Aerospace Measurement and Experimental System Development Characterization

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Overview

• Aerospace measurement and experimental system development characterization for research and development presents opportunities for
  – innovative applications of existing statistical methods
  – impetus for statistical research

Highlighted Methods
• Inverse Regression
• Response Surface Methodology for Characterization
• Iterative, Inverse Prediction and Prediction Intervals
• Process optimization
## Aerospace R&D Characterization vs. Classical Calibration

<table>
<thead>
<tr>
<th>Aerospace R&amp;D</th>
<th>Calibration</th>
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<tbody>
<tr>
<td>One-of-a-kind, application specific measurement system</td>
<td>Common, off-the-shelf instrument</td>
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<tr>
<td>Tested in a unique environmental simulation facility, used in flight or flight-like conditions</td>
<td>Tested in laboratory controlled conditions, used in secondary controlled laboratory</td>
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<td>Known, traceable standards are often not available</td>
<td>Physical, NIST traceable standards</td>
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<tr>
<td>Multiple-sensing device</td>
<td>Element, measuring one property</td>
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<tr>
<td>Multi-dimensional response surface</td>
<td>Simple linear regression</td>
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<tr>
<td>Inverse regression broadly used by engineers due to simplicity</td>
<td>Classical regression, inverse solution</td>
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Simple Linear, Single Measurement
1 Factor, 1 Response, 1\textsuperscript{st}-Order

- Airborne Subscale Transport Aircraft Research (AirSTAR)
- Dynamically scaled, commercial aircraft to
  - study control-upset conditions
  - improve pilot training

- Pressure measurement system for altitude and airspeed on wing tips of vehicle
- System testing performed in laboratory, used in open-air flight

\[ V = b_0 + b_1P + e \]
Classical and Inverse Regression
1 Factor, 1 Response, 1st-Order

Classical Regression Model

\[ y = \beta_0 + \beta_1 x + e \]

Inverse Application

\[ \hat{x} = \frac{y - \hat{\beta}_0}{\hat{\beta}_1} \]

Inverse Regression Model

\[ x = \beta_0 + \beta_1 y + e \]

- Reversing the role of the \( x \) and \( y \) is commonly done in practice
- Both approaches lead to biased predictions, classical is slightly better
- Prediction intervals have essentially correct coverage probabilities
- Inverse interval width is slightly smaller and less variable

Multi-dimensional Response Surface
3 Factors, 1 Response, 2nd-Order

- Pressure measurement system is sensitive to temperature
- Signal (V) as a function of pressure and two temperatures
- Second-order Response Surface

\[ V = f(P, T_1, T_2) + e \]

- Reduce uncertainty in pre-flight landing ellipse estimation through measurements during Mars entry
- Pressure measurements during extreme atmospheric entry temperature conditions

Response Surface for Characterization
3 Factors, 1 Response, 2\textsuperscript{nd}-Order

- Response Surface Methods for a non-traditional application
  - Characterization, not optimization
  - Efficiency in achieving absolute predication variance, not per point
  - Mathematical model delivered, not optimized factor settings
  - Confirmation points to test the model over the entire design space, not sensitivity to the location of optimum performance

- Inverse Prediction of Second-Order Response Surface
  - Iterative procedure employed, (direct, quadratic formula issues)
  \[ \hat{P} = \hat{f}(V, T_1, T_2) = f_{\hat{b}}, V, T_1, T_2, \quad \frac{11}{\hat{P}^2}, \quad \frac{12}{\hat{P}xT_1}, \quad \div \]
  - Approximate inverse prediction intervals from the Delta Method
  \[ \left( \hat{P}(\hat{x}) \right)^T \left( X' \hat{X} \right)^{-1} \left( \hat{P}(\hat{x}) \right) \]

Multi-input, Multi-output, Higher Order
6 Factors, 6 Responses, 2\textsuperscript{nd} and higher

- Multi-component force transducers used in aerospace research and development
- Sensing 3 forces and 3 moments, simultaneously
- No system calibration standards

Modeling

\[
y_i = f_i ([x_1 \cdots x_6]) + \epsilon_i
\]

\[
\hat{x}_1 \cdots \hat{x}_6 = \hat{F} ([y_1 \cdots y_6])
\]


**Multivariable Response Surface**

*6 Factors, 6 Responses, 2\textsuperscript{nd} and higher*

- Internationally, some use inverse regression
  - simplified, direct solution – properties are not well-defined
- Multivariate version of Delta Method, inverse prediction intervals

\[
\begin{bmatrix}
\text{var}(\hat{x}_1) & \cdots & \text{var}(\hat{x}_6)
\end{bmatrix}^T = \begin{bmatrix}
\text{var}(\hat{y}_1(\hat{x})) & \cdots & \text{var}(\hat{y}_6(\hat{x}))
\end{bmatrix}^T \frac{\hat{F}(\hat{x})}{x}
\]

Response surfaces of cubic or higher are feasible

- Cubic designs based on combining two second-order designs
- Design, modeling, inverse prediction extended to higher order

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Experimental System Development

Example: Rapid Test of Aeronautics Concepts (RapidTAC)
- Process optimization with unique experiments
- Quantification of research value and complexity
Concluding Remarks

• Aerospace measurement and experimental system development characterization for research and development
  – similar to classical calibration in concept
  – requires adaption and extension of existing statistical methods

• Methods highlighted
  – Inverse Regression
  – RSM for Characterization
  – Inverse Prediction and Intervals
  – Process optimization