly, implies that the spillovers $\Omega^B_{i,i}(s)$ now form a basis to price the cost of policies that increase liquidations. This is reflected in the optimal tax rates.

At the same time, the domestic planner prefers sufficiently stringent regulation to prevent foreign banks from contributing to domestic fire sales. This is equivalent to requiring foreign banks to maintain domestic allocations that set $L^L_{ji} = 0$.

Next, we can characterize regulatory policy under the optimal cooperative agreement (global planning).

Proposition 21. *Optimal cooperative policy consists of taxes on investment scale and debt, given by*

$$\tau_{ij}^{D} = E\left[\left[\Omega_{j,j}^{B}(s) + \int_{i'} \Omega_{i',j}^{B}(s) di'\right] \frac{\partial L_{ij}(s)}{\partial A_{ij}^{1}(s)}\right] \tag{37}$$

$$\tau_{ij}^{I} = -E\left[\left[\Omega_{j,j}^{B}(s) + \int_{i'} \Omega_{i',j}^{B}(s)\right] \frac{\partial L_{ij}(s)}{\partial I_{ij}}\right] \tag{38}$$

The intuition of Proposition 21 is analogous to the intuition of Proposition 1. Globally optimal policy accounts for the full set of spillovers. Note that cooperative policy no longer features equal treatment in tax rates, to the extent that the responses of different banks' liquidation rules are different on the margin. There is equal treatment in the sense that the basis of spillover effects $\Omega_{i,j}(s)$ are the same, independent of which country generates the spillover.

Finally, we can characterize the optimal tax collection and bailout sharing rules of the cooperative agreement.

Proposition 22. *Globally optimal tax collection and bailout sharing are as follows.*

1. *Optimal cross-country bailout sharing is given by*

$$\omega_i \omega_i^T \frac{\partial V_i^T(\mathcal{T}_i)}{\partial \mathcal{T}_i} = \omega_j \omega_j^T \frac{\partial V_j^T(\mathcal{T}_j)}{\partial \mathcal{T}_j} \quad \forall i,j \tag{39}$$

2. *Any bailout sharing rule* $\left(T_{i,ij}^{1}(s), T_{j,ij}^{1}(s)\right)$ *satisfying* $T_{ij}^{1}(s) = T_{i,ij}^{1}(s) + T_{j,ij}^{1}(s)$ *can be used to implement the globally optimal allocation. Different bailout sharing rules differ in the initial distribution of tax revenue claims* G_i*. We can set* $T_{i,i'j}^{1}(s) = 0$ *whenever* $i \notin \{i', j\}$ *without loss of generality.*

The bailout sharing rule (39) implies that tax burdens of bailouts are smoothed across countries in an average sense but not state-by-state at date 1, so that some countries may be net contributors or recepients of bailouts in any given state s.[79] Bailout sharing rule irrelevance describes equivalent set of bailout sharing rules, and implies that in principle bailout obligations can be delegated entirely to one country (or to one international organization).[80]

[79] For example, if countries have the same indirect utility functions and are equally weighted globally, then expected tax burdens are the same across countries.

[80] For example, the responsibility for deposit insurance can be entirely vested in a single entity (the host country,

B.6 Bailout Proofs

B.6.1 Proof of Lemma 18

The Lagrangian of the country i bank problem is given by

$$\mathcal{L}_i = \int_s c_i(s) f(s) ds + \lambda_i^0 \left[A_i + D_i - T_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right]$$
$$+ \int_s \lambda_i^1(s) \left[A_i^1(s) + (\gamma_i(s) - 1) L_{ii}(s) + R_i(s) I_i + \int_j \left((\gamma_j(s) - 1) L_{ij}(s) + R_j(s) I_j \right) dj - c_i(s) \right] f(s) ds$$

where we have implicitly internalized the demand liquidation function $L_{ij}(s) = \max\{0, -\frac{1}{h_j(s)\gamma_j(s)} A_{ij}^1(s) - \frac{(1-h_j(s))}{h_j(s)} R_j(s) I_{ij}(s)\}$. Taking the FOC is I_{ij} and rearranging, we obtain

$$\tau_{j,ij}^I = -\tau_{i,ij}^I - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda_i^0} E\left[\lambda_i^1(s) \left((\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s) \right) \right].$$

Similarly, taking the FOC for $x_{ij}(s)$ and rearranging, we obtain

$$\tau_{j,ij}^D = -\tau_{i,ij}^D - E\left[\frac{1}{\lambda_i^0} \lambda_i^1(s) \left(1 + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} \right) \right].$$

B.6.2 Proof of Propositions 19 and 20

As in the baseline model, the implementing tax rates of Lemma 18 do not otherwise appear in the country i planning problem. These constraints simply determine these tax rates, for the chosen allocation.

Now, consider the decision problem of the country i planner. The only twist is that the liquidation discount is now given by the equation

$$\gamma_i(s) = \gamma_i \left(L_{ii}(s) + \int_j L_{ji}(s) dj, s \right),$$

where we have adopted the shorthand $\gamma_i = \frac{\partial \mathcal{F}_i}{\partial I_i^A}$. From here, we characterize the response of the liquidation price to an increase ε in total liquidations. Totally differentiating the above equation in

the home country, or an international deposit guarantee scheme). Once the bailout authority has been delegated to a single entity, the goal of the global planner will be to ensure that that entity chooses bailouts optimally. In practice, imperfectly controllable political economy distortions may lead to bailout funds being misused. See Foarta (2018).

total liquidations, we have

$$\frac{\partial \gamma_i(s)}{\partial \varepsilon} = \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \left[1 + \frac{\partial [L_{ii}(s) + \int_j L_{ji}(s) dj]}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon} \right],$$

where $L_{ii}(s)$ and $L_{ij}(s)$ depend on $\gamma_i(s)$ due to the collateral constraint. Rearranging from here, we obtain the equilibrium country i price response

$$\frac{\partial \gamma_i(s)}{\partial \varepsilon} = \frac{1}{1 - \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \frac{\partial L_i^A(s)}{\partial \gamma_i(s)}} \frac{\partial \gamma_i(s)}{\partial L_i^A(s)}.$$

This characterization is useful, since externalities in this proof arise from changes in total liquidations.

Now, consider the Lagrangian of the country i planner. The Lagrangian of the planner can be written as

$$\mathcal{L}_i^{SP} = \mathcal{L}_i + \omega_i^T V_i^T(\mathcal{T}_i) + \lambda_i^T \left[G_i + \mathcal{T}_i - \int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) + \int_j T_{i,ij}^1(s) \right] f(s) ds \right.$$

where $\mathcal{L}_i$ internalizes the liquidation response and liquidation price relationships.

We first characterize the regulatory policies (Proposition 20), and then characterize the bailout policies (Proposition 19).

Regulatory Policies. Consider first the domestic allocations of domestic banks. For foreign allocations and consumption of the bank, the planner and bank derivatives coincide, and no wedges are applied, that is $\tau_{i,ij}^D = \tau_{i,ij}^I = 0$ for all $j \neq i$.

For domestic investment, the planner's derivative is

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial I_{ii}} = \frac{\partial \mathcal{L}_i}{\partial I_{ii}} + \int_s \underbrace{\frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon}}_{\equiv \lambda_i^0 \Omega_{i,i}^B(s) f(s)} \frac{\partial L_{ii}(s)}{\partial I_{ii}} ds$$

so that the domestic tax on domestic investment scale is given by

$$\tau_{i,ii}^I = -E \left[\Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial I_{ii}} \right],$$

which is simply the expected spillover effect. Next, we can apply the same argument to taxes on domestic state-contingent securities D_{ii}. We have

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial D_{ii}} = \frac{\partial \mathcal{L}_i}{\partial D_{ii}} + \int_s \frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon} \frac{\partial L_{ii}(s)}{\partial D_{ii}}$$

so that the required tax rate is

$$\tau_{i,ii}^D = E\left[\Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}\right].$$

Finally, considering domestic allocations of foreign banks, we only have the price spillover effect. This implies that there is a liquidation ban whenever there is an adverse price spillover, $-\Omega_{i,i}^B(s) > 0$.

Note that we can formally characterize the spillover effect $\Omega_{i,j}^B(s)$ by evaluating

$$\frac{\partial \mathcal{L}_i}{\partial \gamma_j(s)} = \lambda_i^1(s)\left[L_{ij}(s) + (\gamma_j(s) - 1)\frac{\partial L_{ij}(s)}{\partial \gamma_j(s)}\right] f(s)$$

so that we have

$$\Omega_{i,j}^B(s) = \frac{\frac{\partial \mathcal{L}_i}{\partial \gamma_j(s)} \frac{\partial \gamma_j(s)}{\partial \varepsilon}}{\lambda_i^0 f(s)} = \frac{\lambda_i^1(s)}{\lambda_i^0}\left[L_{ij}(s) + (\gamma_j(s) - 1)\frac{\partial L_{ij}(s)}{\partial \gamma_j(s)}\right] \frac{\partial \gamma_j(s)}{\partial \varepsilon}.$$

Bailout Policies. We next characterize the optimal bailout policies. Consider first the bailout rule for domestic activities of domestic banks, where we have

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial T_{i,ii}^1(s)} = \underbrace{\frac{\partial \mathcal{L}_i}{\partial A_{ii}^1(s)}}_{\equiv \lambda_i^0 B_{ii}^1(s) f(s)} + \frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon} \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)} - \lambda_i^T f(s).$$

Now, the FOC for tax collection tells us that $\lambda_i^T = -\omega_i^T \frac{\partial V_i^T}{\partial \mathfrak{T}_i}$. Noting that $\frac{\partial V_i^T}{\partial \mathfrak{T}_i} < 0$, we rearrange and obtain the bailout rule

$$\frac{\omega_i^T}{\lambda_i^0}\left|\frac{\partial V_i^T}{\partial \mathfrak{T}_i}\right| \geq B_{ii}^1(s) + \Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}.$$

The remaining two equations follow simply by noting that the spillover term does not appear in the FOC for bailouts of foreign activities of domestic banks, while the bank benefit term does not appear in the FOC for bailouts of domestic activities of foreign banks.

B.6.3 Proof of Propositions 21 and 22

The Lagrangian of the global planner is given by

$$\mathcal{L}_i^G = \int_i \left[\mathcal{L}_i + \omega_i^T V_i^T(\mathfrak{T}_i) + \lambda_i^T \left[G_i + \mathfrak{T}_i - \int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) + \int_j T_{i,ij}^1(s) \right] f(s) ds \right] + \int_i \left[\lambda^0 T_i + \lambda^T G_i \right] di$$

where the last terms reflect the set of lump sum transfers. The FOC for G_i implies $\lambda^T = \lambda_i^T$ while the FOC for T_i implies $\lambda^0 = \lambda_i^0$. From here, the regulation and bailout rules follow by the same steps as in the non-cooperative equilibrium, except that now the full set of spillovers appear, and the benefits to banks of bailouts are always accounted for.

Next, the relationship $\lambda^T = \lambda_i^T$ gives the tax sharing rule. Bailout irrelevance arises by setting $G_i = \int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) dj + \int_j T_{i,ij}^1(s) dj \right] dj - \mathfrak{T}_i$, for the desired bailout rule.

B.6.4 Proof of Propositions 16

The country planner Lagrangian is the same as under regulation, except that there is now also tax revenue collected from foreign banks.

The tax revenue collected by country j from country i banks is given by

$$T_{j,ij}^* = \tau_{j,ij} I_{ij} + \tau_{j,ij}^D D_{ij}$$

so that differentiating in total liquidations in state s, we have

$$\frac{\partial T_{j,ij}^*}{\partial \varepsilon} = \frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda_i^1(s)}{\lambda_i^0} (\gamma_j(s) - 1) \left[\frac{\partial L_{ij}(s)}{\partial I_{ij}} I_{ij} - \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} D_{ij} \right] \right] \frac{\partial \gamma_j(s)}{\partial \varepsilon} f(s)$$

from here, we note that $L_{ij}(s)$ is homogeneous of degree 1 in $(I_{ij}, A_{ij}^1(s))$, given γ_j, so that we can write

$$\begin{aligned} \frac{\partial T_{j,ij}^*}{\partial \gamma_j(s)} &= \frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda_i^1(s)}{\lambda_i^0} (\gamma_j(s) - 1) \left[L_{ij}(s) - \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} T_{ij}^1(s) \right] \right] \frac{\partial \gamma_j(s)}{\partial \varepsilon} f(s) \\ &= \Omega_{j,i}^B(s) f(s) + \Delta_{ij}^T(s) T_{ij}^1(s) f(s) \end{aligned}$$

where we have defined $\Delta_{ij}^{T}(s) = -\frac{\partial}{\partial \gamma_j(s)}\left[\frac{\lambda_i^1(s)}{\lambda_i^0}\left(\gamma_j(s)-1\right)\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}T_{ij}^1(s)\right]\frac{\partial \gamma_j(s)}{\partial \varepsilon}$.

From here, results on regulation follow by the usual steps. Moreover, results on bailouts also follow the usual steps, noting the bailout has indirect effects on tax rates through the liquidation price, but does not have direct effects due to the linear nature of $L_{ij}(s)$.

C General Framework: Non-Cooperative

This Appendix presents the results of the non-cooperative problem in the general framework of Section 6.

C.1 Non-Cooperative Setup

The setup of the non-cooperative problem is analogous to the setup in the baseline model. Country planner i maximizes the welfare of domestic agents using a complete set of wedges $\tau_{i,i} = \{\tau_{i,ij}(m)\}_{j,m}$ on the actions of domestic agents, and wedges $\tau_{i,ji} = \{\tau_{i,ji}(m)\}_m$ on domestic actions of foreign agents. The total tax burden faced by country i agents from the domestic planner (excluding remissions) is therefore $T_{i,i} = \tau_{i,ii}a_{i,ii} + \int_j \tau_{i,ij}a_{i,ij}dj$, while the total tax burden from foreign planner j is given by $T_{j,ij} = \tau_{j,ij}a_{i,ij}$. These taxes appear in the wealth of the multinational agent.

As in the baseline model, under *quantity regulation* wedges are revenue-neutral, while under *Pigouvian taxation* wedges generate revenues from foreign banks.

Implementability. As in the baseline model, the approach to implementability is standard for domestic agents. Moreover, an implementability result analogous to Lemma 2 holds in the general environment, allowing us to apply the standard approach for domestic actions of foreign agents.

Lemma 23. *The domestic actions of foreign agents can be chosen by the domestic planner, with implementing wedges*

$$\tau_{i,ji}(m) = -\tau_{j,ji}(m) + \frac{1}{\lambda_j^0}\left[\omega_j \frac{\partial U_j}{\partial u_j}\frac{\partial u_{ji}}{\partial a_{ji}(m)} + \omega_j \frac{\partial U_j}{\partial u_j^A}\frac{\partial u_{ji}^A}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j}\frac{\partial \phi_{ji}}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j^A}\frac{\partial \phi_{ji}^A}{\partial a_{ji}(m)}\right]$$

where $\tau_{j,ji}$, λ_j^0, Λ_j, $\frac{\partial U_j}{\partial u_j}$, $\frac{\partial U_j}{\partial u_j^A}$, $\frac{\partial \Gamma_j}{\partial \phi_j}$, and $\frac{\partial \Gamma_j}{\partial \phi_j^A}$ are constants from the perspective of country planner i.

The intuition behind these implementability conditions is analogous to the baseline model: the planner first unwinds the wedge placed by the foreign planner, and then sets the residual wedge equal to the benefit to foreign agents of conducting that activity.

C.2 Non-Cooperative Quantity Regulation

We now characterize the non-cooperative equilibrium under quantity regulation, where wedges are revenue neutral. We obtain the following characterization of the non-cooperative equilibrium.

Proposition 24. *Under non-cooperative quantity regulation, the equilibrium has the following features.*

1. *The domestic wedges on domestic activities of domestic agents are*

$$\tau_{i,ii}(m) = -\Omega_{i,i}(m)$$

while the domestic wedges on foreign activities of domestic agents are 0.

2. *The domestic wedges on foreign banks generate an allocation rule*

$$\Omega_{i,i}(m)a_{ji}(m) = 0$$

so that foreign activities are allowed only up to the point they increase domestic welfare.

Proposition 24 reflects logic closely related to the baseline model. On the one hand, regulatory policies applied to domestic agents account for spillovers to domestic agents, but not to foreign agents. On the other hand, regulatory policies applied to foreign agents' domestic activities do not account for benefits to foreign agents of domestic activities. Foreign agents are allowed to conduct domestic activities only to the extent the *domestic* benefits of those activities outweigh *domestic* costs.

This characterization leads to a generic inefficiency result in the presence of cross-border activities. We say that there are *cross border activities* if $\exists M' \subset M$ and $I, J \subset [0,1]$ such that $a_{ii}(m), a_{ji}(m) > 0 \; \forall m \in M', i \in I, j \in J$.

Proposition 25. *Suppose that a globally efficient allocation features cross border activities over a triple (M', I, J). The non-cooperative equilibrium under quantity regulation generates this globally efficient allocation* only if *the globally efficient allocation features $\Omega_{i,i}(m) = \int_{i'} \Omega_{i',i}(m)di' = 0$ $\forall m \in M', i \in I$.*

Proposition 25 provides a strong and generic result that quantity regulation does not generate an efficient allocation when there are regulated cross-border activities. In particular, cross-border activities must generate no *net* domestic externality to avoid the problem of unequal treatment, and cross-border activities must generate no *net* foreign externalities to avoid the problem of uninternalized foreign spillovers. Notice that efficient under Proposition 25 requires no regulation of cross-border activities in the globally efficient policy.

C.3 Non-Cooperative Pigouvian Taxation

Finally, we characterize non-cooperative Pigouvian taxation and its optimality.

Proposition 26. *Suppose Assumption 9 holds. The equilibrium under non-cooperative Pigouvian taxation has the following features.*

1. *The domestic wedges on domestic activities of domestic agents are*

$$\tau_{i,ii}(m) = -\Omega_{i,i}(m) - \int_j \Omega_{j,i}(m)dj$$

while domestic wedges on foreign activities of domestic agents are 0*.*

2. *The domestic wedges on domestic activities of foreign agents are*

$$\tau_{i,ji}(m) = \tau_{i,ii}(m) - \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$$

As in the baseline model, the derivatives of foreign tax revenue in domestic liquidation prices yield the foreign spillovers, so that planners account for these effects in designing policy. However, revenue collection generates a monopolistic distortion. The generalized problem therefore reflects the same logic as Proposition 4, with the only difference being the nature of the spillovers and of the monopolistic distortion. As in the baseline model, when this monopolist distortion is zero, non-cooperative Pigouvian taxation results in a globally efficient allocation.

Proposition 27. *Suppose Assumption 9 holds, and suppose that $u_{ij}, \phi_{ij}, u^A_{ij}, \phi^A_{ij}$ are linear in a_{ij} (given a^A_j) for all i and $j \neq i$. Then the non-cooperative equilibrium under taxation is globally efficient, and there is no scope for cooperation.*

Proof. Observe that when $u_{ij}, \phi_{ij}, u^A_{ij}, \phi^A_{ij}$ are linear in a_{ij} (given a^A_j) for all i and $j \neq i$, then the non-cooperative tax rates align with the cooperative ones, resulting in an efficient allocation. ■

Non-cooperative taxation is globally efficient if Assumption 9 holds, and if the monopolistic distortions are zero. The assumption of linearity on $u_{ij}, \phi_{ij}, u^A_{ij}, \phi^A_{ij}$ ensures that (partial equilibrium) elasticities of foreign activities with respect to tax rates are infinite, so that monopolistic distortions are zero. This reflects the same notion of sufficient substitutability as in the baseline model, and generalizes Proposition 5 to a broader class of problems.

As in the baseline model, Proposition 27 provides a limiting case of exactly efficiency. Comparing Propositions 26 and 8 reveals that even without exact efficiency, there are three appealing properties of Pigouvian taxation. The first is that the need for cooperation is restricted to foreign activities of multinational agents. The second is that cooperation is needed only to correct bilateral monopolist problems. The third is that the information needed to determine the magnitude of these problems is a set of partial equilibrium elasticities. This provides a potential method to evaluate the need for cooperation in practice.

C.4 Proofs

C.4.1 Proof of Lemma 23

Taking the Lagrangian of bank i

$$\mathcal{L}_i = \omega_i U_i\left(u_i(a_i), u^A_i(a_i, a^A))\right) + \Lambda_i \Gamma_i\left(A_i - T_i, \phi_i(a_i), \phi^A_i(a_i, a^A)\right)$$

and taking the first order condition in $a_{ij}(m)$, we obtain

$$0=\omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)}+\omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_{ij}(m)}+\Lambda_i\frac{\partial \Gamma_i}{\partial W_i}\left(-\tau_{i,ij}(m)-\tau_{j,ij}(m)\right)+\Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)}+\Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)}.$$

Defining $\lambda_i^0=\Lambda_i\frac{\partial \Gamma_i}{\partial W_i}$ and rearranging, we obtain

$$\tau_{j,ij}(m)=-\tau_{i,ij}(m)+\frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)}+\omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_{ij}(m)}+\Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)}+\Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)}\right]$$

giving the relevant equation. From here, notice that the allocations a_{ij} and aggregates a_j^A appear to first order only in the tax rate equations in country j. As a result, considering any candidate equilibrium, the first order conditions for optimality for allocations by country i banks outside of country j are not affected (to first order) by policies in country j, and so continue to hold independent of a_{ij} and a_j^A. As a result, any allocation a_{ij} can be implemented with the above tax rates. The implementability result follows.

C.4.2 Proof of Proposition 24

Substituting in the equilibrium tax revenue, the optimization problem of the country i social planner is

$$\max_{a_i,\{a_{ji}\}}\ \omega_i U_i\left(u_i(a_i),u_i^A(a_i,a^A))\right)\qquad \text{s.t.}\quad \Gamma_i\left(A_i-\int_j \tau_{j,ij}a_{ij}dj,\phi_i(a_i),\phi_i^A(a_i,a^A)\right)\geq 0$$

and subject to the implementability conditions of Lemma 23. Note that the wedges rates $\tau_{i,ji}$ do not appear except in the implementability conditions, meaning that they are set to clear implementability but do not contribute to welfare. As a result, the Lagrange multipliers on implementability are 0, and the Lagrangian of planner i is given by

$$\mathcal{L}_i^{SP}=\omega_i U_i\left(u_i(a_i),u_i^A(a_i,a^A))\right)+\Lambda_i\Gamma_i\left(A_i-\int_j \tau_{j,ij}a_{ij}dj,\phi_i(a_i),\phi_i^A(a_i,a^A)\right).$$

First of all, note that the social planner does not internalize impacts on foreign aggregates. As a result, $\frac{d\mathcal{L}_i^{SP}}{da_{ij}(m)}=\frac{\partial \mathcal{L}_i^{SP}}{\partial a_{ij}(m)}$. Social and private preferences align, and therefore we have $\tau_{i,ij}(m)=0$.

Next, consider a domestic policy $a_{ii}(m)$. Here, we have $\frac{d\mathcal{L}_i^{SP}}{da_{ii}(m)} = \frac{\partial\mathcal{L}_i^{SP}}{\partial a_{ii}(m)} + \frac{\partial\mathcal{L}_i^{SP}}{\partial a_i^A(m)}$. To align preferences, the domestic planner therefore sets

$$\tau_{i,ii}(m) = -\frac{1}{\lambda_i^0}\frac{\partial\mathcal{L}_i^{SP}}{\partial a_i^A(m)} = -\Omega_{i,i}(m)$$

where the final equality follows as in the proof of Proposition 8.

Finally, consider $a_{ji}(m)$. Here, we have $\frac{d\mathcal{L}_i^{SP}}{da_{ji}(m)} = \frac{\partial\mathcal{L}_i^{SP}}{\partial a_i^A(m)} = \lambda_i^0\Omega_{i,i}(m)$, giving the allocation rule.

C.4.3 Proof of Proposition 25

Given a globally efficient allocation with cross border activites over (M', I, J), suppose that the non-cooperative equilibrium under quantity regulation generates this allocation. From Proposition 24, $a_{ji}(m) > 0$ implies that $\Omega_{i,i}(m) = 0$ over (M, I, J). Using Propositions 8 and 24, $\tau_{i,ii}(m) = \tau_{ii}(m)$ and $\Omega_{i,i}(m) = 0$ implies that $\int_{i'} \Omega_{i',i}(m)di' = 0$ over (M, I, J), completing the proof.

C.4.4 Proof of Proposition 26

It is helpful to begin by characterizing the derivative of revenue from foreign agents in the domestic aggregate. Using the implementability conditions of Lemma 23, the revenue collected by planner j from country i agents is

$$T_{j,ij}^* = \tau_{j,ij}a_{ij} = -\tau_{i,ij}a_{ij} + \frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}}a_{ij} + \omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_{ij}}a_{ij} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i}\frac{\partial\phi_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i^A}\frac{\partial\phi_{ij}^A}{\partial a_{ij}}a_{ij}\right].$$

Applying Assumption 9, $\frac{\partial u_{ij}^A}{\partial a_{ij}}a_{ij} = u_{ij}^A$ and $\frac{\partial\phi_{ij}^A}{\partial a_{ij}}a_{ij} = \phi_{ij}^A$, so that we obtain

$$T_{j,ij}^* = -\tau_{i,ij}a_{ij} + \frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}}a_{ij} + \omega_i\frac{\partial U_i}{\partial u_i^A}u_{ij}^A + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i}\frac{\partial\phi_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i^A}\phi_{ij}^A\right].$$

Finally, differentiating in $a_j^A(m)$, we obtain

$$\frac{\partial T_{j,ij}^*}{\partial a_j^A(m)} = \frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_j^A(m)} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i^A}\frac{\partial\phi_{ij}^A}{\partial a_j^A(m)}\right] = \Omega_{i,j}(m)$$

which is the spillover effect. From here, the country i social planner's Lagrangian is given by

$$\mathcal{L}_i^{SP} = \omega_i U_i\left(u_i(a_i), u_i^A(a_i, a^A))\right) + \Lambda_i \Gamma_i\left(A_i - \int_j \tau_{j,ij} a_{ij} dj + \int_j T_{i,ji}^* dj, \phi_i(a_i), \phi_i^A(a_i, a^A)\right).$$

From here, derivation follows as in the proof of Proposition 24, except for the additional derivative in revenue. For $a_{ij}(m)$, there is no additional revenue derivative, and so $\tau_{i,ij}(m) = 0$ as before. For $a_{ii}(m)$, we have following the steps of Proposition 24

$$\tau_{i,ii}(m) = -\frac{1}{\lambda_i^0}\frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)} - \frac{1}{\lambda_i^0}\Lambda_i \frac{\partial \Gamma_i}{\partial W_i}\int_j \frac{\partial T_{i,ji}^*}{\partial a_i^A(m)} dj = -\Omega_{i,i}(m) - \int_j \Omega_{j,i}(m) dj$$

giving the first result.

Finally, considering a foreign allocation a_{ji}, we have

$$0 = \frac{d\mathcal{L}_i^{SP}}{da_{ji}(m)} = \frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)} + \lambda_i^0\left[\frac{dT_{i,ji}^*}{da_{ji}(m)} + \int_j \frac{\partial T_{i,ji}^*}{\partial a_i^A(m)} dj\right].$$

From here, noting that we have $\frac{dT_{i,ji}^*}{da_{ji}(m)} = \tau_{i,ji}(m) + \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$, we obtain

$$0 = -\tau_{i,ii}(m) + \tau_{i,ji}(m) + \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$$

which rearranges to the result.

D Extensions of the Banking Model

In this Appendix, we present extensions to and discussions of the model, as applied to the banking context. To ease exposition, we express all results in this appendix for interior solutions, except for foreign allocations under non-cooperative regulation.

D.1 Dispersed Bank Ownership

Banks in practice are multinational not only in their activities, but also in ownership: even though a bank is headquartered in one country, part of its equity can be owned by foreigners. This invites a natural question: do regulatory incentives change when part of the value of banks accrues to non-domestic agents, and if so does it cause inefficiencies?

This model is a straight-forward extension of the baseline model, and an application of the general theory. In particular, suppose that there is a disconnected set of global "equity" investors, who have preferences given by $U_{e,0}(c_{e,0}) + E[U_{e,1}(c_{e,1} + c_{e,2})]$.[81] These global investors imply that the global (probability-normalized) price of equity payoff in state s is given by

$$q(s) = \frac{U'_{e,1}(c_{e,1} + c_{e,2})}{U'_{e,0}(c_{e,0})},$$

that is the stochastic discount factor of global equity arbitrageurs.

Now, suppose that bank i sells "equity" payoff claims $\alpha_i(s)$ to global investors. It receives total revenue from this equity sale of

$$E_i = \int_s q(s)\alpha_i(s)f(s)ds.$$

Equity issuance may be constrained by some general constraint set, as in Section 6, for example incentive constraints. From here, the results are an application of Appendix E.1. Globally efficient regulation does not regulate issuance of equity, since the global price $q(s)$ only generates distributive externalities that net out in equilibrium.[82] As such, the core results of the baseline model carry through, and the efficiency results are the same. This generalizes the results to include common ownership of cross-border banks.

D.2 Local Capital Goods and Protectionism

Although financial stability and fire sales have been highlighted as justifications for post-crisis regulation, cooperative agreements predate the crisis, including the previous Basel accords. In this context, regulators may care about additional considerations such as domestic spillovers. Additionally, regulators may care about controlling local costs of investment, for example wishing to ensure that (less strictly regulated) foreign banks are not at a competitive advantage over domestic banks while also wishing to allow domestic banks to expand in more-regulated foreign markets

[81] We could alternatively consider a representative equity investor existing within each country.

[82] The key here is not that equity is not regulated, but rather that there is not a welfare-relevant pecuniary externality from the global price.

where lending costs are cheaper. This form of trade-off underlies prior literature on international cooperation, such as Dell'Ariccia and Marquez (2006).[83]

We consider such a motivation by extending the model to include a common domestic investment price. In particular, we augment the model with local capital goods, which are used to produce domestic projects. For simplicity, we rule out all spillovers besides capital good prices in this sections. As a result, there are no fire sales and no extended stakeholder spillovers.

Banks can produce projects using both initial endowment, and with units of a local capital good. Bank i purchases a vector k_i of local capital goods, with k_{ij} being the capital good of country j, at prices p_j. When using k_i of the capital good, it costs an additional $\Phi_{ii}(I_{ii}, k_{ii}) + \int_j \Phi_{ij}(I_{ij}, k_{ij})dj$ to produce the vector I_i of projects. The date 0 budget constraint of bank i is

$$p_i k_{ii} + \int_j p_j k_{ij} dj + \Phi_{ii}(I_{ii}, k_{ii}) + \int_j \Phi_{ij}(I_{ij}, k_{ij}) dj \leq A_i + D_i.$$

The optimization problem of banks is otherwise unchanged, except that k_i is now a choice variable of banks.

In each country, there is a representative capital producing firm. The capital producing firm produces the capital good out of the consumption good with an increasing and weakly convex cost function $\mathcal{K}_i(K_i)$, and so has an optimization problem

$$\max_{K_i} p_i K_i - \mathcal{K}_i(K_i).$$

The resulting equilibrium capital good price in country i is

$$p_i = \frac{\partial \mathcal{K}_i(K_i)}{\partial K_i}, \qquad K_i = k_{ii} + \int_j k_{ji} dj. \tag{40}$$

The local capital producing firm cannot be controlled by country planners, so that equation (40) is an implementability condition of the model. Note that $\frac{\partial p_i}{\partial K_i} \geq 0$.[84]

Finally, the social planner places a welfare weight ω_i^K on the capital producing firm, so that

[83] In this Appendix, our main contribution relative to their paper is to allow for common agency, to study the impacts of Pigouvian taxation, and to relate this mechanism to fire sales.

[84] In order to ensure that firm profits are bounded above, we will assume that $\frac{\partial p_i}{\partial K_i} = 0$ above some point K^*, which amounts to assuming that $\mathcal{K}_i(K_i)$ becomes linear on the margin above K^*.

the social welfare function is

$$V_i^P = \int_s c_i(s) f(s) ds + \omega_i^K \Big[p_i(K_i) K_i - \mathcal{K}_i(K_i) \Big].$$

From here, note that the model is in the form of Section 6 when we interpret profits of the capital producing firm as a utility spillover to the domestic representative bank.

From here, we see that there are spillovers to both domestic and foreign agents from changes in capital purchases, given by

$$\Omega_{i,i}^K = -\frac{\partial p_i}{\partial K_i} k_{ii} + \frac{\omega_i^K}{\lambda_i^0} \frac{\partial p_i}{\partial K_i} K_i$$

$$\Omega_{j,i}^K = -\frac{\partial p_i}{\partial K_i} k_{ji}$$

The spillover from the capital price increase is the additional resource cost to the bank of purchasing their existing level of the capital good. This is closely related to the direct price spillover under fire sales.

Let us suppose that we are in an environment where the domestic planner wishes to subsidize domestic banks by keeping capital cheap. We represent this by the limiting case $\omega_i^K = 0$. In this case, there is a *negative* spillover from increases in the capital price to both domestic and foreign banks, which make capital more expensive.

The globally efficient policy subsidizes capital by limiting capital purchases of all banks. By contrast, non-cooperative quantity regulation is protectionist and bans foreign banks from purchasing the domestic capital. In effect, it shields domestic banks from foreign competition.

Nevertheless, the "pecuniary externality" here falls within the class of problems under Assumption 9. As a result, assuming no monopoly power, the non-cooperative equilibrium under Pigouvian taxation is globally efficient.

Relationship to the Pre-Crisis World. In addition to understanding the Basel accords, this result also helps contextualize the historical aversion to capital control measures or other barriers to capital flows. In a purely non-cooperative environment, countries are tempted to engage in inefficient protectionism to shield domestic banks from foreign competition. Protectionism is inefficient because all countries do so, and so countries benefit from agreements against protectionist policies.

For example, agreements might allow expansion via branches, rather than subsidiaries, in addition to lifting other barriers to capital flows. Our results suggest that although quantity-based measures lead to inefficient protectionist policies, priced-based measures (taxes) do not. This provides another advantage of tax-based policies in the international context.

Differences from Fire Sales. Although the general characterizations in this extension are closely related in a general sense to the characterizations of the main paper under fire sales, there are two important differences.

The first important difference is the form of restrictions on foreign banks. Under fire sales, non-cooperative policies were meant to restrict premature liquidations. This corresponded most naturally to either ring fencing type policies, or to restrictions on capital outflows. By contrast, with local capital prices, non-cooperative policies are meant to restrict investment in the first place, and so
on capital inflows. The motivation under the former is to enhance domestic financial stability, while the motivation under the latter is more protectionist in nature.

The second important distinction is in the implications for cooperation. Under fire sales, cooperation was required among countries who invest across borders *and* who share common crisis states. By contrast under local capital goods, cross border investment alone determines the need for cooperation.

D.3 Quantity Restrictions in the Form of Ceilings

In this appendix, we show how the global optimum (Proposition 1) and the non-cooperative optimum under quantity restrictions (Proposition 3) can be achieved using explicit quantity restrictions, rather than revenue-neutral taxes. Moreover, we show this forms an optimal policy under quantity restrictions. In this manner, we will show that duality between quantity restrictions and revenue-neutral taxes holds in our baseline model.

The argument will proceed in two steps. First ("Step 1"), we will argue that provided we can find an implementation of Propositions 1 and Proposition 3 using explicit quantity restrictions, then that implementation is in fact optimal among all possible quantity restrictions. Second ("Step 2"), we will show how quantity restrictions implement these outcomes.

D.3.1 Global Optimum

We can formally define quantity restrictions set by the global planner to be a set of restrictions $\Phi_i(c_i, D_i, I_i, L_i) \leq 0$ on the bank i contract. For example, possible restrictions include: (i) a ceiling on liquidations, $L_{ij}(s) \leq \overline{L}_{ij}(s)$ (e.g. a quantity-based capital control restricting outflows); and, (ii) a ceiling on debt, $D_i \leq \overline{D}_i$ (e.g. a leverage requirement).

Step 1. To start with the first step of the argument, suppose that we can in fact find set of quantity restrictions Φ_i that implements the cooperative outcomes $(c_i^*, D_i^*, I_i^*, L_i^*)$ of Proposition 1, where we have used the asterisk notation to denote the optimal quantities.[85] It then in fact follows that we have found a global optimum under all possible quantity restrictions. The reason is that Proposition 1 is already the solution to the global constrained efficient planning problem in which the global planner directly chooses the contracts (c_i, D_i, I_i, L_i) of all banks, subject to the same constraints faced by banks. The only difference is in how the global planner implements this Pareto efficient allocation. In Proposition 1, it is implemented via revenue-neutral Pigouvian wedges. Here, we are looking to implement it via quantity restrictions.

Step 2. Now, we argue how the Pareto efficient allocation (global optimum) of Proposition 1 can be implemented using quantity restrictions. In particular, consider a ceiling on liquidations $L_{ij}(s) \leq \overline{L}_{ij}(s) = L_{ij}^*(s)$, that is the ceiling $\overline{L}_{ij}(s)$ is set equal to the globally optimal liquidation rule $L_{ij}^*(s)$. To complete the argument, we need to show that there are non-negative Lagrange multipliers such that the allocation $(c_i^*, D_i^*, I_i^*, L_i^*)$ satisfies the Lagrangian optimality conditions.

Taking the bank Lagrangian in Appendix A.1 and incorporating the quantity ceiling restrictions,

[85] As in Proposition 3, cooperation may also involve date 0 lump-sum transfers $\mathcal{T}_i$ to guarantee Pareto efficiency.

we have,

$$
\begin{aligned}
\mathcal{L}_i =& \omega_i \int_s c_i(s) f(s) ds + \lambda_i^0 \left[A_i + D_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right] \\
&+ \int_s \lambda_i^1(s) \Bigg[\gamma_i(s) L_{ii}(s) + (1 + r_{ii})(R_i(s) I_{ii} - L_{ii}(s)) \\
&\qquad\qquad + \int_j \left[\gamma_j(s) L_{ij}(s) + (1 + r_{ij})(R_j(s) I_{ij} - L_{ij}(s)) \right] dj - c_i(s) - D_i \Bigg] f(s) ds \\
&+ \int_s \Lambda_i^1(s) \left[-D_i + \gamma_i(s) L_{ii}(s) + \int_j \gamma_j(s) L_{ij}(s) dj + (1 - h_i(s)) \mathcal{C}_{ii}(s) + \int_j (1 - h_j(s)) \mathcal{C}_{ij}(s) dj \right] f(s) ds \\
&+ \int_s \left[\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ii}(s)(R_i I_{ii} - L_{ii}(s)) + \int_j \left(\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ij}(s)(R_j I_{ij} - L_{ij}(s)) \right) \right] f(s) ds \\
&+ \int_s \left[\kappa_{ii}(s)(L_{ii}^*(s) - L_{ii}(s)) + \int_j \kappa_{ij}(s)(L_{ij}^*(s) - L_{ij}(s)) dj \right] f(s) ds
\end{aligned}
$$

where $\kappa_{ij}(s)$ is the Lagrange multiplier on the regulatory constraint $L_{ij}(s) \leq L_{ij}^*(s)$. We now construct the multipliers $\lambda_i^0, \Lambda_i^1(s), \Lambda_i^1(s), \underline{\xi}_{ij}(s), \overline{\xi}_{ij}(s), \kappa_{ij}(s)$ so that the allocation $(c_i^*, D_i^*, I_i^*, L_i^*)$ satisfies the optimality conditions. In particular, suppose that we set the Lagrange mulitpliers $\lambda_i^0 = \lambda_i^{0*}$, $\lambda_i^1(s) = \lambda_i^{1*}(s)$, $\Lambda_i^1(s) = \Lambda_i^{1*}(s)$ to coincide with the social planner's Lagrange multiplier values in the constrained efficient planning problem of Proposition 1, and further set $\underline{\xi}_{ij}(s) = \overline{\xi}_{ij}(s) = 0$. It follows as in the proof of Proposition 1 that the optimality conditions now coincide for c_i, I_i, D_i, leaving only optimality of liquidations L_i. Here, we have the optimality condition for $L_{ij}(s)$ given by

$$
0 = \lambda_i^1(s)(\gamma_j(s) - (1 + r_{ij})) + \Lambda_i^1(s)(\gamma_j(s) - (1 - h_j(s))\gamma_j(s)) - \kappa_{ij}(s).
$$

Therefore, construct the multiplier $\kappa_{ij}(s) = \lambda_i^{0*} \tau_{ij}(s)$ using the optimal wedge $\tau_{ij}(s)$ given in Proposition 1. Because the wedge $\tau_{ij}(s)$ is non-negative, the multiplier $\kappa_{ij}(s)$ is also non-negative. Under this multiplier, this optimality condition is the same condition as in Proposition 1 and so holds under the optimal allocation $(c_i^*, D_i^*, I_i^*, L_i^*)$. As a result, we have found non-negative Lagrange multipliers such that the optimality conditions are satisfied under the quantity ceilings $L_{ij}(s) \leq L_{ij}^*(s)$. This shows how the globally optimal allocation of Proposition 1 can be implemented via use of explicit quantity restrictions, rather than revenue-neutral wedges.

Summary. In sum, we have shown how the global planner can implement the global optimum of Proposition 1 using explicit quantity restrictions in the form of ceilings on liquidations, and moreover that this constitutes optimal policy under quantity restrictions. This illustrates more concretely the connection between revenue-neutral Pigouvian wedges and explicit quantity restrictions, showing that duality between them holds in the global planning problem of the baseline model.

In the next subsection, we show that the same applies in the non-cooperative optimum.

D.3.2 Non-Cooperative Optimum.

We will now proceed to show that non-cooperative ceilings on liquidations $L_{i,ji}(s) \leq \overline{L}_{i,ji}(s)$ form an optimal policy, and implement the equilibrium of Proposition 3. The argument will follow in two steps. First, we will consider country planner i, whose banks face quantity restrictions of the form $L_{j,ij}(s) \leq \overline{L}_{j,ij}(s)$ imposed upon them by foreign country planners, where the ceilings are set in according with the equilibrium outcomes under Proposition 3. In this setting, we will show that the domestic planner can do no better than to implement the allocations under Proposition 3. Second, we will argue that quantity restrictions in the form of liquidation ceilings can be used to implement the outcomes of Proposition 3.

Step 1. Consider the allocation achieved under Proposition 3, denoted by $(c_i^*, D_i^*, I_i^*, L_i^*)$. Suppose that foreign planners have set quantity restrictions $L_{j,ij}(s) \leq \overline{L}_{j,ij}(s) \equiv L^*_{j,ij}(s)$, that is they have imposed ceilings on liquidations by bank i, with the ceiling set equal to the allocation under Proposition 3. Note that in any state s in which there was a positive foreign spillover in country j, this implies $\overline{L}_{j,ij}(s) = L^*_{j,ij}(s) = 0$, that is the quantity restriction explicitly bans foreign liquidations (rather than implicitly banning it via a large wedge). Now to proceed with the first step of the argument, suppose we study the following relaxed problem: planner i can directly choose allocations (c_i, D_i, L_i, I_i) of domestic banks and (I_{ji}, L_{ji}) of foreign banks, regardless of whether there is a set of quantity restrictions that implements this allocation, subject to the limitations $L_{j,ij}(s) \leq \overline{L}_{j,ij}(s)$ on domestic banks imposed by foreign planners. If the solution to the relaxed problem can be achieved via quantity restrictions, then it is clearly optimal for planner i. Thus, we will first show that the solution to the relaxed problem is the solution of Proposition 3, and then we will show that it can be implemented by ceilings on liquidations.

First, consider the solution to the relaxed problem. Recall that Lemma 2 provides an implementability result whereby planner i directly chooses allocations, and then backs out implementing wedges. In other words, Lemma 2 and Proposition 3 already study a relaxed problem, but with one key difference. This key difference is that in the environment of Lemma 2 and Proposition 3, bank i faces wedges on liquidations imposed by foreign planners, $\tau^L_{j,ij}(s)$, rather than ceilings $\overline{L}_{j,ij}(s)$. Nevertheless, employing the same strategy as for the global optimum, suppose we now conjecture that planner i faces Lagrange multipliers $\lambda^0_i = \lambda^{0*}_i$, $\lambda^1_i(s) = \lambda^{1*}_i(s)$, $\Lambda^1_i(s) = \Lambda^{1*}_i(s)$ that are the same as the Lagrange multipliers of the planning problem of Proposition 3, and moreover set $\underline{\xi}_{ij}(s) = \overline{\xi}_{ij}(s) = 0$. Suppose finally that we set the Lagrange multiplier $\kappa_{ij}(s)$ on the foreign planner regulatory constraint $L_{j,ij}(s) \leq L^*_{j,ij}(s)$ to be $\kappa_{ij}(s) = \lambda^{0*}_i \tau^{L*}_{j,ij}(s)$, where $\tau^{L*}_{j,ij}(s)$ is the optimal liquidation wedge set by planner j in Proposition 3. Because $\tau^{L*}_{j,ij}(s) \geq 0$, this multiplier is non-negative, and so we have constructed non-negative Lagrange multipliers such that $(c^*_i, D^*_i, I^*_i, \{L^*_{ji}, I^*_{ji}\})$ is a solution of the relaxed problem of planner i who faces the quantity restrictions $L_{j,ij}(s) \leq L^*_{j,ij}(s)$ imposed by foreign planners. Thus, the optimal allocation of Proposition 3 is an optimum of the relaxed problem here.

Step 2. Second, we have to show that the solution of the relaxed problem here can be implemented via quantity restrictions. In particular, define ceilings $L_{i,ii}(s) \leq \overline{L}_{i,ii}(s) = L^*_{i,ii}(s)$ for domestic banks and $L_{i,ji}(s) \leq \overline{L}_{i,ji}(s) = L^*_{i,ji}(s)$ for foreign banks (recall that the foreign planner has imposed the ceilings on foreign liquidations by domestic banks). From here, construction of non-negative Lagrange multipliers of the bank i problem proceeds exactly as before, with $\kappa_{ii}(s) = \lambda^{0*}_i \tau^{L*}_{i,ii}(s)$ and $\kappa_{ij}(s) = \lambda^{0*}_i \tau^{L*}_{j,ij}(s)$, where the wedges are the optimal wedges in Proposition 3, verifying that the bank optimality conditions hold.

D.4 Commitment and Time Consistency

The baseline model of Sections 2-4 assumes that banks and planners operate under commitment when choosing ex-post liquidation policies, L, or wedges on ex-post liquidation policies, τ^L. In this appendix, we study the role of planner commitment over wedges in the baseline model, and show that absent commitment a time consistency problem arises owing to the revenue collection motive. In particular, revenue from taxes on investment is collected at date 0, but this revenue is affected by taxes on liquidations, which are set at date 1. This leads to a time consistency problem as the date 1 planner does not internalize the effect on date 0 revenue. The effect of this time consistency problem is that non-cooperative planners using Pigouvian taxation partially neglect the value of date 0 investment for use as collateral at date 1 when setting date 1 taxes, that is they fail to fully account for the collateral externality that arises from the domestic fire sale. By extension, in the limiting case where there is a full haircut and the domestic asset cannot be used as collateral, Pigouvian efficiency is restored.

Importantly, this appendix also highlights a key difference between the baseline model and the model of Section 5. In Section 5, all regulatory decisions are taken at date 0, and hence no commitment problem exists.

Concretely, suppose that wedges on liquidations are set at date 1 without commitment. Notice that this implies the global planner also lacks commitment, and so we will study whether the date 1 cooperative and non-cooperative solutions coincide. The net debt position of bank i at date 1, accounting for tax burdens and remissions, is given by $D_i + \tau_i^L(s)L_i(s) - \Pi_i^*(s)$, where $\Pi_i^*(s)$ is revenue remissions to bank i. Hence, the consolidated dates 1 and 2 budget constraint of bank i in state s is

$$c_i(s) \leq \mathcal{R}_{ii}(s) + \int_j \mathcal{R}_{ij}(s)dj - \left(D_i + \tau_i^L(s)L_i(s) - \Pi_i^*\right),$$

where $\mathcal{R}_{ij}(s) = \gamma_j(s)L_{ij}(s) + (1+r_{ij})(R_j(s)I_{ij} - L_{ij}(s))$ is the total return on initial investment I_{ij}, as in the baseline model. Similarly, the collateral constraint of bank i in state s is

$$\begin{aligned} D_i + \tau_i^L(s)L_i(s) - \Pi_i^* \leq & h_i(s)\gamma_i(s)L_{ii}(s) + \int_j h_j(s)\gamma_j(s)L_{ij}(s)dj \\ & + (1-h_i(s))\gamma_i(s)R_i(s)I_{ii} + \int_j (1-h_j(s))\gamma_j(s)R_j(s)I_{ij}dj \end{aligned}$$

which is the same as that in the baseline model, except for the addition of the tax burden and revenue remissions. Importantly, notice that the investment portfolio I_i that appears in the second line is taken as given by bank i and by all planners, since it was determined at date 0. However, liquidations $L_(s)i$ are a choice variable at date 1. Thus, the problem of bank i is to choose liquidations $L_i(s)$ in order to maximize its consumption value $c_i(s)$, subject to its collateral constraint and taking as given its investment portfolio I_i and inherited debt D_i.

Global Optimum. Consider the global spillover that arises from liquidations in country i. The spillover of an increase in liquidations $L_i^A(s)$ onto the welfare of country i' banks in wealth equivalent is given by

$$\underbrace{\frac{1}{\lambda_{i'}^1(s)+\Lambda_{i'}^1(s)}}_{\text{Wealth Equivalent}}\underbrace{\frac{\partial\gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}}\left[\underbrace{\lambda_{i'}^1(s)L_{i'i}(s)}_{\text{Distributive Externality}}\quad\underbrace{+\Lambda_{i'}^1(s)\left(\overbrace{h_i(s)L_{i'i}(s)}^{\text{Date 1 Liquidations}}\quad+\overbrace{(1-h_i(s))R_i(s)I_{i'i}}^{\text{Date 0 Investment}}\right)}_{\text{Collateral Externality}}\right] \tag{41}$$

Notice that once again, this spillover is of the same general form as the spillovers $\Omega_{i'i}^L(s)$ in Proposition 1, and combines distributive and collateral externalities. Importantly, there are two components of the collateral externality. The first relates to how date 1 liquidations appear in the collateral constraint. The collateral constraint is relaxed by liquidations proportional to the haircut $h_i(s)$, which is the *excess* date 1 funds that can be raised by liquidating country i assets rather than by using them as collateral. This excess value is relative to the baseline where all date 0 investments are used as collateral. The value of this baseline is the second term of the collateral externality. This second term will be the key source of inefficiency in this model lacking commitment.

Non-Cooperative Optimum. In the problem of bank i accounting for the wedges imposed by country planners, denote $\lambda_i^1(s)$ to be the Lagrange multiplier on the consolidated budget constraint and $\Lambda_i^1(s)$ the Lagrange multiplier on the collateral constraint. Following the same steps as the proof of Lemma 2, we obtain the implementability condition for the domestic liquidations of foreign banks as

$$\tau_{i,ji}^L(s) = -\tau_{j,ji}^L(s) + \frac{\lambda_j^1(s)}{\lambda_j^1(s)+\Lambda_j^1(s)}\Big[\gamma_i(s)-(1+r_{ji})\Big] + \frac{\Lambda_j^1(s)}{\lambda_j^1(s)+\Lambda_j^1(s)}h_i(s)\gamma_i(s).$$

First, let us consider quantity regulation. Under quantity regulation, Π_i^* is taken as given by country planner i, and hence by the same steps as the proof of Proposition 3 we obtain a ban on liquidations of the domestic asset by foreign banks whenever there is an adverse domestic spillover, and further than the wedge set on domestic liquidations by domestic banks only accounts for domestic spillovers. Thus, under quantity regulation, lack of commitment does not affect the qualitative insights of the baseline model with commitment.

By contrast, let us consider Pigouvian taxation. Under non-cooperative Pigouvian taxation, $\Pi_i^* = \int_j \tau_{i,ji}^L L_{ji}(s)dj$ is the total revenue collected from liquidation taxes on foreign banks. Substituting in the implementing tax rate and using $\tau_{j,ji}^L(s) = 0$, we obtain that the tax revenue collected from bank i' is

$$\tau_{i,i'i}^L(s)L_{i'i}(s) = \frac{1}{\lambda_{i'}^1(s) + \Lambda_{i'}^1(s)}\left[\lambda_{i'}^1(s)\left[\gamma_i(s) - (1 + r_{i'i})\right] + \Lambda_{i'}^1(s)h_i(s)\gamma_i(s)\right].$$

Finally, following the proof of Proposition 4, we have the derivative of tax revenue from bank i' in aggregate liquidations $L_i^A(s)$ given by

$$\frac{\partial \tau_{i,i'i}^L(s)L_{i'i}(s)}{\partial L_i^A(s)} = \underbrace{\frac{1}{\lambda_{i'}^1(s) + \Lambda_{i'}^1(s)}}_{\text{Wealth Equivalent}} \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \left[\underbrace{\lambda_{i'}^1(s)L_{i'i}(s)}_{\text{Distributive Externality}} \underbrace{+\Lambda_{i'}^1(s)\left(\overbrace{h_i(s)L_{i'i}(s)}^{\text{Date 1 Liquidations}} \right)}_{\text{Collateral Externality}} \right] \tag{42}$$

In the baseline model, the proof of Proposition 4 relied on showing that the tax revenue derivative for revenues collected from bank i' coincided with the global spillover effect onto bank i', resulting in efficiency. Here, we have conducted the same exercise without commitment, looking to compare the global spillover effect (equation 41) with the tax revenue derivative (equation 42). In this model without commitment, we see that these two expressions differ from each other by the collateral externality term denoted "Date 0 Investment." Assuming investment $I_{i'i}$ is positive, then this term is zero only provided that $h_i(s) = 1$, that is there is a full haircut and debt cannot be rolled over at all. Otherwise it is generally positive, and the tax revenue derivative is generally not equal to the global spillover.

To understand why this spillover is correctly internalized in the model with commitment but not in the model without commitment, consider the problem at date 0. At date 0, banks choose investment scale, anticipating the outcome of the date 1 equilibrium. As a result, the implementability condition on date 0 investment is

$$\tau_{i,ji}^I = -\tau_{j,ji}^I - \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E\left[\frac{\lambda_j^1}{\lambda_j^0}(1 + r_{ji})R_i\right] + \frac{1}{\lambda_j^0}E\left[\Lambda_j^1(1 - h_i)\gamma_i R_i\right].$$

As a result, revenue collected from bank i' *at date 0* is given by

$$\tau^I_{i,i'i} I_{i'i} = -\frac{\partial \Phi_{i'i}}{\partial I_{i'i}} I_{i'i} + E\left[\frac{\lambda^1_{i'}}{\lambda^0_{i'}}(1+r_{i'i})R_i I_{i'i}\right] + \frac{1}{\lambda^0_{i'}} E\left[\Lambda^1_{i'}(1-h_i)\gamma_i R_i I_{i'i}\right]$$

and hence, the revenue derivative at date 0 in date 1 aggregate liquidations $L^A_i(s)$ is

$$\frac{\partial \tau^I_{i,i'i} I_{i'i}}{\partial L^A_i(s)} = \frac{1}{\lambda^0_{i'}} \frac{\partial \gamma_i(s)}{\partial L^A_i(s)} \Lambda^1_{i'}(s)(1-h_i(s))R_i(s) I_{i'i} f(s).$$

In the baseline model with commitment, this was the source of the term missing in equation (42) in the model without commitment. Economically, this term reflects the baseline collateral value of date 0 investment. As the date 1 fire sale worsens, this baseline value falls, and so the tax rate on investment must fall, reducing tax revenue.

In the model with commitment, taxes on investment and liquidation are set simultaneously, and hence planner i internalizes how a decrease in the tax on liquidations leads to a worse fire sale and a forces (via implementability) a lower tax on investment. By contrast, in the model without commitment, the tax on liquidation is set after the tax on investment. Although changes in the liquidation tax at date 1 affects revenue collected from the date 0 investment tax, by the time the date 1 tax is being set the date 0 tax revenue has already been collected. This leads to the time consistency problem and the deviation from efficiency.

D.5 Real Economy or Arbitrageur Spillovers and Quantity Regulation

In the baseline model, the lack of any benefit from foreign banking led to the strong result of Proposition 3 of a ban on liquidations by foreign banks. In this appendix, we study the impact of benefits from foreign banking on the quantity regulation game, adopting an extension of the simple form provided in Examples 1 and 2 in Section 6. In this setting, we show that benefits from foreign banking still lead to under-regulation of domestic banks, but can lead to either under- or over-regulation of foreign banks. In particular, as in the baseline model, over-regulation arises when at the margin the domestic fire sale spillover is greater than the domestic benefit from foreign banking, as was the case by assumption in the baseline model. On the other hand, there is under-regulation (on the margin) when at the global optimum, the marginal domestic economic benefit from foreign banks outweighs the marginal fire sale spillover to domestic banks, but does not outweigh the total fire sale spillover to all banks (domestic and foreign).

In particular, suppose there is a domestic spillover $u_i^A(I_i^A, L_i^A)$ from total bank activities in country i. This could be a real economy spillover from credit extension (Example 1), or alternatively could capture surplus of domestic arbitrageurs (Example 2).

We will focus here on the optimal liquidation rule. Define $\Omega_{ii}^A(s) = \frac{1}{\lambda_i^0}\frac{\partial u_i^A}{\partial L_i^A(s)}$ to be the marginal spillover associated with the real economy/arbitrageur surplus. It follows from the same steps as the proof of Proposition 1 that the globally optimal wedge on liquidations is

$$\tau_{ji}^L(s) = -\Omega_{ii}(s) - \int_{i'} \Omega_{i'i}(s)di' - \Omega_{ii}^A(s).$$

On the other hand, following the steps of the proof of Proposition 3, we obtain that the non-cooperative optimum generates wedges on domestic liquidations of domestic banks of

$$\tau_{ii}^L(s) = -\Omega_{ii}(s) - \Omega_{ii}^A(s),$$

while for foreign banks it generates an allocation rule

$$L_{ji}(s)\left[-\Omega_{ii}(s) - \Omega_{ii}^A(s)\right] = 0.$$

First considering domestic regulation of domestic banks, we have the same under-regulation of domestic banks as in the baseline model, as foreign spillovers are neglected.

Now, let us consider regulation of foreign banks. First, the non-cooperative rule tells us that either it is the case that $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ or otherwise $L_{ji}(s) = 0$. This captures the basic logic of the baseline model: liquidations by foreign banks are allowed only up to the point that they do not contribute adversely to domestic spillovers. In the baseline model with $\Omega_{ii}^A(s) = 0$, this meant it had to be the case that $L_{ji}(s) = 0$ whenever $\Omega_{ii}(s) \neq 0$, resulting in the ban on liquidations. However with additional spillovers (for example, surplus to domestic arbitrageurs) it can arise that $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ and $L_{ji}(s) > 0$, so that the domestic planner allows some liquidation of domestic assets by foreign banks. However, note that $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ does not generally imply foreign banks are unregulated.[86]

The key question is whether the domestic planner is more or less stringent than the global optimum with regards to foreign banks. Consider the marginal incentives. Recall that $\tau^L_{ji}(s)$ is always (by implementability) equal to the marginal benefit to foreign banks of liquidating assets in country i. Therefore, we have from the tax rate

$$\underbrace{\tau^L_{ji}(s) + \int_{i'} \Omega_{i'i}(s) di'}_{\text{Total MB to Foreign Banks}} = \underbrace{-\Omega_{ii}(s) - \Omega^A_{ii}(s)}_{\text{MC to Domestic Banks}} .$$

Suppose first that at the global optimum, $-\Omega_{ii}(s) > \Omega^A_{ii}(s)$. In this case, the net domestic cost of fire sales $-\Omega_{ii}(s)$ exceeds the net domestic benefit of other spillovers $\Omega^A_{ii}(s)$, and hence country planner i on the margin prefers fewer liquidations. However, the positive marginal cost to domestic banks means that there must be an equal and positive marginal benefit to foreign banks, which is neglected by the domestic planner when designing regulation. This leads to over-regulation of foreign banks. The baseline model is the limiting case where $\Omega^A_{ii}(s) = 0$, and hence the marginal cost to domestic banks must be non-negative at the optimum, resulting in the ban.

Conversely if $-\Omega_{ii}(s) < \Omega^A_{ii}(s)$, then the domestic benefit outweighs the domestic cost, and hence country planner i prefers more liquidations on the margin. In contrast to the previous case, the fact that the domestic marginal benefit is positive means that the foreign marginal cost is also positive, due to the foreign fire sale spillover. In this case, the domestic planner designing regulation neglects the net cost to foreign banks, and under-regulates foreign banks relative to the optimum. The logic of this case is economically similar therefore to under-regulation of domestic banks, which is also driven by the domestic planner considering positive domestic benefits but ignoring foreign costs.

[86]A simple example of this is the knife-edge case where the optimum features $\Omega_{ii}(s) + \Omega^A_{ii}(s) = 0$ and $L_{ji}(s) = 0$, that is when equilibrium liquidations by domestic banks put the planner exactly on their first order condition. In this case, implementability implies that $\tau^L_{i,ji}(s) = \frac{\lambda^1_j(s)}{\lambda^0_j}(\gamma_i(s) - (1 + r_{ji})) + \frac{1}{\lambda^0_j}\Lambda^1_j(s)h_i(s)\gamma_i(s)$ and so the wedge on foreign bank liquidations is positive provided that r_{ji} is sufficiently large. Notably, in this case we nevertheless have $\tau^L_{i,ii}(s) = 0$, that is domestic banks are unregulated. Economically, this difference reflects unequal treatment. The domestic planner values domestic banks, and since in equilibrium the domestic spillover is zero then the optimal tax rate is zero. However, suppose that there is a foreign bank which, at $L_{ji}(s) = 0$, places positive value on liquidations, and so $\tau^L_{i,ji}(s) > 0$. If the planner instead allowed foreign banks to liquidate assets, they would set $L_{ji}(s) > 0$ and push the marginal cost of liquidations to be positive, rather than zero. The domestic planner internalizes the social cost this would generate, but not the marginal benefit to foreign banks. Hence, the domestic planner imposes a positive wedge on foreign banks.

Taken together, this extension reflects the same underlying forces as were present in the baseline model, with the same policy implications for under-regulation of domestic banks. However, it caveats and qualifies the implications for over-regulation of foreign banks.

E Extensions of the General Model

In this Appendix, we provide extensions of the general model presented in Section 6.

E.1 World Prices

We now extend the model to incorporate world prices, for example allowing for state contingent securities prices at date 0 to be endogenous. We show that provided that global prices only enter constraints through the wealth level, the problem is unaffected. This result is in line with Korinek (2017) and follows similarly.

Let $x_i = \{x_i(n)\}_{n \in N}$ be a vector of global goods held by country i, so that market clearing implies $\int_i x_i(n) di = 0$. Global goods trade at prices q, so that the wealth level of country i multinational agents is

$$W_i = A_i - T_i - \sum_n q(n) x_i(n).$$

Global goods enter into $u_{ii}, u_{ii}^A, \phi_{ii}, \phi_{ii}^A$, but prices do not enter except through the wealth level. Note that because global goods enter into domestic functions, they do not influence Assumption 9. From here, we obtain the following result.

Proposition 28. *The optimal cooperative wedges are of the same form as Proposition 8, with no wedges on x_i. Pigouvian taxation is efficient under the same conditions as Proposition 27.*

Proposition 28 may apply, for example, to a global market for liabilities at date 0.

E.1.1 Proof of Proposition 28

The global planning problem has a Lagrangian

$$\mathcal{L}^G = \int_i \left[\omega_i U_i \left(u_i(a_i, x_i), u_i^A(a_i, x_i, a_i^A)) \right) + \Lambda_i \Gamma_i \left(A_i + \mathfrak{T}_i, \phi_i(a_i, x_i), \phi_i^A(a_i, x_i, a_i^A) \right) - \lambda^0 \mathfrak{T}_i - \lambda^0 Q x_i \right] di$$

where we have suggestively denoted $Q(n)$ to be the Lagrange multiplier on the global goods market clearing for good n. Differentiating in $x_i(n)$, we obtain

$$0 = \frac{\partial \mathcal{L}_i}{\partial x_i(n)} - \lambda^0 Q(n)$$

so that world prices $q(n) = Q(n)$ form an equilibrium (recall that $\lambda_i^0 = \lambda^0$). Globally efficient policy is as in Proposition 8, with no wedges placed on x_i.

E.2 Local Constraints on Allocations

We extend Section 6 to incorporate local constraints on allocations. Note that such constraints are already available through Γ_i for domestic allocations, but that such constraints are not available in countries $j \neq i$. The extension captures, for example, the constraints $0 \leq L_{ij}(s) \leq R_j(s)I_{ij}$ and $I_{ij} \geq 0$ imposed in the main paper.

Suppose that in country j, there is a vector of linear constraints $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ on allocations, where $\chi_{ij}(a_j^A)$ potentially depends on aggregates in country j and where $b_{ij} \geq 0$.[87] We impose linearity in the spirit of the required conditions for optimality of Pigouvian taxation in Proposition 27. We obtain the following revised implementability result for foreign allocations, which mirrors Lemma 23

Lemma 29. *Any domestic allocation of foreign agents satisfying constraints $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ is optimally implemented with the wedges in Lemma 23.*

Lemma 29 implies that implementability constraints are the same as in Section 6. The only difference is that now the constraint set on local allocations is a constraint of the local planner. Note that this implies that the local planner directly internalizes spillovers of domestic aggregates onto the constraint set $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$, so that such spillovers are not an issue.

From here, all results proceed as in Section 6.[88] Intuitively, the only adjustment we need to make is that $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ is now a constraint set of planner j. Without loss of generality,

[87] We impose $b_{ij} \geq 0$ to ensure that non-participation ($a_{ij} = 0$) is always feasible.

[88] Notice that it is expositionally convenient to define the decentralization of the global optimum in an analogous manner to the corner solutions associated in Lemma 29, where taxes are set to make banks indifferent at the corner solution with Lagrange multipliers of zero on local constraints.

scale the Lagrange multiplier ν_{ij} by λ_i^0, and define the "local constraint spillover" of a change in aggregates by

$$\Omega^{LC}_{j,ij}(m) = -\nu_{ij}\frac{\partial \chi_{ij}}{\partial a^A_j(m)}a_{ij}$$

so that we can define the total domestic local constraint set spillover as

$$\Omega^{LC}_{j}(m) = -\int_i \Omega^{LC}_{j,ij}(m)di$$

From here, it follows that the efficiency results of Section 6 apply, treating the total domestic spillover as $\Omega_{j,j}(m)+\Omega^{LC}_j(m)$.[89]

Note that if the local constraints were non-linear, this would not generally hold, as we would not be able to recover the complementary slackness condition precisely in the above proof. As a result, the domestic planner may have an incentive to manipulate the tax rates that implement corner solutions in order to increase revenue. This would amount to another form of "monopolistic" revenue distortion in the model.

E.2.1 Proof of Lemma 29

For expositional ease, we suppress the notation $\chi_{ij}(a^A_j)$ and simply write χ_{ij}. Let $\nu_{ij} \geq 0$ be the Lagrange multipliers on the local feasibility constraints $b_{ij} - \chi_{ij}a_{ij} \geq 0$. The first order condition for an action m is

$$0 = \omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u^A_i}\frac{\partial u^A_i}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}\left(-\tau_{i,ij}(m) - \tau_{j,ij}(m)\right) + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi^A_i}\frac{\partial \phi^A_i}{\partial a_{ij}(m)}$$
$$- \nu_{ij}\chi_{ij}(m)$$

[89]To see that $\tau_{i,ij}(m) = 0$ constitutes an equilibrium policy for $j \neq i$, suppose that $\tau_{j,ij}(m)$ is set to clear the first-order condition. Then, the first order condition of country planner i for $a_{ij}(m)$ is satisfied with equality, and so we must have $\nu_{ij} = 0$, so that there is no value to country planner i of relaxing the local constraints in country j at the equilibrium. As a result, the preferences of country planner i align with country i agents over actions in country j, and we have $\tau_{i,ij}(m) = 0$.

So that rearranging, we obtain

$$\lambda_i^0 \tau_{j,ij}(m) + \nu_{ij}\chi_{ij}(m) = -\lambda_i^0 \tau_{i,ij}(m) \\ + \omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_i^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_i^A}{\partial a_{ij}(m)}.$$

Notice that the right-hand side is constant for a given allocation, and is the same formula as in Lemma 23. Denote it to be $\lambda_i^0 \tau_{j,ij}^*(m)$, so that we have $\tau_{j,ij}(m) = \tau_{j,ij}^*(m)$ if $\nu_{ij} = 0$. Given corner solutions, there may be multiple vectors of tax rates that implement this allocation. We can express the problem of country planner j therefore maximizing tax revenue collected while implementing the same allocation, that is

$$\max_{\nu_{ij},\tau_{j,ij}} \tau_{j,ij} a_{ij}^* \quad \text{s.t.} \quad \lambda_i^0 \tau_{j,ij}(m) + \nu_{ij}\chi_{ij}(m) = \lambda_i^0 \tau_{j,ij}^*(m), \quad \nu_{ij}\left(b_{ij} - \chi_{ij} a_{ij}^*\right) = 0$$

where the second constraint is complementary slackness. Substituting in for $\tau_{j,ij}$ and substituting in the complementary slackness condition, we obtain

$$\max_{\nu_{ij}\geq 0} \tau_{j,ij}^* a_{ij}^* - \frac{1}{\lambda_i^0}\nu_{ij} b_{ij}$$

Because $b_{ij} \geq 0$, revenue collection is maximized at $\nu_{ij} = 0$, so that we have $\tau_{j,ij} = \tau_{j,ij}^*$. As a result, the implementability conditions of Lemma 23 hold.

E.3 Heterogeneous Agents

We extend the model of Section 6 by allowing for heterogeneous agents within a country. Suppose that in each country, there are $\mathcal{K} = \{1, ..., K\}$ agents, who differ in their utility functions and constraint sets, whom we index i_k. Some agents may not be able to conduct cross-border activities, in which case foreign actions would not appear in their utility function or constraint sets. Agents of type i_k have relative mass μ_{i_k} and are assigned a social welfare weight ω_{i_k}.

It is easy to see that we can treat the problem as if there were a single representative agent in country i. In particular, define $a_i = \{a_{i_k}\}_{k\in K}$, $U_i = \sum_k \mu_{i_k}\omega_{i_k} U_{i_k}$, and $\Gamma_i = (\Gamma_{i_1}, ..., \Gamma_{i_K})$. The problem is as-if we have a single representative agent who solves

$$\max_{a_i} U_i \quad \text{s.t.} \quad \Gamma_i \geq 0,$$

since this decision problem is fully separable in a_{i_k} and yields the optimality conditions of each agent type. The only difference relative to Section 6 is that there are K different measures of wealth, W_{i_k}. Domestic lump sum transfers imply that $\lambda^0_{i_k} = \lambda^0_i$ is independent of k, and the characterization of optimal policy follows as in Section 6.

E.4 Nonlinear Aggregates

In Section 6, we assumed that aggregates are linear, that is $a^A_i(m) = a_{ii}(m) + \int_j a_{ij}(m)dj$. The welfare-relevant aggregates may not necessarily be linear. We can represent this by

$$Z_i\left(z_{ii}(a_{ii}, a^A_i) + \int_j z_{ji}(a_{ji}, a^A_i)dj, a^A_i\right) = 0$$

for some functions z and Z. The key change in the model is that we now have spillover effects that depend on the identity of the country investing, as in the bailouts model. The optimality of non-cooperative Pigouvian taxation follows from the same steps and logic as the baseline model, simply incorporating the change in aggregates that arises through this nonlinear relationship. This clarifies once again that the homogeneity property of Assumption 9 applies to allocations, not to aggregates.

The possibility for non-linear aggregation helps to generalize the results to settings where regulation is set at an initial date, but the economy is not regulated thereafter (Section 5).

E.5 General Government Actions

We extend the model to feature more general government actions, for example bailouts as in Appendix B.4. In particular, country planner i can take actions $g_{i,jk}(m) \geq 0$ (for either $i = j$ or $i = k$), which affect country j agents in the same way as action m in country k. As such, we can define the total domestic action of agent i as

$$\overline{a}_{ii}(m) = a_{ii}(m) + g_{i,ii}(m)$$

and the total foreign action of agent i as

$$\overline{a}_{ij}(m) = a_{ij}(m) + g_{i,ij}(m) + g_{j,ij}(m).$$

This classification allows for a rich set of both agent and government actions. For example, a domestic action m that can only be taken by the government, such as government debt issuance or a bailout, could feature a feasibility constraint $a_{ii}(m) = 0$. From here, the domestic aggregates are given by

$$a_i^A(m) = \overline{a}_{ii}(m) + \int_j \overline{a}_{ji}(m) dj.$$

The flow utility of the country i representative agent is now given by

$$\max_{a_i} U_i\left(u_i(a_i, g_i, \overline{a}_i), u_i^A(a_i, g_i, \overline{a}_i, a_i^A))\right) \quad \text{s.t.} \quad \Gamma_i\left(W_i, \phi_i(a_i, g_i, \overline{a}_i), \phi_i^A(a_i, g_i, \overline{a}_i, a^A)\right) \geq 0,$$

where we have $u_i(a_i, g_i, \overline{a}_i) = u_{ii}(a_{ii}, u_{ii}^g(g_{i,ii}) + \int_j u_{ij}^g(g_{i,ji})dj, \overline{a}_{ii}) + \int_j u_{ij}(a_{ij}, g_{i,ij}, \overline{a}_{ij})dj$ and so on. It simplifies exposition to include in Γ_i any government feasibility constraints, for example government budget constraints. Observe that such constraints would be assigned Lagrange multipliers of 0 by the representative agent, but not by the social planner.

From here, we begin by characterizing the globally efficient allocation. Observe first that the optimal wedges for private actions are still given by the equations in Proposition 8.

Proposition 30. *The globally efficient allocation can be decentralized by the wedges of Proposition 8. The globally efficient government actions $g_{i,jk}$ (for either $i = j$ or $i = k$) are given by*

$$-\underbrace{\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)}}_{\text{Country } i \text{ Cost}} \geq \underbrace{\frac{\partial \mathcal{L}_j}{\partial \overline{a}_{jk}(m)}}_{\text{Country } j \text{ Benefit}} + \underbrace{\frac{\partial \mathcal{L}_k}{\partial a_k^A(m)}}_{\text{Country } k \text{ Spillover}} + \underbrace{\int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_k^A(m)} di'}_{\text{Foreign Spillovers}} \tag{43}$$

where $\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)} = \omega_i \frac{\partial U_i}{dg_{jk}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial g_{jk}(m)}$ *and so on.*

Proof. The proof of the decentralizing wedges follows as in the proof of Proposition 8. The government action rules follow directly from the derivatives of the global Lagrangian. ■

The globally efficient allocation of government actions is a generalization of the optimal bailout rule of Proposition 14, with analogous intuition. Note that for $j \neq i$, we have an action smoothing result: $\frac{\partial \mathcal{L}_i}{\partial g_{i,ji}(m)} = \frac{\partial \mathcal{L}_j}{\partial g_{j,ji}(m)}$, that is the marginal cost of providing the action is smoothed across countries. For example, this corresponds to bailout sharing.

From here, the non-cooperative results on quantity regulation follow as in the baseline model and bailouts section. Taking either $i = j$ or $i = k$, the neglected terms are always the terms that affect other countries, namely the foreign spillovers and either the spillover ($i = j$) or the benefit ($i = k$). For domestic actions, there are neglected foreign spillovers, while for domestic actions on foreign agents there is unequal treatment when the cost of providing the action is held fixed.

On the other hand, suppose that choices of foreign government actions $g_{i,ij}$ and $g_{i,ji}$ are delegated to agents, but can be taxed.[90] Once this is imposed and governments use Pigouvian taxation, these foreign government actions are no different from regular actions from a technical perspective,[91] and the efficiency of Pigouvian taxation is restored.

E.6 Preference Misalignment

We now suppose that there is a difference in preferences between country planners and multinational agents, that is country planners have a utility function $V_i(v_i(a_i), v_i^A(a_i, a^A))$. For example, preference differences may arise due to paternalism, control by special interest groups, or corruption. For simplicity, we incorporate the welfare weights into the planner utility function.

We define efficient policies with respect to those of country planners. This is a natural efficiency benchmark, as country planners agree to cooperative agreements.[92] Under this definition, globally efficient policy can be characterized as follows.

[90] Notice that $g_{i,ij}$ is delegated to country i agents and $g_{i,ji}$ to country j agents.
[91] Excepting that there is a non-linear aggregate arising from u_{ii}^g, which is covered above.
[92] See Korinek (2017) for the same argument.

Proposition 31. *The globally efficient wedges are given by*

$$\tau_{ji}(m) = -\Delta_{ji}(m) - \Omega^{v}_{i,i}(m) - \int_{i'} \Omega^{v}_{i',i}(m) di' \tag{44}$$

where we have

$$\Delta_{ji}(m) = \frac{1}{\lambda_j^0} \left[\frac{\partial V_j}{\partial v_j} \frac{\partial v_{ji}}{\partial a_{ji}(m)} - \frac{\partial U_j}{\partial u_j} \frac{\partial u_{ji}}{\partial a_{ji}(m)} \right]$$

and where $\Omega^{v}_{i,j}$ are defined analogously to $\Omega_{i,j}$, but with the planner utility functions.

Proof. The proof follows as usual by writing country social welfare as $U_i + (V_i - U_i)$ and comparing the planner and agent first order conditions. ■

Globally efficient policy accounts for spillovers onto the welfare of country planners in a standard way. However, it also must correct for the difference in preferences, yielding the first term $\Delta_{ji}(m)$.

From here, characterization of optimal quantity regulation follows as in Section 6, except with the spillovers defined above. Regulation of domestic agents accounts for both the preference difference and spillovers to country planner welfare, but does not account for spillovers to foreign planners. Regulation of foreign agents allows them to conduct activities only to the point that it increases domestic planner welfare. The result is uninternalized spillovers and unequal treatment.

The result for Pigouvian taxation is more subtle. Considering tax revenue collections with no monopolist distortion, we have the tax revenue collection $\tau_{ji}(m)a_{ji}(m)$. Note first that differentiating in $a_{ji}(m)$, we obtain the total revenue impact (assuming no monopoly rents)

$$\tau_{ji}(m) + \int_{i'} \frac{\partial \tau_{i'i}}{\partial a_i^A(m)} a_{i'i}(m) = \tau_{ji}(m) + \int_{i'} \Omega_{i',i}(m) di'$$

where we note that $\tau_{i,ji}(m)$ is now the benefit to the foreign agent net of the wedge placed by the foreign planner, which unwinds the preference difference. This results in the difference $\Delta_{ji}(m)$ being correctly accounted for. However, the spillovers defined above are the spillovers to the agent, not the planner. This implies setting correct policy requires $\Omega^{v}_{j,i} = \Omega_{j,i}$ when $j \neq i$. The simplest way for this requirement to hold is if spillovers onto foreign agents are limited to constraint set spillovers, for example the fire sales of the baseline model.

Finally, it should be noted that these results imply that country planners can achieve the cooperative outcome using Pigouvian taxation. However, this section does not address whether the cooperative outcome is superior to the non-cooperative outcome. This latter claim requires a normative stand on whether the preferences of the planner or the agent are the normatively legitimate preferences, which depends on the source of preference difference. Although interesting for future work, such analysis is beyond the scope of this paper.

E.7 A Finite Country Game

We now consider a game with a finite number of countries, and show that the optimality of Pigouvian taxation is obtained up to a set of new external reoptimization effects. Provided that these external reoptimization effects are negligible, for example in the limit with a large number of countries, the results of the paper are obtained.

Suppose that rather than a continuum of countries, we have a finite set $I = \{1, \ldots I\}$ of countries, each of measure $\frac{1}{I}$. To simplify exposition, we assume that there is a single action $M = \{m\}$ and rule out constraint sets. As a result, we write

$$\max_{a_i} U_i\left(u_i(a_i, I), u_i^A(a_i, a^A, I), W_i\right)$$

where we have

$$u_i(a_i, I) = \sum_{j \in I} u_{ij}(a_{ij}, I)$$

$$a_i^A = \frac{1}{I} \sum_{j \in I} a_{ij}$$

and so on. We use the functional dependency on I to capture scaling as we take the limit $I \to +\infty$, which will allow for home bias and marginal foreign investment.

The following Proposition characterizes the equilibrium under non-cooperative Pigouvian taxation. For expositional purposes, we focus on the domestic tax rate $\tau_{i,ii}$.

Proposition 32. *Suppose that Assumption 9 holds. In the finite country game, the non-cooperative equilibrium under Pigouvian taxation has the following tax rate on the domestic activity of domestic* *agents*

$$\tau_{i,ii} = \underbrace{-\frac{1}{I}\sum_j \Omega_{j,i}}_{\text{Total Spillovers}} \underbrace{- \frac{1}{I}\frac{1}{\lambda_i^0}\sum_{j\neq i}\sum_k \mu_{i,jk}\left[\overbrace{\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\Omega_{j,i}}^{\text{“Wealth Effect”}} + \overbrace{\frac{d}{du_{ji}^A}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\frac{du_{ji}^A}{da_i^A}}^{\text{“Price Effect”}}\right]}_{\text{External Reoptimization Effects}}.$$

where $\mu_{i,jk}$ is a Lagrange multiplier defined in the proof.

In the finite country game, the intuition behind the internalization of foreign spillovers is the same as the baseline model. However, there is also an additional set of *external reoptimization effects* that arise due to global monopoly power: the *entire contracts* of foreign agents are affected to first order by changes in domestic activities and aggregates, including allocations and aggregates outside the domestic economy.

These external reoptimization effects consist of two effects. The “Wealth Effect” arises because taxes on foreign agents reduce their wealth level, impacting their preferences over their entire contract. The “Price Effect” arises because a change in the domestic aggregate affects the benefit foreign agents get from activities, which in turn affects their entire contract. These additional forces amount to an additional form of monopolist distortion. When these monopolist distortions disappear, efficiency is restored.

In the baseline model, we have taken a continuous limit, where the marginal presence in foreign countries implies that the wealth effects and price effects are negligible.

Notice that if we characterized the tax rate $\tau_{i,ij}$ on foreign activities of agents, it would now account for the fact that agents’ contribution to the foreign aggregate spills back to domestic agents. This would result in a form of excessive taxation, because the domestic planner is also taxing this externality. This term would disappear in limit, as the contribution to the foreign aggregate becomes negligible, so that excessive taxation disappears in limit, as in the baseline model.

E.7.1 Proof of Proposition 32

Given this setup, the demand functions of the country i multinational agent are given by the system of equations

$$\tau_{i,ii} = \frac{1}{\lambda_i^0}\frac{dU_i}{da_{ii}}$$

$$\tau_{i,ij} + \tau_{j,ij} = \frac{1}{\lambda_i^0}\frac{dU_i}{da_{ij}}$$

where we have defined $\lambda_i^0 = \frac{\partial U_i}{\partial W_i}$ and $\frac{dU_i}{da_{ij}} = \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}} + \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_{ij}}$.

Now, consider the optimization problem of country planner i, which is given by

$$\max_{a,\tau_i} U_i\left(u_i(a_i,I), u_i^A(a_i,a^A,I), A_i + \sum_{j\neq i}\left[\tau_{i,ji}a_{ji} - \tau_{j,ij}a_{ij}\right]\right)$$

subject to the above implementability conditions in all countries, taking as given τ_{-i}. Notice that tax collections do not need to be scaled by $\frac{1}{I}$ since countries have equal measure. We write the Lagrangian as

$$\mathcal{L}_i = U_i\left(u_i(a_i,I), u_i^A(a_i,a^A,I), A_i + \sum_{j\neq i}\left[\tau_{i,ji}a_{ji} - \tau_{j,ij}a_{ij}\right]\right) + \sum_{j,k}\mu_{i,jk}\mathrm{FOC}_{jk}$$

where $\mu_{i,jk}$ is the Lagrange multiplier on the FOC of agent j for its action in country k.

From here, note that we have $\mu_{i,ik} = 0$, given the complete set of controls on domestic agents. Moreover, the FOC for the tax on foreign agents $\tau_{i,ji}$ is given by

$$\begin{aligned} 0 &= \lambda_i^0 a_{ji} - \mu_{i,ji} + \sum_k \mu_{i,jk}\frac{d\mathrm{FOC}_{jk}}{dW_j}a_{ji} \\ &= \lambda_i^0 a_{ji} - \mu_{i,ji} + \sum_k \mu_{i,jk}\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]a_{ji} \end{aligned}$$

where the final term reflects wealth effects on foreign multinational agent k.

From here, let us take the FOC in the domestic action a_{ii}. We have

$$0 = \frac{dU_i}{da_{ii}} + \frac{1}{I}\frac{dU_i}{da_i^A} + \frac{1}{I}\sum_{j\neq i}\sum_k \mu_{i,jk}\frac{d\mathrm{FOC}_{jk}}{da_i^A}.$$

Taking the derivatives, substituting in for $\mu_{i,ji}$, and applying Assumption 9, we obtain

$$0=\frac{dU_i}{da_{ii}}+\frac{1}{I}\frac{dU_i}{da_i^A}+\frac{1}{I}\sum_{j\neq i}\left[\lambda_i^0+\sum_k \mu_{i,jk}\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\right]\frac{1}{\lambda_j^0}\frac{dU_j}{da_i^A}+\frac{1}{I}\sum_{j\neq i}\sum_k \mu_{i,jk}\frac{d}{du_{ji}^A}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\frac{du_{ji}^A}{da_i^A}$$

and finally, substituting in the tax rate,

$$\tau_{i,ii}=\underbrace{-\frac{1}{I}\sum_j \Omega_{j,i}}_{\text{Total Spillovers}}\underbrace{-\frac{1}{I}\frac{1}{\lambda_i^0}\sum_{j\neq i}\sum_k \mu_{i,jk}\left[\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\Omega_{j,i}+\frac{d}{du_{ji}^A}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\frac{du_{ji}^A}{da_i^A}\right]}_{\text{External Reoptimization Effects}}.$$