Aerospace Measurement and Experimental System Development Characterization

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April 13, 2016

Knowledge Exchange Workshop

Crystal City, Virginia

Overview

- **Aerospace measurement and experimental system development characterization for research and development presents opportunities for**
  - **innovative applications of existing statistical methods**
  - **impetus for statistical research**

Highlighted Methods

- **Inverse Regression**
- **Response Surface Methodology for Characterization**
- **Iterative, Inverse Prediction and Prediction Intervals**
- **Process optimization**

Aerospace R&D Characterization vs. Classical Calibration

Aerospace R&D	Calibration
One-of-a-kind, application specific measurement system	Common, off-the-shelf instrument
Tested in a unique environmental simulation facility, used in flight or flight-like conditions	Tested in laboratory controlled conditions, used in secondary controlled laboratory
Known, traceable standards are often not available	Physical, NIST traceable standards
Multiple-sensing device	Element, measuring one property
Multi-dimensional response surface	Simple linear regression
Inverse regression broadly used by engineers due to simplicity	Classical regression, inverse solution

Simple Linear, Single Measurement
1 Factor, 1 Response, 1st-Order

- **Airborne Subscale Transport Aircraft Research (AirSTAR)**
- **Dynamically scaled, commercial aircraft to**
  - **study control-upset conditions**
  - **improve pilot training**

- **Pressure measurement system for altitude and airspeed on wing tips of vehicle**
- **System testing performed in laboratory, used in open-air flight**

$$V = b_0 + b_1 P + \epsilon$$

Classical and Inverse Regression

1 Factor, 1 Response, 1st-Order

Classical Regression Model

$$y = b_0 + b_1 x + \epsilon$$

Inverse Application

$$\hat{x} = \frac{y - \hat{\beta}_0}{\hat{\beta}_1}$$

Inverse Regression Model

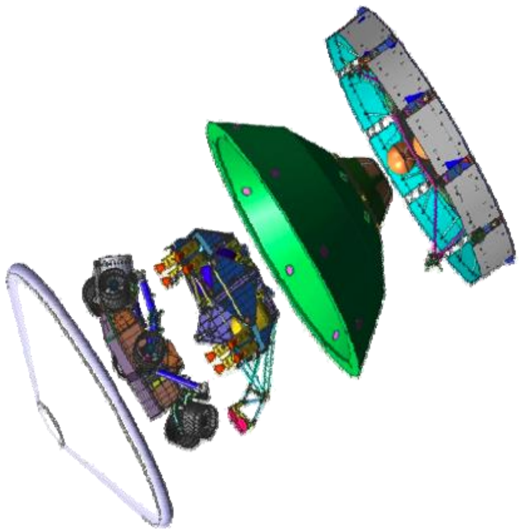
$$x = g_0 + g_1 y + \epsilon$$

- **Reversing the role of the x and y is commonly done in practice**
- **Both approaches lead to biased predictions, classical is slightly better**
- **Prediction intervals have essentially correct coverage probabilities**
- **Inverse interval width is slightly smaller and less variable**

Parker, Vining, Wilson, et al. (2010) "The Prediction Properties of Classical and Inverse Regression for the Simple Linear Calibration Problem," *Journal of Quality Technology*

Multi-dimensional Response Surface

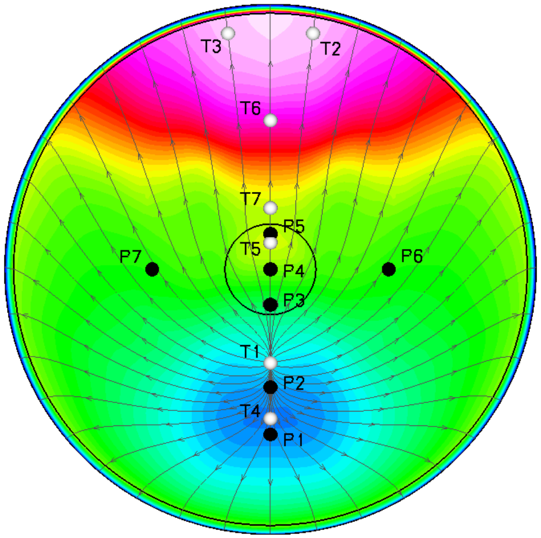
3 Factors, 1 Response, 2nd-Order



- **Reduce uncertainty in pre-flight landing ellipse estimation through measurements during Mars entry**
- **Pressure measurements during extreme atmospheric entry temperature conditions**

- **Pressure measurement system is sensitive to temperature**
- **Signal (V) as a function of pressure and two temperatures**
- **Second-order Response Surface**

$$V = f\left(P,\ T_1,\ T_2\right) + \epsilon$$



Parker, Hutchinson, Mitchell, and Munk (2008) “Strategic Pressure Measurement System Characterization of the Mars Entry Atmospheric Data System,” *6th International Planetary Probe Workshop*

Response Surface for Characterization

3 Factors, 1 Response, 2nd-Order

- **Response Surface Methods for a non-traditional application**
  - **Characterization, not optimization**
  - **Efficiency in achieving absolute predication variance, not per point**
  - **Mathematical model delivered, not optimized factor settings**
  - **Confirmation points to test the model over the entire design space, not sensitivity to the location of optimum performance**
- **Inverse Prediction of Second-Order Response Surface**
  - **Iterative procedure employed, (direct, quadratic formula issues)**

$$\hat{P} = \hat{f}\left(V,\ T_1,\ T_2\right) = f\left(\underline{\hat{\beta}},\ V,\ T_1,\ T_2,\ \ldots,\ \frac{\hat{\beta}_{11}}{\hat{\beta}_1}\hat{P}^2,\ \frac{\hat{\beta}_{12}}{\hat{\beta}_1}\hat{P}xT_1,\ \ldots\right)$$

  - **Approximate inverse prediction intervals from the Delta Method**

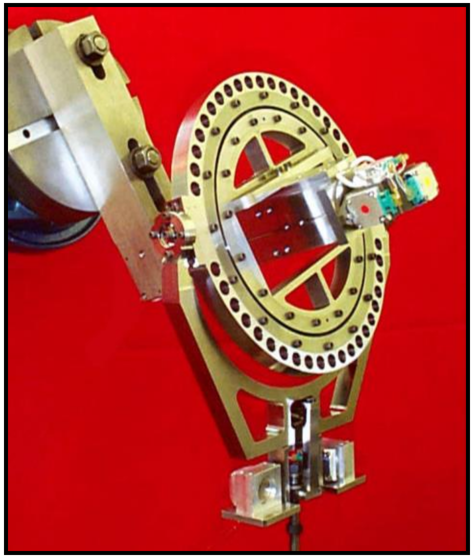
$$\left(\partial\hat{P}(\hat{\mathbf{x}})\right)^T\left(\mathbf{X}'\hat{\Sigma}^{-1}\mathbf{X}\right)^{-1}\left(\partial\hat{P}(\hat{\mathbf{x}})\right)$$

Parker and Commo (2010) "MEADS System Calibration Modeling, Flight Data Reduction, and Pressure Uncertainty," *NASA Engineering Report*

Multi-input, Multi-output, Higher Order

6 Factors, 6 Responses, 2nd and higher

- **Multi-component force transducers used in aerospace research and development**
- **Sensing 3 forces and 3 moments, simultaneously**
- **No system calibration standards**



- **Modeling**

$$\sum_{i=1}^{6} y_i = f_i\left(\left[x_1 \cdots x_6\right]\right) + e_i$$

- **Inverse prediction**

$$\left[\hat{x}_1 \cdots \hat{x}_6\right] = \hat{F}\left(\left[y_1 \cdots y_6\right]\right)$$

Parker and Finley (2010) "Advancements in Aircraft Model Force and Attitude Instrumentation by Integrating Statistical Methods," *AIAA Journal of Aircraft*

Multivariable Response Surface

6 Factors, 6 Responses, 2nd and higher

- **Internationally, some use inverse regression**
  - **simplified, direct solution – properties are not well-defined**
- **Multivariate version of Delta Method, inverse prediction intervals**

$$\left[\operatorname{var}\left(\hat{x}_1\right)\cdots\operatorname{var}\left(\hat{x}_6\right)\right]^T = \left[\operatorname{var}\left(\hat{y}_1(\hat{\mathbf{x}})\right)\cdots\operatorname{var}\left(\hat{y}_6(\hat{\mathbf{x}})\right)\right]^T \left[\frac{\partial \hat{F}(\hat{\mathbf{x}})}{\partial \mathbf{x}}\right]^{-1}$$

Response surfaces of cubic or higher are feasible

- **Cubic designs based on combining two second-order designs**
- **Design, modeling, inverse prediction extended to higher order**

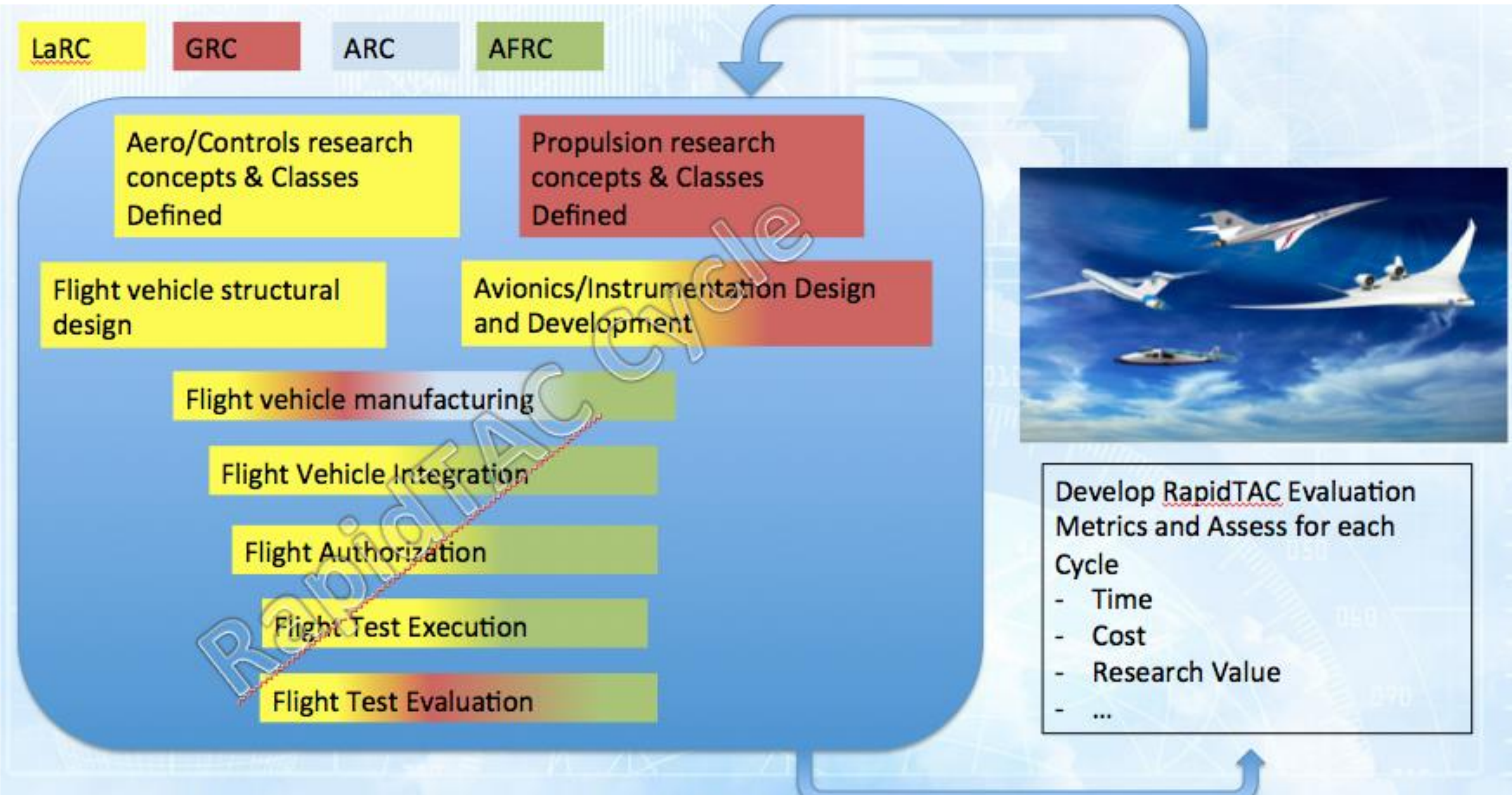
Tripp and Tcheng (1999) "Uncertainty Analysis of Instrument Calibration and Application," *NASA Technical Publication*

Draper (1960) "Third Order Rotatable Designs in Three Dimensions," *The Annals of Mathematical Statistics*

Experimental System Development

Example: Rapid Test of Aeronautics Concepts (RapidTAC)

- Process optimization with unique experiments
- Quantification of research value and complexity



Concluding Remarks

- **Aerospace measurement and experimental system development characterization for research and development**
  - **similar to classical calibration in concept**
  - **requires adaption and extension of existing statistical methods**

- **Methods highlighted**
  - **Inverse Regression**
  - **RSM for Characterization**
  - **Inverse Prediction and Intervals**
  - **Process optimization**