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Organizers: Marcelo Disconzi, Chenyun Luo, Giusy Mazzone, Gieri Simonett

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On the motion of rigid solid particles in visco-elastic liquids

Abstract: This lecture concerns the numerical simulation of the motion of rigid solid particles in a space region filled with an incompressible viscoelastic liquid. Two types of viscoelastic liquids will be considered, namely: (i) Oldroyd-B, and (ii) FENE-CR, a more realistic model (CR being for Chilcott & Rallison, 1988). The multi-physics features of these two-phase non-Newtonian flow problems made them natural candidates for solution methods based on operator-splitting, among other computational ingredients, such as well-suited finite element approximations. The results of numerical experiments will be presented, a particular attention being given to the simulation of particle chaining phenomena. When experimental data are available, the matching between numerical and laboratory experiments is quite remarkable.

Juhi Jang
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Rotating axisymmetric solutions of the Euler-Poisson equations

Abstract: We consider stationary axisymmetric solutions of the Euler-Poisson equations, which govern the internal structure of barotropic gaseous stars. The equation of states is taken to be close to that of the polytropic gaseous stars of the adiabatic exponent ranging from 6/5 to 2. The problem is formulated as a nonlinear integral equation. We present non-variational approaches and also discuss the oblateness of star surface. The talk is based on joint work with Tetu Makino.

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The Euler equations with a free interface

Abstract: We address the local existence of solutions for the water wave problem, which is modelled by the incompressible Euler equations in a domain with a free boundary evolving with the flow. We are particularly interested in the local existence for the initial velocity, which is rotational and belongs to a low regularity Sobolev space. We will review the available existence and uniqueness results for the problem with both, vanishing or nonzero surface tension. The results are joint with M. Disconzi and A. Tuffaha.

Haoxiang Luo
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Computational fluid-structure interaction for biological applications

Abstract: Fluidstructure interaction (FSI) can be found for many biological tissues and organs. Among the examples are insect wings, fish fins, heart valves, and human vocal folds. Although their anatomies, structural components, and mechanical behaviors are quite different from one another, these flexible bodies share the kinematic features of large deformation in the three-dimensional (3D) space, and the time-varying deformations are critical for the bodies to perform their physiological functions. Because of the large deformation and also the intrinsic complexity of the accompanying flow, computational modeling of the 3D FSI for this type of problems is still highly challenging and thus has not been extensively explored. In this talk, I will describe the immersed-boundary approach to solve the governing Navier-Stokes equation and its coupling with a finite-element approach for solving the tissue mechanics. I will present several applications pursued in our group, namely, animal flight, fish swimming, vocal cord vibration, and heart valves.

Anna Mazzucato
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Boundary layers and the vanishing viscosity limit in incompressible flows

Abstract: I will discuss recent results on the analysis of the vanishing viscosity limit, that is, whether solutions of the Navier-Stokes equations converge to solutions of the Euler equations, for incompressible fluids when walls are present. At small viscosity, a viscous boundary layer arise near the walls where large gradients of velocity and vorticity form may propagate in the bulk (if the boundary layer separates). This is thought to be one of the main mechanisms for onset of turbulence. The vanishing viscosity limit and the analysis of the associated viscous boundary layer are poorly understood from a mathematical point of view except in special cases, where the limit can be established rigorously, such as for symmetric flows and for the linearized equations.

Todd Oliynyk
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Dynamical relativistic liquid bodies: local-in-time existence and uniqueness

Abstract: In this talk, I will discuss a new proof that establishes the local-in-time existence and uniqueness of solutions to the relativistic Euler equations representing dynamical liquid bodies. The proof involves three main steps. The first step is to reduce the existence problem for the Euler equations to that for a non-linear system of wave equations with mixed Dirichlet and acoustic boundary conditions. The second step is to develop an appropriate existence theory for systems of variable coefficient linear wave equations with mixed Dirichlet and acoustic boundary conditions. The third and final step is to use a fixed point scheme to obtain the local-in-time existence of solutions to the non-linear problem from the linear theory. During the talk, I will give an overview of the main ideas needed to carry out each of the three steps.

Jared Speck
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Shock formation for multidimensional fluid equations and related multiple speed systems

Abstract: In this talk, I will describe some of my recent work on stable shock formation in solutions to large classes of quasilinear hyperbolic PDE systems in multiple spatial dimensions, including various models for compressible fluids. I will highlight the crucial role that nonlinear geometric optics plays in the proofs, and, in the case of relativistic and non-relativistic fluid mechanics, emphasize the equally crucial role played by new formulations of the equations exhibiting miraculous geo-analytic structures. I will also describe some important open problems and connect the results to the broader goal of obtaining a rigorous mathematical theory modeling the long-time behavior of solutions. Some of the works I will discuss are joint with M. Disconzi, G. Holzegel, J. Luk, and W. Wong.

Chongchun Zeng
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Wind-driven water waves and instability of the Euler equation

Abstract: In this talk, we start with the mathematical theory of wind-generated water waves in the framework of the interface problem between two incompressible inviscid fluids under the influence of gravity. This entails the careful study of the stability of the shear flow solutions to the interface problem of the two-phase Euler equation. We first obtained rigorously the linear instability criterion of Miles due to the presence of the critical layer in the steady shear flows. Our analysis is valid even in the presence of surface tension and a vortex sheet (discontinuity in the tangential velocity across the air–sea interface). We are thus able to give a unified equation including the Kelvin–Helmholtz and quasi-laminar models of wave generation put forward by Miles. While the rigorous nonlinear instability proof is still missing for this problem, we are aiming at a stronger statement – constructing local unstable manifolds of the full nonlinear system of the interface problem of the Euler equation. If time permits, in the second half of the talk, we discuss the unstable manifolds of steady states of the Euler equation on fixed domains. Suppose the linearized equation at a steady state v_* has an exponential dichotomy with a finite dimensional unstable subspace. By rewriting the Euler equation as an ODE on an infinite dimensional manifold in H^k, $k > \frac{n}{2} + 1$, the unstable manifold of v_* is constructed under certain conditions on the Lyapunov exponents of the vector field v_*. In particular, this leads to the desired nonlinear instability of v_* in the sense that small H^k perturbations can lead to L^2 derivation of the solutions.