Crisis Interventions in Corporate Insolvency

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Abstract

We model the optimal resolution of insolvent firms in general equilibrium. Collateral-constrained banks lend to (i) solvent firms to finance investments and (ii) distressed firms to avoid liquidation. Liquidations create negative fire-sale externalities. Liquidations also relieve bank balance-sheet congestion, enabling new firm loans that generate positive collateral externalities by lowering bank borrowing rates. Socially optimal interventions encourage liquidation when firms have high operating losses, high leverage, or low productivity. Surprisingly, larger fire sales promote interventions encouraging more liquidations. We study synergies between insolvency interventions and macroprudential regulation, bailouts, deferred loss recognition, and debt subordination. Our model elucidates historical crisis interventions.

Keywords: Corporate Insolvency, Bankruptcy, Crisis Intervention, Zombie Lending

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1 Introduction

The COVID-19 pandemic motivated a wave of proposals for government interventions into existing processes for resolving insolvent firms. Most proposals aimed to preserve firms that would otherwise be liquidated under existing insolvency laws.[1] Insolvency interventions, however, are not COVID-specific and have not always promoted reorganization. For example, in an attempt to end the problem of nonperforming loans impairing bank balance sheets, Japan in the 2000s implemented policies more closely resembling subsidies for liquidation under the "Takenaka Plan."[2] This raises an important policy question of whether a planner should intervene in the corporate insolvency process and, if so, under what conditions should policy promote reorganization or promote liquidation.

This paper provides a simple framework to study optimal interventions in existing corporate insolvency systems. Our model embeds both a fire-sale externality – additional liquidations reduce liquidation prices – and a collateral externality operating through bank borrowing rates. The collateral externality arises because distressed loans congest bank balance sheets. This constrains

[1] To paraphrase Greenwood, Iverson, and Thesmar (2020): (i) Hanson, Stein, Sunderam, and Zwick (2020b) recommend keeping firms solvent by funding fixed obligations like rent; (ii) Saez and Zucman (2020) recommend keeping firms solvent by funding all expenses; (iii) Brunnermeier and Krishnamurthy (2020) recommend subsidizing refinancing for small firms; (iv) Greenwood and Thesmar (2020) recommend extending a tax credit to claimants (i.e., landlords) that accept a haircut on loan obligations; (v) Iverson, Ellias, and Roe (2020) recommend hiring additional bankruptcy judges; (vi) Skeel (2020) recommend creating a standard "prepacked" restructuring process; (vii) Blanchard, Philippon, and Pisani-Ferry (2020) recommend the government accept larger losses than creditors in reorganizations; (viii) DeMarzo, Krishnamurthy, and Rauh (2020) recommend a government-funded vehicle extend debtor-in-possession financing; (ix) the Bankruptcy and COVID-19 Working Group recommends extending deadlines for small businesses in Chapter 11.

[2] The Takenaka Plan (referring to the minister of the Japanese Financial Services Agency) entailed a forceful effort to end the non-performing loans problem. Takenaka forced banks to "make more rigorous evaluation of assets using discounted expected cash flows or market prices of non-performing loans... This stopped the process of evergrowing non-performing loans" (Hoshi and Kashyap, 2010). Indeed, the Takenaka plan led to an overall reduction in nonperforming loans on bank balance sheets by 50%. See also Hoshi and Kashyap (2011).

Japan has also implemented policies encouraging reorganizations. In response to slow economic growth in the 1990's and a proliferation of nonperforming loans on bank balance sheets, "The Resolution and Collection Corporation (RCC), a government asset management company that already existed, also shifted their activities to put much more emphasis on reorganizing troubled borrowers."

financing for solvent firms and forces firms to borrow more directly from households. This ties up more household resources and increases bank borrowing rates. As we will explain, higher rates tighten bank collateral constraints through lower collateral valuations, further depressing lending by all banks. We find that the socially optimal intervention discourages liquidation if the fire-sale externality dominates this collateral externality. Conversely, optimal intervention encourages liquidations if the collateral externality dominates. Specifically, we show that greater corporate distress – measured by high operating losses, low productivity, and high leverage – leads a social planner to intervene to promote *liquidation* over reorganization, due to increased costs of balance-sheet congestion relative to fire sales. Surprisingly, *larger* liquidation discounts also lead the planner to promote liquidation. Intuitively, larger discounts raise marginal congestion costs relative to marginal fire-sale costs. We use our framework to study synergies between insolvency interventions and other policies, and to ask how different policies target the different externalities present in our model.

In our two-period baseline model, firms enter a crisis with assets in place and debt owed to banks. Some firms are solvent. These firms have a high long-run value and a new investment opportunity requiring financing. Both banks and households lend to solvent firms, but banks have a comparative advantage due to a monitoring technology (Diamond, 1984). Other firms are insolvent. Banks choose whether to liquidate or reorganize insolvent firms. In reorganization, banks receive the stochastic long-run value of the firm in exchange for funding a current operating loss. In liquidation, third-party arbitrageurs buy the insolvent firm's assets. Banks borrow from households and use liquidation proceeds to fund new solvent-firm loans and distressed reorganizations.

Banks face a collateral constraint when borrowing from households: households refuse to lend more than they expect to recover if the bank were to hypothetically fail. Due to an agency friction, banks can only pledge a fraction of their period-two cashflows as collateral, which households discount at the household-to-bank interest rate to determine bank borrowing capacity. A decline in the household-to-bank borrowing rate thus improves collateral values, improving bank borrowing capacity. In equilibrium, the collateral constraint binds because banks borrow as much as possible: the comparative advantage of banks in firm lending allows them to earn an excess return by borrowing from households and lending to firms.

We begin in Section 3 by characterizing the privately optimal insolvency rule chosen by banks for resolving distressed firms. Formally, an insolvency rule is a probability of liquidating a distressed firm of a given long-run value. We show that the privately optimal liquidation rule is a threshold rule for liquidation: firms with long-run value above a threshold are reorganized, while firms with long-run value below the threshold are liquidated. The optimal threshold for liquidation equalizes on the margin the value to the bank of reorganizing or liquidating the firm. Liquidations produce immediate cashflows for banks. The value of this cashflow to the bank depends on the bank's *effective return*: the bank's profit from making a new loan and using it as collateral to borrow more. As collateral constraints tighten, the effective return increases. This occurs because scarce bank lending pushes up the firm borrowing rate, increasing the excess return banks earn from using loans as collateral to borrow and lend more. Privately optimizing banks thus liquidate more when they are collateral constrained because the larger effective return from liquidation proceeds increases the opportunity cost of reorganizing a given firm.

Section 4 provides the main results of the paper: characterizing socially optimal insolvency rules. We study the problem of a social planner who chooses an insolvency rule to maximize social welfare, internalizing equilibrium price impacts but otherwise respecting constraints faced by private agents. We show that the socially optimal insolvency rule is also a threshold rule for liquidation. However, the socially optimal threshold differs from the privately optimal threshold for two reasons. The first is the fire-sale externality: additional liquidations depress the fire-sale price, which reduces bank recovery. Fire sales are a transfer from banks to arbitrageurs, but they lower welfare because the collateral constraint leads banks to have a high marginal social value of date-one funds. The second reason is the collateral externality. Banks do not internalize that reorganizations use up funds that could have gone to solvent-firm lending, pushing up the rate solvent firms are willing to pay for loans. As a result, households are motivated to lend more to solvent firms instead of banks, pushing up the interest rate banks have to pay on borrowing and reducing their collateral values. In contrast to the fire-sale externality, the collateral externality leads the planner to prefer *more* liquidations. By encouraging liquidations, the planner can redeploy bank funds to healthy firms, which reduces interest rates and positively revalues collateral.

The model features significant interactions between the two externalities. On the one hand, larger collateral externalities increase the marginal cost of a contraction in loanable funds, and so increase the cost of the fire-sale externality that contracts loanable funds. On the other hand, larger fire sales tighten bank collateral constraints, and so contract loanable funds. This increases excess returns on lending and so exacerbates the collateral externality. This significant interaction makes the direction of intervention – more or fewer liquidations – a priori uncertain.

Our model delivers concrete insights on when optimal policy promotes more or fewer liquidations. We show that the social optimum can be decentralized with a simple tax or subsidy on liquidations. The tax/subsidy is not contingent on the long-term value of a firm, meaning the planner does not need specific knowledge of a specific insolvent firm's characteristics to implement the social optimum. We show that optimal policy (ceteris paribus) favors liquidation *subsidies* when nonfinancial firms are in greater distress: when firm operating losses are higher, when firm productivity is lower, and when firm leverage is higher. Most surprisingly, we show that *lower* liquidation prices – that is, larger fire sales – can also lead to liquidation subsidies being optimal. Although at first surprising, the result is intuitive: under constant elasticities, already-low liquidation prices make the marginal impact of further liquidations on total recovery relatively low. Thus the marginal impact of fire sales on total recovery falls even as its absolute magnitude rises. Although the larger fire sale reduces loanable funds obtained from liquidation, liquidation also saves banks the cost of covering the firms' operating losses. This leads the collateral externality to dominate when fire sales are large, making liquidation subsidies desirable.

In Section 5, we extend our analysis to consider several policies that can potentially complement or substitute for insolvency interventions. In Section 5.1, we incorporate an initial firm-bank borrowing-lending problem and study macroprudential regulation of bank balance sheets. Interestingly, optimal macroprudential regulation is tailored entirely to the collateral externality and not the fire-sale externality. Macroprudential regulation is largest when the social value of loanable funds is largest, that is when liquidation subsidies are desirable. In Section 5.2, we study the optimal sector for a planner to target bailouts. We show that bailouts to banks always dominate bailouts to solvent firms because banks can capitalize on a multiplier effect of collateralizing loans. Interestingly, the value of bailouts is determined by the collateral externality, not the prevention of liquidations, a result of liquidations being optimally chosen by a social planner. Bailouts are more valuable as the social value of loanable funds rises, suggesting a role for non-revenue-neutral liquidation (or reorganization) subsidies, which serve a dual role of both recapitalizing banks and correcting their insolvency rule, over unconditional bailouts.[3] In Section 5.3, we study the possibility that banks can boost loanable funds by avoiding recognizing losses on loans to insolvent firms: banks can pledge "zombie loans" at full collateral value. While this leads both the bank and the planner to prefer fewer liquidations, we show that the optimal intervention is qualitatively similar to the baseline model. In Section 5.4, we study intervention when banks are heterogeneous

[3]This "double dividend" of Pigouvian subsidies – Pigouvian subsidies both correct externalities and generate bailouts for banks – is analogous to the double dividend that Pigouvian taxes both correct externalities and generate revenues for the government (Tullock, 1967; Clayton and Schaab, 2022b).

in their lending capabilities. We show that the planner bifurcates banks into two separate functions: banks with small collateral haircuts are “secured” creditors that receive payoffs from liquidations, while banks with large collateral haircuts are distressed lenders that reorganize distressed firms. This promotes more reorganizations, suggesting that seniority structure interventions can partially substitute for interventions promoting liquidation. Interestingly, we also show that bailouts should be targeted toward the secured creditors, because they use liquidation payoffs to lend to solvent firms. That is, the value of targeting bailouts is summarized by how it can promote new lending. Building upon this analysis, in Section 6, we further discuss how our two externalities interact and how the presence of both externalities along with insolvency interventions changes how traditional policy interventions are used.

Finally, in Section 7, we characterize and compare the two crises mentioned above: the US COVID crisis and Japan’s nonperforming loan crisis. We use aggregate statistics to argue that, relative to the US crisis, Japan’s crisis featured: higher corporate leverage, lower firm productivity (e.g., a more permanent shock), a higher reliance on bank lending in the corporate sector, and lower profitability among firms. Using our model comparative statics, we show that all four of these facts imply a social planner would have optimally subsidized liquidation in Japan and subsidized reorganization in the US. This prediction aligns with observed policy responses. We conclude by providing testable implications from our model for future empirical work.

Related literature. This paper contributes to the theoretical literature studying the optimal resolution of insolvent firms. In early seminal work, Shleifer and Vishny (1992) show that fire-sale externalities create a motive for social planners to avoid liquidations. Corbae and D’Erasmo (2021a) estimate a general-equilibrium model with both reorganization and liquidation in bankruptcy, but do not consider fire-sale externalities or collateral constraints. Corbae and D’Erasmo (2021b) estimate a general-equilibrium model with a constrained banking sector to study government interventions, but do not study nonfinancial corporate insolvency.[4] Li and Li (2021) show that public liquidity interventions during crises preserve low-quality firms and mitigate the cleansing effect of crises. Li (2019) studies government interventions in a general equilibrium model but focuses on failing banks rather than nonfinancial firms. Hanson, Stein, Sunderman, and Zwick (2020a) model an economy in which extending credit to otherwise insolvent firms helps to mitigate aggregate demand externalities, promoting reorganization. Philippon (2020) models a mechanism-design problem in which the government seeks to prevent inefficient liquidations without resorting to an indiscriminate bailout. Philippon and Wang (2022) propose resolving ex ante moral hazard by targeting bailouts to better performing banks and liquidating worse performing banks. Chari and Kehoe (2016) study optimal policy with time inconsistent bailouts in a costly state verification framework, and show that restrictions on debt and size constitute optimal policy and that intervention in resolution is not required. Clayton and Schaab (2022a) show that an orderly bank resolution (bail-in) regime promoting reorganization is a socially optimal policy in an optimal contracting model with an incentive problem and fire sales. Colliard and Gromb (2018) and Keister and Mitkov (2021) study how the prospect of bailouts distorts the incentives of banks to privately bail-in or renegotiate with their creditors. Glode and Opp (2021) study complementarities in debt renegotiation when businesses are connected in a debt chain, and study how government interventions can prevent waves of defaults. Donaldson, Morrison, Piacentino, and Yu (2020) study complementarities between bankruptcy and out-of-court restructurings, and use their model to analyze recent proposed interventions in firm insolvency. Our paper contributes to this literature by showing in a problem of optimal insolvency rule design that optimal intervention can favor either liquidation or continuation, depending on the strength of externalities in the banking sector versus

[4] Corbae and Quintin (2015) study the interaction between bank choices and household defaults.

the nonfinancial sector.

This paper also relates to the literature on zombie loans: subsidized bank loans to insolvent firms.[5] Caballero, Hoshi, and Kashyap (2008) and Acharya, Crosignani, Eisert, and Eufinger (2020) show theoretically and empirically that, by keeping insolvent firms alive to compete in product markets, zombie loans lead to lower product prices and markups, reducing entry and productivity. Acharya, Lenzu, and Wang (2021) study the role of monetary policy in exacerbating zombie lending, focusing on a congestion externality arising from zombie loans. In related empirical work, Iverson (2018) shows that congested courts create deadweight losses and Hotchkiss (1995) shows that successfully reorganized firms subsequently perform poorly. We contribute to this literature by studying optimal insolvency intervention with both (i) a fire-sale externality from inefficient liquidation and (ii) an opposing "zombie lending" collateral externality from the misallocation of credit arising due to socially suboptimal firm preservation.[6] Our results provide guidance on the interaction between the externalities, reveal conditions under which each externality dominates the other, and show how insolvency interventions affect use of other traditional policy tools such as macroprudential regulation and bailouts.

Additionally, we make a theory contribution to the literature studying how banks impose externalities on other banks through changes in aggregate interest rates. This literature commonly features a collateral externality that arises because excessive ex ante borrowing forces ex post deleveraging that impairs collateral values through lower asset prices or higher interest rates (Kiyotaki and Moore, 1997; Caballero and Krishnamurthy, 2001; Lorenzoni, 2008; Bianchi and Mendoza, 2018; Dávila and Korinek, 2018; Stein, 2012). In contrast, our paper studies how ex post balance sheet

[5]For empirical evidence of zombie lending and the economic impact of zombie loans, see Caballero, Hoshi, and Kashyap (2008); Acharya, Eisert, Eufinger, and Hirsch (2019); Blattner, Farinha, and Rebelo (2019); Acharya, Borchert, Jager, and Steffen (2021) and Acharya, Crosignani, Eisert, and Eufinger (2020).

[6]See Antill (2022) and Ayotte and Morrison (2009) for empirical evidence of inefficient liquidations and a review of the empirical literature on inefficient liquidations.

congestion raises interest rates and devalues collateral when banks tie up capital supporting insolvent firms. In a related paper, Diamond and Rajan (2005) model failing banks that prematurely call loans to repay running depositors. Failing banks and their depositors do not internalize that forcing early loan repayment reduces total liquidity, resulting in higher interest rates that lower asset valuations and force other banks into insolvency. Differing from this work, we model a bank collateral externality that operates through solvent banks' allocations of funds between solvent and insolvent *firms*, rather than operating through bank runs. Even in the many crises where bank runs do not occur, we show that solvent banks impose negative externalities by restructuring distressed firms: solvent banks do not internalize that restructuring distressed firms congests balance sheets, pushing up interest rates and inefficiently tightening collateral constraints for other banks. Lanteri and Rampini (2021) model heterogeneous firms (absent banks) and show new investment *subsidies* that *lower* future capital prices can be optimal. In their model, a lower capital price generates a positive distributive externality from reallocating resources from low marginal product capital sellers to high marginal product capital buyers. In our model, liquidating distressed firms provides loanable funds to constrained banks, resulting in loans to solvent firms that support the lending price and induce a positive collateral externality. Finally, reducing balance sheet congestion in our model by liquidating distressed firms acts as an internal recapitalization mechanism, complementing the literatures on recapitalization by bailouts or bail-ins (Farhi and Tirole, 2012; Bianchi, 2016; Chari and Kehoe, 2016; Clayton and Schaab, 2022a).

We make another theory contribution by jointly examining our collateral externality and a fire-sale externality. While these externalities push the socially optimal liquidation rule in opposite directions, they also amplify one another: fire sales lower available funds, making the collateral externality more relevant; similarly, the social cost of fire sales rises as the collateral constraint tightens through interest-rate externalities. Moreover, we show that in the presence of optimal insolvency interventions, the use of traditional instruments of macroprudential regulation and bailouts revolves around the collateral externality (value of loanable funds), and not directly around preventing fire sales. We thus show that regulators need to jointly consider the banking and nonfinancial sector when designing crisis responses, as well as to consider what policies best target which externalities. Otherwise, an intervention aimed at mitigating fire sales could backfire if it tightens bank collateral constraints.

2 Model

There are two dates, date one and date two. The economy consists of four types of agents: firms, banks, arbitrageurs, and households. There is a unit continuum of each type of agent. All agents do not discount the future.

2.1 Firms

Firms enter date one with an inherited project and long-term debt d_0 due at date two. In our baseline model, d_0 is exogenous.[7] Inherited debt d_0 is held entirely by banks. We use lowercase for exogenous model parameters and uppercase for endogenous model quantities.

At date one, firms are either *solvent* with probability p or *insolvent* with probability $1-p$.[8] Solvent firms have a constant exogenous high payoff v_S on their outstanding project, which realizes at date two. They can also invest in a new technology, whereby $I_S \geq 0$ of the consumption good at date one produces $g_S(I_S)$ units of the consumption good at date two. We assume the exogenous function $g_S(\cdot)$ is increasing, concave, and differentiable.

[7] Section 5.1 extends the model with an endogenous d_0 choice.

[8] We endogenize p in an extension in Appendix B and find qualitatively similar results.

Solvent firms can borrow from both banks and households to finance their investment. We let Q_S denote the endogenous price of loans for solvent firms at date one, so that Q_S^{-1} is the gross interest rate paid by firms. Solvent firms choose I_S to maximize date-two cashflows (net of debt repayment),

$$\max_{I_S} \; g_S(I_S) - \frac{I_S}{Q_S} + v_S - d_0, \tag{1}$$

where I_S/Q_S is the required date-two debt repayment to raise I_S from banks and households. In other words, I_S is the date-one market value of the loans demanded by solvent firms at date one. The solvent-firm investment choice is thus

$$g_S'(I_S) = Q_S^{-1}. \tag{2}$$

Insolvent firms have an idiosyncratic date-two payoff $v \in [\underline{v}, \overline{v}]$ on their outstanding project, with density f. Uncertainty is resolved at date one, so insolvent firms know what their date-two payoff will be at the beginning of date one. We assume insolvent firms are distressed and do not have a new investment opportunity. However, they experience an exogenous deterministic current operating loss $c \geq 0$ at date one. This cost c must be paid at date one in order to maintain viability. If it is not paid, the insolvent firm can be liquidated in a process described below. These firms are insolvent in the sense that liabilities exceed expected future cashflows, $\overline{v} < d_0 + c$.[9] Insolvent firms are resolved by their banks – their initial creditors – in a process which we describe below, and have no residual equity value.

[9] For simplicity, our analysis assumes environments with $Q_S \leq 1$ so that the interest rate is nonnegative.

2.2 Banks

Banks enter date one owning debt claims with date-two value d_0. These claims come from earlier lending to both solvent and insolvent firms. For expositional simplicity, we assume that banks enter date one with no existing debts to households.[10] All of our results hold if we instead assume that banks enter date one owing deposits to households.

Banks at date one choose: (i) new borrowing B_1 from households, at an endogenous price Q_B; (ii) new loans D_1 to solvent firms, at endogenous price Q_S; and (iii) how to resolve insolvent firms.

2.2.1 Resolving insolvent firms

The insolvency-resolution process is a rule for sorting firms into one of two outcomes: liquidation or continuation. Formally, it is a probability $\rho(v) \in [0,1]$ that an insolvent firm with viability v is liquidated, with probability $1-\rho(v)$ of continuation. *Liquidation* entails a sale of firm assets to arbitrageurs[11] at the endogenous market price γ, giving banks γ per unit of liquidated assets. *Continuation* entails the bank paying the firm's date-one expenses in exchange for the firm's future cashflows.[12] The bank thus pays c at date one to get a claim worth v at date two. For technical reasons, we assume that a small fraction of the worst firms ($v \leq \hat{v}$) are nonviable and must be

[10]Because banks borrow at date one to cover operating costs and new lending, collateral constraints may still bind despite the absence of inherited debt owed by banks to households.

[11]The arbitrageur may be interpreted as a strategic buyer that will use the liquidated assets in its business operations. Because the arbitrageur uses the assets in its own business, the liquidation payoff does not depend on the insolvent firm's viability or expenses. See Section 2.3 for details.

[12]Liquidation could be achieved through either an out-of-court foreclosure and sale or through a bankruptcy filing under Chapter 7 or Chapter 11 of the US bankruptcy code. Continuation could represent a "zombie loan" in which a bank lends to a firm at a subsidized rate, an out-of-court restructuring, or a reorganization under Chapter 11 of the US bankruptcy code. Since all uncertainty in our model is resolved at date one, it is irrelevant in our model whether the bank receives debt or equity from the insolvent firm in a continuation. To limit notation, we do not model the various deadweight losses that arise in liquidation or continuation; Appendix G shows that the inclusion of distinct liquidation costs or continuation costs does not change our results.

liquidated.[13]

In our baseline model, we omit the possibility of acquisitions for ease of exposition. In the appendix, we consider extensions where insolvent firms can be acquired by either arbitrageurs (Appendix E) or by solvent firms (Appendix F). We find results that are qualitatively similar to our baseline-model results.

2.2.2 The bank objective

The bank's date-one budget constraint is given by

$$\underbrace{pD_1}_{\text{New Loans}} + \underbrace{(1-p)\int(1-\rho(v))cf(v)dv}_{\text{Continuation Operating Losses}} \leq \underbrace{B_1}_{\text{Household Borrowing}} + \underbrace{(1-p)\int\rho(v)\gamma f(v)dv}_{\text{Liquidation Proceeds}} \tag{3}$$

so that total funds lent out to solvent and distressed firms equals resources obtained from liquidating distressed firms and (net) borrowing from households.

We assume that an agency friction limits banks' borrowing from households. While banks never default *in equilibrium* in our model, households refuse to lend more than the amount they expect to recover if a bank were to default at date two. Banks have an incentive to report inflated asset values in an attempt to borrow more. We assume that households can only verify a fraction ϕ of banks' date-two cash flows from solvent-firm loans. For simplicity, households cannot verify reorganized-firm values.[14] This leads to a "verifiable collateral" constraint for bank borrowing:

[13] This bounds the liquidation price even with CRRA arbitrageur preferences, as in Assumption 1. Formally, we assume there is an exogenous cutoff $\hat{v} \in (\underline{v}, \overline{v})$ such that firms with $v < \hat{v}$ must be liquidated. The cutoff $\hat{v}$ can be arbitrarily close to $\underline{v}$.

[14] We obtain qualitatively similar results assuming reorganized-firm value can be partially verified.

$$B_1 \leq \phi\, Q_B \quad \underbrace{p\,(d_0 + \frac{D_1}{Q_S})}_{\text{Solvent Firm Loans}} \quad . \tag{4}$$

This collateral constraint features a haircut $1-\phi$ applied to the value of a bank's collateral: the date-two payoff from solvent-firm loans, discounted to date one. Importantly, the date-two cash-flows are discounted at the household discount rate because the households are the banks' lenders. Thus, the household discount factor Q_B is applied to determine the date-one value of collateral. Equivalently, this constraint restricts the date-two face value of household debt, B_1/Q_B, to be lower than verifiable date-two bank cashflows.

The date-one liquidation price γ does not directly enter the collateral constraint because house-holds limit lending based on the portion of date-two cashflows that they can verify would be avail-able in a date-two bank default. Nevertheless, the liquidation price appears implicitly through its important role in endogenously determining Q_B. Because γ appears in the banks' date-one budget constraint, it drives bank decisions that endogenously impact Q_B, as we will explain.

Banks choose (B_1, D_1, ρ) in order to maximize their final value,

$$\max_{B_1, D_1, \rho} \quad \underbrace{p\left(\frac{D_1}{Q_S} + d_0\right)}_{\text{Solvent Loans}} + \underbrace{(1-p)\int (1-\rho(v))vf(v)dv}_{\text{Distressed Continuations}} - \frac{B_1}{Q_B}, \tag{5}$$

subject to their budget constraint (3) and their collateral constraint (4). Recall that ρ is a function of v.

2.3 Arbitrageurs

Arbitrageurs are the second-best users of firms' liquidated investment projects. We interpret ar-bitrageurs as strategic buyers, such as new entrants, who repurpose firms' liquidated assets for another business. Arbitrageurs have a technology that converts L dollars of liquidated assets into $a(L)$ units of consumption. We assume that $a(\cdot)$ is increasing, concave, and differentiable. Given an equilibrium liquidation price γ, arbitrageurs solve

$$\max_L a(L) - \gamma L$$

resulting in demand for liquidated assets given implicitly by

$$a'(L) = \gamma. \tag{6}$$

2.4 Households

Households enter date one with an exogenous endowment e of the consumption good. At date one, households choose how much of this endowment to invest and how much to consume.

Households invest in two Walrasian markets. First, they invest in a market for bank deposits. In this market, households are the only supplier of funds. Only banks demand deposits. The endogenous loan price Q_B in this market is determined to equate supply and demand. Households earn a gross return Q_B^{-1} for lending to banks. We let B_H denote date-one household-to-bank lending.

Second, households participate in a market for firm lending. In this market, both households and banks supply loans to firms. Only solvent firms demand loans in this market. The loan price Q_S is determined to equate supply and demand. Firms thus pay the same loan price (interest rate) whether they borrow from banks or households.

Importantly, we assume that households are less efficient than banks at investing in firms. We model this with an exogenous parameter $t > 0$, which captures transaction costs that households face when lending to firms. This transaction cost may be interpreted as a reduced-form approach to modeling banks' superior firm-monitoring technology (Diamond, 1984). Formally, we assume that households must spend $pD_H(1+t)$ to make a date-one loan of pD_H to solvent firms. Once households make the loan, they face the same Walrasian loan price Q_S (with gross return Q_S^{-1}) as banks, with total date-two payoff $pD_H Q_S^{-1}$ from solvent-firm loans.

Formally, households solve

$$\max_{B_H,D_H} u\left(e - B_H - p(1+t)D_H \right) + \frac{B_H}{Q_B} + p\frac{D_H}{Q_S}, \tag{7}$$

where $u(\cdot)$ is an exogenous function capturing the utility households receive from date-one consumption. We assume that $u(\cdot)$ is increasing, concave, and differentiable.

For the remainder of the paper, we focus on cases with interior solutions: households optimally supply a strictly positive quantity to both firms and banks. Intuitively, an interior solution arises when the return to firm lending is sufficiently high for households to willingly incur the transaction cost t. The return to firm lending is high when banks are sufficiently constrained, limiting their supply of funds to firms.

In an interior solution, the households' optimal choices B_H, D_H are characterized by the first-order conditions for (7), given by:

$$u'\left(e - B_H - p(1+t)D_H \right) = \frac{1}{Q_B} \tag{8}$$

$$u'\left(e - B_H - p(1+t)D_H \right) = \frac{1}{(1+t)Q_S}. \tag{9}$$

Equation (8) implies that Q_B is the equilibrium household discount rate: in any equilibrium, households are indifferent between receiving one unit of date-one consumption or receiving $1/Q_B$ units of date-two consumption. In this sense, bank collateral is discounted at the household discount rate in equation (4).

2.5 Market clearing and competitive equilibrium

Our model has three equilibrium prices at date one: the price Q_S of loans to solvent firms, the price Q_B of household loans to banks, and the price γ of real liquidated assets. All three markets must clear at date one:

$$D_1 + D_H = I_S \tag{10}$$

$$(1-p)\int \rho(v)f(v)dv = L \tag{11}$$

$$B_H = B_1. \tag{12}$$

A competitive equilibrium of the model is a collection of prices (Q_S, Q_B, γ) and allocations $(D_1, D_H, I_S, \rho, L, B_H, B_1)$ such that: (i) solvent firms choose I_S to maximize utility; (ii) banks choose (B_1, D_1, ρ) to maximize utility; (iii) arbitrageurs choose L to maximize utility; (iv) households choose D_H, B_H to maximize utility; (v) markets clear.

3 Privately optimal liquidation rules

We begin by characterizing the privately optimal liquidation rule chosen by banks in a competitive equilibrium. The policies (B_1, D_1) can then be determined using the liquidation rule, constraints, and market prices.

Recall that banks face a budget constraint (3). Let δ_P denote the Lagrange multiplier on this budget constraint in the bank's problem of choosing a privately optimal policy, that is the private marginal value of date-one wealth to banks expressed in date-two consumption. We can thus interpret δ_P as the bank's *effective return* at date two from receiving an extra dollar at date one.

The following proposition characterizes this effective return and the bank's liquidation rule:

Proposition 1. *In any competitive equilibrium,*

1. *The bank's privately optimal liquidation rule is a threshold rule* $\rho(v) = \mathbf{1}\left(v \leq V_P\right)$*, where*[15]

$$V_P = \delta_P \left(\gamma + c \right). \tag{13}$$

2. *If* $Q_S^{-1} \geq Q_B^{-1}$*, the bank's effective return from receiving a dollar at date one is*

$$\delta_P = \underbrace{\frac{1}{Q_S}}_{\text{Direct Return}} + \underbrace{\frac{\phi \frac{Q_B}{Q_S}}{1 - \phi \frac{Q_B}{Q_S}}}_{\text{Collateral Multiplier}} \times \underbrace{\left(\frac{1}{Q_S} - \frac{1}{Q_B} \right)}_{\text{Excess Return}}. \tag{14}$$

All proofs appear in Appendix A. Proposition 1 implies that banks optimally liquidate firms according to a threshold rule. In principle, banks could choose arbitrary rules ρ. However, we show it is always optimal for banks to start by liquidating firms with lower long-run values.

Additionally, equation (13) implies that banks choose the liquidation rule that maximizes their ex-post recovery. By definition, the value to a bank from an additional dollar at date one is δ_P. If a bank chooses continuation for a firm with viability v, it gets a cashflow of v at date two and it pays c dollars at date one to cover the firm's operating cost. This has a total payoff of $v - \delta_P c$ for the bank. If the bank chooses liquidation for the firm, it gets γ extra dollars at date one, which the bank values at $\delta_P \gamma$. From the definition of V_P, we see that the threshold firm that is liquidated is precisely the one where the bank's date-two value from liquidation, $\delta_P \gamma$, is equated with the bank's date-two value from continuation, $V_P - \delta_P c$.

[15] Unless explicitly mentioned, we focus attention on cases with interior solutions $V_P \in (\underline{v}, \overline{v})$ and $V_P > \hat{v}$.

To build intuition for the effective return δ_P, we first consider the case in which $Q_S = Q_B$. In this case, if a bank receives an extra dollar at date one, it can (i) lend it to firms for a return $Q_S^{-1} = Q_B^{-1}$ or (ii) reduce its borrowing from households, saving an interest payment of Q_B^{-1}. Either way, the value to the bank is $\delta_P = Q_B^{-1}$. If the bank chooses to make a new loan to a firm, it can use this loan as collateral. However, this is irrelevant because the bank collateral constraint does not bind when Q_S is equal to Q_B — there is no excess return to borrowing from households to lend to firms.[16] In summary, the effective return is simply $\delta_P = Q_S^{-1} = Q_B^{-1}$.

Next, we consider the case in which $Q_S^{-1} > Q_B^{-1}$. In this case,[17] banks borrow as much as possible because they earn an excess return of $Q_S^{-1} - Q_B^{-1}$ by borrowing from households and lending to firms. As a result, the bank collateral constraint binds. In this case, if banks receive an extra dollar at date one, they lend it to firms for a date-two payoff of Q_S^{-1}. This is the *direct* return on an additional dollar of date-one funds. Additionally, since banks are collateral constrained, they use the new loan as collateral to borrow more from households. The new loan has a date-two value of Q_S^{-1}, so it can be pledged to borrow $\phi Q_B \times Q_S^{-1}$ at date one. This additional borrowing allows banks to lend and get an excess return of $Q_S^{-1} - Q_B^{-1}$. The loan created by this additional borrowing can also be used as collateral. Iterating this, if banks get an additional dollar to make a loan worth Q_S^{-1}, the ability to collateralize loans gives them a payoff of

$$\phi Q_B Q_S^{-1} \left(Q_S^{-1} - Q_B^{-1} \right) + \phi Q_B \left(\phi Q_B Q_S^{-1} \times Q_S^{-1} \right) \left(Q_S^{-1} - Q_B^{-1} \right) + \ldots \tag{15}$$

which converges to the second term in equation (14). Note that we must implicitly assume that $\phi Q_B / Q_S < 1$ or else this argument implies bank value is infinite so the problem is not well defined.

[16] We show formally in Appendix A that the collateral constraint binds if and only if $Q_S > Q_B$.

[17] The case in which $Q_S^{-1} < Q_B^{-1}$ is trivial and unrealistic — banks do not borrow from households since the cost of household-to-bank borrowing exceeds the return on bank-to-firm lending, and as a result households also do not lend directly to firms.

Equation (15) represents the *shadow* value of an additional date-one dollar. This shadow value reflects the fact that an additional dollar lets banks make loans that can be used as collateral for further loans. Proposition 1 shows that the effective return δ_P is the sum of the direct and shadow values of receiving another dollar.

In sum, Proposition 1 represents a form of creditor-recovery-maximizing liquidation decisions, whereby the bank privately trades off the total (direct + shadow) value of a reorganized firm against the value of having funds to redeploy to new loans. The optimal liquidation rule sets equal on the margin these two values, and liquidates firms when the opportunity cost of reorganization is larger than the total value of reorganization.

4 Socially optimal liquidation rules

We next study the socially optimal liquidation rule that is designed by a social planner who internalizes the determination of equilibrium prices, but must otherwise respect the same constraints faced by private agents. The planner has a complete set of Pigouvian wedges τ on the decisions of banks, but must take as given the decisions of solvent firms, households, and arbitrageurs. Given complete wedges, we can adopt the primal approach whereby the planner directly chooses (B_1, D_1, ρ) for banks, but must take the following as given: the constraints faced by private banks (3)- (4); the equilibrium pricing conditions of firms (2), arbitrageurs (6), and households (8)- (9); and the market clearing conditions (10) - (12).

The social planner has a utilitarian objective function, where as usual we interpret the planner as achieving Pareto efficiency via lump sum transfers — either at date zero (Section 5.1) or at date two. The utilitarian objective function, summed across all agents, is given by

$$\underbrace{p\left(g_S(I_S)+v_S\right)+(1-p)\int(1-\rho(v))vf(v)dv}_{\text{Surplus from firms}}+\underbrace{a(L)-\gamma L}_{\text{Surplus from arbitrageurs}}+\underbrace{u\left(e-B_1-p(1+t)D_H\right)}_{\text{Surplus from households}}.$$

We derive this objective in Appendix A.2. The utilitarian objective nets out transfers between arbitrageurs, banks, firms, and households. As a result, the objective captures the three sources of surplus in the economy: surplus from firm production, surplus from arbitrageur purchases, and surplus from household consumption. Loans between banks, firms, and households drop out because they reflect intermediated resources. Note that γL appears in the surplus equation because it represents the diversion of consumption resources towards production.[18]

In the results to come, it will be helpful to introduce the following notation:

$$\sigma_S \equiv \frac{-g_S''(I_S)}{g_S'(I_S)}, \qquad \sigma_H \equiv \frac{-u''\left(e-B_1-p(1+t)D_H\right)}{u'\left(e-B_1-p(1+t)D_H\right)}, \qquad \xi_\gamma \equiv -\frac{L}{\gamma}\frac{\partial\gamma}{\partial L}. \tag{16}$$

The elasticity ξ_γ and risk aversions σ_S, σ_H are not required to be constant. The elasticity ξ_γ is characterized using the demand function of arbitrageurs. Note that, by definition, $\sigma_H, \sigma_S, \xi_\gamma \geq 0$.

We are now ready to solve the social planner's problem. Let δ_* denote the Lagrange multiplier on the bank's budget constraint in the social planner's optimization. This represents the social planner's effective return at date two from receiving an extra dollar at date one. The following proposition characterizes the socially optimal liquidation rule and the planner's effective return δ_*.

[18] Specifically, γL represents funds from arbitrageurs intermediated through banks via liquidation sales.

Proposition 2. *The socially optimal liquidation rule is a threshold rule* $\rho(v) = \mathbf{1}\left(v \leq V_*\right)$, *where*

$$V_* = \underbrace{\delta_*(\gamma + c)}_{\text{Bigger than } V_P} - \underbrace{(\delta_* - 1)\xi_\gamma \gamma.}_{\text{Fire-Sale Externality}>0} \tag{17}$$

The social planner's effective return is $\delta_* = M\delta_P$, *where* δ_P *is the banks' private effective return and* $M \geq 1$ *is a multiplier:*

$$M = \left[1 - \frac{\sigma_S \sigma_H \phi t}{\left(p(1+t)\sigma_H + \sigma_S\right)\left(1 - \phi(1+t)\right)} p Q_B d_0\right]^{-1} \geq 1. \tag{18}$$

Proposition 2 shows that like the privately optimal rule, the socially optimal rule is a threshold rule for liquidation: firms with low long-run values $v \leq V_*$ are liquidated, while firms with higher long-run values $v \geq V_*$ are reorganized.

If $\delta_* = \delta_P = 1$, then the privately and socially optimal liquidation rules align. Intuitively, if the collateral constraint does not bind in the private equilibrium, then the planner has no need to intervene. Absent a binding collateral constraint, the first welfare theorem implies that the private equilibrium is socially efficient; fire sales and other price impacts are simply transfers between agents.[19] In this case, the planner's solution will feature creditor-recovery maximization just as in Proposition 1.

Proposition 2 shows that when the collateral constraint binds, the socially optimal rule differs from the privately optimal rule in two key manners. The first force driving the planner's solution is the fire sale of real assets. Banks do not internalize that liquidating a firm pushes down the price of liquidated assets, lowering liquidation recoveries for all banks. This lower liquidation price leads

[19] For example, in this case of nonbinding constraints, a depressed liquidation price γ is simply redistribution between buyers and sellers (a welfare-irrelevant pecuniary externality).

to a transfer from banks to arbitrageurs. However, when collateral constraints bind, the marginal value of wealth at date one is higher for banks than for arbitrageurs, giving rise to a welfare-relevant pecuniary externality. In particular, fire sales reduce surplus because lower liquidation prices reduce all banks' loanable funds. This effect scales based on the social effective return δ_*. This force leads the planner to prefer *fewer* liquidations than private banks.

The second difference between the planner's solution and the private solution arises due to the difference between the private and social effective returns from date-one funds. In particular, the social effective return δ_* differs from the private effective return δ_P by a multiplier M, and is always *larger*. Intuitively, an optimizing bank does not internalize that its loans to solvent firms can change the prices Q_S, Q_B. In particular, bank lending creates a positive collateral externality: (1) more lending increases Q_S, since firms must be willing to undertake the new loans; (2) a higher Q_S leads households to allocate more funds to banks rather than firms; (3) the increase in household-to-bank lending supply increases Q_B; (4) a higher Q_B increases collateral values; (5) higher collateral values let banks borrow and lend more, increasing production.[20] While banks do not internalize this externality, the social planner does. This externality leads the social effective return to exceed the private effective return and motivates the planner to intervene to increase loanable funds. Because liquidations free up funds for lending to solvent firms, this force leads the planner to prefer *more* liquidations than private banks.

Equation (18) displays the multiplier M. When M is larger, the gap between the social and private effective returns grows. Intuitively, M reflects the extent to which an additional loan to a solvent firm helps other banks borrow. Holding all else equal, M is larger when there is more collateral to revalue (higher d_0), or when collateral haircuts are smaller (higher ϕ). This highlights

[20] In this chain of events, households lend less to firms and banks lend more to firms; this increases total funds available to firms because of the transaction cost t that households face. In other words, a looser collateral constraint allows the planner to shift more firm financing activity to banks, the more efficient firm lenders.

the importance of collateral revaluation for the planner's motive to incentivize liquidations: if there are no outstanding loans, $d_0 = 0$, or outstanding loans cannot be collateralized, $\phi = 0$, then $M = 1$. In either of these cases, $\delta_* = \delta_P$ and only the fire-sale externality is present. The multiplier also reflects the extent to which loan prices respond to loan supply. Holding all else equal, M increases with σ_H and σ_S. Intuitively, when σ_H or σ_S is large, a small increase in loan supply leads to a large decline in gross interest rates to equate supply and demand. This means the planner achieves a larger collateral revaluation from each dollar of loanable funds (through price changes), and thus a greater welfare improvement.

Proposition 2 nests two limiting cases where only one externality is present and the direction of intervention is unambiguous. First, suppose $\xi_\gamma = 0$ so there is no fire sale of real assets (i.e., $a(L)$ is affine so $\gamma = a'(L)$ is an exogenous constant). In this case, the fact that $\delta_* \geq \delta_P$ implies that on the margin optimal policy *encourages* liquidations. Second, suppose $\sigma_S = 0$ or $\sigma_H = 0$ (i.e., g_S or u is affine so Q_S or Q_B is fixed exogenously). In this case, banks are borrowing constrained but there are no interest rate or loan price impacts for the planner to internalize. In this latter case, $M = 1$ and $\delta_* = \delta_P$, so that there is no collateral externality. A binding borrowing constraint still means banks have a higher marginal value of wealth than arbitrageurs ($\delta_P \geq 1$), so that optimal policy in this second case *discourages* liquidations to mitigate the fire-sale externality.

4.1 Liquidation taxes or liquidation subsidies?

Propositions 1 and 2 provide the privately and socially optimal insolvency rules. We can now conduct the exercise of asking what is the optimal tax (or subsidy) on liquidations that would decentralize the socially optimal outcome.

Formally, we introduce a tax τ (per unit of liquidated assets) on liquidations. Taxes are paid out of banks' final payout at date two. This means that if a bank liquidates a distressed firm, it gets the usual γ units of loanable funds at date one, but must make a payment τ to the government out of its final equity value at date two. If $\tau < 0$, then the policy is a liquidation subsidy (the bank receives payment at date two). The tax/subsidy proceeds are remitted lump sum at the end of date two, with the lump sum rebate taken as given in decision rules. As a result, the privately optimal value the bank receives from liquidating a firm is $\delta_P(\gamma + c) - \tau$ per unit, while the value of continuation is v. We thus obtain the following characterization.

Proposition 3. *The liquidation tax/subsidy τ that decentralizes the social optimum of Proposition 2 is given by*

$$\tau = \underbrace{(\gamma + c)\delta_P(1 - M)}_{\text{Subsidy}} + \underbrace{(M\delta_P - 1)\xi_\gamma \gamma}_{\text{Tax}}. \tag{19}$$

Proposition 3 provides a simple decentralization of the social optimum using a uniform tax/subsidy on liquidating a firm. The tax/subsidy is independent of firm long-term value, and incentivizes banks to move their optimal threshold V_P to coincide with the socially optimal threshold, that is to set $V_P = V_*$. Intuitively, since both banks and the planner choose threshold rules, a constant tax/subsidy suffices to align the private and social optimum, without the need to specify v-contingent taxes or subsidies. This means that implementing the efficient rule in this context requires no knowledge from the planner about the long-term value of any specific individual firm. This allows the planner to achieve the social optimum with a simple tool, without the need to tailor intervention towards specific firms. In Appendix H, we describe realistic policies that could be used to implement a tax or subsidy on liquidations.[21]

Intuitively, the optimal tax/subsidy on liquidation trades off two effects. On the one hand, a

[21] For example, the IRS could agree to accept zero recovery on tax-related debts in any restructuring reaching the desired outcome. Appendix H provides details.

liquidation increases loanable funds by $\gamma + c$, which enhances loan prices Q_B, Q_S and creates a positive collateral externality. This promotes a liquidation subsidy. On the other hand, it exacerbates the fire sale, $\gamma\xi_\gamma$. This promotes a liquidation tax. The first effect is given a relative weight $\frac{\delta_P(M-1)}{M\delta_P - 1} \geq 0$, which functions akin to a relative Pareto weight on this effect.

Proposition 3 provides strong intuition on the direction of intervention, that is whether a liquidation tax or subsidy is optimal. All else held equal, higher operating losses c and a higher social multiplier M both promote liquidation subsidies, $\tau < 0$. This is intuitive, as both increase the desirability of loanable funds by either directly increasing the demand for them or by increasing their social value. Also intuitively, a higher fire-sale elasticity promotes a liquidation tax by increasing the cost of the fire sale. Interestingly, this relative importance of this effect is *dampened* as the value of loanable funds rises, reflecting the increased desire of the planner to obtain loanable funds even at the cost of a higher price impact.

Proposition 3 further highlights a surprising result that optimal intervention targets the *less* distressed market: it is at relatively low liquidation values γ (high levels of liquidation L) that intervention promotes liquidation. Put another way, optimal intervention promotes reorganization unless the liquidation fire sale has become sufficiently severe. The intuition comes from the combination of liquidation discounts and sunk costs. When liquidation discounts $1-\gamma$ are already large, there is little additional cost to exacerbating the fire sale and depressing liquidation values, but there is still a large cost to keeping afloat a loss-making firm. This results in liquidation subsidies that promote further liquidations being desirable when a large number of firms are already being liquidated.

Comparative Statics. The discussion of Propositions 2 and 3 so far is based on equilibrium objects. For example, M is endogenously determined. To shed further light on the problem, we now make the following assumptions, which enable us to directly characterize the direction of intervention in terms of comparative statics on date-one exogenous objects.

Assumption 1. *Assume that $g_S(I_S) = \bar{g}\log(I_S)$, that $u(x) = \bar{u}\log(x)$, and that $a(L) = \frac{1}{1-\bar{\xi}_\gamma}L^{1-\bar{\xi}_\gamma}$, where $\bar{g}, \bar{u}, \bar{\xi}_\gamma > 0$ are exogenous parameters. Finally, assume $1 > \phi(1+t)$.*[22]

We maintain Assumption 1 for the rest of this section and focus on the case where $Q_S^{-1} > Q_B^{-1}$. Assumption 1 implies constant elasticities; in particular, $\xi_\gamma = \bar{\xi}_\gamma \geq 0$. It also implies that there is a highest feasible value $\overline{\gamma}$ of the liquidation price γ.[23] Under Assumption 1, we obtain the following characterization.

Proposition 4. *Under Assumption 1, a sufficient condition for $\tau < 0$ is*

$$\bar{\xi}_\gamma < (1 + \frac{c}{\overline{\gamma}})\frac{\phi t}{(p\overline{g} + \overline{u})(1 - \phi(1+t))}pd_0 \tag{20}$$

Proposition 4 provides a simple sufficient condition on date-one exogenous objects under which the optimal intervention takes the form of a liquidation subsidy. The intuition for this result parallels the intuitions discussed above. The left side of (20) is the extent to which additional liquidations reduce liquidation prices. The right side of (20) represents the planner's internalized multiplier effect of additional loans. Proposition 4 thus shows that comparative statics with respect to (date-one) exogenous model parameters confirm the intuitions behind Propositions 2 and 3.

Proposition 4 shows that d_0, the leverage of the nonfinancial corporate sector, is an important determinant of an optimal government response. As d_0 increases, there is more collateral for banks

[22] This final assumption is a technical assumption to guarantee that a loan does not generate more than one unit of loanable funds through collateral.

[23] Recall that we assume a firm must be liquidated if $v < \hat{v}$ for $\hat{v} \approx \underline{v}$. Together with Assumption 1, this implies γ must be less than $\overline{\gamma} \equiv ((1-p)\int_{v \leq \hat{v}} f(v)dv)^{-\bar{\xi}_\gamma}$.

to pledge when borrowing from households. As a result, the marginal social value of a liquidation increases: liquidations lower the household discount rate in equilibrium, improving the value of a larger collateral pool. An increase in d_0 thus makes it more likely that a social planner will subsidize liquidations. By the same logic, liquidation subsidies are more attractive when ϕ is high because the collateral pool is larger.

Proposition 4 also shows that higher operating losses c encourage liquidation subsidies. Intuitively, the planner values each unit of loanable funds more than a privately optimizing bank. As c grows, more resources are tied up in restructuring distressed companies. Since the planner values the lost units of loanable funds more than private banks, the planner values liquidations that free up a greater amount of loanable funds by avoiding covering the operating loss. Higher c thus encourages liquidation subsidies. Note that high c can also be interpreted as low profitability, since the ratio of an insolvent firm's profits to its revenue (absent discounting) is given by $(v-c)/v$.

Further, Proposition 4 shows that liquidation subsidies are more likely when $\bar{g}$ is low: when solvent firms have poor growth prospects due to weaker investment opportunities. Intuitively, when a bank liquidates a firm, it supplies more funds to solvent firms. In equilibrium, Q_S increases so that I_S rises to meet this supply. Equation (2) implies that $\bar{g}^{-1}I_S = Q_S$, so the impact of a liquidation on the equilibrium loan price Q_S is declining in $\bar{g}$. Put differently, firms with weak growth prospects are relatively insensitive to changes in interest rates because their investment opportunities aren't appealing anyway. Low-growth firms thus require a substantial shift in interest rates to invest more. In a low-growth economy, an increase in the supply of bank lending thus induces a substantial change in interest rates (and thus collateral values) to make firm demand catch up with supply. By this logic, liquidation subsidies have a greater impact on collateral values and thus a greater social benefit in a low-growth economy facing a permanent shock. A parallel argument applies to $\overline{u}$, reflecting households' opportunity costs of consumption and their responsiveness to interest rate changes.

Proposition 4 shows that liquidation subsidies are more likely when t is high: when banks have a strong comparative advantage at lending to firms. Intuitively, the planner can create value by making banks intermediate firm loans, avoiding household-to-firm loans. This intermediation avoids the transaction cost paid by households when lending to firms: one unit of firm lending costs t dollars more when supplied by a household. A liquidation increases the supply of bank-to-firm lending and lowers the interest rate paid by firms, causing households to allocate more funds to banks. This in turn boosts bank collateral values through lower household-to-bank interest rates, letting banks borrow and intermediate more loans. Since the social payoff to bank intermediation rises with t, liquidation subsidies are more appealing when t is high.

Finally, Proposition 4 shows that liquidation subsidies are more likely when $\overline{\gamma}$ is lower — when fire sales are more severe.[24] The intuition parallels that above: as fire-sale prices decline, banks recover less $c + \gamma$ from a given liquidation, but the cost of the fire sale on recovery from other liquidations also falls, $\overline{\xi}\gamma$. In other words, larger discounts imply the majority of funds recovered in liquidation are from saving operating losses c, pushing the planner to favor liquidation.

In sum, the results of this section show that the efficient insolvency intervention takes a simple form, involving a uniform tax or subsidy on liquidation that does not depend on information about the long-term prospects of an individual firm. This allows the planner to intervene to promote efficiency without deep involvement in the insolvency process of an individual firm. We also showed that a number of measures of corporate distress – high operating losses, high corporate leverage, low future productivity, and high liquidation *discounts* (low liquidation values) – promote liquidation *subsidies* by increasing the amount or value of loanable funds obtained through

[24]Note that the bound $\overline{\gamma}$ is not technically a parameter. The bound $\overline{\gamma}$ depends on the exogenous $\hat{v}$. Varying $\hat{v}$ would thus produce variation in $\overline{\gamma}$. Moreover, if we were to add an affine coefficient in front of $a(L)$, then $\overline{\gamma}$ would depend on that coefficient. Varying the coefficient, which can be interpreted as the ease with which assets are deployed elsewhere, would produce variation in $\overline{\gamma}$.

liquidation.

5 Regulation and policy interventions

In this section, we consider how insolvency interventions (liquidation taxes and subsidies) interact with other regulations aimed at mitigating crises, and how different interventions target the two different externalities in the model. We extend our model to study ex ante macroprudential regulation, ex post fiscal interventions, impaired loan recognition by banks, and interventions in creditor seniority structures with heterogeneous banks.

5.1 Ex ante macroprudential regulation

We study the role for macroprudential regulation of banks in conjunction with ex post insolvency intervention. To do so, we introduce the ex ante borrowing/lending decision at date zero. We streamline the date-zero firm and household problems to focus on date-zero bank-balance-sheet regulation.[25]

5.1.1 Date zero setup

Firms. At date zero, there is a large number (measure $N > 1$) of identical firm managers. Each firm manager is penniless and has the ability to undertake a project of fixed scale, normalized to one. A firm manager faces an agency friction that limits her pledgeable income to d_0.[26] Each firm manager offers banks a contract that promises repayment of $D_0 \leq d_0$ in exchange for one unit of funding. Any firm manager whose contract is accepted gets financed and forms a firm, then

[25] We abstract away from aggregate uncertainty for simplicity. Results of this extension are easily extended with aggregate uncertainty.

[26] One simple microfoundation is unobservable effort (e.g., Holmstrom and Tirole (1997)).

proceeds to date one. These firm managers receive indirect utility as described in Section 2. Firm managers whose contracts are not accepted at date zero exit the economy.

Households. Households have overlapping generations. The generation born at date one are as described in Section 2. For simplicity, the households born at date zero have linear utility $c_0 + c_1$, so that households are willing to lend to banks at a loan price of one. We make a further simplifying assumption that all date-zero loans to firms are made by banks.

Banks. A representative bank has a finite lending capability normalized to one.[27] The bank can finance these loans either by raising debt B_0 from households at a price of one per unit, or by raising costly inside equity A_0 at separable utility cost $\Psi(A_0)$. Thus, the financing constraint of banks is $A_0 + B_0 = 1$. Their utility is the final value specified in Section 2, once analysis is extended to allow for existing bank debt, net of utility costs $\Psi(A_0)$ at date zero from costly equity. Specifically, the date one problem for banks is identical except that the budget constraint becomes

$$pD_1 + (1-p)\int (1-\rho(v))cf(v)dv \leq \underbrace{B_1 - B_0}_{\text{Household Borrowing Net of Repayment}} + (1-p)\int \rho(v)\gamma f(v)dv \tag{21}$$

At date zero, the bank accepts contracts from measure one of the N firm managers in descending order by offered debt level, with potentially random allocations among firms offering the same contract. There is thus a symmetric equilibrium in which the bank allocates funding with uniform probability $1/N$ among managers that offer $D_0 = d_0$, and all managers offer contracts $D_0 = d_0$.[28]

[27] Formally, the bank's marginal cost function is 1 for $I \leq 1$ and $1+\kappa$ for $I > 1$, where κ is large.

[28] A manager that deviates from the conjectured equilibrium is rejected with probability one and earns no surplus, whereas a manager that offers the equilibrium contract is accepted with probability $1/N$ and earns positive expected surplus.

In summary, letting $\mathscr{V}^P(B_0, Q_S, Q_B, \gamma)$ denote a bank's indirect utility at date one given anticipated prices and a funding choice B_0, banks solve

$$\max_{B_0, A_0} -\Psi_0(A_0) + \mathscr{V}^P(B_0, Q_S, Q_B, \gamma) \quad \text{s.t.} \quad A_0 + B_0 = 1.$$

5.1.2 Private and social optima

In this environment, the planner's optimum can be described as the optimal choice of ex ante borrowing B_0 and ex post liquidation rule ρ. It is easy to verify that the optimal liquidation rule ρ is analogous to the rule in Section 4, conditional on a financing structure. Thus, letting $\mathscr{V}^*(B_0)$ denote the planner's continuation value at date one, the date-zero planner's objective is simply $-\Psi_0(A_0) + \mathscr{V}^*(B_0)$, where observe that date-zero households have zero net welfare impact from lending to banks.[29] Note that the planner's continuation value is written solely as a function of B_0 because the planner internalizes that equilibrium prices are a function of borrowing. In this environment, we obtain the following privately and socially optimal financing rules.[30]

Proposition 5. *Banks' privately optimal equity issuance satisfies $\Psi'(A_0) = \delta_P$, while the socially optimal equity issuance satisfies $\Psi'(A_0) = \delta_*$. Thus, the date-zero social optimum can be decentralized with a tax on debt given by $\tau_0^b = (M-1)\delta_P$.*

Intuitively, banks' privately optimal debt-versus-equity decision trades off the higher cost of equity financing, $\Psi'(A_0)$, against the indirect cost of debt financing through the contraction in loanable funds, δ_P. Banks therefore adopt a balance sheet that uses more debt when the private value of loanable funds at date one is lower. The intuition is that banks recognize that more debt

[29] They receive utility $-B_0 + B_0 = 0$ from lending.

[30] Although Proposition 5 implements the social optimum with a tax on debt, it is immediate that the social optimum could also be implemented with an equity capital requirement $A_0/B_0 \leq \varphi^*$, where $\varphi^* = A_0^*/B_0^*$ and A_0^* is socially optimal equity, or with a leverage cap.

reduces their lending capacity at date one, and so they prefer to incur greater equity costs when lending capacity is valuable.

Proposition 5 also shows that the social optimum features more equity issuance and less debt issuance than the private optimum, that is $\tau_0^b > 0$, a standard result in the macroprudential policy literature. In our model, it derives from the planner's higher value of bank loanable funds at date zero, $M-1$. Under the special case of Assumption 1, we had $M = \left(1 - \frac{\phi p d_0 t}{(p\bar{g}+\bar{u})(1-\phi(1+t))}\right)^{-1}$. Thus the debt tax is larger when ϕ and d_0 are large, when t is large, and $\bar{u}$ and $\bar{g}$ are small. Interestingly, this implies that macroprudential regulation of *bank* liabilities is particularly valuable when the nonfinancial corporate sector is more heavily indebted, or when nonfinancial firms are less profitable in the future.

A further implication of Proposition 5 is that ex ante regulation is tied to the collateral externality of the interest rate and not directly to the liquidation fire sale. This is a result of Envelope Theorem: an increase in bank debt reduces date-one loanable funds but does not directly force the bank to increase liquidations, which are chosen optimally by the social planner ex post using insolvency intervention.[31] Notably, both macroprudential interventions and liquidation subsidies become more attractive as M increases. Intuitively, strong balance-sheet regulation is desirable when the social value of loanable funds M at date one is high, which is precisely when intervention promoting liquidation is attractive. Interestingly, this suggests that macroprudential interventions and liquidation subsidies can go hand in hand.

In principle, stronger balance-sheet regulation can provide a substitute for insolvency interventions: choosing even higher values of A_0 would serve to increase loanable funds and reduce δ_*. However in the lens of the model, this constitutes a third-best intervention. Insolvency interven-

[31] If the planner used macroprudential regulation but did not intervene in the ex post insolvency system, the planner would have to account for how macroprudential regulation changed the banks' privately optimal ex post insolvency decisions.

tions in our model provide the planner an additional tool for boosting loanable funds at date one, which is separate from the macroprudential tool. Moreover, our model highlights that there is a natural synergy between the two tools, both of which boost loanable funds.

5.2 Ex post fiscal interventions

Section 4 illustrates how interventions in the insolvency rule can be a desirable method of boosting loanable funds at the interim date (either by directly increasing loanable funds or by increasing the liquidation price). We now consider fiscal interventions ("bailouts") that directly boost loanable funds. This question is interesting in particular because, given there are multiple agents (banks, solvent firms, distressed firms), it is not a priori obvious to which agent a social planner would want to allocate bailout funds, how it might interact with insolvency interventions, or how the planner would weight the two externalities in choosing bailouts. We briefly characterize how bailouts and insolvency interventions go hand in hand.

We study the following exercise: suppose that the social planner was endowed with a marginal unit of bailout funds that it could allocate to any agent at date one, in conjunction with its insolvency intervention. As our focus is on where the benefit is highest, we abstract away from the costs of raising bailout funds. The following proposition characterizes the marginal social welfare impact of allocating this unit of funds to different agents. It would be straight-forward to extend analysis to study optimal bailout rules.

Proposition 6. *The marginal social welfare benefit of a bailout is:*

1. δ_* *when transferred to either banks or distressed firms*
2. $\left(1 - \frac{\phi t}{1-\phi}\right)\delta_*$ *when transferred to solvent firms.*

Proposition 6 reveals that bailing out banks (weakly) dominates bailing out firms, and that bailing out distressed firms is better than bailing out solvent ones. Intuitively, bailing out a solvent firm directly creates a loan. This reduces the need for firms to borrow from households, increasing household consumption at date one. Bailing out a bank creates this same benefit, since the bank extends a loan to solvent firms. However, the bank obtains an additional benefit: the bank receives an extra unit of collateral it can borrow against. It is thus better for the planner to supply the bank with funds to lend, rather than to directly lend to nonfinancials.

On the other hand, a bailout of a bank and a distressed firm are equivalent in welfare terms. To understand why, consider a distressed firm that is being reorganized (the intuition for a liquidation is similar). The bank has to cover the operating loss c for the reorganized firm, and the government bailout covers part of that operating loss. The bank then ties up less funds in the reorganization, and can use the saved funds for lending. Thus, bailing out a distressed firm is equivalent to bailing out a bank, conditional on an insolvency rule for that firm.

Proposition 6 reveals that the social value of a bailout is proportional to the social value of loanable funds, δ_*, and so increases in M. Interestingly, this suggests a synergy between bailouts and liquidation subsidies: both serve to increase loanable funds when loanable funds are especially valuable. One interesting manifestation of this idea is to use Pigouvian interventions that generate revenues for banks, and so simultaneously correct incentives and recapitalize banks. For example, a revenue-negative liquidation subsidy (i.e., that transfers resources to banks) achieves the dual benefit of encouraging banks to adopt the optimal insolvency rule (Proposition 3) and of providing bailouts to banks (Proposition 6). Given that liquidation subsidies are more attractive when bailouts are also attractive – that is, M is large – this suggests that liquidation subsidies are actually more efficient than unconditional bailouts. In a similar fashion, a planner that wished to promote reorganization could offer subsidized DIP loans to distressed firms that were reorganized,[32] achieving the dual benefit of a Pigouvian intervention in the insolvency decision along with an indirect bailout transfer to banks.

5.3 Bank writedown avoidance and zombie loans

The literature studying zombie loans notes that when a bank recognizes a nonperforming loan, it will likely have to write off existing capital, tightening minimum capital constraints (Caballero, Hoshi, and Kashyap, 2008). This creates an incentive for banks to offer credit to an insolvent firm, even if the firm's liquidation value exceeds its going-concern value, to avoid recognizing a loss.

In Appendix D, we extend the model to consider delayed loss recognition. Specifically, we assume that if a bank chooses continuation for a particular distressed firm, it can pledge the loan as collateral as if the firm were solvent. Formally, the collateral constraint in this extension is

$$B_1 \leq \phi\, Q_B \left[\underbrace{p\,(d_0 + \frac{D_1}{Q_S})}_{\text{Solvent Firm Loans}} + \underbrace{d_0(1-p)\int (1-\rho(v))f(v)dv}_{\text{Unrecognized Nonperforming Loans}} \right] \tag{22}$$

In this setting, we derive analogs of Propositions 1 and 2 (see Appendix D). All of the model forces in our baseline model play the same roles in this setting. In particular, banks consider both direct recovery and the value of loanable funds when choosing liquidations. The planner trades off the same externalities. However, this setting produces two noteworthy novel results. First, privately optimizing banks are more inclined to reorganize firms in this extension. In our baseline setting, the shadow value of funds produced by liquidations always pushes for more liquidations relative to a rule maximizing direct creditor recovery. In this setting, the incentive to avoid writedowns pushes in the other direction, and as a result banks can find it optimal to choose more continuations or more liquidations, relative to a rule maximizing direct creditor recovery.

[32] See Dahiya, John, Puri, and Ramırez (2003) for a description of DIP loans.

Second, comparing the planner solution to the private solution, we show that bank writedown avoidance amplifies the existing tradeoffs. Bank writedown avoidance increases the size of the collateral pool that gets revalued by discount rate changes, which in turn increases the social multiplier M and pushes for *more* liquidations. On the other hand, fire sales become more costly because each dollar lost through fire sales could have had an even higher social effective return. This pushes for *fewer* liquidations. Delayed loss recognition thus serves to amplify the existing trade-offs. Appendix D provides further details.

5.4 Heterogeneous banks

In Appendix C, we extend our model to consider the effect of heterogeneous bank creditors. In this extension, multiple creditors indexed by $b = 1, 2, ..., n_B$ differ in the extent to which their collateral constraints bind: the parameter ϕ_b varies across banks. We study the design of the socially optimal liquidation rule, as well as the socially optimal seniority structure among creditors.

We show that a social planner can improve welfare by strategically subordinating the claims of some banks based on their idiosyncratic collateral constraints. Specifically, the planner bifurcates banks into two groups. Banks that can easily collateralize loans to solvent firms (i.e., high ϕ_b) become "secured lenders:" they receive seniority in liquidations, allowing them to lend more to solvent firms. Banks that have a hard time collateralizing loans (i.e., low ϕ_b) become "distressed lenders:" they receive seniority in continuations and provide the necessary capital because their opportunity cost is lower. Interestingly, the social planner chooses a seniority structure with the potential for allocating all recovery to just two banks: the ones with the highest and lowest ϕ_b values in liquidations and continuations, respectively.[33]

[33] Although beyond our model, such a seniority structure could cause problems in practice such as a too-big-to-fail dilemma in future crises.

In this extension, the socially optimal liquidation rule now depends on the extent to which secured and distressed lenders are constrained. For expositional purposes, suppose there are two banks, $b = 1, 2$, with $\phi_1 < \phi_2$. We show that the socially optimal liquidation rule is

$$V_* = \delta_*^1 c + \delta_*^2 \gamma - \left(\delta_*^2 - 1 \right) \gamma \xi_\gamma \tag{23}$$

where δ_*^b is the social effective return the planner receives from giving a dollar to bank b at date one. We show that $\delta_*^2 > \delta_*^1$ because $\phi_2 > \phi_1$. When the planner chooses reorganization, it ties up resources c of bank one (distressed lender), who has a lower social return from date one funds. Conversely, if the planner chooses liquidation, it frees up resources for bank two (secured lender), who has a higher social return from date-one funds. Taken together, this suggests a greater tendency towards reorganization when seniority interventions allow the planner to allocate the proceeds/costs of liquidation/reorganization towards the bank with a comparative advantage in handling that bankruptcy outcome. Note, however, that these statements are all comparative statements on the planner's solution, and not the difference between the planner and private solutions.

While we do not explicitly model private equilibrium in this setting (as an initial financing decision is not specified under which to obtain it), heterogeneous collateral constraints have interesting implications for banks' private incentives. In particular, some banks might find it optimal to restructure a distressed firm while other banks would find it optimal to liquidate the same firm. This conflict arises because banks with tighter funding constraints place a higher premium on the immediate availability of liquidation proceeds. This suggests an interesting friction in the bankruptcy process even between two banks in the same creditor class, as the two banks have different relative values of reorganization and liquidation.

Finally, we can revisit the social benefit of a bailout to different banks. Parallel to Proposition

6, the marginal social welfare benefit of a bailout to bank b is exactly δ_*^b. When there are two banks as above with $\phi_1 < \phi_2$, then $\delta_*^2 > \delta_*^1$ and therefore the planner prefers bailouts of bank two on the margin. Interestingly, bank two was responsible for new lending while bank one was responsible for resolving distressed firms. Therefore, the planner prefers using bailouts to promote new lending rather than resolving distressed firms.

6 Interaction between the two externalities

As discussed above, Proposition 2 can be specialized to the case where only one externality is present. The model not only features meaningful interactions between the two externalities, but modeling both together also sheds new light on optimal ex ante and ex post interventions.

Proposition 2 reveals significant interactions between the two externalities in determining the optimal pattern of intervention. The fire-sale externality, all else equal, scales up in δ_*, which in turn rises in the collateral multiplier M. This means that the collateral externality exacerbates the cost of the fire sale: the social value of loanable funds increases, pushing the planner to be even more adverse to the fire-sale cost associated with liquidations. At the same time, larger fire sales also feed into the collateral externality: a larger fire sale reduces bank net worth (3), which contracts lending to solvent firms and forces households to redeploy resources from consumption to firms. This drives up the household discount rate and exacerbates the adverse collateral revaluation, incentivizing the planner to value loanable funds. The fact that the externalities not only coexist but also interact makes it a priori difficult to know which externality dominates from the case where each externality exists in isolation.

One conjecture would be that the attractiveness of liquidation taxes versus subsidies depends on the severity of the fire sale. Propositions 2-4 caveat this statement by giving a central role to the source of the fire sale. When the fire-sale price *elasticity* ξ_γ is high — additional liquidations have a larger effect on the fire-sale price — Proposition 4 indeed reveals that the planner is more likely to favor liquidation taxes to prevent larger deteriorations of liquidation values. On the other hand, when a fire sale is severe because assets are fundamentally hard to redeploy, that is $\overline{\gamma}$ is small, liquidation subsidies actually become more attractive. Put another way, the model highlights the importance of the *source* of the opportunity cost of reorganization. When recovery values are low, the primary opportunity cost of reorganizing a distressed firm is the operating cost c that needs to be covered, whereas the direct recovery $\overline{\gamma}$ and the further fire-sale impact become secondary considerations. This motivates liquidation subsidies. In contrast, when the opportunity cost of reorganization primarily comes from the value of redeploying assets — c is small relative to $\overline{\gamma}$ — liquidation subsidies become less attractive, because the funds recovered in a liquidation are also those being directly devalued by the liquidation.

Finally, modeling both externalities allows us to consider how optimal insolvency interventions affect the role of traditional instruments, such as macroprudential regulation and bailouts (see Section 5). In a standard model of fire-sale externalities, macroprudential regulation directly targets the fire sale.[34] Intuitively, forcing banks to reduce debt levels decreases their ex post liquidations, and which mitigates the fire sale. As Proposition 5 makes clear, in our model optimal insolvency interventions only directly target our collateral externality, and have no direct dependency on the fire-sale externality. The result is from Envelope Theorem: given optimal insolvency intervention, the social welfare consequences of a change in date-one bank debt on date-one bank liquidations are second order, whereas the effect on contraction in loanable funds is first order. Our framework therefore clarifies the roles for different instruments: insolvency interventions manage the trade-off between the fire sale and interest rate externalities, whereas macroprudential regulation

[34] See Lorenzoni (2008) and Dávila and Korinek (2018), among many others.

targets the excess social value of bank net worth, M, which operates through interest rate collateral revaluation. Since bailouts are another means of increasing bank funds at date one, a parallel argument reveals that bailouts also exclusively target interest rate collateral revaluation; the impact of bailouts on bank liquidations and fire sales is second order.

This also has important implications for how bailout funds should be distributed among heterogeneous banks within the banking system (see Section 5.4). In a standard model of fire sales, bailouts are best allocated to banks where the most asset liquidations can be prevented. In our model, we allow the planner not only to consider insolvency interventions, but also seniority interventions that allocate to some banks the role of reorganizing distressed firms and to some banks the role of liquidating distressed firms and undertaking new lending. As we show in Section 5.4, the marginal value of bailouts is actually higher to the banks responsible for liquidating and lending, that is bailouts are given to banks that are expected to liquidate bad assets. The intuition is again from Envelope Theorem: given optimal interventions, the benefit of bailouts in preventing liquidations is second order. Thus the role of bailouts is correcting the collateral externality, which is accomplished by new lending to solvent firms. Bailouts are therefore given to banks that are expected to redeploy resources, both from bailouts and asset liquidations, to new lending.

7 Characterizing historical crises and testable implications

7.1 Characterizing historical crises

In this section, we characterize and compare two crises: the COVID-19 public health emergency in the US and the nonperforming loan crisis in Japan. We use our model to provide normative recommendations on how each government should have responded to each crisis.

For the purpose of this exercise, we define the US COVID-19 crisis to cover the period March

2020 - May 2023, when the public health emergency in the US officially ended.[35] We define the Japan nonperforming loan crisis to cover the period 1990-2005, an approximate range consistent with Caballero, Hoshi, and Kashyap (2008).

The ideal approach would be to calibrate or structurally estimate our model to match empirical moments from each crisis. We could then directly compare the model-implied optimal interventions. However, such an exercise would rely on extreme assumptions. Specifically, our static stylized model is unlikely to realistically capture the dynamics of an entire economy. The model is designed to give clear intuition for socially optimal policy interventions. Adding features such as dynamic state variables would complicate the interpretation and intuition behind our results and expand the set of parameters, making it difficult to compellingly identify parameters.

Instead, our second-best approach uses aggregate statistics to loosely map our model into empirical predictions about optimal government responses. Our model delivers general comparative statics with respect to how the optimal liquidation tax or subsidy varies with changes in parameters. We can thus make relative statements about how optimal policy responses should vary across crises without exact parameter estimates.

Specifically, we consider four aggregate statistics in each crisis. For each statistic, we argue that a cross-crisis comparison suggests a certain parameter value would be higher in one crisis than another. Our model comparative statics then yield normative implications for how each government should have optimally responded to each crisis. Combining our model results with aggregate statistics in this manner, we find the following result: consistent with observed policies, an optimizing social planner would have subsidized liquidation in Japan and subsidized reorganization in the US. This model prediction follows from the fact that, relative to the Japan crisis, the US crisis featured: (i) lower corporate leverage; (ii) a smaller share of corporate borrowing coming from

[35] See https://www.cdc.gov/coronavirus/2019-ncov/your-health/end-of-phe.html.

banks, rather than bond markets, (iii) a higher rate of GDP growth, suggesting a more temporary shock, and (iv) higher realized profitability for corporations. We now provide details.

7.1.1 Leverage

In our model, the parameter d_0 determines the leverage of the nonfinancial corporate sector entering a crisis. As we explain in Section 4.1 and formalize in Proposition 4, an increase in d_0 makes it more likely that a social planner will optimally subsidize liquidations.

We now turn to data. We download aggregate statistics at the country-year level from the International Monetary Fund website.[36] This allows us to observe the ratio of total nonfinancial corporate debt to GDP. We average these country-year statistics across years for each crisis: 2020-2022, the most recent year in the data, for the US and 1990-2005 for Japan. Table 1 displays the result. On average during Japan's crisis, corporate debt was equal to 126.8% of GDP. On average during the US crisis, this ratio was 80.9%. This suggests that the parameter d_0 was higher for Japan's crisis than for the US crisis. By the intuition described in Section 4.1, our model thus predicts that liquidation subsidies were more likely to be optimal for Japan than for the US.

7.1.2 Profitability

Next, our model parameter c captures firm operating costs. When c is high, the ratio of firm profits $v-c$ to firm revenue v is low. Proposition 4 implies that liquidation subsidies are more desirable when c is high (i.e., when firm profitability is low).

To compare c across crises, we download firm-year data on large US and Japanese firms from Compustat North America and Compustat Global. We sum across firms and years to calculate aggregate EBIT and aggregate revenue during each crisis period. Table 1 shows that the ratio of

[36] See https://www.imf.org/external/datamapper/NFC_LS@GDD/CHN/USA/GBR/ESP/KOR/JPN/ITA/IRL/DEU/FRA/CAN.

aggregate EBIT to aggregate revenue was 5.5% in Japan's crisis compared to 14% in the US crisis. The higher profitability of firms during the US crisis suggests that c was higher in Japan. Our model then implies once again that liquidation subsidies were more likely to be optimal for Japan (see Section 4.1 for intuition).

7.1.3 Bank lending

Our model parameter t determines the comparative advantage of banks in corporate lending. Proposition 4 shows that a social planner optimally subsidizes liquidations when t is high.

We argue that t was higher in Japan than in the US using aggregate data on corporate borrowing. We obtain data at the country-year-quarter level from the Federal Reserve Bank of St. Louis (FRED).[37] We measure total corporate borrowing from banks and total corporate borrowing from bonds. In each country and each quarter, we calculate the ratio of (i) total corporate borrowing from banks to (ii) total corporate borrowing from banks or bonds. We average across year-quarters and report the result in Table 1. On average, banks accounted for 83.5% of corporate borrowing in Japan but only 38% of corporate borrowing in the US.[38] Through the lens of our model, this suggests that banks had a higher comparative advantage in lending in Japan, relative to the US, so t was higher in Japan's crisis. Again, by the intuition in Section 4.1, our model thus predicts that liquidation subsidies were more likely to be socially optimal in Japan's crisis.

[37] See `https://fred.stlouisfed.org/series/NCBDBIQ027S` and `https://fred.stlouisfed.org/series/BCNSDODNS` and `https://fred.stlouisfed.org/series/DSAMRIAONCJP` and `https://fred.stlouisfed.org/series/QJPPBMUSDA`. Note this data ends in April 2023.

[38] We emphasize that the denominator is total borrowing from banks and bonds. This leads both of these ratios to appear higher than similar statistics quoted elsewhere, which often use denominators that include other sources of financing. See, for example, `https://www.ft.com/content/1b2bfc57-ac1e-4d80-b2eb-a20a998082cb`.

7.1.4 Growth and temporary versus permanent shocks

In our model, the production function g_S determines the efficacy of investment by solvent firms. This drives the growth between date one and date two in our model. In Assumption 1, we parameterize the function g_S as $\bar{g}\log(\cdot)$ for a parameter $\bar{g} > 0$. A high value of $\bar{g}$ thus corresponds to a higher growth rate for the economy. For example, a high value of $\bar{g}$ could represent that the crisis is a temporary shock rather than a permanent one. We show in Proposition 4 that liquidation subsidies are more likely when $\bar{g}$ is low.

To compare $\bar{g}$ across crises, we download GDP at the country-year-quarter level from FRED for Japan and the US.[39] We construct quarterly GDP growth rates and average across all crisis quarters. Table 1 shows that the average quarterly GDP growth rate was 0.1% during Japan's crisis, compared to 1.8% during the US crisis. This evidence suggests the US COVID shock was more temporary than the shock that disrupted Japan's economy. In the context of our model, $\bar{g}$ was higher in the US. By the logic described in Section 4.1, this again implies that liquidation subsidies were more likely to be optimal for Japan.

7.2 Testable implications

Finally, we present two testable implications from our model that can be investigated in future empirical work. First, equation (4) implies that bank collateral constraints are more likely to bind when bank borrowing rates are high. Combining equations (8) and (9), our model likewise implies that bank collateral constraints are more likely to bind when corporate interest rates are high. To the extent that bank capital requirements correspond to the pledgeability microfounded collateral constraints in our model, this implies the testable implication that bank capital requirements are

[39] See https://fred.stlouisfed.org/series/JPNNGDP and https://fred.stlouisfed.org/series/JPNNGDP. This data ends in April 2023.

tighter when corporate interest rates are higher. Second, in our heterogeneous-banks extension, banks with higher ϕ_b have assets that enjoy lower hair cuts. Our model predicts these banks are more likely to liquidate a given asset in a crisis.

8 Conclusion

We study policies that mitigate crises by altering the process for resolving insolvent firms. In a general equilibrium environment, we show that crisis interventions can improve welfare relative to existing rules like the best-interest-of-the-creditors test (11 U.S.C. §1129(a)7), which prohibits a Chapter 11 reorganization whenever a liquidation would improve creditor recovery. However, our model reveals that liquidation-preventing policies are not always beneficial: a social planner can find it optimal to *encourage* liquidations. If crisis conditions constrain bank lending, such an intervention improves welfare by reallocating scarce capital to stronger firms. We show that various measures of corporate distress – low productivity, high operating loss, and high leverage – tend to promote liquidation subsidies. Surprisingly, an optimal policy response to extreme fire-sale externalities sometimes calls for even more liquidations, since banks harmed by these externalities must conserve capital for strong firms. Our results demonstrate that policymakers should jointly consider externalities in the banking and nonfinancial sectors when responding to crises.

For tractability and parsimony, our model omits many relevant features of crises. This allows us to cleanly characterize optimal interventions in corporate insolvency systems and provide clear intuition. However, our results are limited by these omissions. For example, our choice of a static framework precludes a rigorous analysis of temporary versus permanent shocks to an economy. Moreover, in a more general dynamic framework, the future liquidation values of firms could determine the value of bank assets, creating additional channels by which liquidations impact welfare. Likewise, our static framework makes it difficult to assess how policy interventions in one period can affect the fundamentals and solvency of firms in future periods. We leave it to future work to model how these important considerations impact optimal government interventions.

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Table 1: Stylized empirical facts about US and Japan crises

This table displays statistics about the US COVID health emergency (2020-2023) and Japan's nonperforming loan crisis (1990-2005). We obtain the ratio of total nonfinancial corporate debt to GDP from the IMF. We average these annual numbers across crisis years and present the averages in the first row. We obtain the total amount of bank lending to corporations and the total amount corporate bonds outstanding from FRED. We divide the total bank lending by the sum of total bank lending and bonds outstanding. The second row displays the average across crisis years. We obtain quarterly GDP from FRED and calculate quarterly growth rates. The third column displays the average across quarters in the crisis period. We obtain EBIT and Revenue for large US and Japanese firms from Compustat North America and Compustat Global. For each country, we sum EBIT and Revenue across all firms and crisis years and report the ratio in the fourth row.

Statistic	Japan (1990-2005)	US (2020-2023)
Corporate debt / GDP	126.8%	80.9%
Bank share of corporate lending	83.5%	38%
Quarterly GDP growth	.1%	1.8%
EBIT / Revenue	5.5%	14%

Internet Appendix

A Proofs

This appendix presents proofs for all of the results in the paper. To make our results more general, in all of our appendices, we assume that banks enter date one with deposits b_0 that they owe to households. This extra parameter enters the bank's budget constraint and does not appear anywhere else in the model. For simplicity, the main text focuses on the case $b_0 = 0$. We show that all of our results hold for arbitrary $b_0 \geq 0$.[40]

Also, for convenience, we use $F(v)$ in the appendices to refer to the CDF of v. Note that this CDF is exogenous.

[40] Following the date-zero model extension in Section 5.1, debt is owed to the date-zero generation of households. It is straight-forward to extend results if it is owed to date-one households (who also lent at date zero).

Table IA.A1: Parameter definitions

	Definition
	Exogenous parameters (lower case)
d_0	Value of long-term debt issued at date zero due at date two (date-one value = date-two value)
p	Probability of a firm being solvent
v_S	Solvent-firm long-term value from assets in place
$\overline{v}$, $\underline{v}$	Highest (lowest) value of a distressed firm
$g_S(\cdot)$	Production function for solvent firms
v	Heterogeneous date-two value of insolvent firm
$f(v)$	pdf of v
c	Cost of restructuring an insolvent firm
e	Household endowment parameter
$a(\cdot)$	Exogenous arbitrageur production function
$u(\cdot)$	Household utility from date-one consumption
t	Transaction cost from households lending to firms
ϕ	Pledgeability constraint parameter
$\hat{v}$	Lowest value such that a firm can be reorganized
b_0	(Appendix only) Date-one value of deposits banks owe households at date one
	Endogenous model objects (upper case)
Q_S	Loan price for solvent-firm borrowing
Q_B	Loan price for bank borrowing from households
γ	Liquidation price
I_S	Solvent-firm investment
B_1	Debt owed by banks to households
B_H	Debt supplied by households to banks
D_H	Debt owed by solvent firms to households
D_1	Debt owed by solvent firms to banks
$\rho(v)$	Probability of liquidating insolvent firm with viability v
V_*	Welfare-maximizing liquidation threshold
V_P	Private liquidation threshold
L	Quantity of liquidated assets demanded by arbitrageurs
	Lagrange multipliers and elasticities (Greek)
σ_H	Household absolute risk aversion
σ_S	Solvent-firm absolute risk aversion
δ_P	Private Langrange multiplier on budget constraint
δ_*	Welfare-maximizing Langrange multiplier on budget constraint
τ	Decentralizing liquidation tax ($\tau > 0$) or subsidy ($\tau < 0$)

A.1 Proof of Proposition 1

Banks choose (B_1, D_1, ρ) in order to maximize their final value,

$$\max_{B_1, D_1, \rho} \quad p\left(\frac{D_1}{Q_S} + d_0\right) + (1-p)\int (1-\rho(v))v dF(v) - \frac{B_1}{Q_B}, \tag{IA.A.1}$$

subject to

$$pD_1 + (1-p)\int (1-\rho(v))c dF(v) \leq B_1 - b_0 + (1-p)\int \rho(v)\gamma dF(v) \tag{IA.A.2}$$

and

$$B_1 \leq \phi \; Q_B \; p \; (d_0 + \frac{D_1}{Q_S}). \tag{IA.A.3}$$

Finally, recall the constraints $0 \leq \rho(v) \leq 1$.

We begin with a proof by contradiction that the optimum involves a threshold rule. Suppose that the optimum did not involve a threshold rule, so that there are two points $v_1 < v_2$ such that $\rho(v_1) < 1$ and $\rho(v_2) > 0$.[41] Consider a perturbation whereby the bank increases $\rho(v_1)$ by $\frac{\varepsilon}{f(v_1)}$ and decreases $\rho(v_2)$ by $\frac{\varepsilon}{f(v_2)}$. This perturbation increases the objective function since $v_2 > v_1$. This perturbation has no impact on the budget constraint, which depends on v only through $f(v)$, or the collateral constraint. Thus, this is a feasible perturbation that improves the objective and has no impact on constraints, a contradiction that the rule was optimal. We therefore have a threshold rule.

Given a threshold rule is optimal, we can redefine the bank's problem over the threshold V_P,

[41] To see that violation of this condition implies a threshold rule, suppose no two such points exist. If $\rho(v) \in \{0, 1\}$ for all v, a threshold rule follows. We need only to rule out that $0 < \rho(v) < 1$ over a positive measure set. Suppose there is a $\underline{v} < \hat{v} < \overline{v}$ such that $0 < \rho(\hat{v}) < 1$. Thus for any $v_1 < \hat{v}$, $\rho(v_1) = 1$, while for any $v_2 > \hat{v}$, $\rho(v_2) = 0$. Thus we have a threshold rule.

rather than over the entire rule ρ. The private bank Lagrangian (substituting in the threshold rule) is

$$\begin{aligned}\mathscr{L} = p\left(\frac{D_1}{Q_S}+d_0\right)+(1-p)\int_{v\geq V_P} v dF(v) - \frac{B_1}{Q_B} + \theta_P\left(\phi Q_B p(d_0+\frac{D_1}{Q_S}) - B_1\right) \\ + \delta_P\left(B_1 - b_0 + (1-p)\gamma F(V_P) - \left[pD_1 + (1-p)\left(1-F(V_P)\right)c\right]\right).\end{aligned}$$

Differentiating with respect to B_1,

$$0 = -Q_B^{-1} - \theta_P + \delta_P. \tag{IA.A.4}$$

Differentiating with respect to D_1,

$$0 = pQ_S^{-1} + \theta_P \phi Q_B p Q_S^{-1} - \delta_P p. \tag{IA.A.5}$$

Combining these equations,

$$\begin{aligned}\delta_P &= Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1} \\ \theta_P &= -Q_B^{-1} + Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1} \\ \theta_P &= \frac{1}{1-\phi\frac{Q_B}{Q_S}}\left(\frac{1}{Q_S} - \frac{1}{Q_B}\right).\end{aligned}$$

Finally, differentiating with respect to V_P, setting the derivative equal to zero, dividing by

$(1-p)f(V_P)$, and adding V_P to both sides,

$$V_P = \delta_P(\gamma + c) = (\frac{1}{Q_B} + \theta_P)(\gamma + c). \tag{IA.A.6}$$

We conclude by rearranging to the above equations to see that

$$\begin{aligned}
\delta_P &= Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1} \\
&= Q_S^{-1} + \Big(\frac{1}{1-\phi \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right) \Big) \phi Q_B Q_S^{-1} \\
&= Q_S^{-1} + \frac{\phi Q_B Q_S^{-1}}{1-\phi \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right)
\end{aligned}$$

This completes the proof.

A.2 Deriving the planner's objective and Lagrangian

The planner has a utilitarian objective. It optimizes the sum of the objective functions of the firms, banks, households, and arbitrageurs. By definition, insolvent firms receive no value. The planner's objective is thus

$$\begin{aligned}
&\underbrace{p\left(g_S(I_S) - \frac{I_S}{Q_S} + v_s - d_0 \right)}_{\text{Solvent firms}} + \underbrace{p\left(\frac{D_1}{Q_S} + d_0\right) + (1-p)\int (1-\rho(v))v dF(v) - \frac{B_1}{Q_B}}_{\text{Banks}} \\
&+ \underbrace{a(L) - \gamma L}_{\text{Arbitrageurs}} + \underbrace{u\left(e - B_H - p(1+t)D_H \right) + \frac{B_H}{Q_B} + p\frac{D_H}{Q_S}}_{\text{Households}}.
\end{aligned}$$

Imposing the market clearing conditions ($B_H = B_1$, $I_S = D_H + D_1$), several terms cancel. This simplifies to

$$p\left(g_S(I_S)+v_S\right)+(1-p)\int(1-\rho(v))vdF(v)+a(L)-\gamma L+u\left(e-B_1-p(1+t)D_H\right).$$

where we have used the market clearing conditions. Applying the market clearing conditions and ignoring v_S, the social Lagrangian is

$$\begin{aligned}
\mathscr{L} = {} & pg_S(I_S)+(1-p)\int(1-\rho(v))vdF(v)+u\left(e-B_1-p(1+t)D_H\right) \\
& +a((1-p)\int\rho(v)dF(v))-\gamma(1-p)\int\rho(v)dF(v) \\
& +\theta_*\left(\phi pQ_B\left[d_0+\frac{I_S-D_H}{Q_S}\right]-B_1\right) \\
& +\delta_*\left(B_1-b_0+(1-p)\gamma\int\rho(v)dF(v)-(1-p)c\int(1-\rho(v))dF(v)-p\left(I_S-D_H\right)\right) \\
& +\kappa(1-Q_S g_S'(I_S))+\eta_1\left(u'\left(e-B_1-p(1+t)D_H\right)-\frac{1}{Q_B}\right) \\
& +\eta_2\left(p(1+t)u'\left(e-B_1-p(1+t)D_H\right)-\frac{p}{Q_S}\right)+\xi\left(a'((1-p)\int\rho(v)dF(v))-\gamma\right).
\end{aligned}$$

Combining the household first-order conditions with respect to B_H and D_H, it must be that:

$$\frac{Q_B}{1+t}=Q_S. \tag{IA.A.7}$$

Substituting this in, we automatically satisfy one household first-order condition and get

$$
\begin{aligned}
\mathscr{L} = & \ p g_S(I_S) + (1-p)\int (1-\rho(v)) v dF(v) + u\left(e - B_1 - p(1+t)D_H\right) \\
& + a((1-p)\int \rho(v) dF(v)) - \gamma(1-p)\int \rho(v) dF(v) \\
& + \theta_* \left(\phi p Q_B \left[d_0 + \frac{(1+t)(I_S - D_H)}{Q_B}\right] - B_1\right) \\
& + \delta_* \left(B_1 - b_0 + (1-p)\gamma \int \rho(v) dF(v) - (1-p)c\int (1-\rho(v)) dF(v) - p\left(I_S - D_H\right)\right) \\
& + \kappa(1 - \frac{Q_B}{1+t} g_S'(I_S)) + \eta\left(u'\left(e - B_1 - p(1+t)D_H\right) - \frac{1}{Q_B}\right) \\
& + \xi\left(a'((1-p)\int \rho(v) dF(v)) - \gamma\right).
\end{aligned}
$$

Finally, note that (IA.A.7) also implies that

$$
\begin{aligned}
\theta_P &= \frac{1}{1 - \phi \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B}\right) \\
&= \frac{1}{1 - \phi(1+t)} \frac{t}{Q_B},
\end{aligned}
$$

and

$$
\begin{aligned}
\delta_P &= Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1} \\
&= \frac{1+t}{Q_B} + \phi(1+t)\frac{1}{1-\phi(1+t)}\frac{t}{Q_B} \\
&= \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}.
\end{aligned}
$$

A.3 Proof of Proposition 2

As we just derived, the planner's Lagrangian is:

$$
\begin{aligned}
\mathscr{L} = & \, pg_S(I_S) + (1-p)\int(1-\rho(v))v dF(v) + u\left(e - B_1 - p(1+t)D_H\right) \\
& + a((1-p)\int\rho(v)dF(v)) - \gamma(1-p)\int\rho(v)dF(v) \\
& + \theta_*\left(\phi p Q_B\left[d_0 + \frac{(1+t)(I_S - D_H)}{Q_B}\right] - B_1\right) \\
& + \delta_*\left(B_1 - b_0 + (1-p)\gamma\int\rho(v)dF(v) - (1-p)c\int(1-\rho(v))dF(v) - p\left(I_S - D_H\right)\right) \\
& + \kappa(1 - \frac{Q_B}{1+t}g'_S(I_S)) + \eta\left(u'\left(e - B_1 - p(1+t)D_H\right) - \frac{1}{Q_B}\right) \\
& + \xi\left(a'((1-p)\int\rho(v)dF(v)) - \gamma\right).
\end{aligned}
$$

As in the proof of Proposition 1, we begin with a proof by contradiction that the social optimum involves a threshold rule for ρ. As in that proof, if the social optimum did not involve a threshold rule, there are points $v_1 < v_2$ such that $\rho(v_1) < 1$ and $\rho(v_2) > 0$. Consider the same perturbation of increasing $\rho(v_1)$ by $\frac{\varepsilon}{f(v_1)}$ and decreasing $\rho(v_2)$ by $\frac{\varepsilon}{f(v_2)}$. Note that this perturbation has no impact on total liquidations $L \equiv (1-p)\int\rho(v)dF$ and hence no impact on equilibrium prices. Therefore as in the private optimum, this perturbation increases the objective since $v_2 > v_1$, and it has no impact on the constraints. Thus, this is a feasible perturbation that improves welfare, a contradiction that the rule was optimal. We therefore have a threshold rule with threshold V_*. Substituting this in:

$$
\begin{aligned}
\mathscr{L} = & \, pg_S(I_S) + (1-p)\int_{v \geq V_*} v dF(v) + u\left(e - B_1 - p(1+t)D_H \right) \\
& + a((1-p)F(V_*)) - \gamma(1-p)F(V_*) \\
& + \theta_* \left(\phi p Q_B \big[d_0 + \frac{(1+t)(I_S - D_H)}{Q_B} \big] - B_1 \right) \\
& + \delta_* \left(B_1 - b_0 + (1-p)\gamma F(V_*) - (1-p)c(1-F(V_*)) - p\left(I_S - D_H \right) \right) \\
& + \kappa(1 - \frac{Q_B}{1+t} g_S'(I_S)) + \eta \left(u'\left(e - B_1 - p(1+t)D_H \right) - \frac{1}{Q_B} \right) \\
& + \xi \left(a'((1-p)F(V_*)) - \gamma \right).
\end{aligned}
$$

We now take derivatives with respect to the planner's control variables:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial I_S} &= pg_S'(I_S) + \theta_* \phi p(1+t) - \delta_* p - \kappa \frac{Q_B}{1+t} g_S''(I_S) \\
\frac{\partial \mathscr{L}}{\partial V_*} &= (1-p)f(V_*)\Big[-V_* + \mathbf{0} + \delta_* \left(\gamma + c \right) + \xi a''((1-p)F(V_*)) \Big] \\
\frac{\partial \mathscr{L}}{\partial B_1} &= -u'\left(e - B_1 - p(1+t)D_H \right) - \theta_* + \delta_* - \eta u''\left(e - B_1 - p(1+t)D_H \right) \\
\frac{\partial \mathscr{L}}{\partial D_H} &= -p(1+t)u'\left(e - B_1 - p(1+t)D_H \right) - \theta_* \phi p(1+t) + \delta_* p - \eta p(1+t)u''\left(e - B_1 - p(1+t)D_H \right) \\
\frac{\partial \mathscr{L}}{\partial Q_B} &= \theta_* \phi p d_0 - \kappa \frac{1}{1+t} g_S'(I_S) + \eta \frac{1}{Q_B^2} \\
\frac{\partial \mathscr{L}}{\partial \gamma} &= (1-p)F(V_*)\big[-1 + \delta_* \big] - \xi
\end{aligned}
$$

Now, we introduce shorthand:

$$C^U \equiv e - B_1 - p(1+t)D_H$$

$$\sigma_H \equiv \frac{-u''(C^U)}{u'(C^U)} > 0$$

$$\sigma_S \equiv \frac{-g_S''(I_S)}{g_S'(I_S)}.$$

Gathering terms, simplifying, and applying $g_S' = (1+t)/Q_B$, $u'(C^U) = Q_B^{-1}$,

$$0 = \frac{1+t}{Q_B} - \delta_* + \theta_* \phi (1+t) + \frac{\kappa \sigma_S}{p}$$

$$0 = -V_* + \delta_* \left(\gamma + c \right) + \xi a''(L)$$

$$0 = -Q_B^{-1} - \theta_* + \delta_* + \eta \frac{\sigma_H}{Q_B}$$

$$0 = -Q_B^{-1} - \theta_* \phi + \frac{\delta_*}{1+t} + \eta \frac{\sigma_H}{Q_B}$$

$$0 = \theta_* \phi p d_0 Q_B - \kappa + \eta \frac{1}{Q_B}$$

Multiply the fourth equation by -1 and add it to the third:

$$0 = -\theta_*(1-\phi) + \frac{t}{1+t}\delta_* \Rightarrow \theta_* = \frac{t}{(1+t)(1-\phi)}\delta_*. \quad \text{(IA.A.8)}$$

Plugging into the first equation and rearranging,

$$\kappa = \frac{p}{\sigma_S}\Big[-\frac{1+t}{Q_B} + \delta_* \left(1 - \frac{\phi t}{1-\phi} \right) \Big] \quad \text{(IA.A.9)}$$

Plugging into the third equation and rearranging,

$$\eta = \frac{Q_B}{\sigma_H}\big[Q_B^{-1} + \delta_* \left(-1 + \frac{t}{(1+t)(1-\phi)} \right)\big]. \tag{IA.A.10}$$

Plugging into the final equation,

$$0 = \delta_* \frac{\phi p d_0 Q_B t}{(1+t)(1-\phi)} - \frac{p}{\sigma_S}\big[-\frac{1+t}{Q_B} + \delta_* \left(1 - \frac{\phi t}{1-\phi} \right)\big] + \frac{1}{\sigma_H}\big[Q_B^{-1} + \delta_* \left(-1 + \frac{t}{(1+t)(1-\phi)} \right)\big] \tag{IA.A.11}$$

$$0 = \delta_* \frac{\phi p d_0 Q_B t}{(1+t)(1-\phi)} - \frac{p}{\sigma_S}\big[-\frac{1+t}{Q_B} + \delta_* \left(\frac{1-\phi(1+t)}{1-\phi} \right)\big] + \frac{1}{\sigma_H}\big[Q_B^{-1} + \delta_* \left(\frac{-(1-\phi(1+t))}{(1+t)(1-\phi)} \right)\big]. \tag{IA.A.12}$$

At this point, recall that

$$\delta_P = \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}.$$

Define $M \equiv \delta_*/\delta_P$. Then

$$0 = M \frac{\phi p d_0 Q_B t}{Q_B(1-\phi(1+t))} - \frac{p}{\sigma_S}\big[-\frac{1+t}{Q_B} + M \left(\frac{1+t}{Q_B} \right)\big] + \frac{1}{\sigma_H}\big[Q_B^{-1} + M \left(\frac{-1}{Q_B} \right)\big]. \tag{IA.A.13}$$

Cleaning up and rearranging to a common denominator,

$$0 = M\frac{\sigma_S\sigma_H\phi p d_0 Q_B t - \left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}{\sigma_S\sigma_H Q_B(1-\phi(1+t))} \tag{IA.A.14}$$

$$+\frac{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}{\sigma_S\sigma_H Q_B(1-\phi(1+t))}. \tag{IA.A.15}$$

Moving M to the left and dividing by the fraction multiplying M,

$$M = \frac{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))-\sigma_S\sigma_H\phi p d_0 Q_B t} \geq 1. \tag{IA.A.16}$$

Alternatively,

$$M = \left(1-\frac{\sigma_S\sigma_H\phi p d_0 Q_B t}{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}\right)^{-1} \geq 1. \tag{IA.A.17}$$

Finally, we solve for the liquidation threshold. From $\partial\mathscr{L}/\partial\gamma$,

$$\xi = (1-p)F(V_*)\left[\delta_*-1\right] = \left[\delta_*-1\right]L. \tag{IA.A.18}$$

Plugging this into $\partial\mathscr{L}/\partial V_*$,

$$V_* = \delta_*\left(\gamma+c\right)+\left[\delta_*-1\right]La''(L). \tag{IA.A.19}$$

Recall from the arbitrageur first-order condition that

$$a'(L)-\gamma = 0 \Rightarrow \frac{\partial\gamma}{\partial L} = a''(L), \tag{IA.A.20}$$

so by definition,

$$\xi_\gamma = \frac{-L}{\gamma} a''(L), \tag{IA.A.21}$$

and thus

$$V_* = \delta_* \left(\gamma + c \right) - \left[\delta_* - 1 \right] \gamma \xi_\gamma. \tag{IA.A.22}$$

Note that

$$\delta_P = \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))} > 1$$

and $\delta_* = M\delta_P > \delta_P$, so $\delta_* \left(\gamma + c \right) > \delta_P(\gamma + c)$ and $\left[\delta_* - 1 \right] \gamma \xi_\gamma > 0$. This completes the proof.

A.4 Proof of Proposition 3

Given a tax τ, the private bank problem is:

$$\max_{B_1, D_1, \rho} p \left(\frac{D_1}{Q_S} + d_0 \right) + (1-p) \int (1-\rho(v)) v dF(v) - \frac{B_1}{Q_B} - \tau (1-p) \int \rho(v) dF(v), \tag{IA.A.23}$$

subject to the same conditions as before. Following the same argument as the proof of Proposition 1 (and noting that the perturbation has no impact on the tax burden), a threshold rule is privately optimal. Differentiating with respect to V_P, dividing by $(1-p)f(V_P)$ and rearranging,

$$V_P = -\tau + \delta_P(\gamma + c). \tag{IA.A.24}$$

Following the same steps as in the proof of Proposition 1, we see δ_P is the same as before, unaffected by the introduction of τ. We thus need to find τ such that $V_P = V_*$, or

$$-\tau + \delta_P(\gamma + c) = \delta_*(\gamma + c) - (\delta_* - 1)\xi_\gamma \gamma. \tag{IA.A.25}$$

Rearranging,

$$\tau = (\gamma + c)\delta_P(1 - M) + (M\delta_P - 1)\xi_\gamma \gamma. \tag{IA.A.26}$$

A.5 Proof of Proposition 4

Under Assumption 1,

$$a(L) = \frac{1}{1 - \bar{\xi}_\gamma} L^{1-\bar{\xi}_\gamma} \tag{IA.A.27}$$

$$a'(L) = L^{-\bar{\xi}_\gamma} = \gamma \tag{IA.A.28}$$

$$a''(L) = -\bar{\xi}_\gamma L^{-1-\bar{\xi}_\gamma} = -\bar{\xi}_\gamma \frac{\gamma}{L} \tag{IA.A.29}$$

$$\xi_\gamma = -\frac{L}{\gamma}\frac{\partial \gamma}{\partial L} = \bar{\xi}_\gamma. \tag{IA.A.30}$$

We also have

$$u(C^U) = \bar{u}\log(C^U) \tag{IA.A.31}$$

$$u'(C^U) = \bar{u}(C^U)^{-1} \tag{IA.A.32}$$

$$u''(C^U) = -\bar{u}(C^U)^{-2} \tag{IA.A.33}$$

$$\sigma_H = (C^U)^{-1} = u'(C^U)\bar{u}^{-1}. \tag{IA.A.34}$$

Assuming $g_S(I_S) = \bar{g}\log(I_S)$, identical steps show $\sigma_S = \bar{g}^{-1}g_S'(I_S)$. Recall from the first-order conditions that $g_S'(I_S) = Q_S^{-1} = (1+t)Q_B^{-1}$ and $u'(C^U) = Q_B^{-1}$. We thus have

$$\sigma_H = \bar{u}^{-1}Q_B^{-1} \tag{IA.A.35}$$

$$\sigma_S = \bar{g}^{-1}(1+t)Q_B^{-1}. \tag{IA.A.36}$$

Plugging these into the definition of M,

$$M = \left(1 - \frac{\sigma_S\sigma_H\phi p d_0 Q_B t}{\left[p(1+t)\sigma_H + \sigma_S\right](1-\phi(1+t))}\right)^{-1} \geq 1. \tag{IA.A.37}$$

$$M = \left(1 - \frac{\bar{g}^{-1}\bar{u}^{-1}\phi p d_0 t}{\left[p\bar{u}^{-1} + \bar{g}^{-1}\right](1-\phi(1+t))}\right)^{-1} \geq 1. \tag{IA.A.38}$$

$$M = \left(1 - \frac{\phi p d_0 t}{\left[p\bar{g} + \bar{u}\right](1-\phi(1+t))} \right)^{-1} \geq 1. \tag{IA.A.39}$$

Recall that

$$\tau = (\gamma + c)\delta_P(1-M) + (M\delta_P - 1)\xi_\gamma \gamma. \tag{IA.A.40}$$

We see $\tau < 0$ if and only if

$$(1 + c\gamma^{-1})\delta_P(1-M) + (M\delta_P - 1)\bar{\xi}_\gamma < 0 \tag{IA.A.41}$$

if and only if

$$\bar{\xi}_\gamma < (1 + c\gamma^{-1})\frac{\delta_P(M-1)}{M\delta_P - 1}. \tag{IA.A.42}$$

Recall there exists a highest feasible value $\bar{\gamma}$ of γ. We see that if

$$\bar{\xi}_\gamma < (1 + c\bar{\gamma}^{-1})\frac{M-1}{M}, \tag{IA.A.43}$$

then the above holds, because

$$(1 + c\bar{\gamma}^{-1})\frac{M-1}{M} < (1 + c\gamma^{-1})\frac{M-1}{M} \tag{IA.A.44}$$

$$= (1 + c\gamma^{-1})\frac{\delta_P(M-1)}{M\delta_P} < (1 + c\gamma^{-1})\frac{\delta_P(M-1)}{M\delta_P - 1} \tag{IA.A.45}$$

We have thus shown that a sufficient condition for $\tau < 0$ is

$$\bar{\xi}_\gamma < (1+c\bar{\gamma}^{-1})\frac{M-1}{M}. \tag{IA.A.46}$$

Finally, we have

$$\frac{M-1}{M} = \frac{1-\left(1-\frac{\phi p d_0 t}{\left[p\bar{g}+\bar{u}\right](1-\phi(1+t))}\right)}{1} \tag{IA.A.47}$$

$$= \frac{\phi p d_0 t}{\left[p\bar{g}+\bar{u}\right](1-\phi(1+t))} \tag{IA.A.48}$$

The condition is thus

$$\bar{\xi}_\gamma < (1+c\bar{\gamma}^{-1})\frac{\phi p d_0 t}{\left[p\bar{g}+\bar{u}\right](1-\phi(1+t))}. \tag{IA.A.49}$$

A.6 Proof of Proposition 5

The bank's first order condition for optimality is

$$\frac{\partial \mathscr{V}^P}{\partial B_0} = -\Psi'(A_0). \tag{IA.A.50}$$

At date one, B_0 appears negatively in the bank's budget constraint, so by Envelope Theorem $\frac{\partial \mathscr{V}^P}{\partial B_0} = -\delta_P$. Therefore, we have

$$\Psi'(A_0) = \delta_P.$$

Next, consider the social planner. The social planner's first order condition, by parallel argument, is

$$-\Psi'(A_0) = \frac{\partial \mathscr{V}^*}{\partial B_0}.$$

Again at date one, B_0 appears negatively in the bank's budget constraint (recall it is a wash for date-zero households), so again by Envelope Theorem $\frac{\partial \mathscr{V}^*}{\partial B_0} = -\delta_*$. Therefore

$$\Psi'(A_0) = \delta_*$$

which completes the proof. The tax rate definition follows immediately. To decentralize this outcome, the planner places a tax τ_0^B on bank date-zero debt, so that its objective function is

$$-\Psi(1-B_0) - \tau_0^B B_0 + \mathscr{V}^P(B_0).$$

Differentiating with respect to B_0, the bank chooses B_0^* such that

$$\Psi'(1-B_0^*) - \tau_0^B + (\mathscr{V}^P)'(B_0^*) = \Psi'(1-B_0^*) - \tau_0^B - \delta_P = 0.$$

To achieve efficiency, we need the bank to set

$$\Psi'(1-B_0^*) = \delta_*,$$

implying that $\tau_0^B = \delta_* - \delta_P$, giving the result.

A.7 Proof of Proposition 6

Consider first a bank bailout, which appears as a positive wedge in the bank's budget constraint. Therefore, it is equivalent to a reduction in debt, and following the proof of Proposition 5 the marginal value of a bank bailout is $-\frac{\partial \mathscr{V}^*}{\partial B_0} = \delta_*$.

Next, consider a bailout of an insolvent firm that does not depend on resolution choice. This also appears solely in the bank's budget constraint, hence its marginal value is the same as for a bank bailout.

Finally, consider a bailout Δ of a solvent firm. This appears in the production function, and can be represented in the Lagrangian by

$$
\begin{aligned}
\mathscr{L} &= pg_S(I_S+\Delta)+(1-p)\int_{v\geq V_*} vdF(v)+u\left(\, e-B_1-p(1+t)D_H \,\right) \\
&+a((1-p)F(V_*))-\gamma(1-p)F(V_*) \\
&+\theta_*\left(\, \phi pQ_B\left[d_0+\frac{(1+t)(I_S-D_H)}{Q_B}\right]-B_1 \,\right) \\
&+\delta_*\left(\, B_1-b_0+(1-p)\gamma F(V_*)-(1-p)c(1-F(V_*))-p\left(\, I_S-D_H \,\right)\right) \\
&+\kappa(1-\frac{Q_B}{1+t}g_S'(I_S+\Delta))+\eta\left(\, u'\left(\, e-B_1-p(1+t)D_H \,\right)-\frac{1}{Q_B} \,\right) \\
&+\xi\left(\, a'((1-p)F(V_*))-\gamma \,\right).
\end{aligned}
$$

Therefore by Envelope Theorem,

$$\frac{\partial \mathscr{L}}{\partial \Delta} = pg_S' - \kappa\frac{Q_B}{1+t}g_S''.$$

From the proof of Proposition 2,

$$p\delta_* - \theta_*\phi p(1+t) = pg_S' - \kappa\frac{Q_B}{1+t}g_S''$$

and therefore we have

$$\frac{\partial \mathscr{L}}{\partial \Delta} = p\left(1 - \frac{\phi t}{1-\phi}\right)\delta_*$$

We then obtain the result by renormalizing by $\frac{1}{p}$ (targeting an individual firm).

B Endogenizing the fraction of solvent firms

In this appendix, we endogenize the fraction p of firms that are solvent. Specifically, we assume that a firm is solvent if and only if its date-one present value exceeds the date-one value of its debt. In this extension, any forces that change discount rates also change the fraction of solvent firms. In this sense, policy changes aimed at subsidizing or taxing liquidations endogenously shift the pool of distressed firms that could potentially be liquidated.

We show that our main results still hold. Privately and socially optimal liquidation rules still take the same basic forms as in the baseline model:

$$V_P = \delta_P(\gamma + c) \tag{IA.B.1}$$

$$V_* = \delta_*(\gamma + c) - (\delta_* - 1)\xi_\gamma \gamma, \tag{IA.B.2}$$

where $\delta_* = M\delta_P$ for a multiplier M. We show that δ_P is the same as in our baseline model. The definition of M changes, as it also includes externalities from the fact that increases in the interest rate push more firms into insolvency. We show that $M \geq 1$, meaning that the collateral externality still pushes for more liquidations.

B.1 Firms

Firms have long-run value $v \in [\underline{v}, \overline{v}]$ and an operating cost c. Firms can only invest in a new project if they are solvent. Thus, a solvent firm solves

$$\max_{I_S} \; g_S(I_S) - \frac{I_S}{Q_S} + v - \frac{c}{Q_S} - d_0. \tag{IA.B.3}$$

Let $G^*(Q_S) = \max_{I_S} g_S(I_S) - \frac{I_S}{Q_S}$. There is then a solvency threshold

$$V_S = \frac{1}{Q_S}c + d_0 - G^*(Q_S) \tag{IA.B.4}$$

such that a firm is solvent if and only if $v \geq V_S$. The equity value of solvent firms is then

$$\int_{V_S}^{\overline{v}} \left[g_S(I_S) - \frac{I_S}{Q_S} + v - \frac{1}{Q_S}c - d_0 \right] dF(v) \tag{IA.B.5}$$

and the first-order condition is the same as before, $g'_S(I_S) = Q_S^{-1}$. Total demand for new loans by solvent firms to cover investment is $\Big(1 - F(V_S)\Big) I_S$. Solvent firms also demand $(1 - F(V_S))c$ to cover operating losses, which we track separately.

B.2 Banks

Assume $V_P \leq V_S$. Then the bank budget constraint is

$$D_1 + c\Big(1 - F(V_P)\Big) \leq B_1 - b_0 + \gamma F(V_P). \tag{IA.B.6}$$

The collateral constraint is

$$\frac{1}{Q_B} B_1 \leq \phi \left((1 - F(V_S)) d_0 + \frac{D_1}{Q_S} \right) \tag{IA.B.7}$$

The bank objective function is

$$\frac{D_1}{Q_S} + d_0(1 - F(V_S)) + \int_{V_P}^{V_S} v dF(v) - \frac{B_1}{Q_B} + \frac{c}{Q_S}(1 - F(V_S)) \tag{IA.B.8}$$

where the last term is bank profit from loans to cover operating losses. This separation of D_1

and $c(1-F(V_S))$ is artificial, but is legitimate provided that $D_1 > 0$ in equilibrium, which it will be under an Inada condition.

B.3 Arbitrageurs

Arbitrageurs are unchanged.

B.4 Households

Households are almost unchanged: they solve

$$\max_{B_H, D_H} u\left(e - B_H - (1+t)D_H\right) + \frac{B_H}{Q_B} + \frac{D_H}{Q_S}, \tag{IA.B.9}$$

so we have

$$u'\left(e - B_H - (1+t)D_H\right) = \frac{1}{Q_B} \tag{IA.B.10}$$

$$u'\left(e - B_H - (1+t)D_H\right) = \frac{1}{(1+t)Q_S}. \tag{IA.B.11}$$

B.5 Market clearing and competitive equilibrium

The new market clearing conditions in this extension are:

$$D_1 + D_H = (1 - F(V_S))I_S \tag{IA.B.12}$$

$$F(V_P) = L \tag{IA.B.13}$$

$$B_H = B_1. \tag{IA.B.14}$$

B.6 Privately optimal liquidation rules

The Lagrangian for the bank problem is

$$\mathscr{L}^p = \frac{D_1}{Q_S} + d_0(1 - F(V_S)) + \int_{V_P}^{V_S} v dF(v) - \frac{B_1}{Q_B} + \frac{c}{Q_S}(1 - F(V_S))$$
$$+ \theta_P \left[\phi Q_B \left((1 - F(V_S))d_0 + \frac{D_1}{Q_S} \right) - B_1 \right] + \delta_P \left[B_1 - b_0 + \gamma F(V_P) - D_1 - c \left(1 - F(V_P) \right) \right]$$

The first-order conditions for V_P, D_1, and B_1 are

$$V_P = \delta_P(\gamma + c)$$

$$0 = \frac{1}{Q_S} + \theta_P \phi \frac{Q_B}{Q_S} - \delta_P$$
$$0 = -\frac{1}{Q_B} - \theta_P + \delta_P.$$

Therefore, we obtain

$$\theta_P = \frac{1}{1 - \phi \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right),$$

and $\delta_P = \frac{1}{Q_B} + \theta_P$. So, the privately optimal liquidation rule is the same as in our baseline model. Likewise, δ_P and θ_P are the same as in our baseline model.

B.7 Socially optimal liquidation rules

Social welfare in this model is

$$\underbrace{\int_{V_S}^{\overline{v}} \left(g_S(I_S) + v \right) dF + \int_{V_*}^{V_S} v dF}_{\text{Surplus from firms}} + \underbrace{a(L) - \gamma L}_{\text{Surplus from arbitrageurs}} + \underbrace{u \left(e - B_1 - (1+t) D_H \right)}_{\text{Surplus from households}}.$$

where we note that operating losses are folded into resource constraints.

Employing the short-hand $p^S = 1 - F(V_S)$, and using $Q_B = (1+t)Q_S$, $D_1 = p^S I_S - D_H$, and $B_H = B_1$, the social planner's Lagrangian is

$$\begin{aligned}
\mathscr{L} = & \int_{V_S}^{\overline{v}} \left(g_S(I_S) + v \right) dF + \int_{V_*}^{V_S} v dF + a(F(V_*)) - \gamma F(V_*) + u \left(e - B_1 - (1+t) D_H \right). \\
& + \theta \left(\phi Q_B p^S d_0 + \phi p^S (1+t) I_S - \phi D_H (1+t) - B_1 \right) \\
& + \delta \left(B_1 - b_0 + \gamma F(V_*) - p^S I_S + D_H - c \left(1 - F(V_*) \right) \right) \\
& + \kappa \left(1 - \frac{Q_B}{1+t} g_S'(I_S) \right) \\
& + \eta \left(u' \left(e - B_1 - (1+t) D_H \right) - \frac{1}{Q_B} \right) \\
& + \xi \left(a'(F(V_*)) - \gamma \right) \\
& + \zeta \left(V_S - \frac{1+t}{Q_B} c - d_0 + G^*\left(\frac{Q_B}{1+t}\right) \right).
\end{aligned}$$

The first-order conditions are:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial I_S} &= p^S g_S'(I_S) + \theta\phi p^S(1+t) - \delta p^S - \kappa \frac{Q_B}{1+t} g_S''(I_S) \\
\frac{\partial \mathscr{L}}{\partial V_*} &= -V_* f(V_*) + 0 + \delta(\gamma + c) f(V_*) + \xi a''(F(V_*)) f(V_*) \\
\frac{\partial \mathscr{L}}{\partial B_1} &= -u' - \theta + \delta - \eta u'' \\
\frac{\partial \mathscr{L}}{\partial D_H} &= -(1+t)u' - \theta\phi(1+t) + \delta - \eta(1+t)u'' \\
\frac{\partial \mathscr{L}}{\partial Q_B} &= \theta\phi p^S d_0 - \kappa \frac{1}{1+t} g_S'(I_S) + \eta \frac{1}{Q_B^2} + \zeta\left(\frac{1+t}{Q_B^2} c + G^{*\prime}\left(\frac{Q_B}{1+t}\right)\frac{1}{1+t}\right) \\
\frac{\partial \mathscr{L}}{\partial \gamma} &= (\delta - 1) F(V_*) - \xi \\
\frac{\partial \mathscr{L}}{\partial V_S} &= -\left(g_S(I_S) + V_S\right) f(V_S) + V_S f(V_S) + \theta\left(\phi Q_B d_0 + \phi(1+t) I_S\right)\frac{\partial p^S}{\partial V_S} - \delta \frac{dp^S}{dV_S} I_S + \zeta
\end{aligned}
$$

First using the equations for V_* and γ,

$$V_* = \delta(\gamma + c) - (\delta - 1)\xi_\gamma \gamma.$$

Then using the equations for B_1 and D_H,

$$\delta = \theta + u' + \eta u''$$

$$\delta = \theta\phi(1+t) + (1+t)u' + \eta(1+t)u''.$$

Following the steps used in the proof of Proposition 2, these yield

$$\theta = \frac{t}{(1+t)(1-\phi)}\delta$$

$$\frac{1-\phi(1+t)}{(1+t)(1-\phi)}\delta = \frac{1}{Q_B} - \frac{1}{Q_B}\eta\sigma_H.$$

These express θ and η in terms of δ.

Next, using the equations for V_S, we have

$$\zeta = \left(g_S(I_S)+V_S\right)f(V_S) - V_S f(V_S) - \theta\left(\phi Q_B d_0 + \phi(1+t)I_S\right)\frac{\partial p^S}{\partial V_S} + \delta\frac{dp^S}{dV_S}I_S$$

and using $\frac{\partial p^S}{\partial V_S} = -f(V_S)$, we get

$$\zeta = \left(g_S(I_S)+V_S\right)f(V_S) - V_S f(V_S) + \theta\left(\phi Q_B d_0 + \phi(1+t)I_S\right)f(V_S) - \delta I_S f(V_S)$$

and substituting in $\theta = \frac{t}{(1+t)(1-\phi)}\delta$, we get

$$\frac{1}{f(V_S)}\zeta = g_S(I_S) + \frac{1-\phi(1+t)}{1-\phi}\left[\frac{1}{1-\phi(1+t)}\frac{t\phi}{1+t}Q_B d_0 - I_S\right]\delta.$$

Next, we can combine the first-order conditions for I_S and Q_B to get

$$-Q_B\sigma_S = \frac{p^S g_S'(I_S) + \theta\phi p^S(1+t) - \delta p^S}{\theta\phi p^S d_0 + \eta\frac{1}{Q_B^2} + \zeta\left(\frac{1+t}{Q_B^2}c + G^{*\prime}\left(\frac{Q_B}{1+t}\right)\frac{1}{1+t}\right)}$$

By Envelope Theorem, $G^{*\prime}(Q_S) = \frac{I_S}{Q_S^2}$, so we obtain

$$-Q_B\sigma_S\left[\theta\phi p^S d_0 + \eta\frac{1}{Q_B^2} + \zeta\frac{1+t}{Q_B^2}\left(c+I_S\right)\right] = p^S g_S'(I_S) + \theta\phi p^S(1+t) - \delta p^S.$$

Substituting in above, we obtain

$$\left[\frac{1-\phi(1+t)}{1-\phi}-\frac{t}{(1+t)(1-\phi)}Q_B\sigma_S\phi d_0\right]p^S\delta-\sigma_S\eta\frac{1}{Q_B}-\sigma_S\zeta\frac{1+t}{Q_B}\left(c+I_S\right)=p^S\frac{1+t}{Q_B}$$

Substituting further,

$$\eta\sigma_H=1-\frac{1-\phi(1+t)}{(1+t)(1-\phi)}Q_B\delta$$

which yields

$$\left[\frac{1-\phi(1+t)}{1-\phi}p^S+\frac{\sigma_S}{\sigma_H}\frac{1-\phi(1+t)}{(1+t)(1-\phi)}-\frac{t}{(1+t)(1-\phi)}Q_B\sigma_S\phi d_0 p^S\right]\delta$$
$$-f(V_S)\sigma_S\frac{1+t}{Q_B}\left(c+I_S\right)\left[g_S(I_S)+\frac{1-\phi(1+t)}{1-\phi}\left[\frac{1}{1-\phi(1+t)}\frac{t\phi}{1+t}Q_B d_0-I_S\right]\delta\right]=p^S\frac{1+t}{Q_B}+\frac{\sigma_S}{\sigma_H}\frac{1}{Q_B}$$

and therefore, we get

$$\delta=\frac{1}{Q_B}\left(p^S(1+t)+\frac{\sigma_S}{\sigma_H}+f(V_S)\sigma_S(1+t)\left(c+I_S\right)g_S(I_S)\right)\Gamma^{-1}, \tag{IA.B.15}$$

where

$$\Gamma\equiv\frac{1-\phi(1+t)}{1-\phi}p^S+\frac{\sigma_S}{\sigma_H}\frac{1-\phi(1+t)}{(1+t)(1-\phi)}-\frac{t}{(1+t)(1-\phi)}Q_B\sigma_S\phi d_0 p^S \tag{IA.B.16}$$
$$-f(V_S)\sigma_S\frac{1+t}{Q_B}\left(c+I_S\right)\frac{1-\phi(1+t)}{1-\phi}\left[\frac{1}{1-\phi(1+t)}\frac{t\phi}{1+t}Q_B d_0-I_S\right] \tag{IA.B.17}$$

Now, plugging $Q_B=(1+t)Q_S$ into δ_P, we know that

$$\delta_P=\frac{(1+t)(1-\phi)}{1-\phi(1+t)}\frac{1}{Q_B}$$

Therefore, we can write

$$\delta = M\delta_P$$

where we have

$$M = \frac{1-\phi(1+t)}{(1+t)(1-\phi)} \left(p^S(1+t) + \frac{\sigma_S}{\sigma_H} + f(V_S)\sigma_S(1+t)\Big(c+I_S\Big)g_S(I_S) \right) \Gamma^{-1}. \quad \text{(IA.B.18)}$$

Simplifying,

$$M = \frac{1-\phi(1+t)}{1-\phi} \left(p^S + \frac{1}{1+t}\frac{\sigma_S}{\sigma_H} + f(V_S)\sigma_S\Big(c+I_S\Big)g_S(I_S) \right) \Gamma^{-1}. \quad \text{(IA.B.19)}$$

Note

$$\begin{aligned}\frac{1-\phi}{1-\phi(1+t)}\Gamma = {} & p^S + \frac{\sigma_S}{\sigma_H}\frac{1}{1+t} - \frac{t}{(1+t)(1-\phi(1+t))}Q_B\sigma_S\phi d_0 p^S \\ & - f(V_S)\sigma_S\frac{1+t}{Q_B}\Big(c+I_S\Big)\left[\frac{1}{1-\phi(1+t)}\frac{t\phi}{1+t}Q_B d_0 - I_S\right],\end{aligned}$$

and using $Q_B = (1+t)Q_S$, the last line simplifies and this is

$$p^S + \frac{\sigma_S}{\sigma_H}\frac{1}{1+t} - \frac{t}{(1+t)(1-\phi(1+t))}Q_B\sigma_S\phi d_0 p^S - f(V_S)\sigma_S\Big(c+I_S\Big)\left[\frac{1}{1-\phi(1+t)}t\phi d_0 - I_S Q_S^{-1}\right].$$

Plugging this back into M,

$$M=\frac{p^S+\frac{1}{1+t}\frac{\sigma_S}{\sigma_H}+f(V_S)\sigma_S\left(c+I_S\right)g_S(I_S)}{p^S+\frac{\sigma_S}{\sigma_H}\frac{1}{1+t}-\frac{t}{(1+t)(1-\phi(1+t))}Q_B\sigma_S\phi d_0 p^S-f(V_S)\sigma_S\left(c+I_S\right)\left[\frac{1}{1-\phi(1+t)}t\phi d_0-I_S Q_S^{-1}\right]}$$

$$M=\frac{p^S+\frac{1}{1+t}\frac{\sigma_S}{\sigma_H}+f(V_S)\sigma_S\left(c+I_S\right)g_S(I_S)}{p^S+\frac{1}{1+t}\frac{\sigma_S}{\sigma_H}-\frac{t}{(1+t)\left[1-\phi(1+t)\right]}Q_B\sigma_S\phi d_0 p^S-f(V_S)\sigma_S\left(c+I_S\right)\left[\frac{t\phi}{1-\phi(1+t)}d_0-\frac{1}{Q_S}I_S\right]}$$

It is now straight-forward to show that $M\geq 1$. In particular guessing and verifying, we have $M\geq 1$ if

$$f(V_S)\sigma_S\left(c+I_S\right)g_S(I_S)\geq -\frac{t}{(1+t)\left[1-\phi(1+t)\right]}Q_B\sigma_S\phi d_0 p^S-f(V_S)\sigma_S\left(c+I_S\right)\left[\frac{t\phi}{1-\phi(1+t)}d_0-\frac{1}{Q_S}I_S\right]$$

$$f(V_S)\sigma_S\left(c+I_S\right)\left[g_S(I_S)-\frac{1}{Q_S}I_S\right]+f(V_S)\sigma_S\left(c+I_S\right)\frac{t\phi}{1-\phi(1+t)}d_0+\frac{t}{(1+t)\left[1-\phi(1+t)\right]}Q_B\sigma_S\phi d_0 p^S\geq 0$$

Note that all three terms must be positive. The first term is positive by optimization by firms. Thus, $M>1$.

C Heterogeneous Banks

In this appendix, we study the effect of multiple heterogeneous bank creditors facing different collateral constraints in the design of the liquidation rule as well as the optimal seniority structure among creditors.

Our results are qualitatively similar to our baseline model in the following sense. The socially optimal liquidation rule is a threshold rule with threshold V_*. The threshold can be higher or lower than the threshold chosen by privately optimizing banks because (i) there are fire-sale externalities and (ii) the social effective return is greater than the private effective return for each bank: $\delta_*^b > \delta_P^b$.

We find additional results. We show that the ranking of effective returns δ_*^b corresponds to the ranking of ϕ_b. Banks that can easily collateralize loans thus have a higher private and social effective return. We show that a social planner can improve welfare in liquidations by subordinating claims of banks that, because of their idiosyncratic collateral constraints, struggle to collateralize loans created with the liquidation proceeds. Moreover, planner intervention in the seniority structure can serve as a partial substitute for intervention in liquidation decisions.

Because we explicitly model seniority, our leading application for this section is to a traditional bankruptcy process, such as Chapter 11, with changes in the liquidation rule reflecting different outcomes of the bankruptcy process. Nevertheless, we preserve the notation and terminology of previous sections for consistency.

C.1 Novel assumptions in the heterogeneous-banks extension

We assume that there are n_B distinct banks, each of equal measure, indexed by $b = 1, 2, ..., n_B$. For simplicity, banks are identical at date zero, so each bank has b_0 in deposits and makes an identical quantity of loans d_0/n_B. For simplicity, we assume that each dollar that bank b lends is divided equally among all firms, ensuring there are multiple creditors for each firm.

However, banks differ in their ability to pledge collateral: each bank has its own ϕ_b. At date one, bank b raises debt B_1^b from households to lend to firms. Additionally, banks differ in their seniority. For simplicity, we assume bank b receives a fraction $S^b(v)$ of any cashflows associated with a bankruptcy for a firm with viability v.

Given a liquidation rule ρ, bank b's budget constraint is

$$pD_1^b + (1-p)\int(1-\rho(v))S^b(v)cdF(v) \leq B_1^b - b_0 + (1-p)\int\rho(v)S^b(v)\gamma dF(v) \tag{IA.C.1}$$

We require that $\sum_b S^b(v) = 1$ to avoid double counting recovery in any bankruptcy. Bank b's collateral constraint is:

$$B_1^b \leq \phi_b Q_B p(\frac{d_0}{n_B} + \frac{D_1^b}{Q_S}). \tag{IA.C.2}$$

The objective is

$$\max_{B_1^b, D_1^b, \rho} p\left(\frac{D_1^b}{Q_S} + \frac{d_0}{n_B}\right) + (1-p)\int(1-\rho(v))vS^b(v)dF(v) - \frac{B_1^b}{Q_B}, \tag{IA.C.3}$$

Finally, the market clearing conditions become

$$\sum_b D_1^b + D_H = I_S \tag{IA.C.4}$$

$$B_H = \sum_b B_1^b. \tag{IA.C.5}$$

Otherwise, the model is identical to the one in the main text. We find the following results.

C.2 Private liquidation rules

It is immediate that any bank will find it optimal to give itself 100% seniority. We thus abstract from seniority for the private bank's problem, instead characterizing the privately optimal liquidation rule when $S^b = 1$.

Proposition C.1. *The privately optimal bank liquidation rule is a threshold rule with*

$$V_P = \delta_P^b(\gamma + c) \tag{IA.C.6}$$

where

$$\delta_P^b = Q_B^{-1} + \frac{1}{1 - \phi_b \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right). \tag{IA.C.7}$$

Proof: Following an identical argument to the one used in the proof of Proposition 1, a threshold liquidation rule is optimal and the Lagrangian is thus

$$\begin{aligned}\mathscr{L} = p\left(\frac{D_1^b}{Q_S} + \frac{d_0}{n_B}\right) + (1-p)\int_{v \geq V_P} v dF(v) - \frac{B_1^b}{Q_B} + \theta_P^b \left(\phi_b Q_B p (d_0/n_B + \frac{D_1^b}{Q_S}) - B_1^b \right) \\ + \delta_P^b \left(B_1^b - b_0 + (1-p)\gamma F(V_P) - \left[p D_1^b + (1-p) \left(1 - F(V_P) \right) c \right] \right).\end{aligned}$$

Differentiating with respect to B_1^b,

$$0 = -Q_B^{-1} - \theta_P^b + \delta_P^b. \tag{IA.C.8}$$

Differentiating with respect to D_1^b,

$$0 = pQ_S^{-1} + \theta_P^b \phi_b Q_B p Q_S^{-1} - \delta_P^b p. \tag{IA.C.9}$$

Combining these equations,

$$\begin{aligned}
\delta_P^b &= Q_S^{-1} + \theta_P^b \phi_b Q_B Q_S^{-1} \\
\theta_P^b &= -Q_B^{-1} + Q_S^{-1} + \theta_P^b \phi_b Q_B Q_S^{-1} \\
\theta_P^b &= \frac{1}{1 - \phi_b \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right).
\end{aligned}$$

Finally, differentiating with respect to V_P, setting the derivative equal to zero, dividing by $(1-p)f(V_P)$, and adding V_P to both sides,

$$V_P = \delta_P^b(\gamma + c) = (\frac{1}{Q_B} + \theta_P^b)(\gamma + c). \tag{IA.C.10}$$

This concludes the derivation, which shows that different banks would choose different liquidation rules given their heterogeneous ϕ_b parameters.

C.3 Socially optimal liquidation

We now consider a planner that optimizes over bank specific borrowing, lending, and seniority, as well as the liquidation rule and prices: the planner chooses $\{B_1^b, D_1^b, S^b\}$ in addition to the usual control variables.

Proposition C.2. *The socially optimal seniority policy is*

$$S^b(v) = \mathbf{1}(b = argmax_z \delta_*^z)\mathbf{1}(v \leq V_*) + \mathbf{1}(b = argmin_z \delta_*^z)\mathbf{1}(v > V_*) \tag{IA.C.11}$$

The socially optimal liquidation rule is a threshold rule $\rho(v) = \mathbf{1}\left(v \leq V_*\right)$*, where*

$$V_* = [\max_b \delta_*^b]\gamma + [\min_b \delta_*^b]c - \left[[\max_b \delta_*^b] - 1\right]\xi_\gamma \gamma. \tag{IA.C.12}$$

For all b, we have $\delta_*^b > \delta_P^b$*. Finally, banks with higher values of* ϕ_b *have higher values of* δ_*^b*.*

Proof: Summing across bank objectives delivers the aggregate bank objective

$$p\left(\frac{\sum_b D_1^b}{Q_S} + d_0\right) + (1-p)\int(1-\rho(v))vdF(v) - \frac{\sum_b B_1^b}{Q_B}, \tag{IA.C.13}$$

where we have used $\sum_b S^b(v) = 1$. Using the new market clearing conditions

$$\sum_b D_1^b + D_H = I_S \tag{IA.C.14}$$

$$B_H = \sum_b B_1^b, \tag{IA.C.15}$$

we see the planner objective is the same as in our baseline model:

$$p\left(g_S(I_S) + v_S\right) + (1-p)\int(1-\rho(v))vdF(v) + a(L) - \gamma L + u\left(e - \sum_b B_1^b - p(1+t)D_H\right).$$

We thus have the same Lagrangian except with heterogeneous bank constraints. An identical argument to the one used before shows threshold rules are optimal. Given the more complicated market clearing condition, we now consider the planner's problem with controls $\{D_1^b\}$ and D_H

rather than I_S and D_H. Otherwise, the Lagrangian is the same:

$$
\begin{aligned}
\mathscr{L} = & \; p g_S(D_H + \sum_b D_1^b) + (1-p)\int_{v \geq V_*} v dF(v) + u\left(e - \sum_b B_1^b - p(1+t)D_H \right) \\
& + a((1-p)F(V_*)) - \gamma(1-p)F(V_*) \\
& + \sum_b \theta_*^b \left(\phi_b p Q_B \left[d_0/n_B + \frac{(1+t)D_1^b}{Q_B} \right] - B_1^b \right) \\
& + \sum_b \delta_*^b \left(B_1^b - b_0 + (1-p)\gamma \int_{v \leq V_*} S^b(v) dF(v) - (1-p)c \int_{v \geq V_*} S^b(v) dF(v) - pD_1^b \right) \\
& + \kappa(1 - \frac{Q_B}{1+t} g_S'(D_H + \sum_b D_1^b)) + \eta \left(u'\left(e - \sum_b B_1^b - p(1+t)D_H \right) - \frac{1}{Q_B} \right) \\
& + \xi \left(a'((1-p)F(V_*)) - \gamma \right).
\end{aligned}
$$

We see S^b only enters into the bank budget constraints. We thus have that $S^b(v) = \mathbf{1}(b = \operatorname{argmax}_z \delta_*^z)$ for $v \leq V_*$ and $S^b(v) = \mathbf{1}(b = \operatorname{argmin}_z \delta_*^z)$ for $v > V_*$. We now take derivatives with respect to the planner's control variables:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial D_1^b} &= pg_S'(D_H+\sum_b D_1^b)+\theta_*^b\phi_b p(1+t)-\delta_*^b p-\kappa\frac{Q_B}{1+t}g_S''(D_H+\sum_b D_1^b)\\
\frac{\partial \mathscr{L}}{\partial V_*} &= (1-p)f(V_*)\Big[-V_*+\mathbf{0}+[\max_b \delta_*^b]\gamma+[\min_b \delta_*^b]c+\xi a''((1-p)F(V_*))\Big]\\
\frac{\partial \mathscr{L}}{\partial B_1^b} &= -u'\left(e-\sum_b B_1^b-p(1+t)D_H\right)-\theta_*^b+\delta_*^b-\eta u''\left(e-\sum_b B_1^b-p(1+t)D_H\right)\\
\frac{\partial \mathscr{L}}{\partial D_H} &= pg_S'(D_H+\sum_b D_1^b)-p(1+t)u'\left(e-\sum_b B_1^b-p(1+t)D_H\right)\\
&\quad -\kappa\frac{Q_B}{1+t}g_S''(D_H+\sum_b D_1^b)-\eta p(1+t)u''\left(e-\sum_b B_1^b-p(1+t)D_H\right)\\
\frac{\partial \mathscr{L}}{\partial Q_B} &= \sum_b \theta_*^b\phi_b p d_0/n_B-\kappa\frac{1}{1+t}g_S'(D_H+\sum_b D_1^b)+\eta\frac{1}{Q_B^2}\\
\frac{\partial \mathscr{L}}{\partial \gamma} &= (1-p)F(V_*)\big[-1+[\max_b \delta_*^b]\big]-\xi.
\end{aligned}
$$

We see immediately from $\frac{\partial \mathscr{L}}{\partial \gamma}$ that

$$\xi=(1-p)F(V_*)\big[-1+[\max_b \delta_*^b]\big]. \tag{IA.C.16}$$

Plugging into $\frac{\partial \mathscr{L}}{\partial V_*}$,

$$V_*=[\max_b \delta_*^b]\gamma+[\min_b \delta_*^b]c+(1-p)F(V_*)\big[-1+[\max_b \delta_*^b]\big]a''((1-p)F(V_*)). \tag{IA.C.17}$$

Recalling that $L=(1-p)F(V_*)$ and $\zeta_\gamma=-La''(L)/\gamma$,

$$V_*=[\max_b \delta_*^b]\gamma+[\min_b \delta_*^b]c-\big[[\max_b \delta_*^b]-1\big]\xi_\gamma\gamma. \tag{IA.C.18}$$

Now, as before, we have $g'_S = Q_S^{-1} = (1+t)Q_B^{-1}$ and $u' = Q_B^{-1}$, so we can simplify:

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial D_1^b} &= p(1+t)Q_B^{-1} + \theta_*^b \phi_b p(1+t) - \delta_*^b p + \kappa \sigma_S \\
\frac{\partial \mathscr{L}}{\partial B_1^b} &= -Q_B^{-1} - \theta_*^b + \delta_*^b + \eta Q_B^{-1} \sigma_H \\
\frac{\partial \mathscr{L}}{\partial D_H} &= p(1+t)Q_B^{-1} - p(1+t)Q_B^{-1} + \kappa \sigma_S + \eta p(1+t)Q_B^{-1} \sigma_H \\
\frac{\partial \mathscr{L}}{\partial Q_B} &= \sum_b \theta_*^b \phi_b p d_0 / n_B - \kappa Q_B^{-1} + \eta \frac{1}{Q_B^2}
\end{aligned}$$

The third equation implies that

$$\kappa \sigma_S = -\eta p(1+t)Q_B^{-1} \sigma_H \tag{IA.C.19}$$

Plugging this in,

$$\begin{aligned}
0 &= p(1+t)Q_B^{-1} + \theta_*^b \phi_b p(1+t) - \delta_*^b p - \eta p(1+t)Q_B^{-1} \sigma_H \\
0 &= -Q_B^{-1} - \theta_*^b + \delta_*^b + \eta Q_B^{-1} \sigma_H \\
0 &= \sum_b \theta_*^b \phi_b p d_0 / n_B + \eta p(1+t)Q_B^{-1} \frac{\sigma_H}{\sigma_S} Q_B^{-1} + \eta \frac{1}{Q_B^2}
\end{aligned}$$

Multiplying the second equation by $p(1+t)$ and adding it to the first,

$$0 = \theta_*^b p(1+t)(\phi_b - 1) + \delta_*^b pt \Rightarrow \theta_*^b = \frac{t}{(1+t)(1-\phi_b)} \delta_*^b. \tag{IA.C.20}$$

Plugging this back into the second equation,

$$\eta Q_B^{-1}\sigma_H = Q_B^{-1} + \delta_*^b\big[\frac{t}{(1+t)(1-\phi_b)} - 1\big] \tag{IA.C.21}$$

Since this holds for all b it holds for the sum:

$$n_B\eta Q_B^{-1}\sigma_H = n_B Q_B^{-1} + \sum_b \delta_*^b\big[\frac{t}{(1+t)(1-\phi_b)} - 1\big]. \tag{IA.C.22}$$

Plugging the expression $\theta_*^b = \frac{t}{(1+t)(1-\phi_b)}\delta_*^b$ into the third equation above, and multiplying by $n_B Q_B^2$,

$$0 = Q_B^2 \sum_b \frac{t}{(1+t)(1-\phi_b)}\delta_*^b \phi_b p d_0 + n_B\eta\left[1 + p(1+t)\frac{\sigma_H}{\sigma_S}\right]. \tag{IA.C.23}$$

We thus see that

$$\eta = -\frac{Q_B^2 \sum_b \frac{t}{(1+t)(1-\phi_b)}\delta_*^b \phi_b p d_0}{n_B\left[1 + p(1+t)\frac{\sigma_H}{\sigma_S}\right]} < 0. \tag{IA.C.24}$$

Now, recall from above that

$$\delta_*^b = Q_B^{-1} + \theta_*^b - \eta Q_B^{-1}\sigma_H. \tag{IA.C.25}$$

Using $\theta_*^b = \frac{t}{(1+t)(1-\phi_b)}\delta_*^b$, this is

$$\delta_*^b\big[1 - \frac{t}{(1+t)(1-\phi_b)}\big] = \delta_*^b \frac{1-\phi_b(1+t)}{(1+t)(1-\phi_b)} = Q_B^{-1} - \eta Q_B^{-1}\sigma_H. \tag{IA.C.26}$$

We thus have

$$\delta_*^b = \frac{(1+t)(1-\phi_b)}{1-\phi_b(1+t)} \left(Q_B^{-1} - \eta Q_B^{-1}\sigma_H \right) . \tag{IA.C.27}$$

Note that $Q_B^{-1} - \eta Q_B^{-1}\sigma_H$ does not depend on b. We thus see that banks with higher ϕ_b will have higher δ_*^b since

$$\frac{\partial}{\partial \phi_b}\frac{(1+t)(1-\phi_b)}{1-\phi_b(1+t)} = \frac{-(1+t)}{1-\phi_b(1+t)} + (1+t)\frac{(1+t)(1-\phi_b)}{(1-\phi_b(1+t))^2} \tag{IA.C.28}$$

$$= \left[-(1-\phi_b(1+t)) + (1+t)(1-\phi_b)\right]\frac{(1+t)}{(1-\phi_b(1+t))^2} > 0. \tag{IA.C.29}$$

Finally, note that as before, $Q_S^{-1} = (1+t)Q_B^{-1}$ implies that

$$\theta_P^b = \frac{t}{1-\phi_b(1+t)} Q_B^{-1} \tag{IA.C.30}$$

$$\delta_P^b = \frac{(1+t)(1-\phi_b)}{1-\phi_b(1+t)} Q_B^{-1}, \tag{IA.C.31}$$

so $\eta < 0$ implies that $\delta_*^b > \delta_P^b$, completing the proof.

Note that the analysis of bailouts in this context follows identical steps to the ones used in the proof of Proposition 6. For brevity, we thus omit the proof that a bailout to bank b has a marginal social benefit of δ_*^b.

D Extension with Bank Writedown Avoidance

In this appendix, we consider a model extension in which banks have an incentive to avoid writing down loans on their balance sheet. Specifically, we assume that if a bank chooses continuation for a particular distressed firm, it can pledge the face value of that firm's debt d_0 as collateral, even though the firm is truly worth less than that. Specifically, the bank's budget constraint is unchanged, but the collateral constraint now reflects the ability of banks to fool households by not writing down distressed loans in continuations:

$$B_1 \leq \phi\, Q_B \left[\underbrace{p\,(d_0 + \frac{D_1}{Q_S})}_{\text{Solvent Firm Loans}} + \underbrace{(1-p)\int (1-\rho(v))dF(v)d_0}_{\text{Zombies}} \right] \tag{IA.D.1}$$

Otherwise, the model is identical to the one in the main text. All of the model forces in our baseline model play the same roles in this setting. In particular, banks consider both direct recovery and the value of loanable funds when choosing liquidations. The planner trades off the same externalities.

We now prove analogs of Propositions 1 - 2. Our results are qualitatively similar in the following sense. Banks choose a threshold liquidation policy to maximize total recovery, which includes a direct and shadow component. The planner has a higher effective return $\delta_* > \delta_P$ from date one funds, relative to banks. The planner thus chooses a threshold V_* that can be (i) higher than the bank's threshold because $\delta_* > \delta_P$ or (ii) lower than the bank's threshold because of fire sales.

We also derive two new results. First, the incentive to disguise insolvent firms for the purposes of pledging collateral makes banks more hesitant to liquidate firms. Both the planner and privately optimizing banks liquidate fewer firms in this extension, as can be seen from lower thresholds. Second, while the private effective return δ_P is unchanged, the social multiplier M is larger in this extension because there is more pledgeable collateral to be revalued by changing discount rates.

This higher multiplier makes fire sales more salient ($\xi_\gamma \gamma$ is multiplied by a larger value). It also makes the collateral externality more salient. In this sense, the planner's tradeoff is amplified.

D.1 Privately optimal liquidation rules

Proposition D.1. *The bank's solution has the same δ_P, θ_P, but the liquidation threshold is lower:*

$$V_P = \delta_P(\gamma + c) - \theta_P \phi Q_B d_0 = (\frac{1}{Q_B} + \theta_P)(\gamma + c) - \theta_P \phi Q_B d_0. \tag{IA.D.2}$$

Proof: Banks choose (B_1, D_1, ρ) in order to maximize their final value,

$$\max_{B_1, D_1, \rho} \quad p\left(\frac{D_1}{Q_S} + d_0\right) + (1-p)\int (1-\rho(v))v dF(v) - \frac{B_1}{Q_B}, \tag{IA.D.3}$$

subject to

$$pD_1 + (1-p)\int (1-\rho(v))c dF(v) \leq B_1 - b_0 + (1-p)\int \rho(v)\gamma dF(v) \tag{IA.D.4}$$

and (IA.D.1). An identical argument to the one used in the proof of Proposition 1 shows that a threshold rule is optimal. The Lagrangian is thus

$$\mathscr{L} = p\left(\frac{D_1}{Q_S} + d_0\right) + (1-p)\int_{v \geq V_P} v dF(v) - \frac{B_1}{Q_B} + \theta_P\left(\phi Q_B\left[p(d_0 + \frac{D_1}{Q_S}) + (1-p)(1-F(V_P))d_0\right] - B_1\right)$$
$$+ \delta_P\left(B_1 - b_0 + (1-p)\gamma F(V_P) - \left[pD_1 + (1-p)\left(1 - F(V_P)\right)c\right]\right).$$

Differentiating with respect to B_1,

$$0 = -Q_B^{-1} - \theta_P + \delta_P. \tag{IA.D.5}$$

Differentiating with respect to D_1,

$$0 = pQ_S^{-1} + \theta_P \phi Q_B p Q_S^{-1} - \delta_P p. \tag{IA.D.6}$$

Combining these equations,

$$\begin{aligned}
\delta_P &= Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1} \\
\theta_P &= -Q_B^{-1} + Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1} \\
\theta_P &= \frac{1}{1 - \phi \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right).
\end{aligned}$$

Finally, differentiating with respect to V_P, setting the derivative equal to zero, dividing by $(1-p)f(V_P)$, and adding V_P to both sides,

$$V_P = \delta_P(\gamma + c) - \theta_P \phi Q_B d_0 = (\frac{1}{Q_B} + \theta_P)(\gamma + c) - \theta_P \phi Q_B d_0. \tag{IA.D.7}$$

We see this is strictly lower than in the main model because banks have an additional incentive to avoid liquidation: continuations create more collateral, giving banks the shadow value of collateral.

D.2 Socially optimal liquidation rules

Proposition D.2. *The social planner's solution features a lower liquidation threshold than before, but it can be higher or lower than the private threshold:*

$$V_* = \underbrace{\delta_* \left(\gamma + c - \frac{t\phi Q_B d_0}{(1+t)(1-\phi)} \right)}_{\textit{Larger than } V_P} - \underbrace{\left[\delta_* - 1\right]\gamma\xi_\gamma}_{\textit{Fire sale}>0}. \tag{IA.D.8}$$

The multiplier M is larger than in the baseline model:

$$M = \left(1 - \frac{\sigma_S \sigma_H \phi d_0 Q_B t \left[p + (1-p)(1-F(V_*))\right]}{\left[p(1+t)\sigma_H + \sigma_S\right](1-\phi(1+t))} \right)^{-1} \geq 1. \tag{IA.D.9}$$

Proof: None of the objectives have changed, so just as before the planner's objective is

$$p\left(g_S(I_S) + v_S \right) + (1-p)\int (1-\rho(v))v dF(v) + a(L) - \gamma L + u\left(e - B_1 - p(1+t)D_H \right).$$

Just as before, the household first-order conditions imply that $\frac{Q_B}{1+t} = Q_S$. Since δ_P and θ_P are the same as in the baseline model, we once again have

$$\theta_P = \frac{1}{1-\phi(1+t)} \frac{t}{Q_B}$$
$$\delta_P = \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}.$$

An identical argument to the one used before shows that the planner uses a threshold rule, so the social Lagrangian is

$$\mathscr{L} = pg_S(I_S) + (1-p)\int_{v \geq V_*} v dF(v) + u\left(e - B_1 - p(1+t)D_H\right)$$
$$+ a((1-p)F(V_*)) - \gamma(1-p)F(V_*)$$
$$+ \theta_*\left(\phi Q_B\left[p\left[d_0 + \frac{(1+t)(I_S - D_H)}{Q_B}\right] + (1-p)(1-F(V_*))d_0\right] - B_1\right)$$
$$+ \delta_*\left(B_1 - b_0 + (1-p)\gamma F(V_*) - (1-p)c(1-F(V_*)) - p\left(I_S - D_H\right)\right)$$
$$+ \kappa(1 - \frac{Q_B}{1+t} g_S'(I_S)) + \eta\left(u'\left(e - B_1 - p(1+t)D_H\right) - \frac{1}{Q_B}\right)$$
$$+ \xi\left(a'((1-p)F(V_*)) - \gamma\right).$$

In other words, the Lagrangian is the same except we have added $\theta_*\phi Q_B(1-p)(1-F(V_*))d_0$. The derivatives with respect to the planner's control variables are thus identical except for V_* and Q_B:

$$\frac{\partial \mathscr{L}}{\partial I_S} = pg_S'(I_S) + \theta_*\phi p(1+t) - \delta_* p - \kappa\frac{Q_B}{1+t}g_S''(I_S)$$
$$\frac{\partial \mathscr{L}}{\partial V_*} = (1-p)f(V_*)\left[-V_* + \mathbf{0} - \underline{\theta_*\phi Q_B d_0} + \delta_*\left(\gamma + c\right) + \xi a''((1-p)F(V_*))\right]$$
$$\frac{\partial \mathscr{L}}{\partial B_1} = -u'\left(e - B_1 - p(1+t)D_H\right) - \theta_* + \delta_* - \eta u''\left(e - B_1 - p(1+t)D_H\right)$$
$$\frac{\partial \mathscr{L}}{\partial D_H} = -p(1+t)u'\left(e - B_1 - p(1+t)D_H\right) - \theta_*\phi p(1+t) + \delta_* p - \eta p(1+t)u''\left(e - B_1 - p(1+t)D_H\right)$$
$$\frac{\partial \mathscr{L}}{\partial Q_B} = \theta_*\phi p d_0 + \underline{\theta_*\phi(1-p)(1-F(V_*))d_0} - \kappa\frac{1}{1+t}g_S'(I_S) + \eta\frac{1}{Q_B^2}$$
$$\frac{\partial \mathscr{L}}{\partial \gamma} = (1-p)F(V_*)\left[-1 + \delta_*\right] - \xi$$

We once again introduce shorthand C^U, σ_H, σ_S defined as before. Gathering terms, simplifying, and applying $g'_S = (1+t)/Q_B$, $u'(C^U) = Q_B^{-1}$,

$$
\begin{aligned}
0 &= \frac{1+t}{Q_B} - \delta_* + \theta_* \phi (1+t) + \frac{\kappa \sigma_S}{p} \\
0 &= -V_* - \underline{\theta_* \phi Q_B d_0} + \delta_* \left(\gamma + c \right) + \xi a''(L) \\
0 &= -Q_B^{-1} - \theta_* + \delta_* + \eta \frac{\sigma_H}{Q_B} \\
0 &= -Q_B^{-1} - \theta_* \phi + \frac{\delta_*}{1+t} + \eta \frac{\sigma_H}{Q_B} \\
0 &= \theta_* \phi p d_0 Q_B + \underline{\theta_* \phi (1-p)(1-F(V_*)) d_0 Q_B} - \kappa + \eta \frac{1}{Q_B}
\end{aligned}
$$

Multiply the fourth equation by -1 and add it to the third:

$$
0 = -\theta_*(1-\phi) + \frac{t}{1+t}\delta_* \Rightarrow \theta_* = \frac{t}{(1+t)(1-\phi)}\delta_*. \qquad \text{(IA.D.10)}
$$

Plugging into the first equation and rearranging,

$$
\kappa = \frac{p}{\sigma_S}\left[-\frac{1+t}{Q_B} + \delta_* \left(1 - \frac{\phi t}{1-\phi} \right) \right] \qquad \text{(IA.D.11)}
$$

Plugging into the third equation and rearranging,

$$
\eta = \frac{Q_B}{\sigma_H}\left[Q_B^{-1} + \delta_* \left(-1 + \frac{t}{(1+t)(1-\phi)} \right) \right]. \qquad \text{(IA.D.12)}
$$

Plugging into the final equation,

$$0 = \delta_* \frac{\phi d_0 Q_B t \left[p + \underline{(1-p)(1-F(V_*))}\right]}{(1+t)(1-\phi)} \tag{IA.D.13}$$

$$- \frac{p}{\sigma_S}\left[-\frac{1+t}{Q_B} + \delta_* \left(1 - \frac{\phi t}{1-\phi}\right)\right] + \frac{1}{\sigma_H}\left[Q_B^{-1} + \delta_* \left(-1 + \frac{t}{(1+t)(1-\phi)}\right)\right] \tag{IA.D.14}$$

$$0 = \delta_* \frac{\phi d_0 Q_B t \left[p + \underline{(1-p)(1-F(V_*))}\right]}{(1+t)(1-\phi)} \tag{IA.D.15}$$

$$- \frac{p}{\sigma_S}\left[-\frac{1+t}{Q_B} + \delta_* \left(\frac{1-\phi(1+t)}{1-\phi}\right)\right] + \frac{1}{\sigma_H}\left[Q_B^{-1} + \delta_* \left(\frac{-(1-\phi(1+t))}{(1+t)(1-\phi)}\right)\right]. \tag{IA.D.16}$$

At this point, recall that

$$\delta_P = \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}.$$

Define $M \equiv \delta_*/\delta_P$. Then

$$0 = M \frac{\phi d_0 Q_B t \left[p + \underline{(1-p)(1-F(V_*))}\right]}{Q_B(1-\phi(1+t))} - \frac{p}{\sigma_S}\left[-\frac{1+t}{Q_B} + M\left(\frac{1+t}{Q_B}\right)\right] + \frac{1}{\sigma_H}\left[Q_B^{-1} + M\left(\frac{-1}{Q_B}\right)\right]. \tag{IA.D.17}$$

Cleaning up and rearranging to a common denominator,

$$0 = M\frac{\sigma_S\sigma_H\phi d_0 Q_B t\left[p+\underline{(1-p)(1-F(V_*))}\right]-\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}{\sigma_S\sigma_H Q_B(1-\phi(1+t))} \tag{IA.D.18}$$

$$+\frac{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}{\sigma_S\sigma_H Q_B(1-\phi(1+t))}. \tag{IA.D.19}$$

Moving M to the left and dividing by the fraction multiplying M,

$$M = \frac{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))-\sigma_S\sigma_H\phi d_0 Q_B t\left[p+\underline{(1-p)(1-F(V_*))}\right]} \geq 1. \tag{IA.D.20}$$

Alternatively,

$$M = \left(1-\frac{\sigma_S\sigma_H\phi d_0 Q_B t\left[p+\underline{(1-p)(1-F(V_*))}\right]}{\left[p(1+t)\sigma_H+\sigma_S\right](1-\phi(1+t))}\right)^{-1} \geq 1. \tag{IA.D.21}$$

Finally, we solve for the liquidation threshold. From $\partial\mathscr{L}/\partial\gamma$,

$$\xi = (1-p)F(V_*)\left[\delta_*-1\right] = \left[\delta_*-1\right]L. \tag{IA.D.22}$$

Plugging this into $\partial\mathscr{L}/\partial V_*$,

$$V_* = \delta_*\left(\gamma+c\right) - \underline{\delta_*\frac{t\phi Q_B d_0}{(1+t)(1-\phi)}} + \left[\delta_*-1\right]La''(L). \tag{IA.D.23}$$

Using the same characterization of ξ_γ,

$$V_* = \delta_* \left(\gamma + c - \frac{t\phi Q_B d_0}{\underline{(1+t)(1-\phi)}} \right) - \left[\delta_* - 1\right] \gamma \xi_\gamma. \tag{IA.D.24}$$

Note that δ_P is the same as in our baseline model so $\delta_* = M\delta_P \geq \delta_P > 1$ and thus $\left[\delta_* - 1\right]\gamma\xi_\gamma > 0$. Likewise, since δ_P, θ_P are unchanged, we again have

$$\delta_P = \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}.$$
$$\theta_P = \frac{1}{1-\phi(1+t)} \frac{t}{Q_B} = \frac{t}{(1+t)(1-\phi)} \delta_P.$$

Finally, recall that in this extension,

$$V_P = \delta_P(\gamma + c) - \theta_P \phi Q_B d_0 = \delta_P \left(\gamma + c - \frac{t\phi Q_B d_0}{(1+t)(1-\phi)} \right). \tag{IA.D.25}$$

Since $\delta_* = M\delta_P > \delta_P$, we have thus shown that

$$V_* = \underbrace{\delta_* \left(\gamma + c - \frac{t\phi Q_B d_0}{(1+t)(1-\phi)} \right)}_{\text{Larger than } V_P} - \underbrace{\left[\delta_* - 1\right] \gamma \xi_\gamma}_{\text{Fire sale}>0}. \tag{IA.D.26}$$

This completes the proof.

E Acquisitions by Arbitrageurs

In our baseline model, we assume that an insolvent firm faces two potential outcomes: reorganization or liquidation. In practice, an insolvent firm might be acquired. Like a liquidation, an acquisition entails an insolvent firm selling all of its assets. Like a continuation, the value of the purchased assets to the acquirer depends on the long-run viability of the insolvent firm, since the acquirer will continue to operate that firm in some form. In this appendix, we consider an extension in which arbitrageurs acquire insolvent firms to continue operating them.

In this extension, we assume that distressed firms can be acquired by arbitrageurs. If an arbitrageur acquires a firm with viability v, the operations can be deployed with enterprise value $1+yv$, where $y \geq 0$ is an exogenous parameter capturing the capabilities of arbitrageurs. The arbitrageurs' technology is the same as before, but the total enterprise value utilized by arbitrageurs is

$$L = (1-p)\int \rho(v)(1+yv)dF(v). \qquad \text{(IA.E.1)}$$

Thus, $y = 0$ reduces to the baseline model.

We derive analogs of Propositions 1 - 2 in this setting. All of the model forces in our baseline model play the same roles in this setting. In particular, banks consider both direct recovery and the value of loanable funds when choosing liquidations. The planner trades off the same externalities. For y not too large, it is clear from inspection that our main results continue to hold.

E.1 Privately optimizing banks

Proposition E.1. *The bank's solution has the same δ_P, θ_P as the solution in the baseline model, but the liquidation threshold is higher:*

$$V_P = \frac{\delta_P}{1-\delta_P \gamma y} \left(\gamma + c \right). \tag{IA.E.2}$$

Proof: Following the same steps used in the proof of Proposition 1, we derive the following private bank Lagrangian:

$$\begin{aligned}\mathscr{L} = p\left(\frac{D_1}{Q_S}+d_0\right)+(1-p)\int_{v\geq V_P} v dF(v)-\frac{B_1}{Q_B}+\theta_P\left(\phi Q_B p(d_0+\frac{D_1}{Q_S})-B_1\right)\\ +\delta_P\left(B_1-b_0+(1-p)\gamma\underline{\int_{v\leq V_P}(1+yv)dF(v)}-\left[pD_1+(1-p)\left(1-F(V_P)\right)c\right]\right).\end{aligned}$$

Differentiating with respect to B_1,

$$0 = -Q_B^{-1} - \theta_P + \delta_P. \tag{IA.E.3}$$

Differentiating with respect to D_1,

$$0 = pQ_S^{-1} + \theta_P \phi Q_B p Q_S^{-1} - \delta_P p. \tag{IA.E.4}$$

Combining these equations,

$$\begin{aligned}\delta_P &= Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1}\\ \theta_P &= -Q_B^{-1} + Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1}\\ \theta_P &= \frac{1}{1-\phi\frac{Q_B}{Q_S}}\left(\frac{1}{Q_S}-\frac{1}{Q_B}\right).\end{aligned}$$

Finally, differentiating with respect to V_P, setting the derivative equal to zero and dividing by $(1-p)f(V_P)$,

$$0 = -V_P + \delta_P \left(\gamma\underline{(1+yV_P)} + c \right), \tag{IA.E.5}$$

so

$$V_P = \frac{\delta_P}{1-\delta_P \gamma y} \left(\gamma + c \right). \tag{IA.E.6}$$

E.2 Socially optimal liquidation

Proposition E.2. *The planner's solution has the same δ_*, θ_*, M as the solution in the baseline model, but the liquidation threshold is:*

$$V_* = \frac{1}{1-\delta_*\gamma y + (\delta_* - 1)\gamma y \xi_\gamma} \left(\delta_*(\gamma + c) - (\delta_* - 1)\gamma \xi_\gamma \right). \tag{IA.E.7}$$

Proof: Since the social planner objective is unchanged and δ_P, θ_P are unchanged, we again see that

$$\delta_P = \frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}$$

and the planner Lagrangian is

$$
\begin{aligned}
\mathscr{L} = & \, p g_S(I_S) + (1-p)\int_{v \geq V_*} v dF(v) + u\left(e - B_1 - p(1+t)D_H\right) \\
& + \underline{a((1-p)\int_{v \leq V_*}(1+yv)dF(v)) - \gamma(1-p)\int_{v \leq V_*}(1+yv)dF(v)} \\
& + \theta_*\left(\phi p Q_B\left[d_0 + \frac{(1+t)(I_S - D_H)}{Q_B}\right] - B_1\right) \\
& + \delta_*\left(B_1 - b_0 + (1-p)\gamma\underline{\int_{v \leq V_*}(1+yv)dF(v)} - (1-p)c(1-F(V_*)) - p\left(I_S - D_H\right)\right) \\
& + \kappa(1 - \frac{Q_B}{1+t} g_S'(I_S)) + \eta\left(u'\left(e - B_1 - p(1+t)D_H\right) - \frac{1}{Q_B}\right) \\
& + \xi\left(a'((1-p)\underline{\int_{v \leq V_*}(1+yv)dF(v)}) - \gamma\right).
\end{aligned}
$$

We now take derivatives with respect to the planner's control variables:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial I_S} &= p g_S'(I_S) + \theta_* \phi p(1+t) - \delta_* p - \kappa \frac{Q_B}{1+t} g_S''(I_S) \\
\frac{\partial \mathscr{L}}{\partial V_*} &= (1-p) f(V_*)\left[-V_* + \mathbf{0} + \delta_*\left(\gamma[\underline{1+yV_*}] + c\right) + \xi a''(\underline{L})[\underline{1+yV_*}]\right] \\
\frac{\partial \mathscr{L}}{\partial B_1} &= -u'\left(e - B_1 - p(1+t)D_H\right) - \theta_* + \delta_* - \eta u''\left(e - B_1 - p(1+t)D_H\right) \\
\frac{\partial \mathscr{L}}{\partial D_H} &= -p(1+t)u'\left(e - B_1 - p(1+t)D_H\right) - \theta_* \phi p(1+t) + \delta_* p - \eta p(1+t) u''\left(e - B_1 - p(1+t)D_H\right) \\
\frac{\partial \mathscr{L}}{\partial Q_B} &= \theta_* \phi p d_0 - \kappa \frac{1}{1+t} g_S'(I_S) + \eta \frac{1}{Q_B^2} \\
\frac{\partial \mathscr{L}}{\partial \gamma} &= (1-p)\underline{\int_{v \leq V_*}(1+yv)dF(v)}\left[-1 + \delta_*\right] - \xi.
\end{aligned}
$$

Since, relative to our baseline model, no derivatives change except for $\frac{\partial \mathscr{L}}{\partial \gamma}$ and $\frac{\partial \mathscr{L}}{\partial V_*}$, we can follow identical steps to the ones used in the proof of Proposition 2 to show M is unchanged:

$$M = \left(1 - \frac{\sigma_S \sigma_H \phi p d_0 Q_B t}{\left[p(1+t)\sigma_H + \sigma_S\right](1-\phi(1+t))} \right)^{-1} \geq 1. \tag{IA.E.8}$$

Finally, we solve for the liquidation threshold. From $\partial \mathscr{L} / \partial \gamma$,

$$\xi = (1-p) \underline{\int_{v \leq V_*} (1+yv) dF(v)} \left[\delta_* - 1\right] = \left[\delta_* - 1\right] L. \tag{IA.E.9}$$

Plugging this into $\partial \mathscr{L} / \partial V_*$,

$$0 = -V_* + \delta_* \left(\gamma[\underline{1+yV_*}] + c \right) + \left[\delta_* - 1\right] L a''(\underline{L})[\underline{1+yV_*}]. \tag{IA.E.10}$$

As before, $\xi_\gamma = \frac{-L}{\gamma} a''(L)$, so this is

$$0 = -V_* + \delta_* \left(\gamma[\underline{1+yV_*}] + c \right) - \left[\delta_* - 1\right][\underline{1+yV_*}]\gamma\xi_\gamma. \tag{IA.E.11}$$

Rearranging,

$$V_* = \frac{1}{1 - \delta_* \gamma y + (\delta_* - 1)\gamma y \xi_\gamma} \left(\delta_*(\gamma + c) - (\delta_* - 1)\gamma\xi_\gamma \right). \tag{IA.E.12}$$

F Acquisitions of Distressed Firms by Solvent Firms

This appendix considers a model in which both solvent firms and arbitrageurs can purchase liquidated assets. Solvent firms choose both investment I_S in their own projects and a quantity L_S of liquidated assets to buy to produce $g_S(I_S) + g_{S,liq}(L_S) + v_S$, where for expositional simplicity we make the technologies separable:

$$\max_{I_S, L_S} g_S(I_S) + g_{S,liq}(L_S) - \frac{I_S + \gamma L_S}{Q_S} + v_S - d_0, \tag{IA.F.1}$$

where $\frac{I_S+\gamma L_S}{Q_S}$ is the required debt repayment to raise $I_S + \gamma L_S$. This implies first order conditions

$$g'_S(I_S) = Q_S^{-1} \tag{IA.F.2}$$

$$g'_{S,liq}(L_S) = \gamma Q_S^{-1}. \tag{IA.F.3}$$

Observe that we therefore have $g'_S/g'_{S,liq} = 1/\gamma$, and so firms shift more towards acquisitions and away from new investments as the liquidation price falls.

Otherwise, the model is the same. In particular, we see the private bank's problem is unchanged, because the bank takes prices as given and is indifferent to the investment composition of the healthy firm (i.e., both projects offer the same collateralizability). We thus obtain the same privately-optimal liquidation rule and multipliers δ_P, θ_P. We now turn to the planner's problem.

F.1 Socially optimal liquidation rule

Proposition F.1. *The planner's socially optimal liquidation threshold is*

$$V_* = \delta_*(\gamma + c) - \hat{\lambda}(\delta_* - 1)\xi_\gamma \gamma. \tag{IA.F.4}$$

where $\hat{\lambda}$ is defined in the proof. If $a'' = 0$, then at the social optimum $V_* > V_P$.

Proof: The planner's objective now incorporates the value produced by solvent firms acquiring distressed firms:

$$p\left(g_S(I_S) + g_{S,liq}(L_S) + v_S\right) + (1-p)\int(1-\rho(v))vdF(v) + a(L) - \gamma L + u\left(e - B_1 - p(1+t)D_H\right).$$

Note that the market clearing conditions are now

$$D_1 + D_H = I_S + \gamma L_S \tag{IA.F.5}$$

$$(1-p)\int \rho(v)dF(v) = L + pL_S \tag{IA.F.6}$$

$$B_H = B_1. \tag{IA.F.7}$$

Plugging in, the planner's Lagrangian is

$$\begin{aligned}
\mathscr{L} = {} & p(g_S(I_S) + \underline{g_{S,liq}(L_S)}) + (1-p)\int_{v \geq V_*} vdF(v) + u\left(e - B_1 - p(1+t)D_H\right) \\
& + a((1-p)F(V_*) - \underline{pL_S}) - \gamma[(1-p)F(V_*) - \underline{pL_S}] \\
& + \theta_*\left(\phi p Q_B\left[d_0 + \frac{(1+t)(I_S + \underline{\gamma L_S} - D_H)}{Q_B}\right] - B_1\right) \\
& + \delta_*\left(B_1 - b_0 + (1-p)\gamma F(V_*) - (1-p)c(1-F(V_*)) - p\left(I_S + \underline{\gamma L_S} - D_H\right)\right) \\
& + \kappa\left(1 - \frac{Q_B}{1+t}g'_S(I_S)\right) + \eta\left(u'\left(e - B_1 - p(1+t)D_H\right) - \frac{1}{Q_B}\right) \\
& + \xi\left(a'((1-p)F(V_*) - \underline{pL_S}) - \gamma\right) + \underline{\upsilon\left(1 - \frac{Q_B}{\gamma(1+t)}g'_{S,liq}(L_S)\right)}.
\end{aligned}$$

We now take derivatives with respect to the planner's control variables:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial I_S} &= pg_S'(I_S) + \theta_* \phi p(1+t) - \delta_* p - \kappa \frac{Q_B}{1+t} g_S''(I_S) \\
\frac{\partial \mathscr{L}}{\partial V_*} &= (1-p) f(V_*) \Big[-V_* + \mathbf{0} + \delta_* \left(\gamma + c \right) + \xi a''((1-p)F(V_*) - \underline{pL_S}) \Big] \\
\frac{\partial \mathscr{L}}{\partial B_1} &= -u' \left(e - B_1 - p(1+t) D_H \right) - \theta_* + \delta_* - \eta u'' \left(e - B_1 - p(1+t) D_H \right) \\
\frac{\partial \mathscr{L}}{\partial D_H} &= -p(1+t) u' \left(e - B_1 - p(1+t) D_H \right) - \theta_* \phi p(1+t) + \delta_* p - \eta p(1+t) u'' \left(e - B_1 - p(1+t) D_H \right) \\
\frac{\partial \mathscr{L}}{\partial Q_B} &= \theta_* \phi p d_0 - \kappa \frac{1}{1+t} g_S'(I_S) - \underline{\upsilon \frac{1}{\gamma(1+t)} g_{S,liq}'(L_S)} + \eta \frac{1}{Q_B^2} \\
\frac{\partial \mathscr{L}}{\partial \gamma} &= (1-p) F(V_*) \big[-1 + \delta_* \big] - \xi + \underline{\theta_* \phi p(1+t) L_S - \delta_* p L_S + \upsilon \frac{Q_B}{\gamma^2 (1+t)} g_{S,liq}'(L_S))} \\
\frac{\partial \mathscr{L}}{\partial L_S} &= \underline{pg_{S,liq}'(L_S) + \mathbf{0} + \theta_* \phi p(1+t) \gamma - \delta_* p \gamma - \upsilon \frac{Q_B}{\gamma(1+t)} g_{S,liq}''(L_S) - \xi pa''((1-p)F(V_*) - pL_S)}.
\end{aligned}
$$

Introducing C^*, σ_H, σ_S as before, applying $g_S' = (1+t)/Q_B$, $u'(C^U) = Q_B^{-1}$, and $g_{S,liq}' = \gamma(1+t)/Q_B$,

$$
\begin{aligned}
0 &= \frac{1+t}{Q_B} + \theta_* \phi (1+t) - \delta_* + \frac{\kappa \sigma_S}{p} \\
0 &= -V_* + \delta_* \left(\gamma + c \right) + \xi a''(L) \\
0 &= -Q_B^{-1} - \theta_* + \delta_* + \eta \frac{\sigma_H}{Q_B} \\
0 &= -Q_B^{-1} - \theta_* \phi + \frac{\delta_*}{1+t} + \eta \frac{\sigma_H}{Q_B} \\
0 &= \theta_* \phi p d_0 Q_B - \kappa - \underline{\upsilon} + \eta \frac{1}{Q_B} \\
0 &= (1-p) F(V_*) \big[-1 + \delta_* \big] - \xi + \underline{\theta_* \phi p (1+t) L_S - \delta_* p L_S + \upsilon \frac{1}{\gamma}} \\
0 &= \underline{p g'_{S,liq}(L_S) + \theta_* \phi p (1+t) \gamma - \delta_* p \gamma - \upsilon \frac{Q_B}{\gamma(1+t)} g''_{S,liq}(L_S) - \xi p a''(L).}
\end{aligned}
$$

Multiply the fourth equation by -1 and add it to the third:

$$
0 = -\theta_*(1-\phi) + \frac{t}{1+t}\delta_* \Rightarrow \theta_* = \frac{t}{(1+t)(1-\phi)}\delta_*. \tag{IA.F.8}
$$

Plugging into the first equation and rearranging,

$$
\kappa = \frac{p}{\sigma_S}\Big[-\frac{1+t}{Q_B} + \delta_* \left(1 - \frac{\phi t}{1-\phi} \right) \Big] \tag{IA.F.9}
$$

Plugging into the third equation and rearranging,

$$
\eta = \frac{Q_B}{\sigma_H}\Big[Q_B^{-1} + \delta_* \left(-1 + \frac{t}{(1+t)(1-\phi)} \right) \Big]. \tag{IA.F.10}
$$

As in the proof of Proposition 2, we have that $a''(L) = \frac{-\gamma}{L}\xi_\gamma$, so we can use the second equation to write

$$V_* = \delta_*(\gamma + c) - \hat{\lambda}(\delta_* - 1)\gamma\xi_\gamma \tag{IA.F.11}$$

for

$$\hat{\lambda} = \frac{\xi}{L(\delta_* - 1)}. \tag{IA.F.12}$$

Now, suppose that $a'' = 0$. Then the last line above implies that

$$\upsilon \frac{Q_B}{\gamma(1+t)} g''_{S,liq}(L_S) = p g'_{S,liq}(L_S) + \theta_* \phi p (1+t)\gamma - \delta_* p \gamma \tag{IA.F.13}$$

$$= p g'_{S,liq}(L_S) + \delta_* p \gamma \left(\frac{t\phi}{(1-\phi)} - 1 \right) \tag{IA.F.14}$$

$$= \frac{p\gamma(1+t)}{Q_B} - \delta_* p \gamma \left(\frac{1 - \phi(1+t)}{(1-\phi)} \right). \tag{IA.F.15}$$

Defining $\sigma_{S,liq} \equiv -g''_{S,liq}/g'_{S,liq}$, we have

$$\upsilon = \frac{p\gamma}{\sigma_{S,liq}} \left(-\frac{1+t}{Q_B} + \delta_* \left(\frac{1 - \phi(1+t)}{(1-\phi)} \right) \right). \tag{IA.F.16}$$

We thus see that

$$\upsilon = \frac{\gamma \sigma_S}{\sigma_{S,liq}} \kappa. \tag{IA.F.17}$$

Plugging this in above,

$$0=\delta_*\frac{\phi p d_0 Q_B t}{(1+t)(1-\phi)}-\frac{p}{\sigma_S}\Big[-\frac{1+t}{Q_B}+\delta_*\Big(1-\frac{\phi t}{1-\phi}\Big)\Big]\underline{\Big(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\Big)}+\frac{1}{\sigma_H}\Big[Q_B^{-1}+\delta_*\Big(-1+\frac{t}{(1+t)(1-\phi)}\Big)\Big] \tag{IA.F.18}$$

$$0=\delta_*\frac{\phi p d_0 Q_B t}{(1+t)(1-\phi)}-\frac{p}{\sigma_S}\Big[-\frac{1+t}{Q_B}+\delta_*\Big(\frac{1-\phi(1+t)}{1-\phi}\Big)\Big]\underline{\Big(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\Big)}+\frac{1}{\sigma_H}\Big[Q_B^{-1}+\delta_*\Big(\frac{-(1-\phi(1+t))}{(1+t)(1-\phi)}\Big)\Big]. \tag{IA.F.19}$$

At this point, recall that

$$\delta_P=\frac{(1+t)(1-\phi)}{Q_B(1-\phi(1+t))}.$$

Define $M\equiv\delta_*/\delta_P$. Then

$$0=M\frac{\phi p d_0 Q_B t}{Q_B(1-\phi(1+t))}-\frac{p}{\sigma_S}\Big[-\frac{1+t}{Q_B}+M\Big(\frac{1+t}{Q_B}\Big)\Big]\underline{\Big(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\Big)}+\frac{1}{\sigma_H}\Big[Q_B^{-1}+M\Big(\frac{-1}{Q_B}\Big)\Big]. \tag{IA.F.20}$$

Cleaning up and rearranging to a common denominator,

$$0=M\frac{\sigma_S\sigma_H\phi p d_0 Q_B t-\Big[p(1+t)\sigma_H\underline{\Big(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\Big)}+\sigma_S\Big](1-\phi(1+t))}{\sigma_S\sigma_H Q_B(1-\phi(1+t))} \tag{IA.F.21}$$

$$+\frac{\Big[p(1+t)\sigma_H\underline{\Big(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\Big)}+\sigma_S\Big](1-\phi(1+t))}{\sigma_S\sigma_H Q_B(1-\phi(1+t))}. \tag{IA.F.22}$$

Moving M to the left and dividing by the fraction multiplying M,

$$M = \frac{\left[p(1+t)\sigma_H \underline{\left(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\right)} + \sigma_S\right](1-\phi(1+t))}{\left[p(1+t)\sigma_H \underline{\left(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\right)} + \sigma_S\right](1-\phi(1+t)) - \sigma_S\sigma_H\phi p d_0 Q_B t} \geq 1. \tag{IA.F.23}$$

Alternatively,

$$M = \left(1 - \frac{\sigma_S\sigma_H\phi p d_0 Q_B t}{\left[p(1+t)\sigma_H \underline{\left(1+\frac{\gamma\sigma_S}{\sigma_{S,liq}}\right)} + \sigma_S\right](1-\phi(1+t))}\right)^{-1} \geq 1. \tag{IA.F.24}$$

This proves that $V_* > V_P$ when $a'' = 0$.

G Differential Bankruptcy Deadweight Losses

This appendix considers a model extension in which liquidations and restructurings lead to deadweight loss. Moreover, we allow for distinct deadweight losses in each outcome.

Specifically, we assume that a fraction ξ_L of the sale proceeds in a liquidation is not received by banks. This might include, for example, the hefty professional fees charged by lawyers in a liquidation (Bris, Welch, and Zhu, 2006; Antill, 2020). Likewise, we assume that a fraction ξ_R of the project value is destroyed in restructuring. This could include direct legal fees, or the indirect financial distress costs of lost consumers or employees (Bris, Welch, and Zhu, 2006; Antill and Hunter, 2022).

The model is otherwise identical to the one described in the main text.

Section G.1 shows that the privately optimal liquidation rule in this setting is:

$$V_P = \delta_P(\gamma\frac{1-\xi_L}{1-\xi_R}+c) = (\frac{1}{Q_B}+\theta_P)(\gamma\frac{1-\xi_L}{1-\xi_R}+c), \tag{IA.G.1}$$

This is intuitive: banks maximize recovery, so when liquidation produces less recovery due to higher relative deadweight losses $\xi_L > \xi_R$, banks liquidate less. The collateral valuation forces are the same as those described in the paper.

Section G.2 shows that the socially optimal liquidation rule in this setting is:

$$V_* = \delta_* \left(\gamma\frac{1-\xi_L}{1-\xi_R}+c \right) - \frac{\delta_*(1-\xi_L)-1}{1-\xi_R}\gamma\xi_\gamma. \tag{IA.G.2}$$

This is equivalent to the socially optimal liquidation rule in the text, except with $(1-\xi_L)/(1-\xi_R)$ terms to capture the change in how liquidations and restructurings affect bank funding constraints. The only other notable difference occurs when liquidation costs are large. As ξ_L approaches 1, liquidations are no longer a meaningful way for banks to recover in insolvency proceedings. At this point, the planner starts pushing for liquidations because fire-sale externalities allow arbitrageurs to obtain assets at low prices with no offsetting cost to banks. This is why the last term can become positive and grow with ξ_γ if ξ_L becomes large. Nonetheless, for reasonable values of ξ_L, our main model insights are robust to including bankruptcy deadweight losses that vary with the bankruptcy resolution approach.

G.1 Privately optimal liquidation

Proposition G.1. *The privately optimal liquidation threshold is*

$$V_P = \delta_P(\gamma\frac{1-\xi_L}{1-\xi_R}+c) = (\frac{1}{Q_B}+\theta_P)(\gamma\frac{1-\xi_L}{1-\xi_R}+c), \tag{IA.G.3}$$

where θ_P, δ_P are the same as the values in the baseline model.

Proof: By the same argument used before, threshold rules are optimal. The private bank Lagrangian is

$$\begin{aligned}\mathscr{L} = {} & p\left(\frac{D_1}{Q_S}+d_0\right)+(1-p)(1-\xi_R)\int_{v\geq V_P} v dF(v)-\frac{B_1}{Q_B}+\theta_P\left(\phi Q_B p(d_0+\frac{D_1}{Q_S})-B_1\right)\\ & +\delta_P\left(B_1-b_0+(1-p)\gamma(1-\xi_L)F(V_P)-\left[pD_1+(1-p)\left(1-F(V_P)\right)(1-\xi_R)c\right]\right).\end{aligned}$$

Differentiating with respect to B_1,

$$0 = -Q_B^{-1}-\theta_P+\delta_P. \tag{IA.G.4}$$

Differentiating with respect to D_1,

$$0 = pQ_S^{-1} + \theta_P \phi Q_B p Q_S^{-1} - \delta_P p. \tag{IA.G.5}$$

Combining these equations, we see that δ_P, θ_P are unchanged.

$$\delta_P = Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1}$$
$$\theta_P = -Q_B^{-1} + Q_S^{-1} + \theta_P \phi Q_B Q_S^{-1}$$
$$\theta_P = \frac{1}{1 - \phi \frac{Q_B}{Q_S}} \left(\frac{1}{Q_S} - \frac{1}{Q_B} \right).$$

Finally, differentiating with respect to V_P, setting the derivative equal to zero, dividing by $(1-p)f(V_P)$, and adding $V_P(1-\xi_R)$ to both sides,

$$V_P(1-\xi_R) = \delta_P(\gamma(1-\xi_L) + c(1-\xi_R)). \tag{IA.G.6}$$

Dividing by $1-\xi_R$,

$$V_P = \delta_P(\gamma \frac{1-\xi_L}{1-\xi_R} + c) = (\frac{1}{Q_B} + \theta_P)(\gamma \frac{1-\xi_L}{1-\xi_R} + c). \tag{IA.G.7}$$

This completes the proof.

G.2 Socially optimal liquidation

Proposition G.2. *The socially optimal liquidation threshold is*

$$V_* = \delta_* \left(\gamma \frac{1-\xi_L}{1-\xi_R} + c \right) - \frac{\delta_*(1-\xi_L) - 1}{1-\xi_R} \gamma \xi_\gamma. \tag{IA.G.8}$$

where $\delta_* = M\delta_P$ for the same M as in the baseline model.

Proof: We assume arbitrageurs do not pay or receive fees, so the social planner objective is the same as before except with $1-\xi_R$ in front of the bank value from restructurings. An identical argument to the one used before shows that threshold rules are optimal, so the social Lagrangian is

$$\begin{aligned}
\mathscr{L} = {} & p g_S(I_S) + \underline{(1-\xi_R)}(1-p)\int_{v\geq V_*} v dF(v) + u\left(e - B_1 - p(1+t)D_H\right) \\
& + a((1-p)F(V_*)) - \gamma(1-p)F(V_*) \\
& + \theta_*\left(\phi p Q_B\left[d_0 + \frac{(1+t)(I_S-D_H)}{Q_B}\right] - B_1\right) \\
& + \delta_*\left(B_1 - b_0 + (1-p)\gamma\underline{(1-\xi_L)}F(V_*) - (1-p)\underline{(1-\xi_R)}c(1-F(V_*)) - p\left(I_S - D_H\right)\right) \\
& + \kappa\left(1 - \frac{Q_B}{1+t}g_S'(I_S)\right) + \eta\left(u'\left(e - B_1 - p(1+t)D_H\right) - \frac{1}{Q_B}\right) \\
& + \xi\left(a'((1-p)F(V_*)) - \gamma\right).
\end{aligned}$$

The last line is the same because the arbitrageurs do not pay fees so fees do not feature in their first-order condition. We now take derivatives with respect to the planner's control variables:

$$\frac{\partial \mathscr{L}}{\partial I_S} = pg_S'(I_S) + \theta_* \phi p(1+t) - \delta_* p - \kappa \frac{Q_B}{1+t} g_S''(I_S)$$
$$\frac{\partial \mathscr{L}}{\partial V_*} = (1-p)f(V_*)\left[-V_* \underline{(1-\xi_R)} + \mathbf{0} + \delta_* \left(\gamma \underline{(1-\xi_L)} + c \underline{(1-\xi_R)} \right) + \xi a''((1-p)F(V_*)) \right]$$
$$\frac{\partial \mathscr{L}}{\partial B_1} = -u' \left(e - B_1 - p(1+t)D_H \right) - \theta_* + \delta_* - \eta u'' \left(e - B_1 - p(1+t)D_H \right)$$
$$\frac{\partial \mathscr{L}}{\partial D_H} = -p(1+t)u' \left(e - B_1 - p(1+t)D_H \right) - \theta_* \phi p(1+t) + \delta_* p - \eta p(1+t)u'' \left(e - B_1 - p(1+t)D_H \right)$$
$$\frac{\partial \mathscr{L}}{\partial Q_B} = \theta_* \phi p d_0 - \kappa \frac{1}{1+t} g_S'(I_S) + \eta \frac{1}{Q_B^2}$$
$$\frac{\partial \mathscr{L}}{\partial \gamma} = (1-p)F(V_*)\left[-1 + \delta_* \underline{(1-\xi_L)} \right] - \xi .$$

Introducing shorthand, gathering terms, simplifying, and applying $g_S' = (1+t)/Q_B$, $u'(C^U) = Q_B^{-1}$,

$$0 = \frac{1+t}{Q_B} - \delta_* + \theta_* \phi (1+t) + \frac{\kappa \sigma_S}{p}$$
$$0 = -V_* \underline{(1-\xi_R)} + \delta_* \left(\gamma \underline{(1-\xi_L)} + c \underline{(1-\xi_R)} \right) + \xi a''(L)$$
$$0 = -Q_B^{-1} - \theta_* + \delta_* + \eta \frac{\sigma_H}{Q_B}$$
$$0 = -Q_B^{-1} - \theta_* \phi + \frac{\delta_*}{1+t} + \eta \frac{\sigma_H}{Q_B}$$
$$0 = \theta_* \phi p d_0 Q_B - \kappa + \eta \frac{1}{Q_B}$$

The first, third, fourth, and fifth equations are unchanged relative to the baseline. Moreover, δ_P is unchanged. We can thus follow identical steps used in the proof of Proposition 2 (See Appendix A) to derive the same M as in the baseline model:

$$M = \left(1 - \frac{\sigma_S \sigma_H \phi p d_0 Q_B t}{\left[p(1+t)\sigma_H + \sigma_S\right](1-\phi(1+t))} \right)^{-1} \geq 1. \tag{IA.G.9}$$

Finally, we solve for the liquidation threshold. From $\partial \mathscr{L} / \partial \gamma$,

$$\xi = (1-p)F(V_*)\left[\delta_* \underline{(1-\xi_L)} - 1\right] = \left[\delta_* \underline{(1-\xi_L)} - 1\right] L. \tag{IA.G.10}$$

Plugging this into $\partial \mathscr{L} / \partial V_*$,

$$V_* \underline{(1-\xi_R)} = \delta_* \left(\gamma \underline{(1-\xi_L)} + c \underline{(1-\xi_R)} \right) + \left[\delta_* \underline{(1-\xi_L)} - 1\right] L a''(L). \tag{IA.G.11}$$

Since arbitrageurs are unaffected by the liquidation cost, we again have $\xi_\gamma = \frac{-L}{\gamma} a''(L)$ and thus

$$V_* \underline{(1-\xi_R)} = \delta_* \left(\gamma \underline{(1-\xi_L)} + c \underline{(1-\xi_R)} \right) - \left[\delta_* \underline{(1-\xi_L)} - 1\right] \gamma \xi_\gamma. \tag{IA.G.12}$$

Dividing by $1 - \xi_R$,

$$V_* = \delta_* \left(\gamma \underline{\frac{1-\xi_L}{1-\xi_R}} + c \right) - \underline{\frac{\delta_*(1-\xi_L) - 1}{1-\xi_R}} \gamma \xi_\gamma. \tag{IA.G.13}$$

H Implementing liquidation subsidies/taxes in practice

Section 4 shows how a social planner can use liquidation taxes or subsidies to achieve the social optimum, depending on the direction of intervention. The planner does not need to observe the long-run viability of individual firms to calculate the optimal subsidy. In this section, we briefly discuss practical methods of implementing such policies. Readers primarily interested in the paper's formal economic analysis may skip this section.[42]

To effectively mitigate the crisis externalities that we model, policymakers need tools that can be quickly implemented. Ideally, a crisis response should not require a lengthy legislative process or new government fundraising, which could create delays. Further, our results imply that policy tools must be able to subsidize liquidation or continuation, depending on the nature of the crisis. We argue below that conditional tax forgiveness for bankrupt firms could feasibly and quickly implement the optimal policy in our model without immediate fundraising.[43]

In response to the COVID-19 pandemic, Blanchard, Philippon, and Pisani-Ferry (2020) proposed that governments could deter liquidations by subsidizing restructurings that allow insolvent firms to continue operating.[44] Specifically, they propose that governments could accept larger write downs or "haircuts" on tax claims than the haircuts accepted by private creditors. Such a policy amounts to subsidizing creditors in any restructuring that results in the continuation of an insolvent firm. However, this form of government subsidy could just as easily be applied to incentivize liquidations: governments accept a haircut on their tax claims in liquidation but not in reorganization. This policy tool is thus an attractive means of implementing the optimal policy of Proposition 3, in which either liquidation or continuation is subsidized. Indeed, the government

[42] In Appendix I, we provide some overview of policies implemented in practice and of related policy proposals.

[43] Of course, forgiven taxes would eventually need to be offset for the government to balance its budget.

[44] Similarly, Greenwood and Thesmar (2020) proposed that the government could create a tax credit for lenders and landlords that agree to a firm-preserving restructuring.

accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3. Interestingly, such a policy also provides the Pigouvian bailout suggested by Section 5.2 and Proposition 6.

Implementing the optimal policy of Proposition 3 through the approach proposed by Blanchard, Philippon, and Pisani-Ferry (2020) would be especially feasible in the US Chapter 11 bankruptcy system. In the US, bankrupt firms frequently owe money to the Internal Revenue Services (IRS) for unpaid taxes.[45] According to 11 U.S.C. §507(a)8, IRS claims receive priority over general unsecured claims. The government could thus increase unsecured creditor recovery by announcing that it would accept plans in which IRS claims receive zero recovery.[46] The government could subsidize a specific bankruptcy outcome, such as continuation or liquidation, by announcing that the subsidy only applies to plans implementing that outcome. In each bankruptcy, the U.S. Trustee could determine whether a plan meets the desired criteria.[47]

The strength of this implementation approach is its flexibility. The government could subsidize liquidation or continuation without novel legislation or fundraising. In the context of our model, these subsidies can improve social welfare in a crisis. However, several caveats are in order. First, targeted tax forgiveness could incentivize nonbankrupt firms to distort their behavior in anticipation of future tax forgiveness.[48] This distortion could lead to suboptimal firm investment and affect government tax revenues. Second, to the extent that tax forgiveness for bankrupt firms must be offset by the government forgoing some future spending, the welfare benefit of the bankruptcy subsidy

[45]For example, the IRS held a $9.5 million claim in Guitar Center's bankruptcy, a $22 million claim in J. Crew's bankruptcy, and a $9.5 million claim in GNC's bankruptcy. See https://cases.primeclerk.com/GNC/Home-ClaimDetails?id=NDQzNDU1MQ==; and https://cases.primeclerk.com/guitarcenter/Home-ClaimDetails?id=NDk2OTYyNQ==; and https://casedocs.omniagentsolutions.com/pocvol1/JCrew/Claim%20Scan/Claims/20-32181/7095000322.pdf.

[46]The government could theoretically do this without hindering plan confirmation because the fair and equitable standard only applies to creditors that do not accept a plan (11 U.S.C. §1129(b)1).

[47]The U.S. Trustee is already tasked with reviewing reorganization plans, see 28 U.S.C. §586(a)3.

[48]For example, this could change the incentives to file for bankruptcy rather than restructuring out of court (Gilson, John, and Lang, 1990; John, Mateti, and Vasudevan, 2013).

could be more than offset by the welfare cost of the future forgone policy. Third and perhaps most importantly, implementable subsidy amounts would be limited by the size of the government's claim in a particular bankruptcy. Thus, while conditional tax forgiveness could feasibly implement the optimal policy in our model, further study is warranted to determine whether the benefits of such a policy outweigh the potential costs.

I Policy Proposal Discussion

I.1 Related policies

In response to previous crises, governments have implemented policies to deter liquidations. In the wake of the COVID-19 pandemic, numerous state and local governments instituted moratoriums on the eviction of commercial tenants. For example, the "COVID-19 Emergency Protect Our Small Businesses Act of 2021," which was signed into law on March 6th 2021, banned evictions and foreclosure actions relating to certain small commercial properties in the state of New York.[49] These moratoriums are analogous to an extreme version of the policy of Proposition 3 in which an infinite tax is levied on liquidations.

Similarly, Section 4013 of the Coronavirus Aid, Relief, and Economic Security (CARES) Act of 2020 encouraged banks to engage in restructurings, rather than liquidations, with distressed borrowers. Prior to this intervention, if banks engaged in troubled debt restructurings (TDRs) with distressed borrowers to avoid liquidations, banks had to categorize the borrowers' loans as impaired.[50] The value of an impaired loan must be revised downward to the expected discounted value of the future cashflows generated by the loan. Such a downward revision of loan value could harm a bank by depleting its regulatory capital, potentially forcing the bank to sell other assets at fire-sale prices to reduce its leverage and satisfy capital requirements (Laux and Leuz, 2010). Thus, the CARES Act encouraged the continuation of insolvent firms by alleviating negative regulatory consequences for banks negotiating with firms that were adversely affected by COVID.[51]

Historically, governments have been more hesitant to subsidize or encourage liquidations dur-

[49] See `https://www.nycourts.gov/eposba/`. A similar rule was passed in the city of San Francisco, see `https://codelibrary.amlegal.com/codes/san_francisco/latest/sf_admin/0-0-0-65019`.

[50] See `https://www.wipfli.com/insights/articles/fi-covid-19-troubled-debt-restructuring`.

[51] The Federal Reserve System and other regulatory institutions provided similar guidance in a statement on April 7, 2020. See `https://www.federalreserve.gov/newsevents/pressreleases/files/bcreg20200407a1.pdf`.

ing a crisis. However, outside of crises, governments have provided incentives for liquidations. For example, asset sales that are part of a liquidating Chapter 11 bankruptcy plan are exempt from any transfer or "stamp" taxes (11 U.S.C. §1146(a)). This tax exemption is effectively a subsidy for liquidations. Other policies have simply made it easier for lenders to liquidate firms. For example, a 2001 reform of Article 9 of the Uniform Commercial Code made it easier for secured lenders to foreclose on assets (Benmelech, Kumar, and Rajan, 2020). Likewise, antirecharacterization passed in states like Texas and Louisiana made it easier for creditors to seize assets associated with bankrupt firms (Ersahin, 2020). Prior to these laws, bankruptcy judges would sometimes recharacterize a debtor's bankrupty remote assets, typically held in a nonbankrupt affiliated special purpose vehicle, as part of the bankrupt firm, subjecting the assets to the automatic stay. By banning this practice of recharacterization, these laws effectively encouraged liquidations.

Internationally, the Czech Republic, France, Germany, Italy, Luxembourg, Portugal, Spain, Switzerland, and Turkey responded to COVID-19 by implementing temporary bankruptcy moratoriums. These interventions prevented creditors from initiating involuntary bankruptcy proceedings against insolvent firms (Gómez et al., 2020). Some of these moratoriums also suspended existing laws imposing personal liability on managers who fail to file for bankruptcy when their firms become insolvent.

I.2 Other Related Policies

The COVID-19 pandemic led to academic proposals for government interventions aimed at mitigating social losses caused by liquidations. For example, as discussed in the main text, Blanchard, Philippon, and Pisani-Ferry (2020) propose that governments could accept lower recovery in restructurings that allow firms to continue operating. Greenwood and Thesmar (2020) propose a tax credit for landlords and lenders that agree to such restructurings. Our model (Proposition 2) implies that such policies are socially optimal if banks are not financially constrained and fire-sale externalities are nontrivial.[52] Given the health of the banking sector during the COVID-19 pandemic (Greenwood, Iverson, and Thesmar, 2020), it is likely that the social planner in our model would enact a similar policy in a crisis resembling the COVID-19 pandemic. More broadly, firms in the US have relied less on bank lending (Crouzet, 2018) in recent years and banks have been well capitalized (Corbae and D'Erasmo, 2021b). According to our model, this bolsters the case for subsidizing continuation in crises. Nonetheless, future crises or crises in different countries could call for the optimal subsidization of liquidation.

DeMarzo, Krishnamurthy, and Rauh (2020) propose that the Federal Reserve and the Treasury create a special purpose vehicle (SPV) to provide DIP loans to bankrupt firms. The SPV would provide highly subsidized loans to bankrupt firms, which would be fully collateralized, priming existing liens if necessary (11 U.S.C. §364(d)). While we do not formally model such a policy, our analysis nonetheless illustrates the potential benefits of the policy proposed by DeMarzo, Krishnamurthy, and Rauh (2020). Recall that when banks are financially constrained, the social planner faces a tradeoff between reorganizing viable firms and preserving funding to solvent firms at reasonable credit spreads. If the government were to exogenously increase the supply of date-1 loans to bankrupt firms at subsidized rates, then more reorganizations could be achieved without exacerbating the loan-price externality, in which privately supplied DIP loans crowd out funding to solvent firms. As such, our model suggests that this proposal may be especially effective when banks are also in financial distress and hence loan price externalities are strong. Outside of our model, it is possible that creditors may force marginally solvent firms into bankruptcy, increasing deadweight losses, in order to take advantage of subsidized government funding. However, this incentive for inefficient behavior would be mitigated by the fact that government DIP loans would

[52] Indeed, the government accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3.

be senior to all unsecured debt, and even potentially secured debt through priming liens.

Finally, other academics proposed policies aimed at paying the debt of all firms to prevent deadweight losses associated with liquidation (Saez and Zucman, 2020; Hanson et al., 2020b). Specifically, Saez and Zucman (2020) recommend expanding unemployment insurance and paying a fraction of the maintenance costs of businesses in sectors that are affected by pandemic-induced shutdowns. Hanson et al. (2020b) propose an intervention in which the government would "provide payment assistance to enable impacted businesses to meet their recurring fixed obligations—including interest, rent, lease, and utility payments." We do not formally model interventions like these that entail paying the expenses of all operating firms in troubled sectors. Such an intervention is analogous to the implementation we describe in Proposition 3, in which the government subsidizes the continuation of insolvent businesses. Unlike the policy described in Proposition 3, these proposed interventions also subsidize the continuation of solvent businesses, making them potentially expensive. However, a policy like ours that makes aid contingent on demonstrated insolvency could incentivize firms to file for bankruptcy, incurring deadweight losses, just to take advantage of the government subsidy. In deciding how to allocate assistance, the government thus faces a tradeoff between the expense of helping solvent firms and the greater incentive for firms to demonstrate their insolvency through value-destroying bankruptcy filings.