

Iterated Simulated Annealing for Combinatorial Optimization

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From Monotonic Cooling to Iterated Operators

Simulated Annealing (SA)

A single monotonic cooling trajectory.



temperature vs. cumulative sweeps (schematic)

From Monotonic Cooling to Iterated Operators

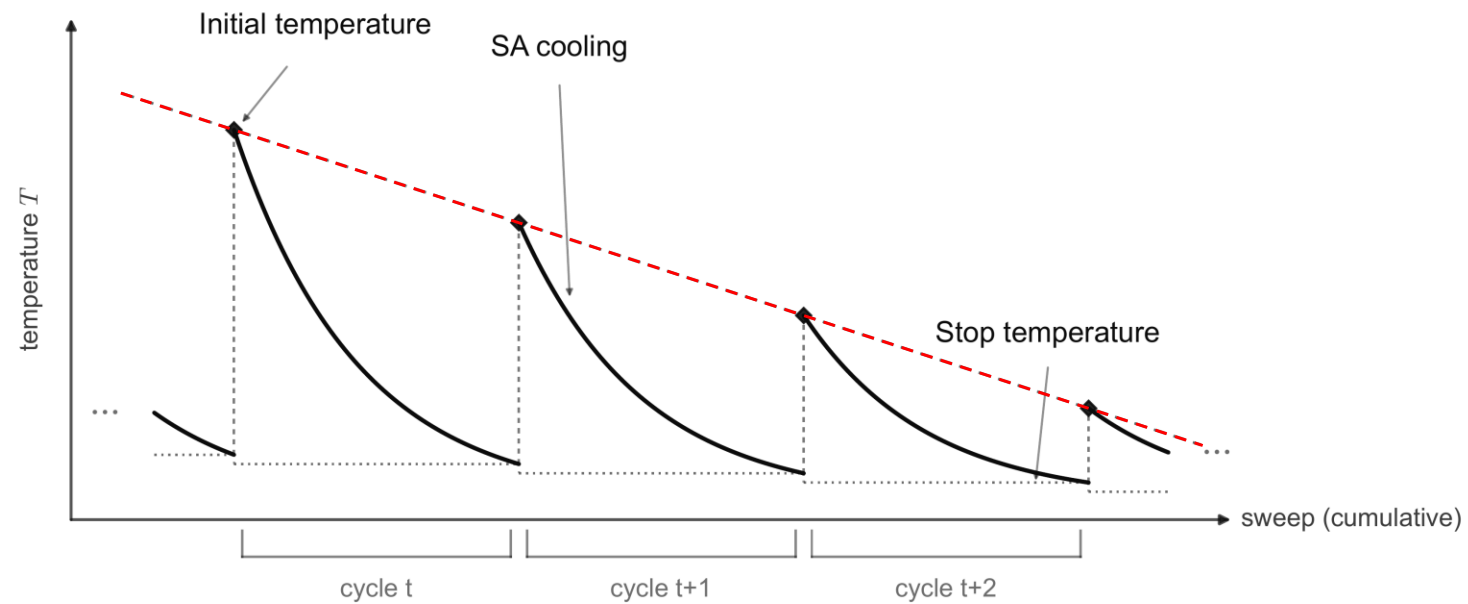
Iterated Simulated Annealing (ISA)

A finite cooling run becomes a repeatable operator.

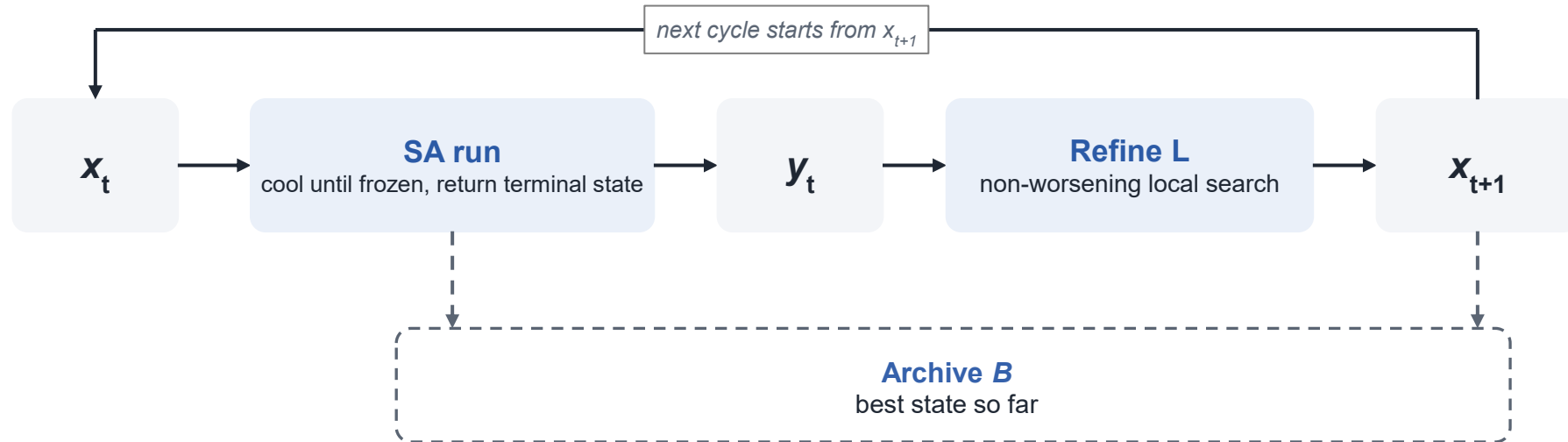
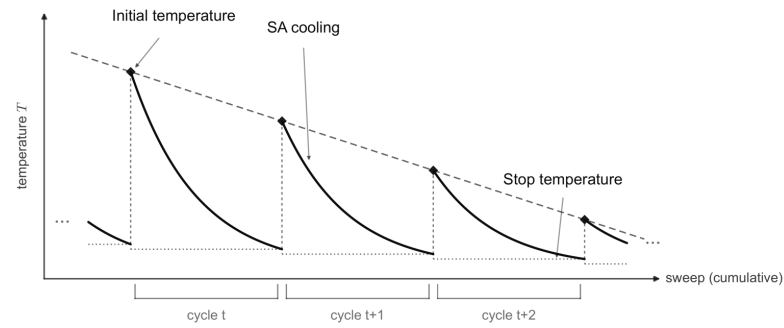


temperature vs. cumulative sweeps (schematic)

ISA Temperature Schedule



ISA Cycle



Two combinatorial optimization problems

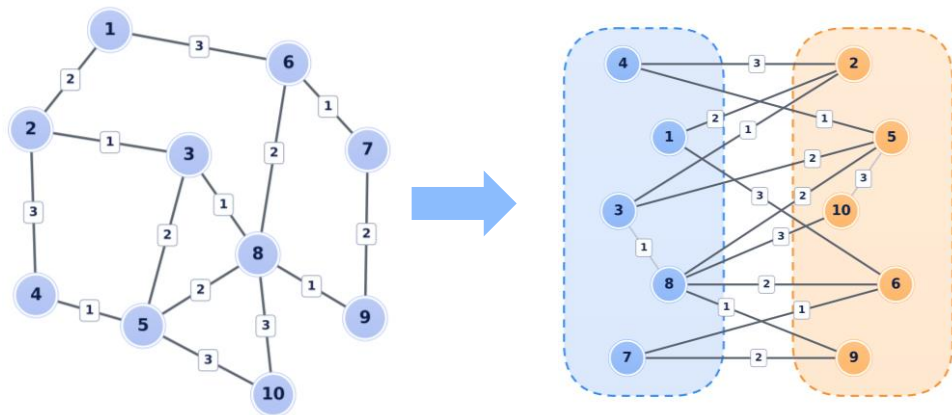
Max-Cut

Objective: Weight of edges cut by a binary labeling

Graphs:

G-set all 71 instances, 800–20,000 vertices, sparse

K2000 dense graph, 2,000 vertices, ~2M edges, weights ± 1



Community Detection

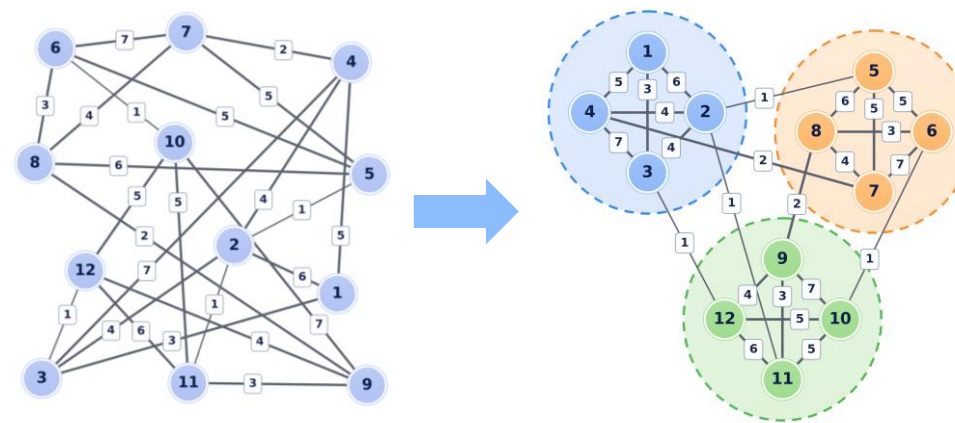
Objective: Newman-Girvan modularity Q

Graphs:

Zachary's Karate Club 34 vertices, 78 edges

IEEE-33 33 vertices, 32 edges

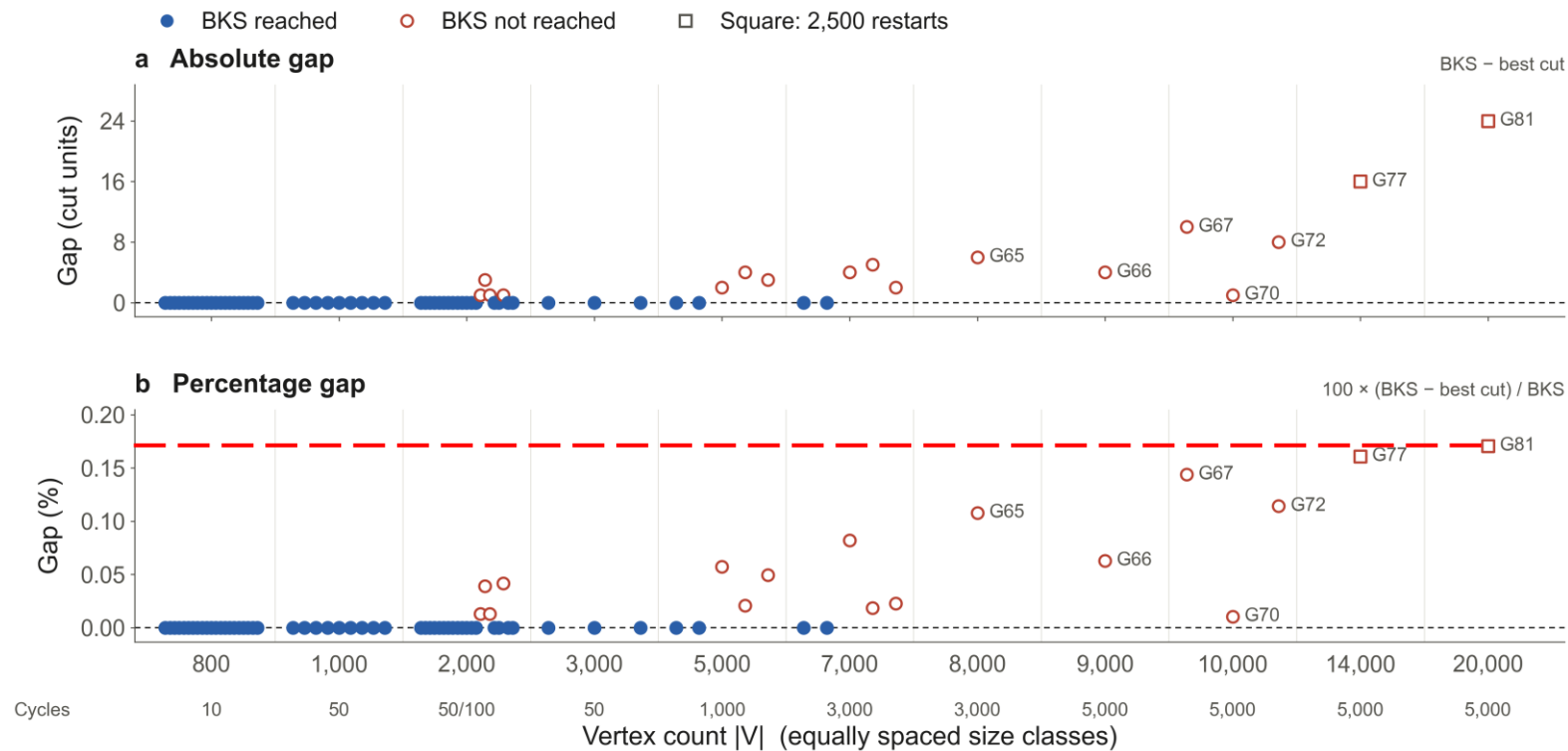
IEEE-118 118 vertices, 179 edges



Mac-Cut results

ISA best-cut gaps across G-set

54 / 71 BKS matches



BKS = best-known solution; circles: 5,000 restarts. Both panels show the best attained cut across restarts.

54 / 71

instances reached BKS.

$\leq 0.17\%$

relative gap of the best cut on every instance.

K2000

reached BKS of 33,337.

Mac-Cut results

		Classical heuristics											Quantum, hybrid, annealing and physics-inspired									
		ISA	BLS	MACUT	TS-UBQP	MAMBP	TSHEA	MOH	LPI†	PR	GESPR (2015)	PF-ESL (2022)	D-Wave Hybrid	SBM	SA1	SA2	QLS3	DAV2	DAV3	Cosmt		
G55	5,000	0	5	0	63	0	0	0	0	34	0		35	10	932	10						
G56	5,000	0	5	1	83	1	0	1	0	36	0		35	9	105	10						
G57	5,000	2	2		34	6	0	0	0	22	0		38	14	58	8	28	24	12			
G58	5,000	4	31		46	18	18	6	0	89	1		73	37	1902	176	78	32	31			
G59	5,000	3	10		69	3	3	1	0	61	2		63	21	196	23						
G60	7,000	0	14	4	133	4	4	0	0	78	2	3	48	22	2070	76						
G61	7,000	0	10		119	3	3	1	0	69	3	7	49	22	354	11				0		
G62	7,000	4	4		50	6	6	4	0	36	2	4	54	28	240	16	44	34	26			
G63	7,000	5	50		84	293	29	14	5	131	2	67	99	61	3296	663	98	35	44			
G64	7,000	2	17		142	21	17	5	0	111	1	26	87	24	594	24						
G65	8,000	6	4	12	44	6	2	2	0	36	0	0	68	30	398	18	60	102	28			
G66	9,000	4	4	12	60	12	0	4	0	50	0	4	82	40	594	14	76		184			
G67	10,000	10	10	16	56	16	6	8	2	48	0	4	82	44	814	26						
G70	10,000	1	54		137	15	47	51	0	132	4	8	79	73	1686	49				0		
G72	10,000	8	10		86	18	18	10	2	62	2	8	94	42	860	24	92		280	0		
G77*	14,000	16	14			40	38	12	12		2	10								0		
G81*	20,000	24	30			82	50	24	18		12	22								0		
Reported, n		17	17	6	15	17	17	17	17	15	17	12	15	15	15	15	7	5	7	5		

Color: relative gap to BKS (%); cell text: absolute deficit BKS – cut
 □ 0 (BKS) □ ≤ 0.05 □ 0.05–0.1 □ 0.1–0.5 □ 0.5–1 □ > 1 □ not reported

ISA ≥ on every covered instance

TS-UBQP, MAMBP,
SBM, DAV3, SA2, D-Wave Hybrid

Mixed

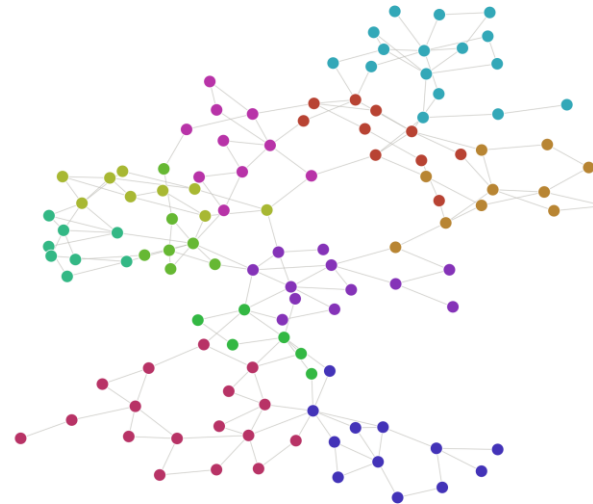
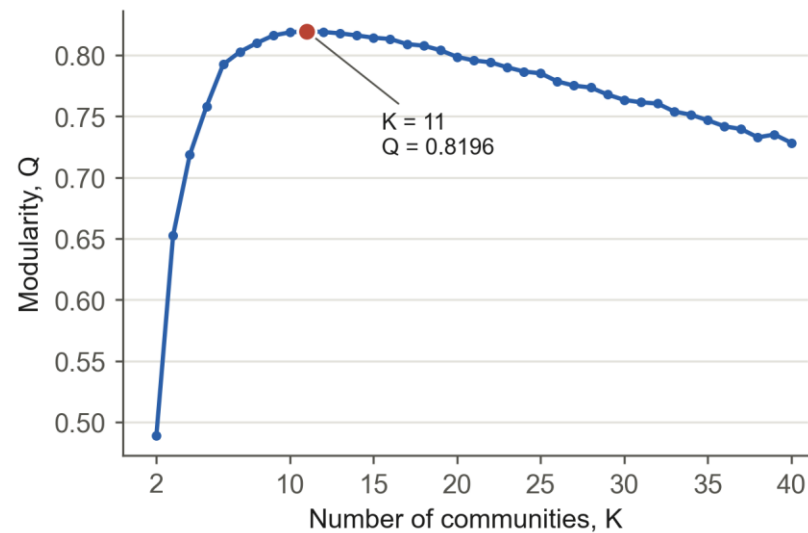
BLS, TSHEA, MOH, PF-ESL

Higher than ISA

LPI and GESPR on 12 of 17 each.
Cosm on 4 of its 5.

Community Detection results

Network	K range	Best K	Highest Q	Published Q	Sweep time
Karate Club	2–8	4	0.444904	0.444904	0.04 s
IEEE 33-bus	2–15	7	0.743158	0.743158	0.08 s
IEEE 118-bus	2–40	11	0.819572	0.819572	2.02 s



ISA's full K-sweep took 2.02 s on a workstation CPU.

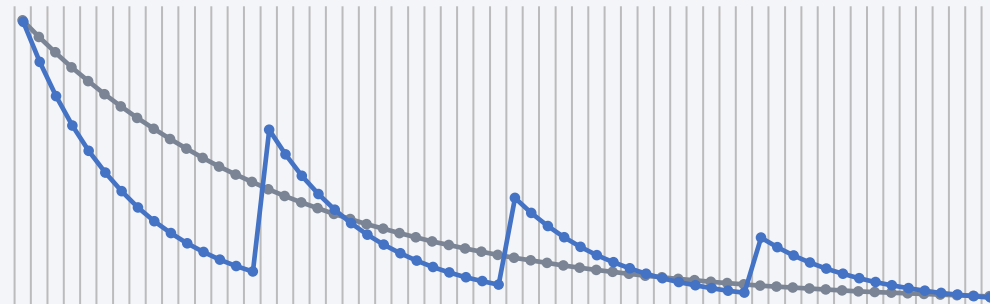
Fujitsu's Digital Annealer reported to solve the IEEE-118 within 12 s (Kao et al., 2023).

IEEE-118: Q vs K curve (left) and the partition at K = 11 (right).

Kao, Y.-T., Liao, J.-L., & Hsu, H.-C. (2023). *Solving Combinatorial Optimization Problems on Fujitsu Digital Annealer* (Version 1). arXiv. <https://doi.org/10.48550/ARXIV.2311.05196>

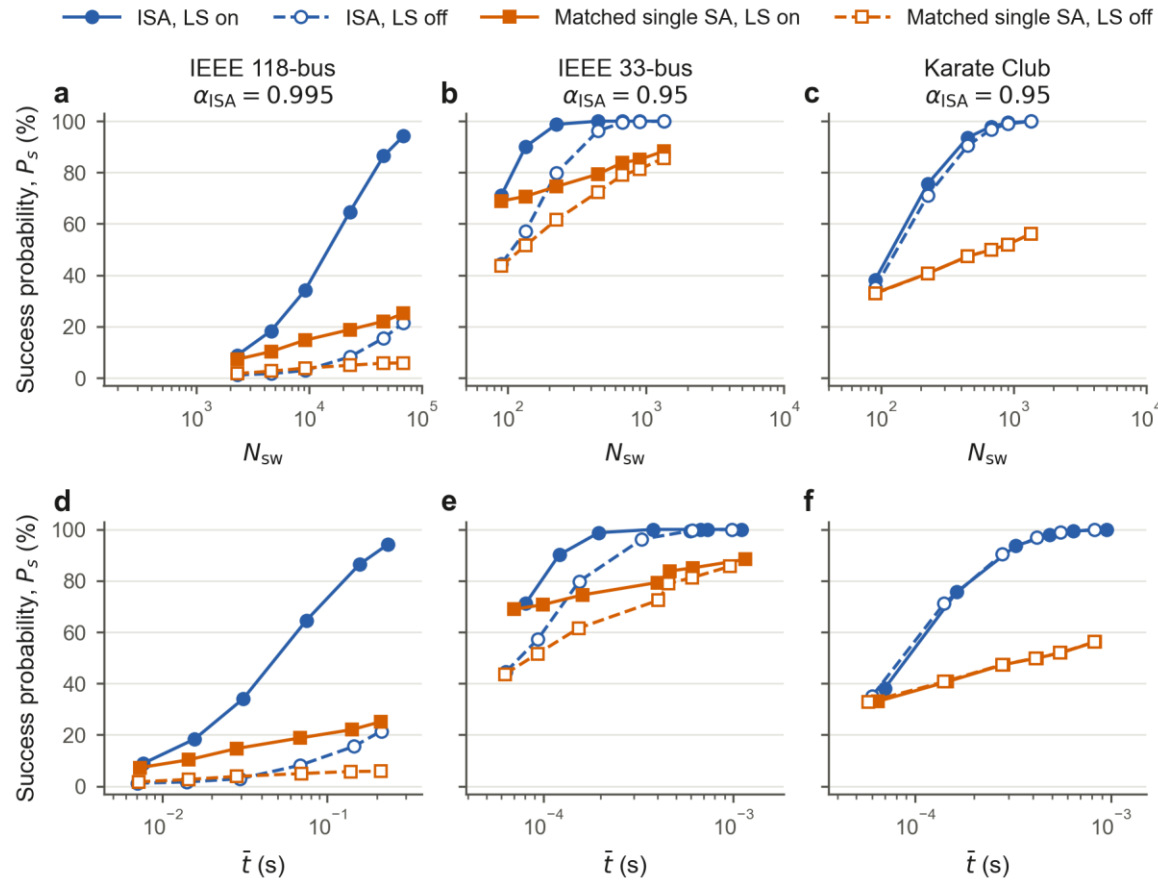
Matched Budget: ISA vs. SA

ISA vs. SA



temperature vs. sweeps (schematic)

Matched Budget: ISA vs. SA



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budget points where ISA is more reliable (LS on).
SA never significantly better

94.20 % vs. 25.12 %

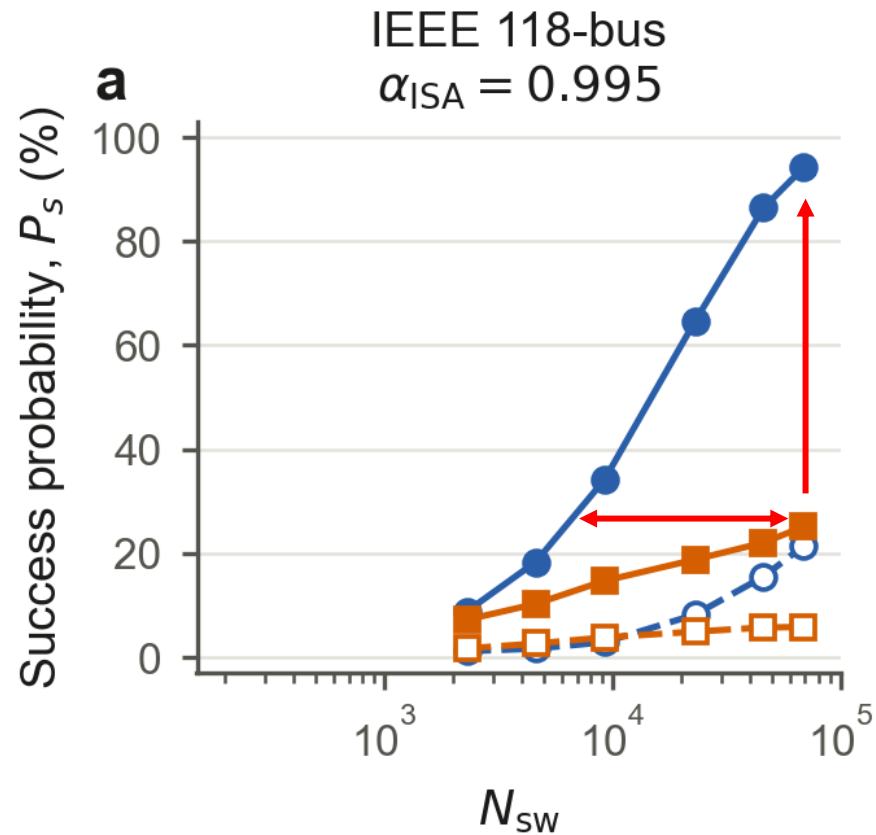
IEEE-118 at the largest budget, ISA vs matched SA

10–43 ×

more sweeps the matched SA spent to reach a lower P_s than ISA

Matched Budget: ISA vs. SA

—●— ISA, LS on —○— ISA, LS off —■— Matched single SA, LS on —□— Matched single SA, LS off



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budget points where ISA is more reliable (LS on).
SA never significantly better

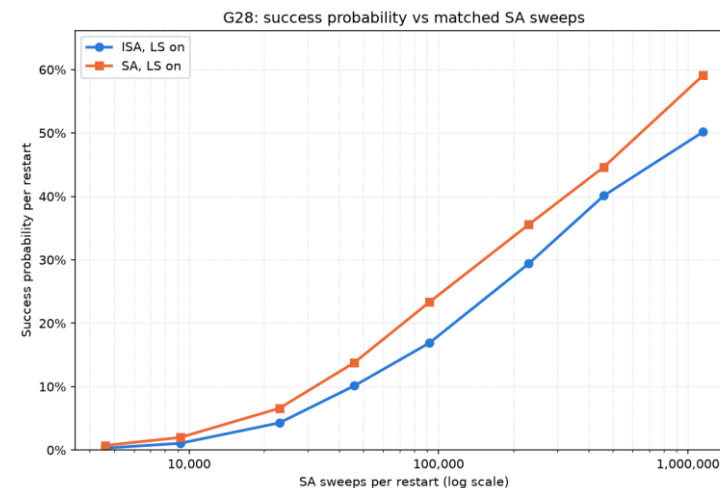
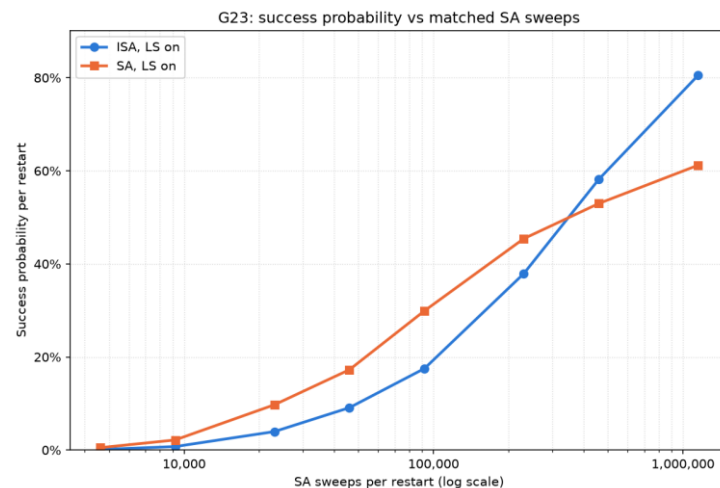
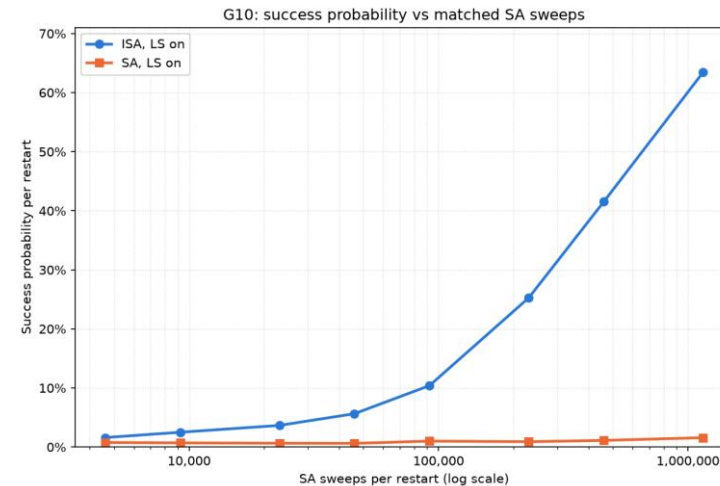
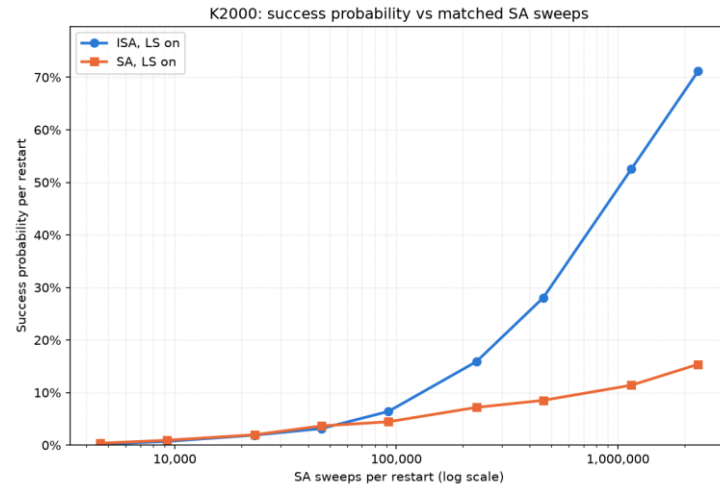
94.20 % vs. 25.12 %

IEEE-118 at the largest budget, ISA vs matched SA

10–43 ×

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Matched Budget: ISA vs. SA



- What structural features drive the performance divergence?
- What types of landscapes fundamentally favor ISA framework?

Thanks

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