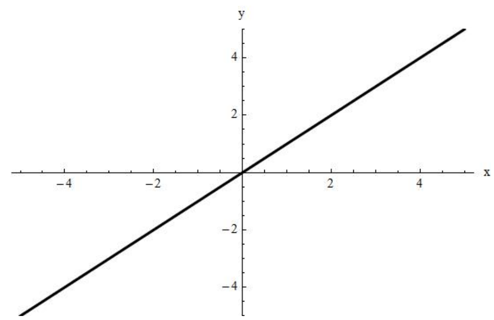
Functions and Lines Solutions

Vertical and Horizontal Shifts:

Suppose $c > 0$. To obtain the graph of:

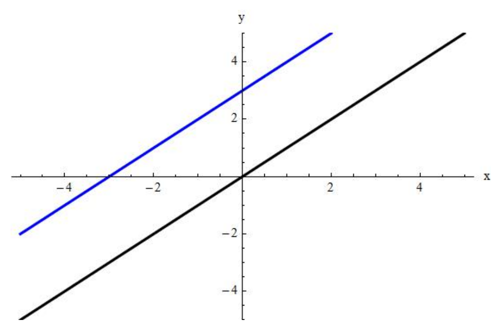
1. $y = f(x) + c$, shift the graph of $y = f(x)$ a distance of c units upward.
2. $y = f(x) - c$, shift the graph of $y = f(x)$ a distance of c units downward.
3. $y = f(x - c)$, shift the graph of $y = f(x)$ a distance of c units to the right.
4. $y = f(x + c)$, shift the graph of $y = f(x)$ a distance of c units to the left.

1. Graph the function $f(x) = x$.

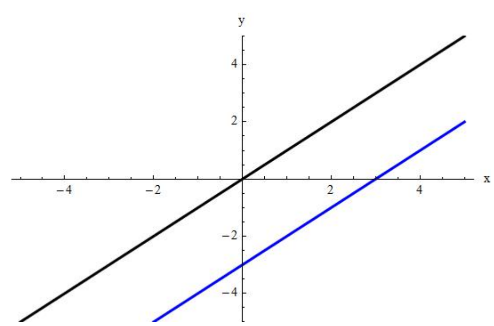


Write the equation that is obtained from the graph of $f(x)$ with the following modifications, and then graph the resulting functions on the same graph with $f(x)$.

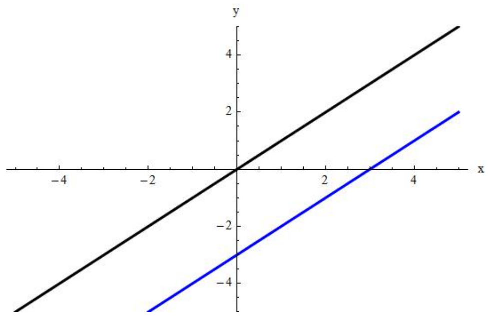
- Shift 3 units upward: $y = f(x) + 3 = x + 3$



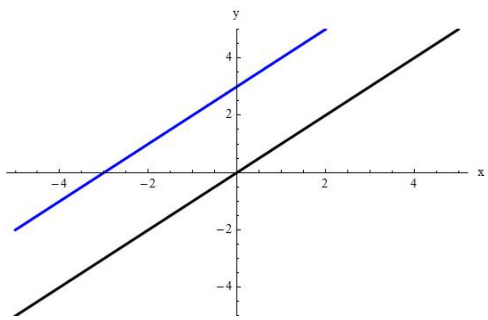
- Shift 3 units downward: $y = f(x) - 3 = x - 3$



- Shift 3 units to the right: $y = f(x-3) = x - 3$



- Shift 3 units to the left: $y = f(x+3) = x + 3$

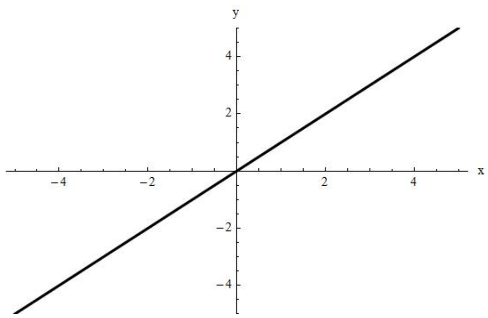


Vertical and Horizontal Stretching and Reflecting:

Suppose $c > 1$. To obtain the graph of:

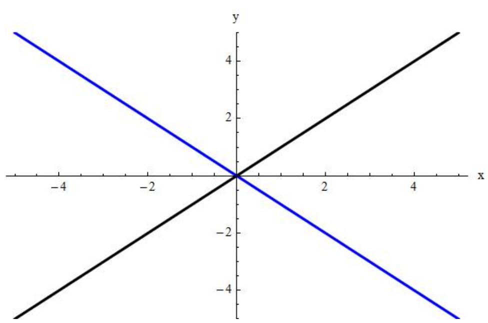
(a) $y = c * f(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c.
(b) $y = \frac{1}{c} * f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c.
(c) $y = f(c * x)$, compress the graph of $y = f(x)$ horizontally by a factor of c.
(d) $y = f\left(\frac{x}{c}\right)$, stretch the graph of $y = f(x)$ horizontally by a factor of c.
(e) $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis.
(f) $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis.

2. Graph the function $f(x) = x$.

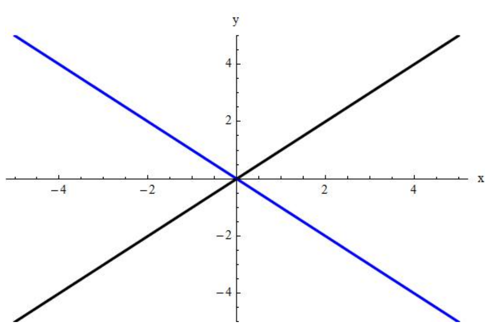


Write the equation that is obtained from the graph of $f(x)$ with the following modifications, and then graph the resulting functions on the same graph with $f(x)$.

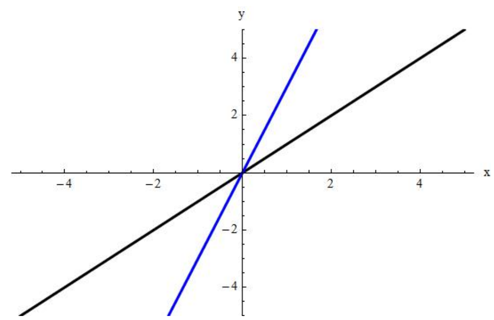
- Reflect about the x-axis: $y = -f(x) = -x$



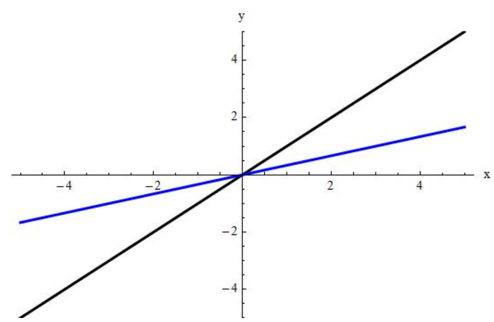
- Reflect about the y-axis: $y = f(-x) = -x$



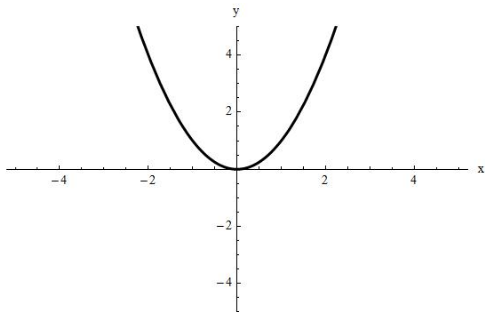
- Stretch vertically by a factor of 3: $y = 3f(x) = 3x$



- Compress vertically by a factor of 3: $y = \frac{1}{3}f(x) = \frac{x}{3}$

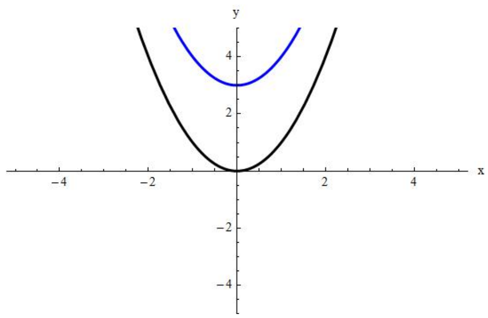


3. Graph the function $g(x) = x^2$.

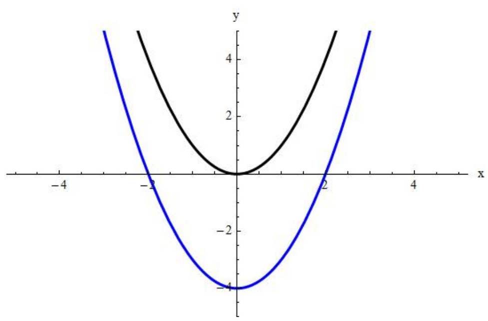


Graph the following functions on the same graph with $g(x)$.

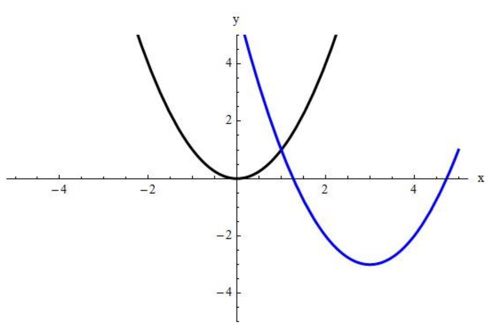
- $y = x^2 + 3$



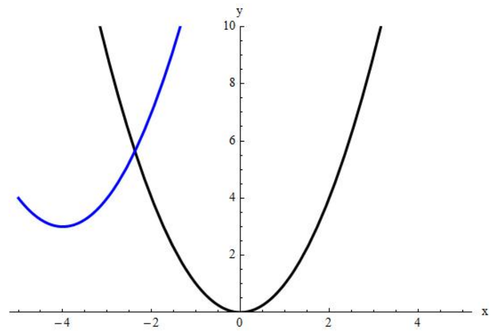
- $y = x^2 - 4$



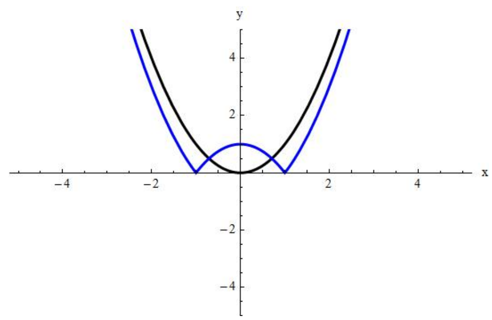
- $y = (x-3)^2 - 3$



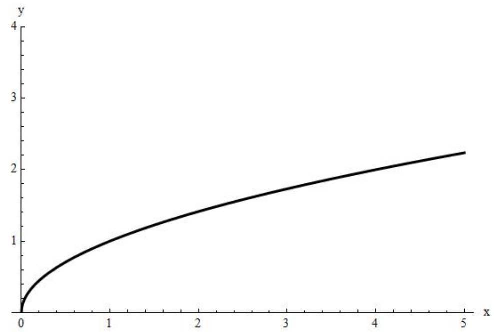
- $y = (x+4)^2 + 3$



- $y = |x^2 - 1|$

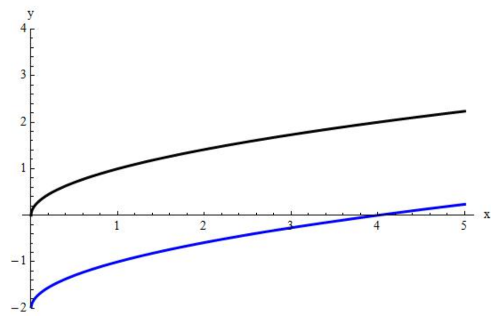


4. Graph the function $h(x) = \sqrt{x}$.

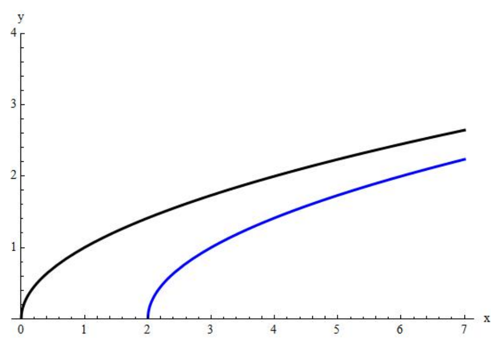


Graph the following functions on the same graph with $h(x)$.

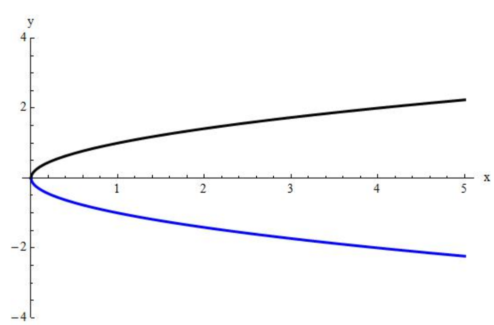
- $y = \sqrt{x} - 2$



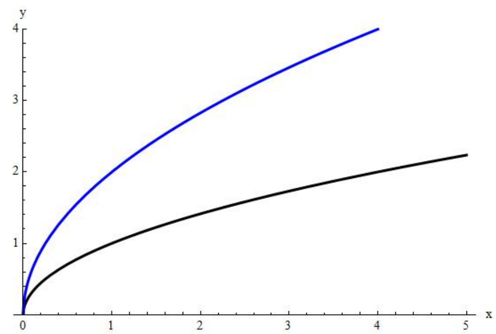
- $y = \sqrt{x - 2}$



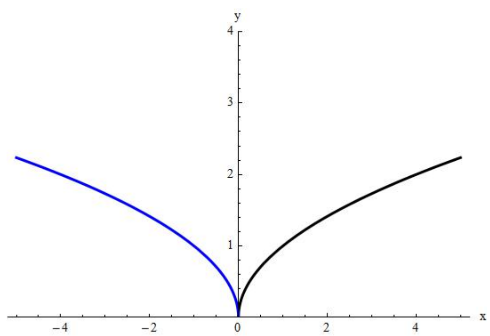
- $y = -\sqrt{x}$



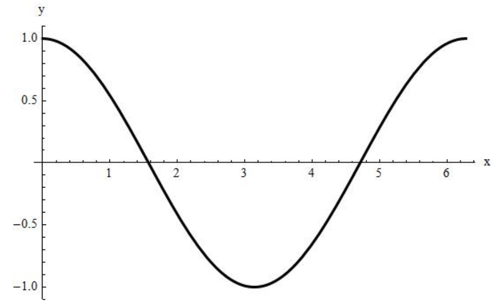
- $y = 2\sqrt{x}$



- $y = \sqrt{-x}$

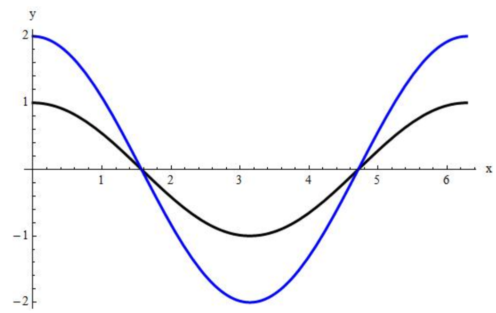


5. Graph the function $k(x) = \cos x$.

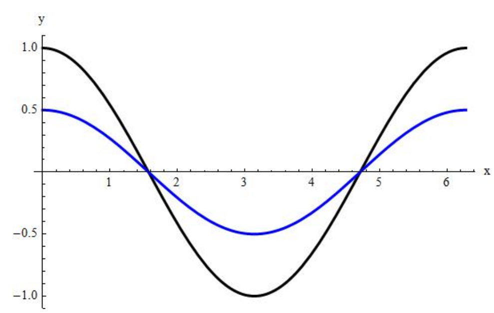


Graph the following functions on the same graph with $k(x)$.

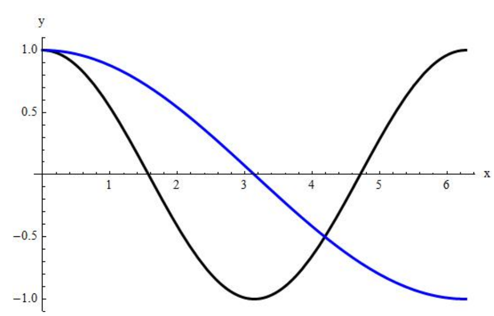
- $y = 2\cos x$



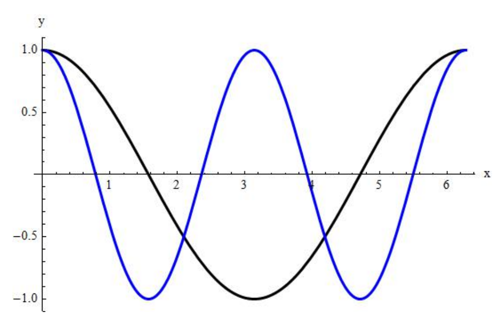
- $y = \frac{1}{2}\cos x$



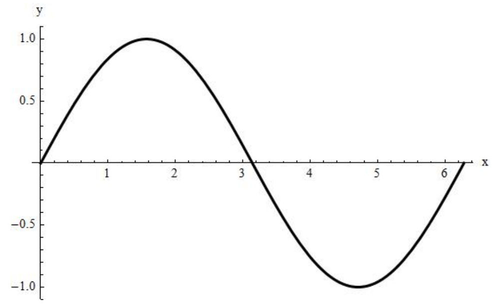
- $y = \cos \frac{x}{2}$



- $y = \cos 2x$

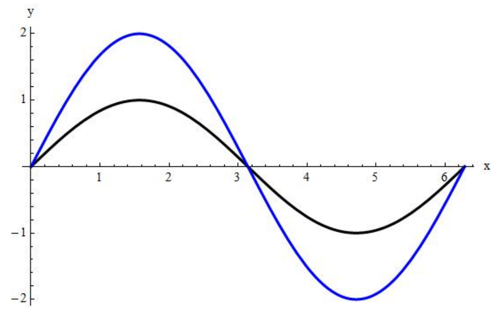


6. Graph the function $l(x) = \sin x$.

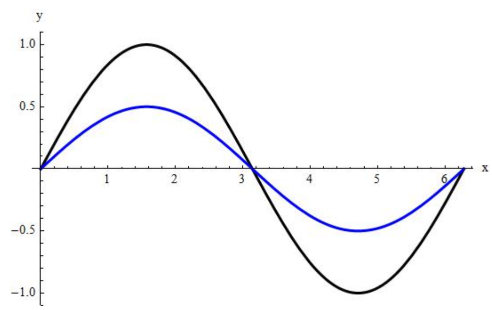


Graph the following functions on the same graph with $l(x)$.

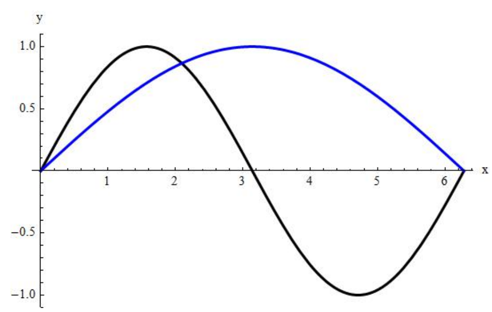
- $y = 2\sin x$



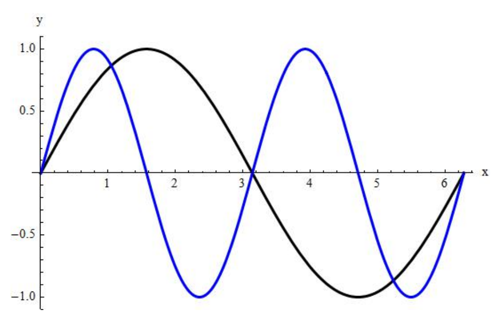
- $y = \frac{1}{2}\sin x$



- $y = \sin\frac{x}{2}$



- $y = \sin 2x$



7. Which of the following is the graph of the function $2y - 3x = 4$?

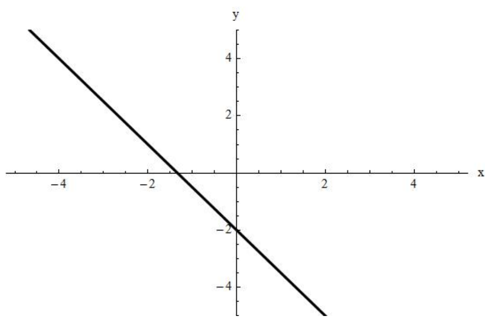


Figure 1: Incorrect

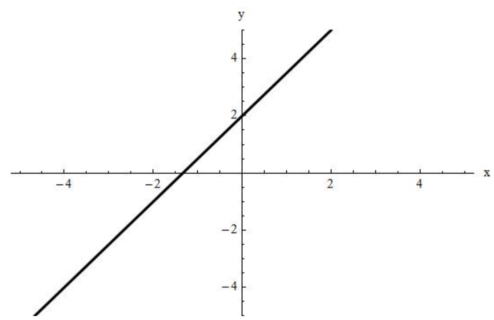


Figure 2: $2y - 3x = 4$

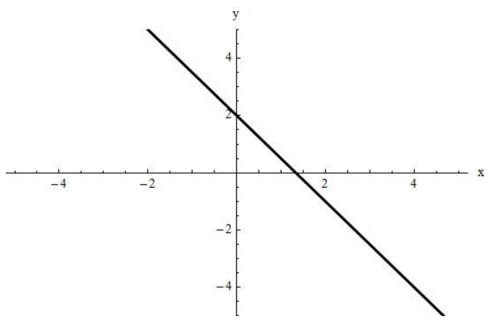


Figure 3: Incorrect

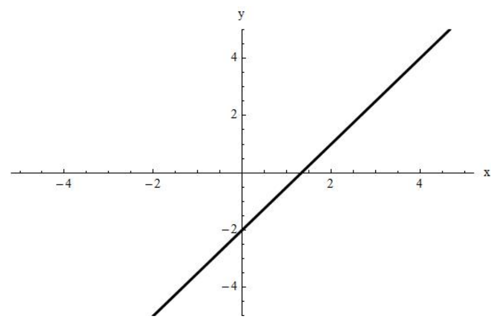
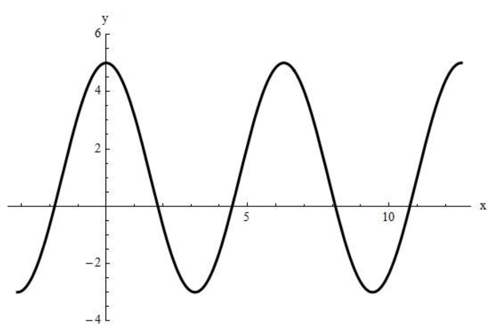


Figure 4: Incorrect

8. What is an equation of the following graph?



- $y = 4\sin x + 1$: Incorrect
- $y = 4\cos x + 1$: Correct
- $y = 5\sin x$: Incorrect
- $y = 5\cos x$: Incorrect

9. Given $f(x) = \sin(x)$, and $g(x) = 1 - \sqrt{x}$, find the functions $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$, and state their domains.

$f \circ g = \sin\left(1 - \sqrt{x}\right); \quad \text{Domain} = \{x | x \geq 0\}$

$g \circ f = 1 - \sin(x); \quad \text{Domain} = \{x | 0 \leq x \leq \pi\}$

$f \circ f = \sin(\sin(x)); \quad \text{Domain} = \{x | x \in \mathbb{R}\}$

$g \circ g = 1 - \sqrt{1 - \sqrt{x}}; \quad \text{Domain} = \{x | 0 \leq x \leq 1\}$

10. Given $f(x) = 1 - 3x$, and $g(x) = 5x^2 + 3x + 2$, find the functions $f \circ g$, and $g \circ f$, and state their domains.

$f \circ g = 1 - 3\left(5x^2 + 3x + 2\right); \quad \text{Domain} = \{x | x \in \mathbb{R}\}$

$g \circ f = 5(1 - 3x)^2 + 3(1 - 3x) + 2; \quad \text{Domain} = \{x | x \in \mathbb{R}\}$

11. Given $f(x) = \sqrt{x - 1}$, $g(x) = x^2 + 2$, and $h(x) = x + 3$, find the function $f \circ g \circ h$, and state its domain.

$f \circ g \circ h = \sqrt{\left((x+3)^2 + 2\right) - 1}; \quad \text{Domain} = \{x | x \in \mathbb{R}\}$