



**UNIVERSITY
OF ICELAND**

14-10-2026

A Finite Element Glacial Isostatic Adjustment Model of Iceland

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Outline



- What it is
- Why we want to model it
- What we need to model it

Glacial Isostatic Adjustment

A GIA model for Iceland

- Iceland as an interesting geological setting
- Our GIA model in Iceland

- Earth parameters
- Magma production

Results

Future work

- Future scenarios for infrastructure planning

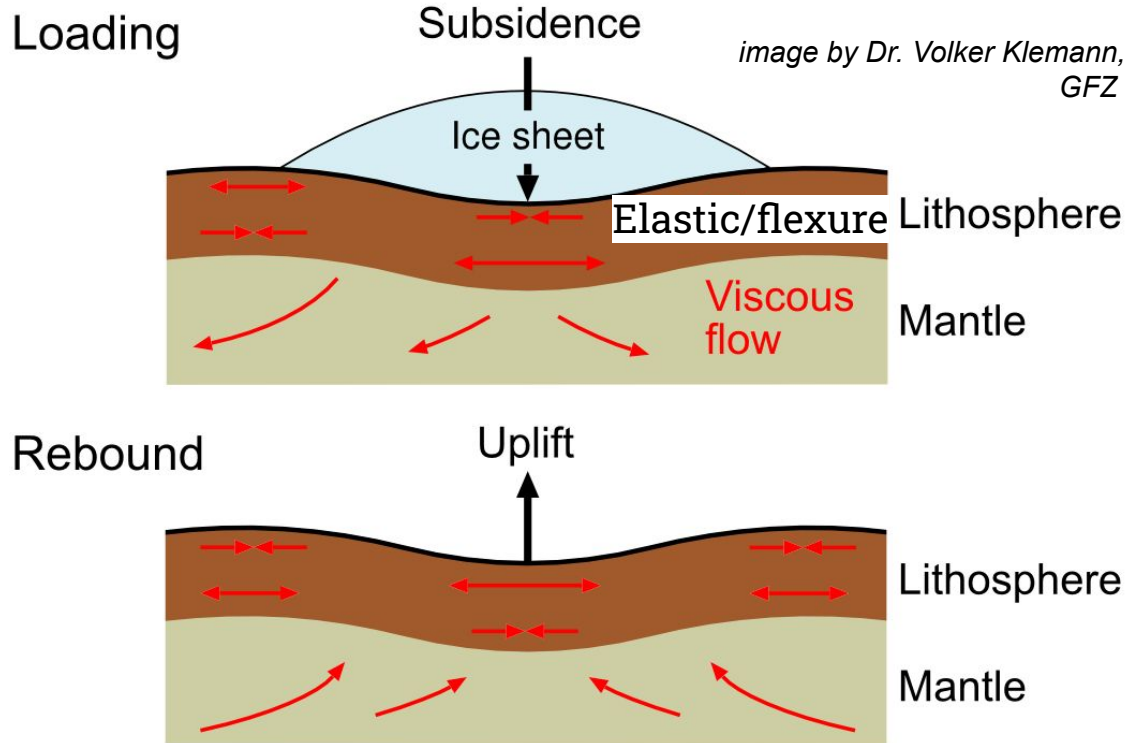
Glacial Isostatic Adjustment



The sofa analogy

Glacial Isostatic Adjustment

GIA is the
response of the
Earth to changes
in loading,
during a glacial
cycle.



GIA modelling: Why?

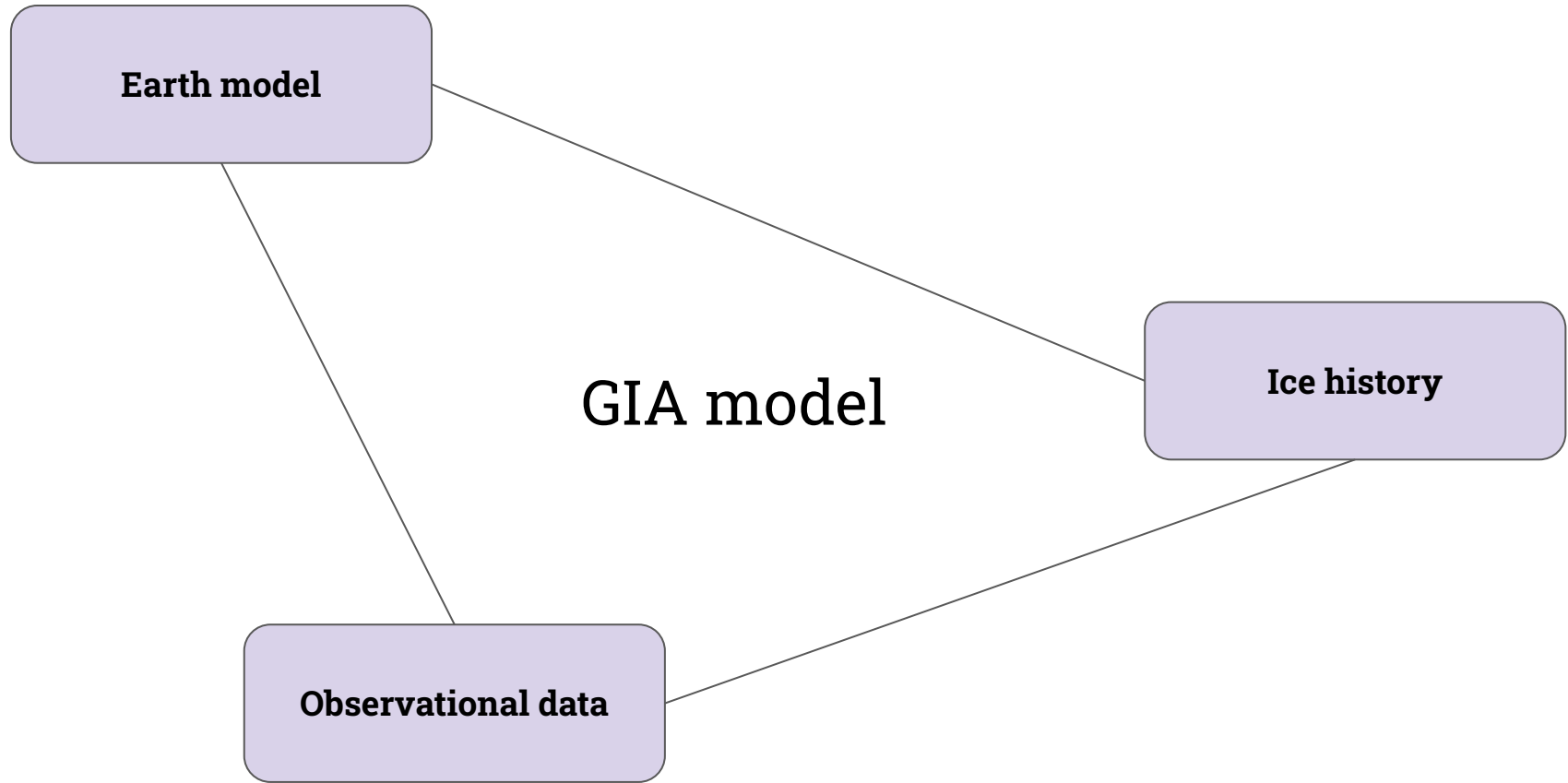
Constrain
mantle
VISCOSITY

To **remove the GIA**
signal from
deformation signals
due to other causes.

Study the effect of the
GIA-induced stress on:

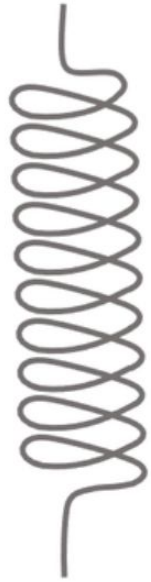
- seismicity,
- magma production,
- magma reservoir instability
- magma propagation
- sea level changes

...and more





Elastic behaviour:
instantaneous
response



mechanical
analogue:
spring

Rheology

ῥέω

flow

λογία

the study of

Viscous
behaviour:
delayed response



mechanical
analogue:
damper

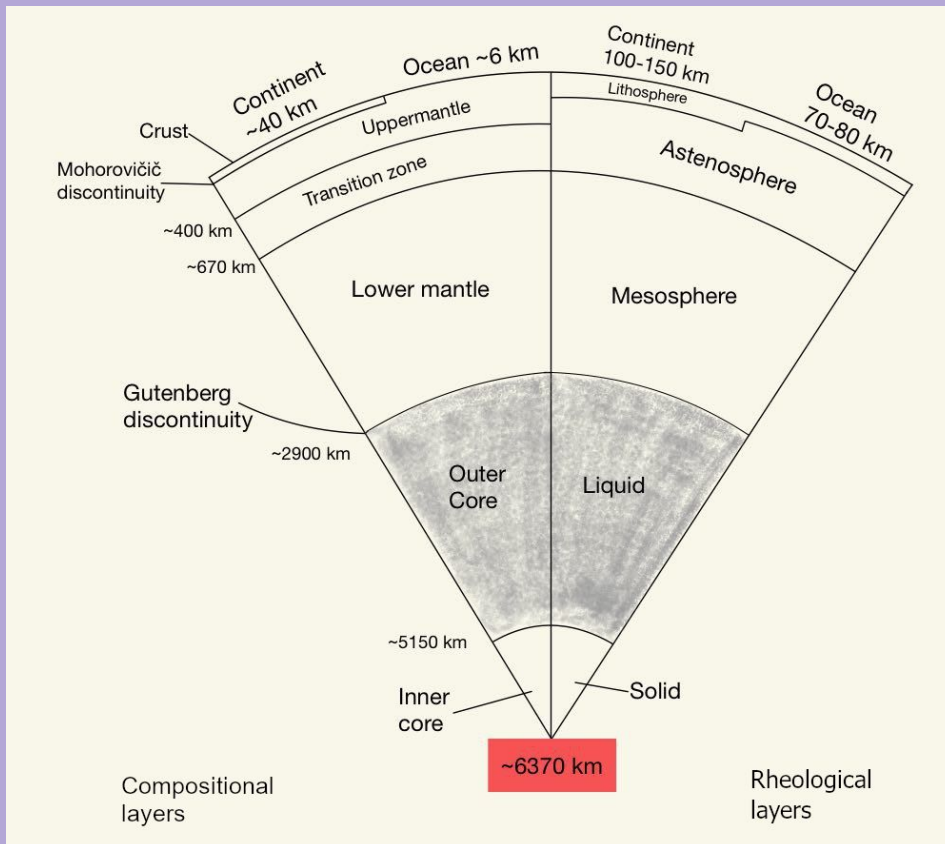
Viscosity

η



The rheology of the Earth

Reproduction
from Van der
Pluijm and
Marshak
(2004)



Mechanical
behaviour
of the Earth

It studies how
the Earth
deforms.

The rheology

Elastic

$$\epsilon_{ij} = \frac{1}{E} [\sigma_{ij}(1 + \nu) - \nu \delta_{ij} \sigma_{kk}]$$

ϵ = strain

$\dot{\epsilon}$ = strain rate

ν = Poisson's ratio

Viscoelastic (Maxwellian)

$$\dot{\epsilon} = \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{E}$$

σ = stress

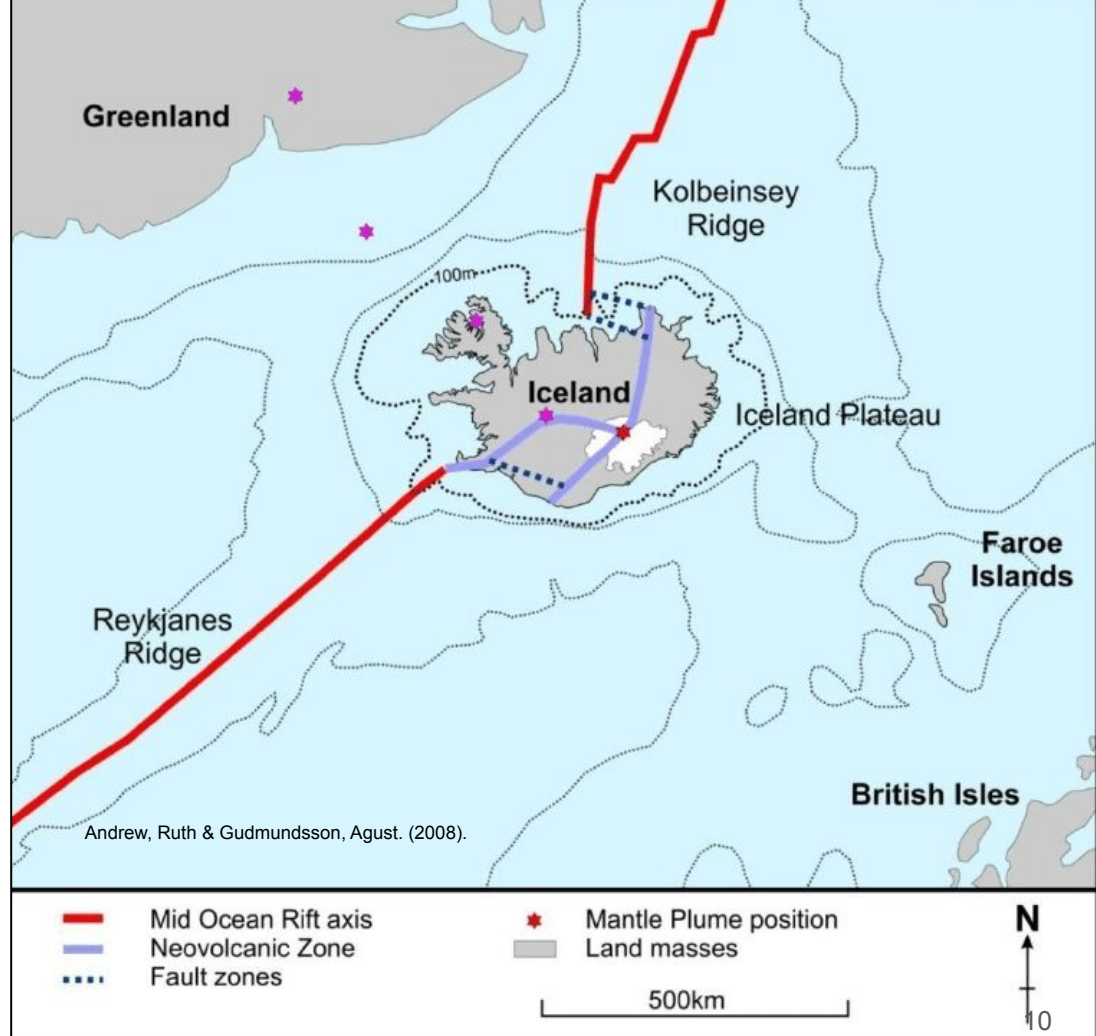
E = Young's modulus

η = viscosity



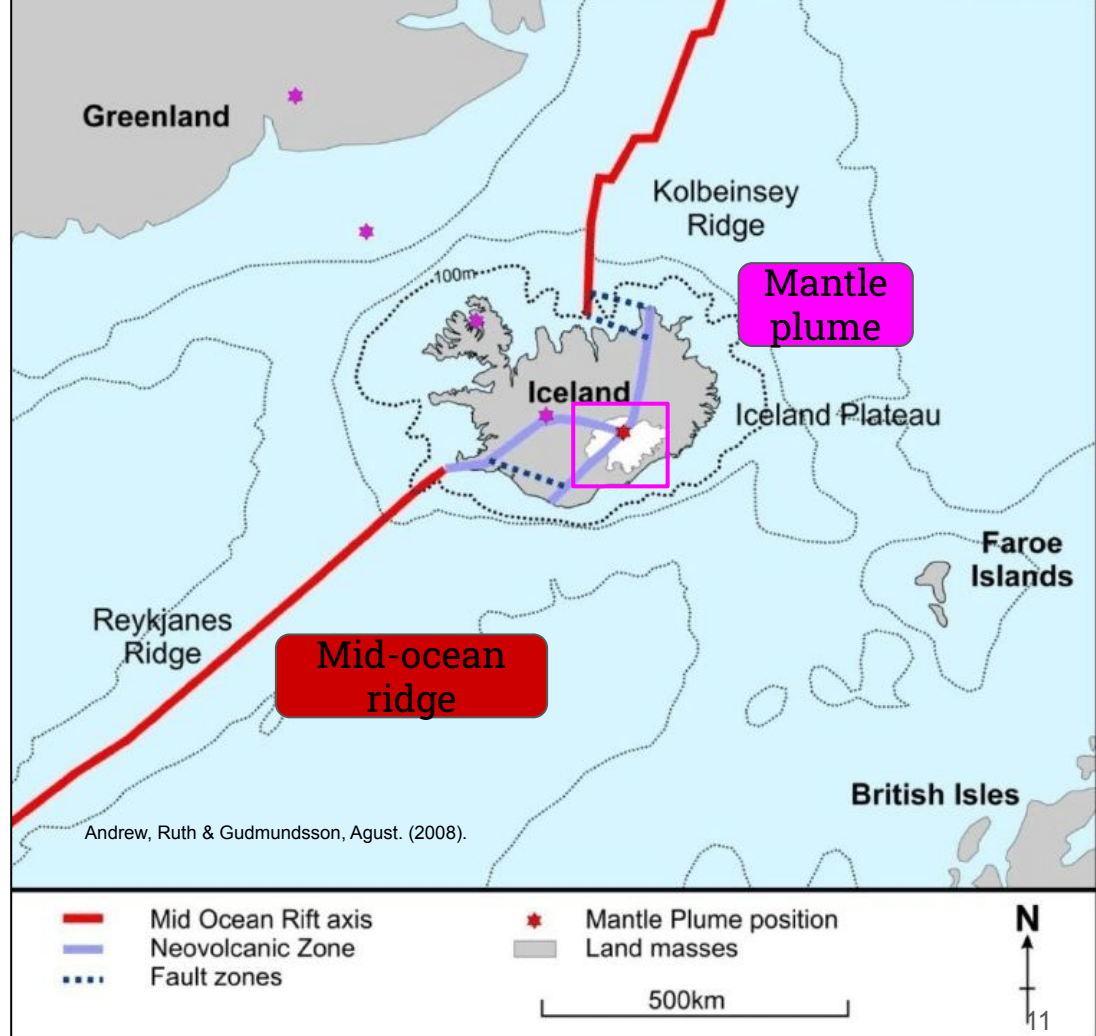
An interesting rheological setting

- Mantle viscosity $< 10^{19}$ Pa s,
(Sigmundsson et al 1991)
- Thin lithosphere
(10-40 km)
- Ongoing deglaciation



An interesting rheological setting

- Mantle viscosity $< 10^{19}$ Pa s,
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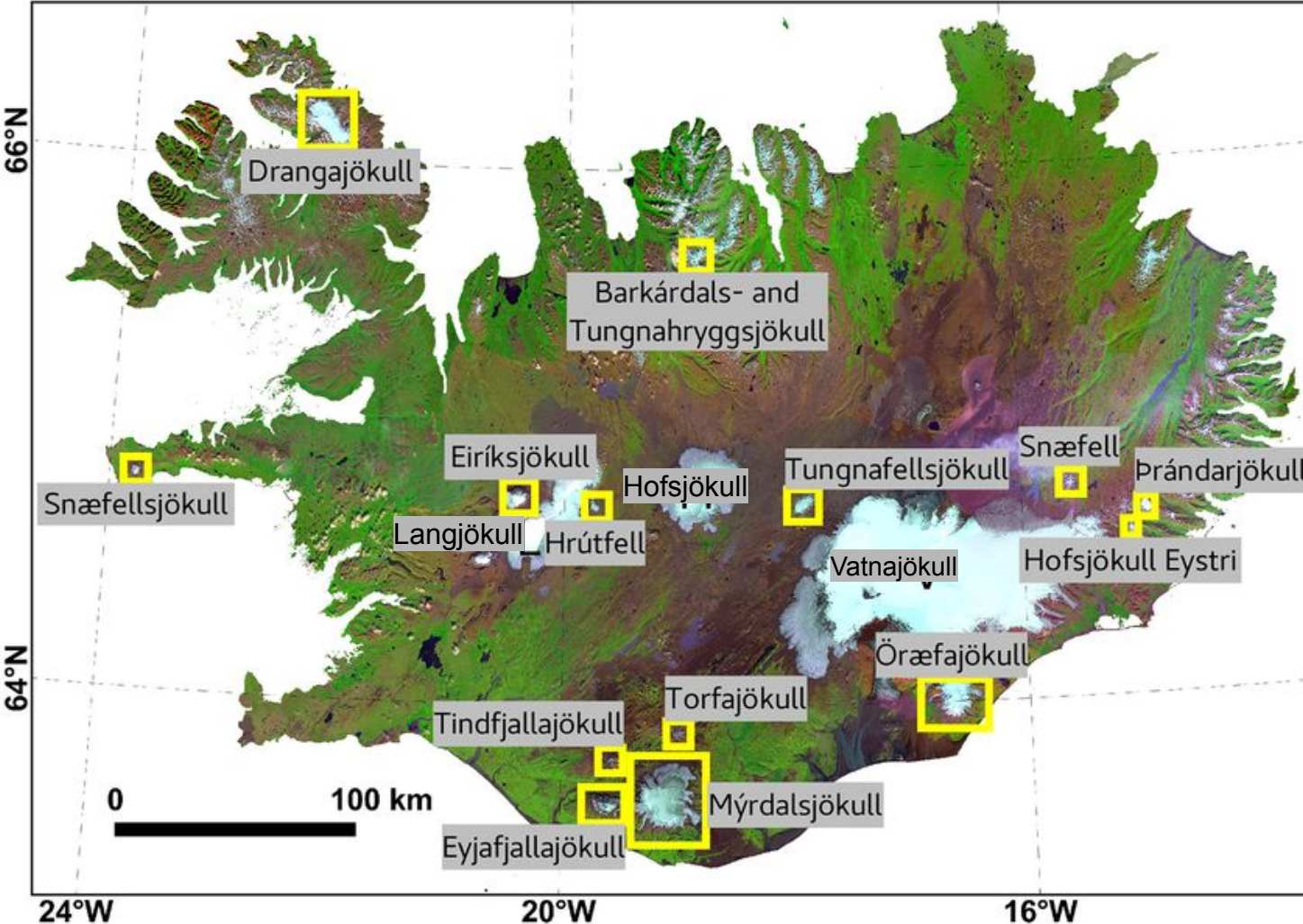
An interesting rheological setting

- Mantle viscosity $< 10^{19}$ Pa s (Sigmundsson et al 1991)
- Thin lithosphere (10-40 km)
- Ongoing deglaciation

Vatnajökull is the largest glacier in Europe



The major glaciers of Iceland

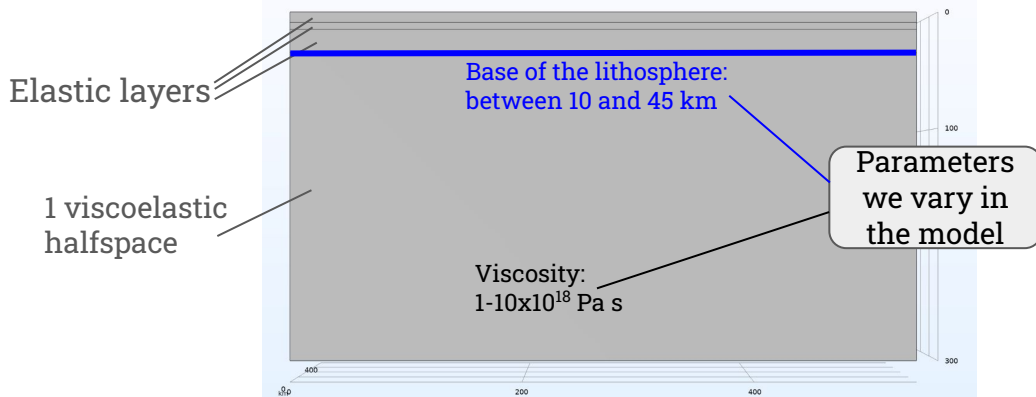
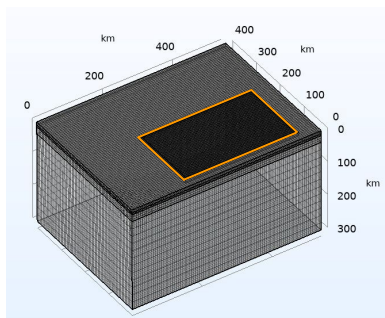


Modified from:
Belart et al
(2020).

A GIA FE model for Iceland

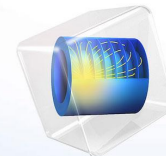
The Earth model

- Flat Earth approximation
- One dimensional
- Elastic + viscoelastic
- Linear rheology (Maxwellian)
- Material compressibility
- Spring foundations
- 600x400 km + expansion area

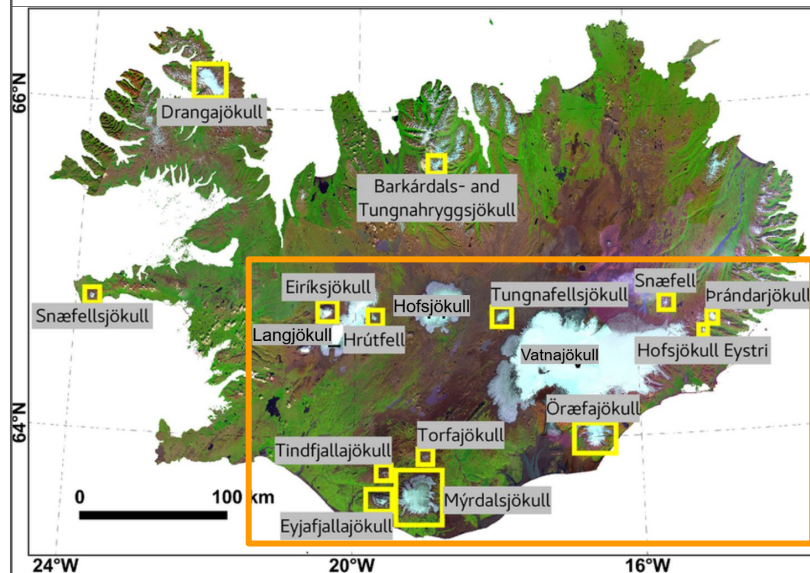


Continuation of
the model by
Schmidt 2012
(Abaqus)

COMSOL
MULTIPHYSICS®



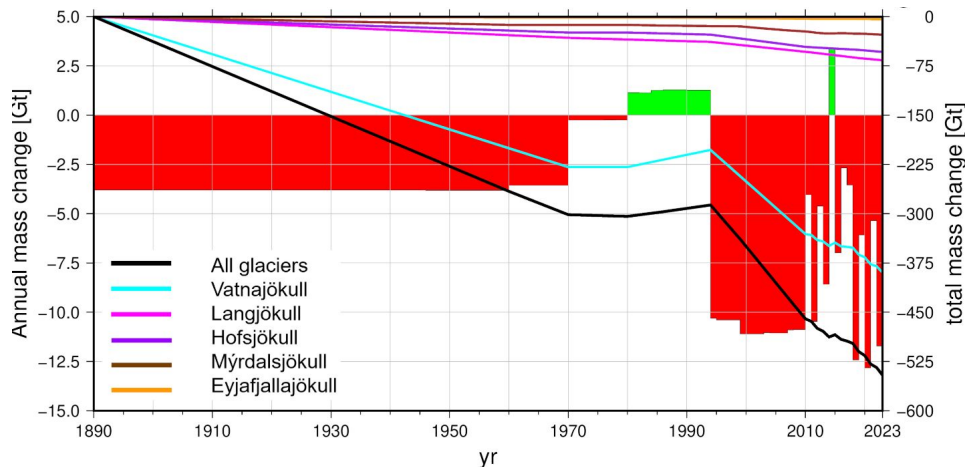
The ice



Belart, Joaquín M.C. & Magnússon, Eyjólfr & Berthier, Etienne & Gunnlaugsson, Ágúst & Pálsson, Finnur & Adalgeirsdóttir, Guðfína & Jóhannesson, Tómas & Thorsteinsson, Thorsteinn & Björnsson, Helgi (2020). Mass Balance of 14 Icelandic Glaciers, 1945–2017: Spatial Variations and Links With Climate. *Frontiers in Earth Science*. 8. 10.3389/feart.2020.00163.

Ice Load History

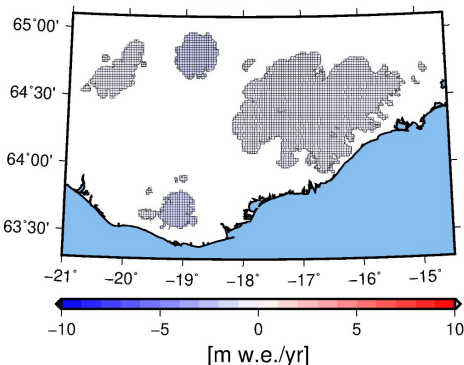
G. Adalgeirsdóttir et al 2020, expanded by E. Magnússon



The ice-load history of Vatnajökull from 2010 to 2023 features spatial resolution.

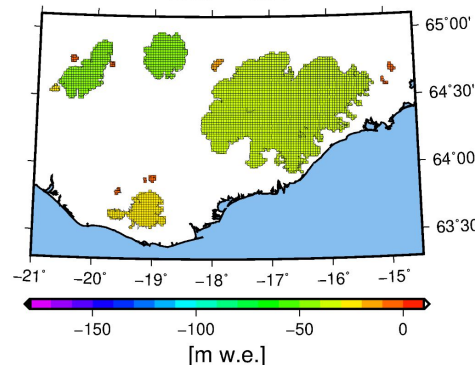
Yearly ice load change

2003 – 2004

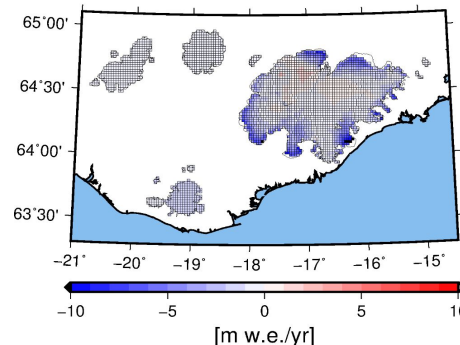


Cumulative ice load change

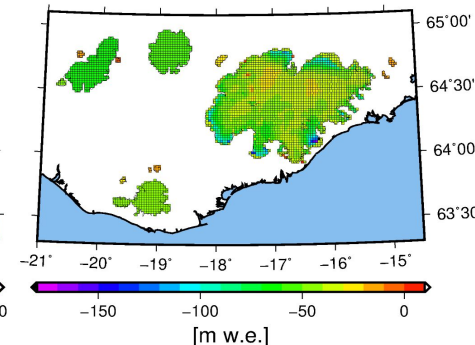
1890 – 2004



2022 – 2023



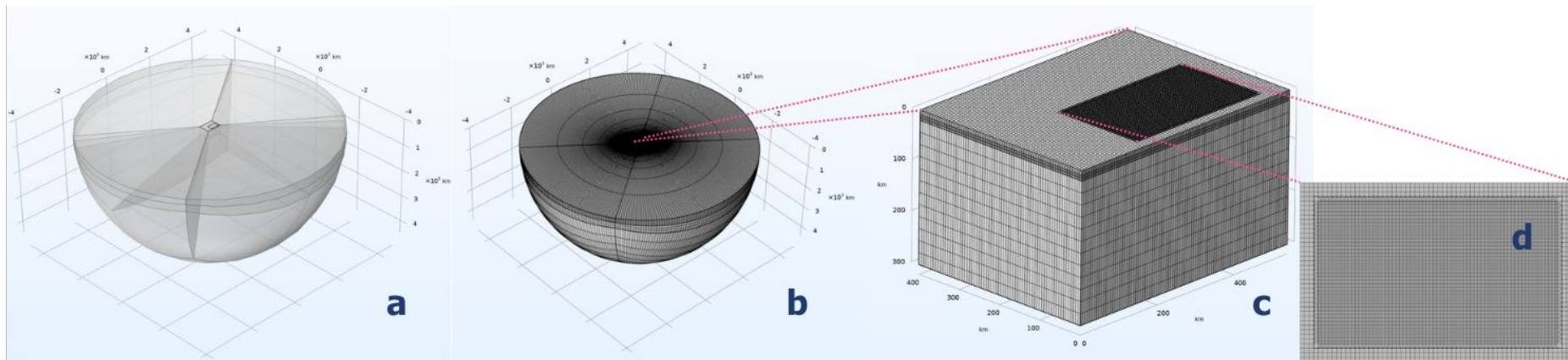
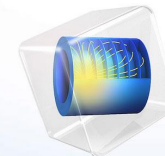
1890 – 2023



A GIA FE model for Iceland: long run time!

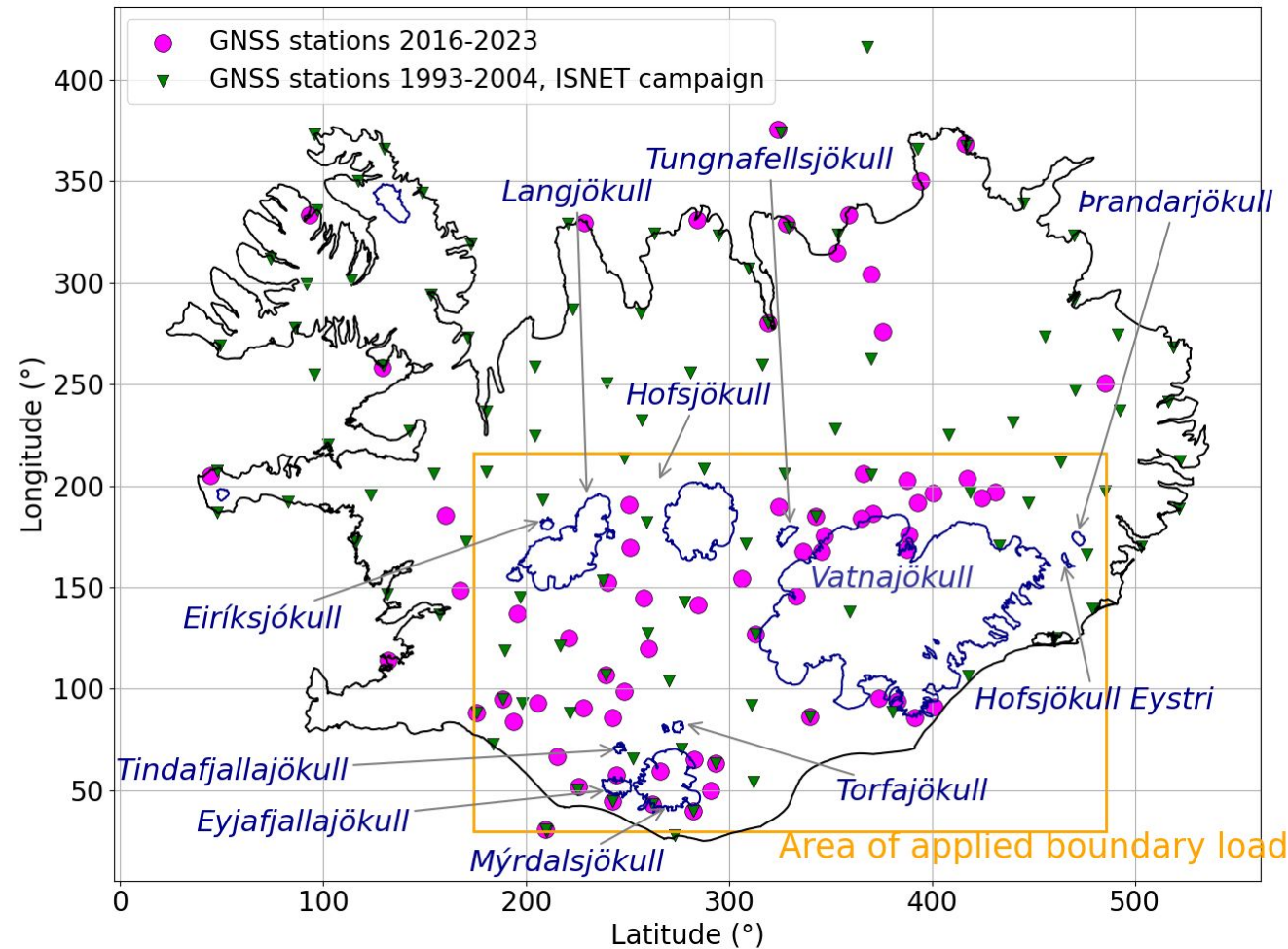
IREI
Icelandic Research
E-Infrastructure

COMSOL
MULTIPHYSICS®



Run time:

- 3-4 h
- 22-24 h for the mantle magma production model (finer mesh)



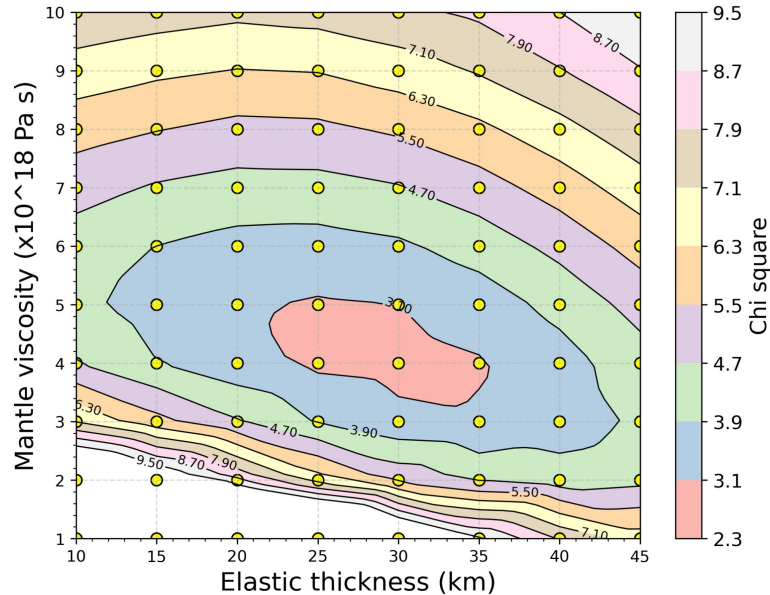
Data comparison: Global Navigation Satellite System Data (GNSS)

Chi-square statistics

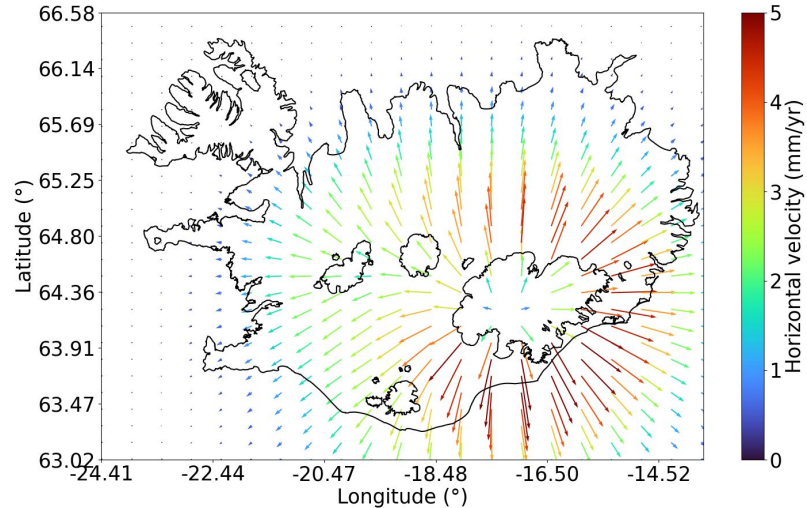
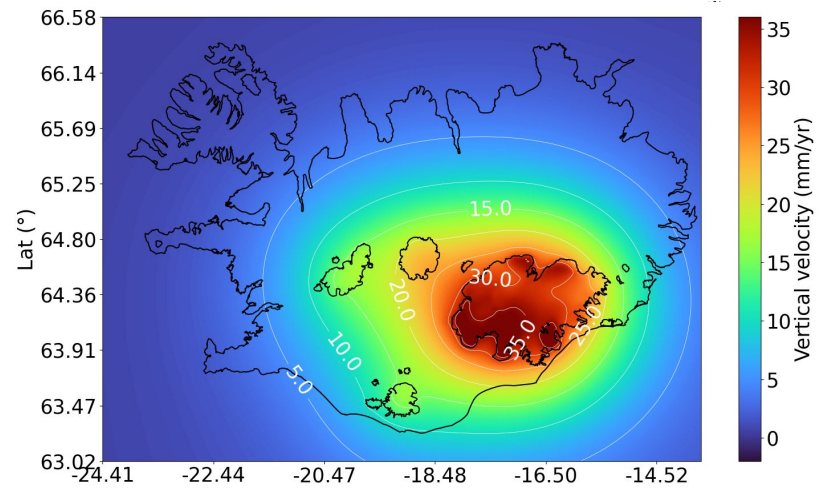
$$\chi^2 = \frac{1}{n-m} \sum \left(\frac{\text{model} - \text{observable}}{\text{uncertainty}} \right)^2$$

Model output and data comparison

Combined dataset



**Best-fit
model:**
Elastic
thickness:
30 km
Viscosity:
 4×10^{18} Pa s



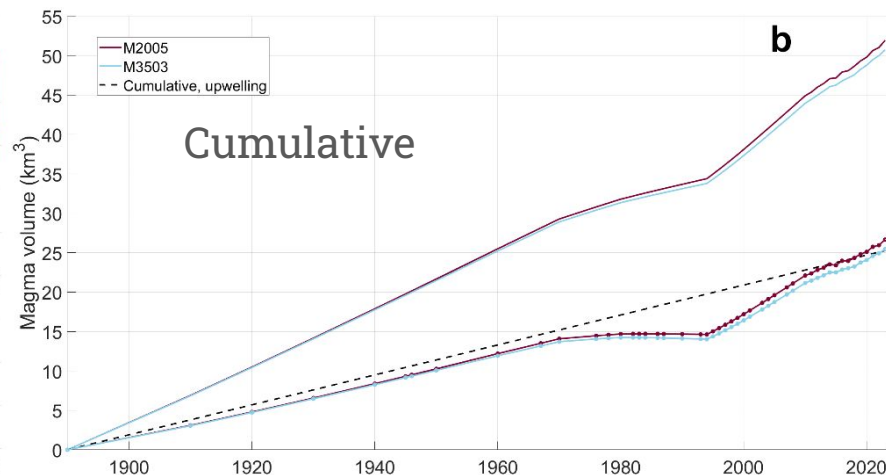
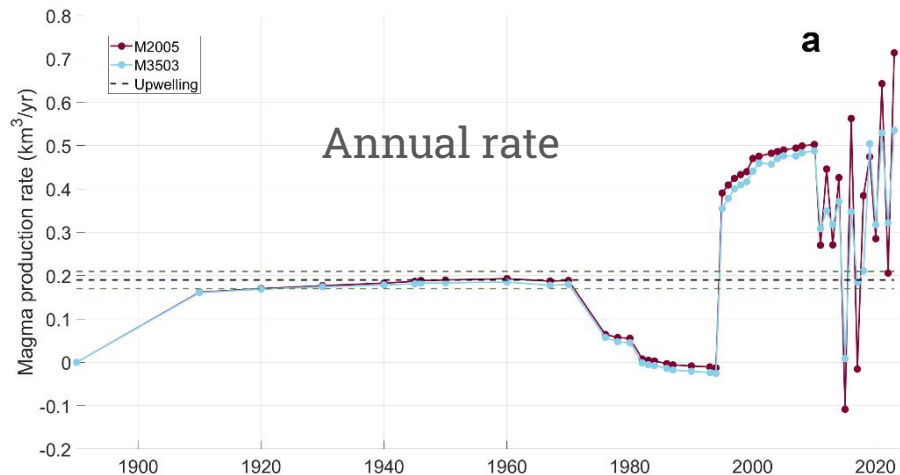
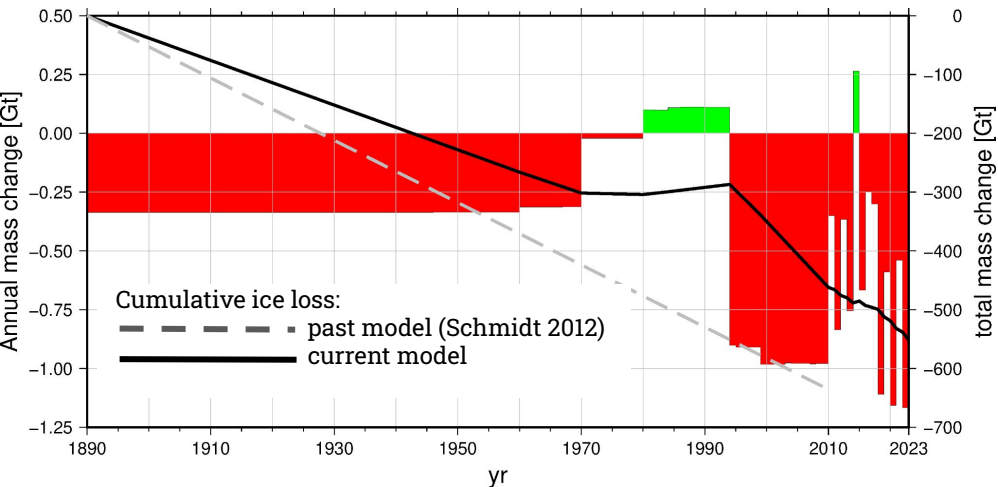
Glacial Isostatic Adjustment

A model for Iceland

Results

Future work

Mantle melt production



Magma volumes of past eruptions

(J. Belart, personal communication)

Barsotti et al (2020)

Þorðarson et al (1992)

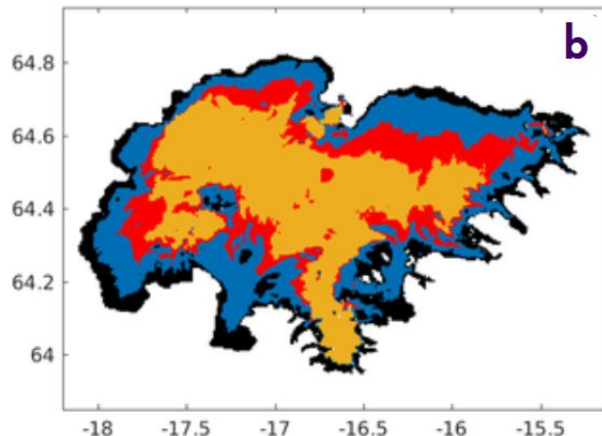
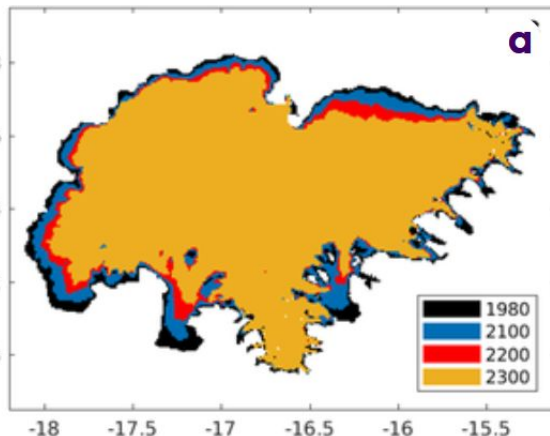
Where:	Svartsengi eruptions, Reykjanes Peninsula	Holuhraun eruption, Bárðarbunga	Laki eruption, Grímsvötn
When:	(December 2023 - April 2025)	(2014-2015)	(1783 -1784)
Lava volume (Bulk):	$\sim 0.21 \text{ km}^3$	$\sim 1.44 \text{ km}^3$	$\sim 15 \text{ km}^3$

Magma generated
through decompression
melting (1890-2023):

$\sim 27 \text{ km}^3$

When and where will this magma erupt?

Future work



From Schmidt et al., 2020. Vatnajökull's outlines for climate scenarios RCP 4.5 (a) and RCP 8.5 (b).

For RCP 4.5:

Volume decrease by 2300: 31–64%

Area loss by 2300: 13–37%

For RCP 8.5:

Volume decrease by 2300: 51–94%

Area decrease by 2300: 24–80%





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Questions? Spurningar?

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Thanks to all the coauthors and contributors!

**Halldór Geirsson, Thomas T. Givens, Peter Schmidt, Michelle Parks,
Eyjólfur Magnússon, Joaquín M. Belart, Freysteinn Sigmundsson,
Benedikt G. Ófeigsson, Vincent Drouin, Holger Steffen, Guðfinna
Aðalgeirsdóttir, Sigrún Hreinsdóttir**

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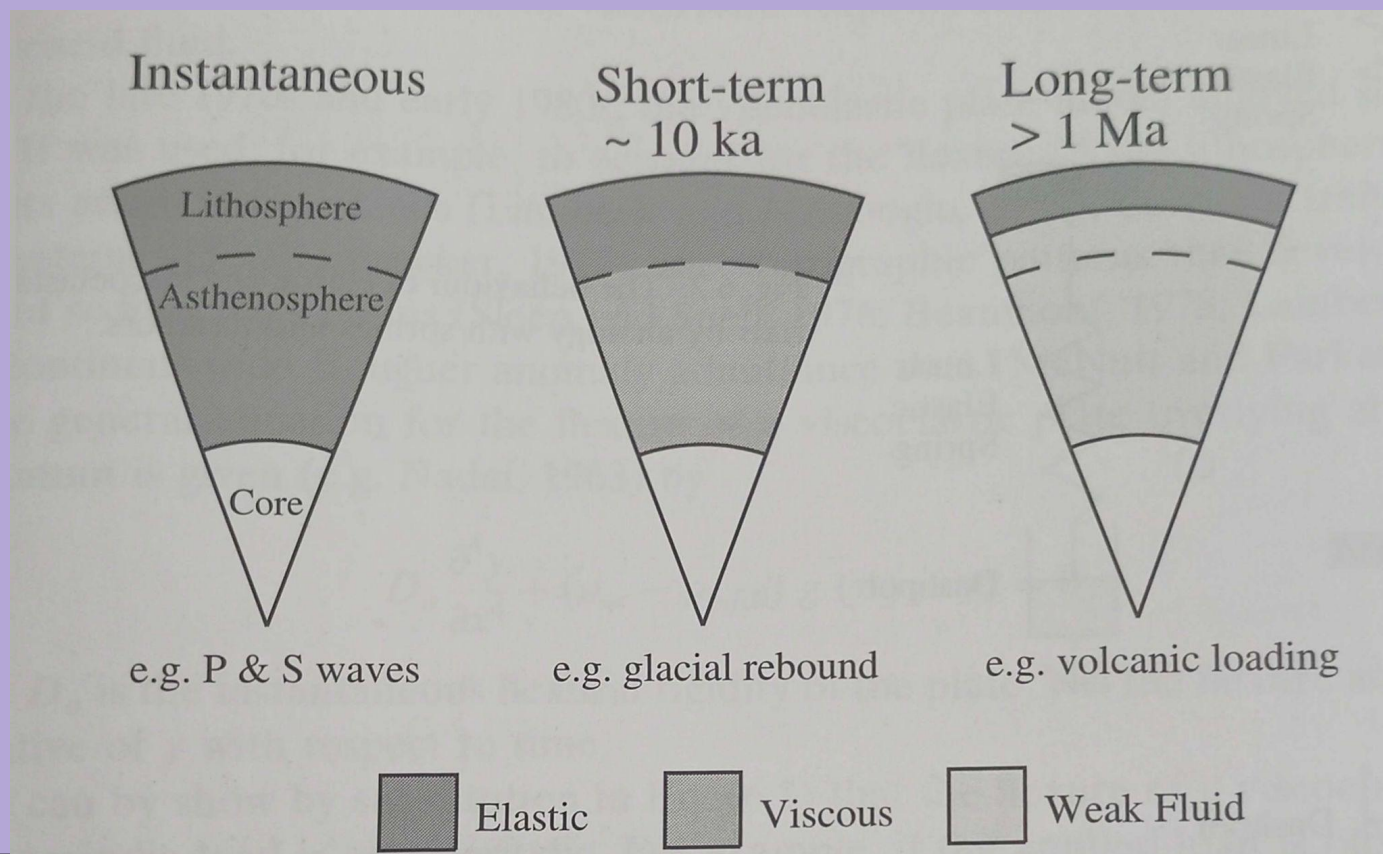


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Backup slides



The rheology of the Earth: timescale



From:
Isostasy
and flexure
of the
lithosphere,
Watts, 2001

The equation of motion for a self-gravitating sphere

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(\mathbf{u} \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \rho_1 g_0 \hat{\mathbf{e}}_r - \rho_0 \nabla \Phi_1 = 0$$

σ = deviatoric stress

$\mathbf{u}$ = displacement vector

Φ_1 = perturbation in the
gravitational potential

g = gravitational acceleration

ρ_0 = unperturbed density

ρ_1 = density perturbation

$\hat{\mathbf{e}}_r$ = radial direction unit vector

Backus (1967),
Cathles (1975)

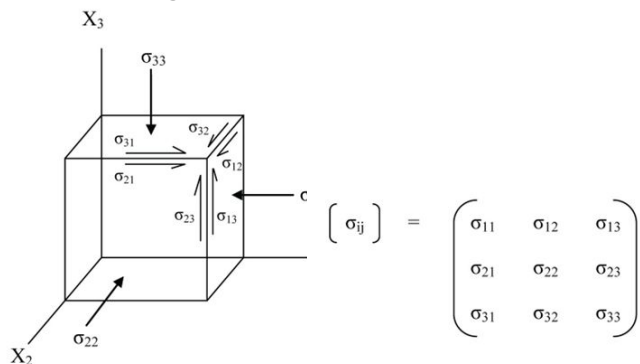
[N/m³]

The equation of motion for a self-gravitating sphere

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(u \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \rho_1 g_0 \hat{\mathbf{e}}_r - \rho_0 \nabla \Phi_1 = 0$$



Divergence of stress

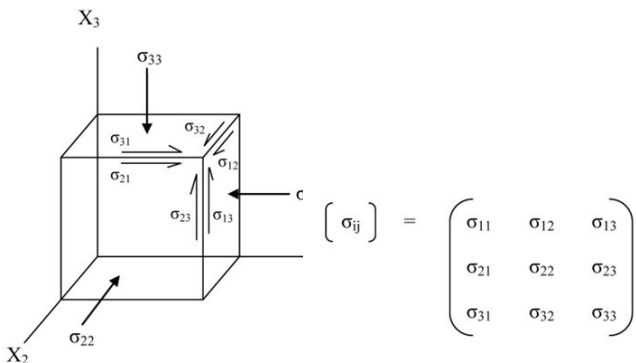


The equation of motion for a self-gravitating sphere

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(\mathbf{u} \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \rho_1 g_0 \hat{\mathbf{e}}_r - \rho_0 \nabla \Phi_1 = 0$$

Pre-stress advection

Divergence of stress



The equation of motion for a self-gravitating sphere

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(\mathbf{u} \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \rho_1 g_0 \hat{\mathbf{e}}_r - \rho_0 \nabla \Phi_1 = 0$$

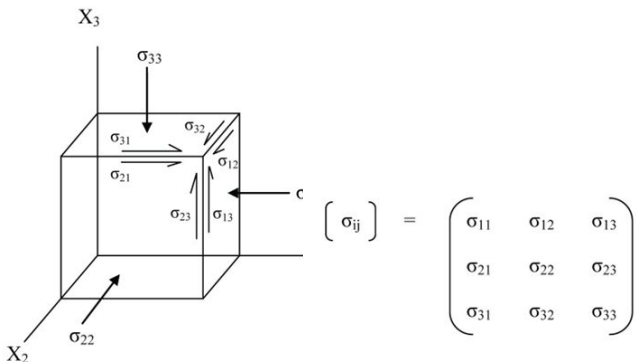
Pre-stress advection

Divergence of stress

Internal buoyancy

Conservation of mass:

$$\rho_1 = -\rho_0 \nabla \cdot \bar{\mathbf{u}} - \bar{\mathbf{u}} \cdot \left(\frac{\partial \rho_0}{\partial r} \right) \mathbf{e}_r$$



The equation of motion for a self-gravitating sphere

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(\mathbf{u} \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \rho_1 g_0 \hat{\mathbf{e}}_r - \rho_0 \nabla \Phi_1 = 0$$

Pre-stress advection

Self-gravitation

Divergence of stress

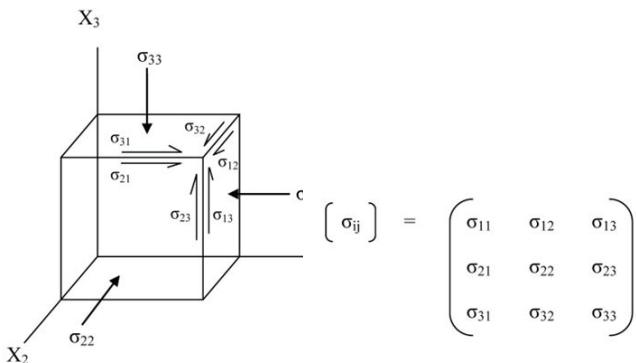
Internal buoyancy

Poisson's equation

Conservation of mass:

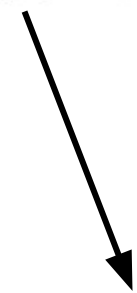
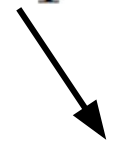
$$\nabla^2 \Phi = 4\pi G \rho_1$$

$$\rho_1 = -\rho_0 \nabla \cdot \bar{\mathbf{u}} - \bar{\mathbf{u}} \cdot \left(\frac{\partial \rho_0}{\partial r} \right) \mathbf{e}_r$$



The equation of motion in this model

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(u \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \cancel{\rho_1 g_0 \hat{\mathbf{e}}_r} - \cancel{\rho_0 \nabla \Phi_1} = 0$$

Incompressible
Earth (material
compressible)

Neglect
self-gravitation:
FLAT EARTH

The equation of motion in this model

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(u \cdot \rho_0 g_0 \hat{\mathbf{e}}_r) - \cancel{\rho_1 g_0 \hat{\mathbf{e}}_r} - \cancel{\rho_0 \nabla \Phi_1} = 0$$

New stress to solve the equation in a commercial software

$$\mathbf{S} = \boldsymbol{\sigma} - \rho_0 g_0 w \mathbf{I}$$

w = vertical displacement component

Incompressible Earth (material compressible)

Neglect self-gravitation: FLAT EARTH

Wu, 2004

Boundary conditions

Before Wu's substitution

at the surface:

$$\sigma_{rr} = \Gamma_l g_0$$

at internal

buoyancy contrasts:

$$[\sigma_{rr}]_-^+ = 0$$

$$[u]_-^+ = 0$$

at all boundaries:

$$[\sigma_{r\theta}]_-^+ = 0$$

After Wu's substitution

$$s_{33} + \rho_0 g_0 u_3 = \Gamma_l g_0$$

$$[s_{33}]_-^+ + (\rho_+ - \rho_-) g_0 u_3 = 0$$

$$[u]_-^+ = 0$$

$$[\sigma_{ij}; i \neq j]_-^+ = 0$$

Boundary conditions

at the surface:

$$s_{33} + \rho_0 g_0 u_3 = \Gamma_l g_0$$

at internal

buoyancy contrasts:

$$[s_{33}]^+_- + (\rho_+ - \rho_-) g_0 u_3 = 0$$

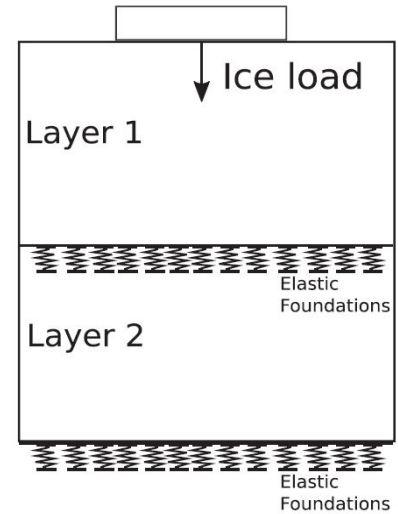
$$[u]^+_- = 0$$

at all boundaries:

$$[\sigma_{ij}; i \neq j]^+_- = 0$$

After Wu's substitution

SPRING
FOUNDATIONS

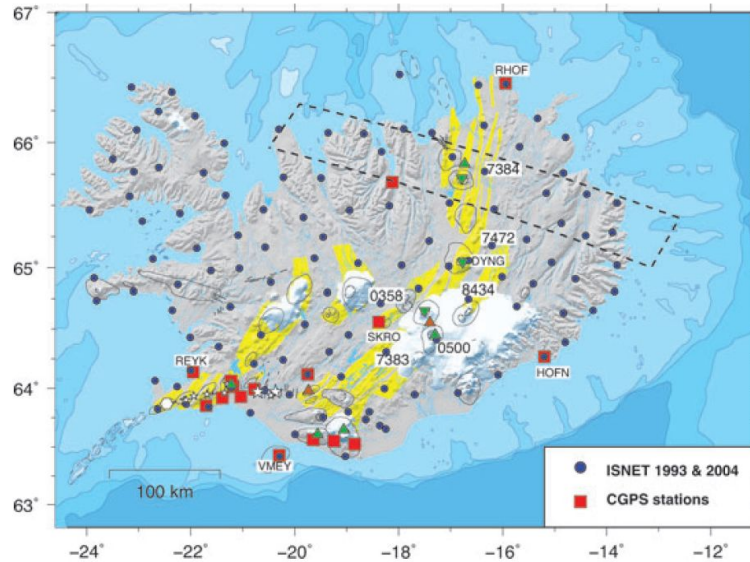


Schematics from Reusen et al, 2023

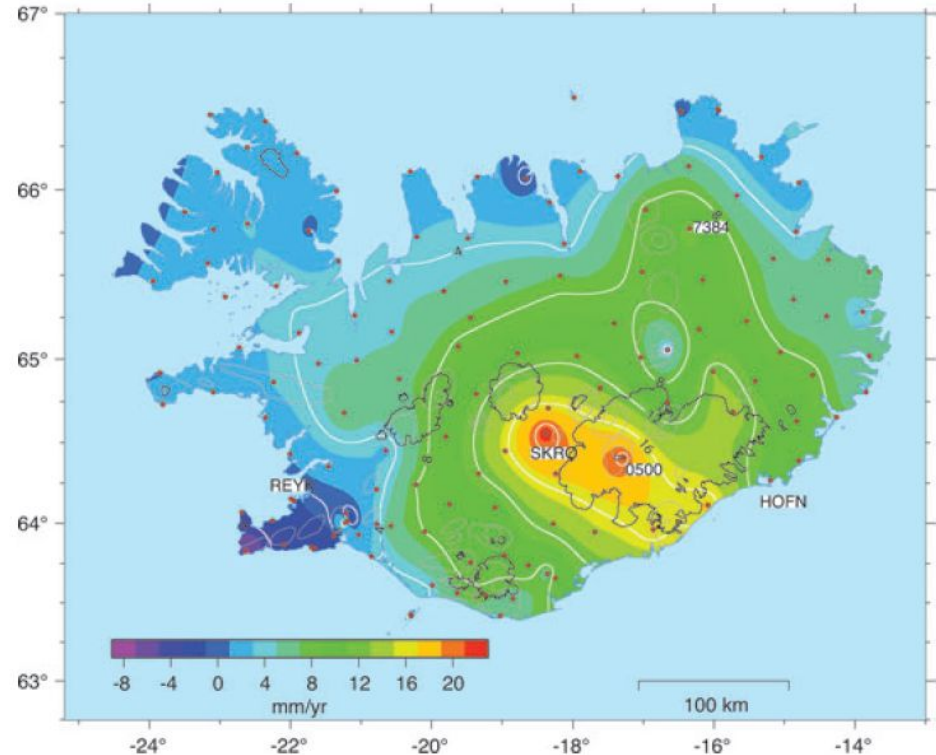
My PhD project

- Implementation of a GIA model for Iceland to obtain horizontal deformation and stresses.
- Time dependency of the GIA response, with deformation data and ice models prior satellite measurements.
- GIA at Grímsvötn: Are GIA effects on stress driving towards a new Laki eruption?

Data comparison: Árnadóttir et al 2009



Isnet GNSS campaign location
and vertical velocities
1993-2004



Magma production through decompression melting

Melt productivity equation

(Eq. D7, McKenzie 1984)

$$\frac{dX}{dP} = \frac{-\frac{C_p}{T} \left(\frac{\partial T}{\partial P} \right)_x + \frac{\alpha_s}{\rho_s} + \left(\frac{\alpha_f}{\rho_f} - \frac{\alpha_s}{\rho_s} \right) X}{\Delta S + \frac{C_p}{T} \left(\frac{\partial T}{\partial X} \right)_p}$$

heat capacity

C_p

T

$\left(\frac{\partial T}{\partial P} \right)_x$

α_s

ρ_s

$\left(\frac{\alpha_f}{\rho_f} - \frac{\alpha_s}{\rho_s} \right) X$

dX

dP

ΔS

$\frac{C_p}{T} \left(\frac{\partial T}{\partial X} \right)_p$

X = melt fraction by weight

Entropy of fusion

α = heat expansion
coefficients
 ρ = densities
f = fusion
s = solid

Magma production through decompression melting

Melt productivity
equation
(Eq. D7, McKenzie 1984)

$$\frac{dX}{dP} = \frac{-\frac{C_p}{T} \left(\frac{\partial T}{\partial P} \right)_x + \frac{\alpha_s}{\rho_s} + \left(\frac{\alpha_f}{\rho_f} - \frac{\alpha_s}{\rho_s} \right) X}{\Delta S + \frac{C_p}{T} \left(\frac{\partial T}{\partial X} \right)_P}$$

heat capacity

X = melt fraction by weight

Entropy of fusion

α = heat expansion coefficients
 ρ = densities
 f = fusion
 s = solid

Melt
parametrization:

$$T = T(X, P)$$

Magma production through decompression melting

Melt productivity
equation
(Eq. D7, McKenzie 1984)

$$\frac{dX}{dP} = \frac{-\frac{C_p}{T} \left(\frac{\partial T}{\partial P} \right)_x + \frac{\alpha_s}{\rho_s} + \left(\frac{\alpha_f}{\rho_f} - \frac{\alpha_s}{\rho_s} \right) X}{\Delta S + \frac{C_p}{T} \left(\frac{\partial T}{\partial X} \right)_P}$$

heat capacity

X = melt fraction by weight

Entropy of fusion

α = heat expansion coefficients
 ρ = densities
 f = fusion
 s = solid

Melt parametrization:
 Katz et al 2003, derived in Schmidt's thesis

$$T = T(X, P)$$

Hydrous melting of a depleted peridotite

Magma production through decompression melting

$$\frac{dX}{dP}$$

**Melt fraction
by weight**
depending
on the
Pressure

Transform the melt
production by weight
in melt production by
volume

$$\Theta(t) = \int_{\Omega} \frac{DX}{Dt} dV$$

**Melt
production
rate**

Magma production through decompression melting

$$\frac{dX}{dP} \longrightarrow \Theta(t) = \int_{\Omega} \frac{DX}{Dt} dV$$

**Melt fraction
by weight**
depending
on the
Pressure

Transform the melt
production by weight
in melt production by
volume

**Melt
production
rate**

$$\Theta(t) = \int_{\Omega} \frac{DX}{Dt} dV = \int_{\Omega} \frac{\rho_s \rho_m}{(\rho_m + X(\rho_s - \rho_m))^2} \left(\frac{\partial X}{\partial P} \right)_s \left(\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla P \right) dV$$

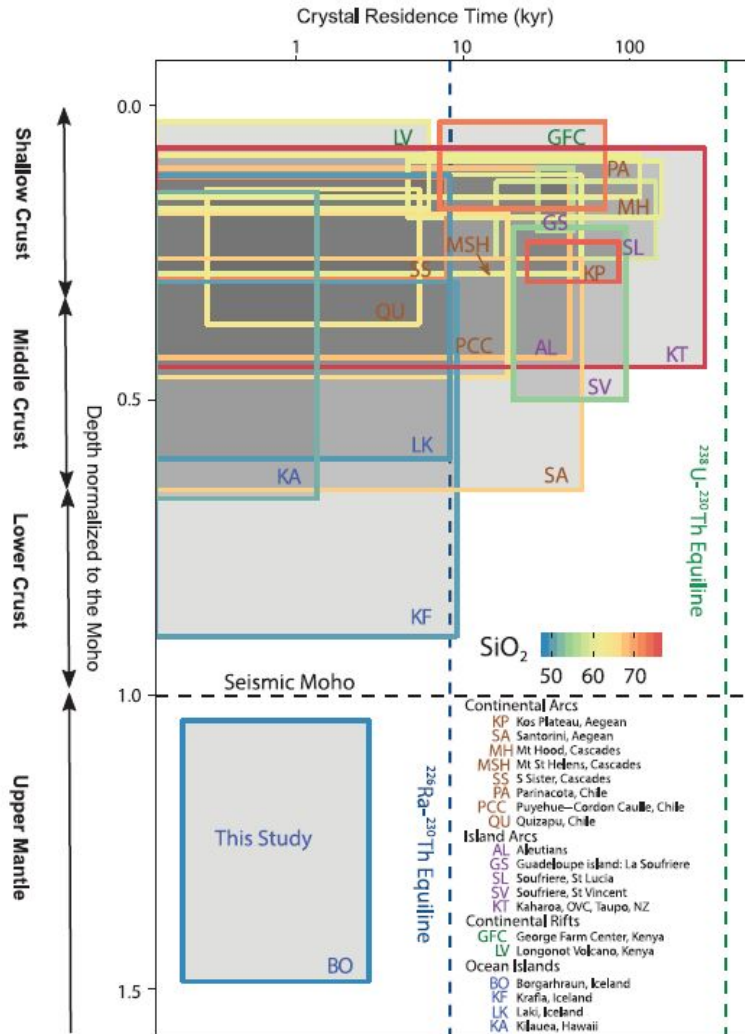
GIA
mantle
upwelling

When will this magma erupt?

Mutch et al, 2019

Magma storage in Peistareykir area
(Borgarhraun)

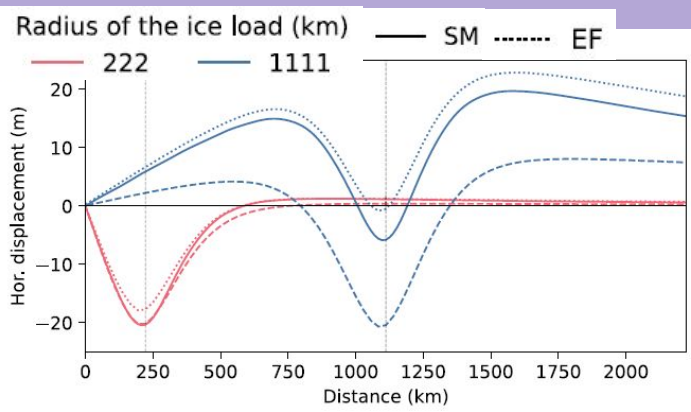
“For ocean island settings, our results indicate that primary magma can be stored near the Moho for thousands of years after initial mixing and crystallization. [...] punctuated magma storage and rapid transport is likely a prominent feature”



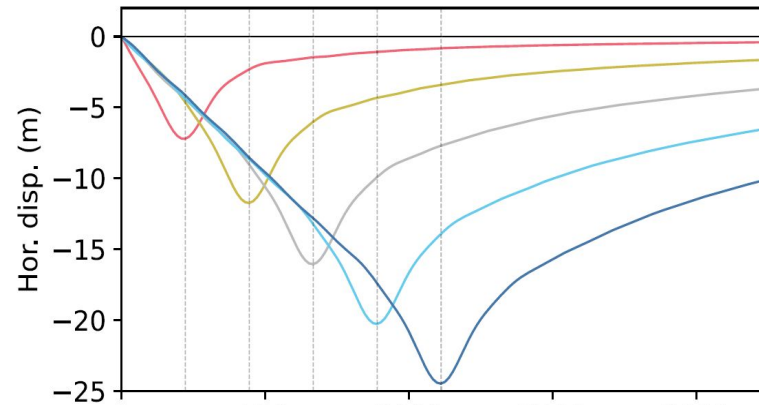


GIA-induced horizontal displacement benchmarking: Reusen et al 2023

- Smallest disc load radius: **222 km** (Iceland: 50 km)
- Lithospheric thickness: **80 km** (Iceland: 35 km)
- Upper mantle viscosity: **10^{21} Pa s** (Iceland: 3×10^{18} Pa s)

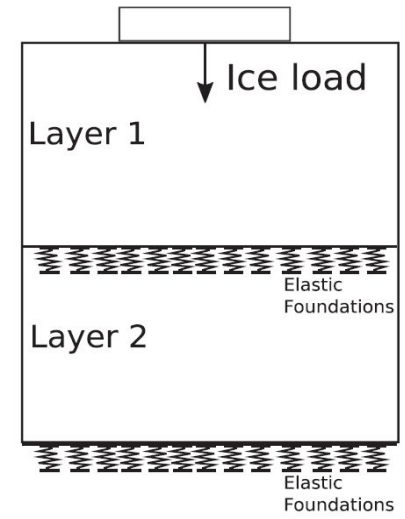
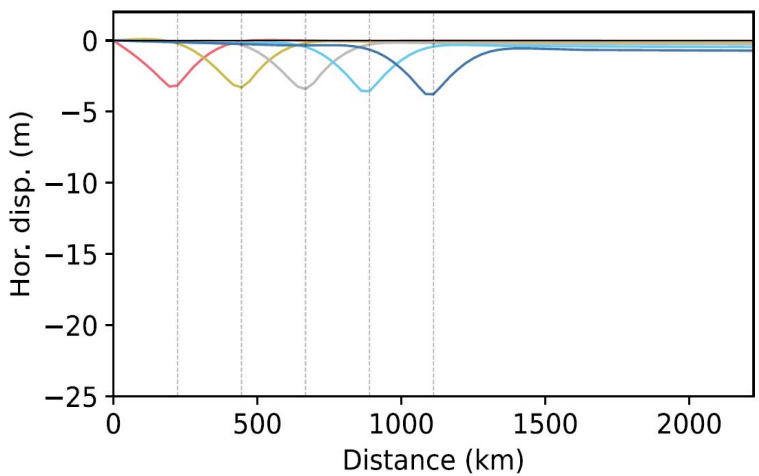


Spherical GIA model based on the normal model method



Abaqus (FEM)
VS ICEAGE
(Normal mode
method by G.
Kaufmann)

FE flat Earth GIA model with elastic foundations



Melt parametrization: Katz et al 2003, Schmidt's thesis derivation

$$\left. \frac{dT}{dP} \right|_F = \begin{cases} F^{1/\beta_1} \left(\frac{\partial T_{liq.}^{lherz}}{\partial P} - \frac{\partial T_{sol.}}{\partial P} \right) + \frac{\partial T_{sol.}}{\partial P} & \left[\begin{array}{l} F \leq F_{cpx-out} \\ X_{H_2O} \leq X_{H_2O}^{sat} \end{array} \right] \\ F^{1/\beta_1} \left(\frac{\partial T_{liq.}^{lherz}}{\partial P} - \frac{\partial T_{sol.}}{\partial P} \right) + \frac{\partial T_{sol.}}{\partial P} - \gamma K (\chi_1 P^\lambda + \chi_2 P)^{\gamma-1} (\lambda \chi_1 P^{\lambda-1} + \chi_2) & \left[\begin{array}{l} F \leq F_{cpx-out} \\ X_{H_2O} > X_{H_2O}^{sat} \end{array} \right] \\ \frac{1}{\beta_2} \left[\frac{F - F_{cpx-out}}{1 - F_{cpx-out}} \right]^{\frac{1-\beta_2}{\beta_2}} \left(\frac{r_2 M_{cpx} (1-F)}{(r_1 + r_2 P)^2 (1 - F_{cpx-out})^2} \right) \times (T_{liq.} - T_{cpx-out}) + \frac{\partial T_{cpx-out}}{\partial P} & [F > F_{cpx-out}] \\ + \left[\frac{F - F_{cpx-out}}{1 - F_{cpx-out}} \right]^{1/\beta_2} \left(\frac{\partial T_{liq.}}{\partial P} - \frac{\partial T_{cpx-out}}{\partial P} \right) & \end{cases}$$

$$\frac{\partial T_{cpx-out}}{\partial P} =$$

$$\left. \frac{dT}{dF} \right|_P = \begin{cases} \frac{F^{\frac{1-\beta_1}{\beta_1}}}{\beta_1} (T_{liq.}^{lherz} - T_{sol.}) + \gamma K \frac{(X_{H_2O}^{bulk})^\gamma (1 - D_{H_2O})}{(D_{H_2O} + (1 - D_{H_2O})F)^{\gamma+1}} & \left[\begin{array}{l} F \leq F_{cpx-out} \\ X_{H_2O} \leq X_{H_2O}^{sat} \end{array} \right] \\ \frac{F^{\frac{1-\beta_1}{\beta_1}}}{\beta_1} (T_{liq.}^{lherz} - T_{sol.}) & \left[\begin{array}{l} F \leq F_{cpx-out} \\ X_{H_2O} > X_{H_2O}^{sat} \end{array} \right] \\ \frac{\left[\frac{F - F_{cpx-out}}{1 - F_{cpx-out}} \right]^{\frac{1-\beta_2}{\beta_2}}}{\beta_2 (1 - F_{cpx-out})} (T_{liq.} - T_{cpx-out}) & [F > F_{cpx-out}] \end{cases}$$

$$\left. \frac{\partial T_{cpx-out}}{\partial P} \right|_F = \begin{cases} \left(\frac{\partial T_{liq.}^{lherz}}{\partial P} - \frac{\partial T_{sol.}}{\partial P} \right) F_{cpx-out}^{1/\beta_1} + \frac{\partial T_{sol.}}{\partial P} + \frac{(T_{sol.} - T_{liq.}^{lherz}) r_2 M_{cpx} F^{\frac{1-\beta_1}{\beta_1}}}{\beta_1 (r_1 + r_2 P)^2} & [X_{H_2O} \leq X_{H_2O}^{sat}] \\ \left(\frac{\partial T_{liq.}^{lherz}}{\partial P} - \frac{\partial T_{sol.}}{\partial P} \right) F_{cpx-out}^{1/\beta_1} + \frac{\partial T_{sol.}}{\partial P} + \frac{(T_{sol.} - T_{liq.}^{lherz}) r_2 M_{cpx} F^{\frac{1-\beta_1}{\beta_1}}}{\beta_1 (r_1 + r_2 P)^2} - \gamma K (\chi_1 P^\lambda + \chi_2 P)^{\gamma-1} (\lambda \chi_1 P^{\lambda-1} + \chi_2) & [X_{H_2O} > X_{H_2O}^{sat}] \end{cases}$$