CORE MATHEMATICS

ACADEMIC YEAR 2022
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Introduction: Core Mathematics at USMA

This document is designed to inform USMA faculty about the core mathematics sequence and high enrollment service courses. Inside you will learn what mathematical skills and concepts you can expect from your cadets as they progress from admission to the end of the core mathematics sequence. You will see some of the philosophy and growth goals that we have adopted in order to develop a diverse group of high school graduates into college juniors who are prepared to succeed in their chosen major. Additional challenges caused by the global COVID-19 pandemic have created an even more diverse group of students and underscores the need to have a flexible program that accounts for potential gaps in knowledge. You will read about our interdisciplinary goal and our Liaison Professor program. These programs aim to achieve a more integrated experience for cadets by promoting coordination and collaboration between the Department of Mathematical Sciences and other academic departments. You will also learn the details of our program for identifying and reinforcing required mathematical skills for entering cadets.

We have included a detailed summary of course objectives for each core mathematics course and select service courses for this academic year. As part of our educational philosophy, we recognize that mastering conceptual knowledge is a difficult process requiring periodic review, practice, and consolidation. Therefore, we recommend that courses which rely heavily on portions of this conceptual material identify those portions to the cadet at the beginning of the course, and then reinforce student understanding as appropriate.

Also of special interest is the list of fundamental mathematical concepts that the MSE Committee has judged should be understood and mastered by each cadet. Within the core mathematics sequence, cadets periodically test their proficiency on this current and accumulated recall knowledge. For MSE courses that rely heavily on some subset of these recall concepts, we again recommend that you identify these to your cadets at the beginning of the course, and then reinforce (and test) them as appropriate.

Technologies are facilitating the teaching of mathematics at the undergraduate level. We try to continually find the appropriate balance between technology and fundamentals as mathematical pedagogy and content evolve. We have identified a list of fundamental concepts both non-technology and technology related for each of our core courses.

We hope you find the information in this booklet useful. We welcome your feedback on how we can better coordinate our programs, and on what information we can include here in order to help you succeed as an instructor who uses the tools from core mathematics. The Core Program Director in the Department of Mathematical Sciences updates this document annually. Please direct any comments to COL Joe Lindquist at joseph.lindquist@westpoint.edu for inclusion in the next edition.

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Educational Philosophy – The Role of Core Mathematics at USMA

*The mind is not a vessel to be filled but a flame to be kindled.* – Plutarch

Core mathematics education at USMA includes both acquiring a body of knowledge and developing thought processes judged fundamental to a cadet’s understanding of basic ideas in mathematics, science, and engineering. Equally important, this educational process in mathematics affords opportunities for cadets to progress in their development as life-long learners who are able to formulate intelligent questions and research answers independently and interactively.

At the mechanical level, the core mathematics sequence seeks to minimize memorization of a disjoint set of facts. The emphasis of the sequence is at the conceptual level, where the goal is for cadets to recognize relationships, similarities, and differences to help internalize the unifying framework of mathematical concepts. To enhance understanding of course objectives, major concepts are presented numerically, graphically, and symbolically. This helps cadets develop a visceral understanding that facilitates the use of these concepts in downstream science and engineering courses.

Concepts are applied to representative problems from science, engineering, and the social sciences. These applications not only develop cadet experience in modeling, but provide immediate motivation for developing a sound mathematical foundation for future studies.

The core mathematics experience at USMA is a vital component in an educational process that enables the cadet to acquire more sophisticated knowledge semi-independently. With Cadet development in mind, we must recognize and provide the cadet time for experimentation, discovery, and reflection. Within this setting, review, practice, reinforcement, and retrieval of mathematical knowledge and concepts are necessary and appropriate, both within the core mathematics sequence and in later science and engineering courses.

Cadets completing the core mathematics sequence will have had the opportunity to develop a degree of proficiency in several modes of thought and habits of the mind. Cadets learn to reason deductively, inductively, algorithmically, by analogy, and with the ability to capture abstractions in models. The cadet who successfully completes the USMA core mathematics sequence will have exposure to fundamental thought processes underlying discrete and continuous processes, linear and nonlinear dynamics, and deterministic and stochastic processes. The goal is for each cadet to possess a curious and experimental disposition, as well as the scholarship to formulate intelligent questions, to seek appropriate references, and to independently and interactively research answers.
Adaptive Curriculum

The Department of Mathematical Sciences is committed to taking the lead in developing an effective curriculum that attempts to foresee the mathematical needs of tomorrow’s students. Our goal is to create a mathematics program that will enhance the mathematical maturity and problem-solving skills of students.

Traditionally, calculus dominated the core mathematics sequence and modeling was used to support the application aspect of the sequence. In accordance with the Dean’s AY22 Annual Guidance to “provide the type of education that prepares our graduates to lead in a new era of war” the current curriculum is built around modeling and inquiry, with calculus and other mathematical topics presented in support. By analogy, whereas the traditional program may have presented a tool, then asked students to find a problem to apply the tool, we now present a problem, then develop the tool to solve the problem.

Placing an emphasis on both discrete and continuous modeling broadens the role of mathematics to include transforming real-world problems into mathematical constructs, performing analysis, and interpreting results. Placing an emphasis on inquiry provides opportunities for student growth in terms of learning how to learn, becoming an exploratory learner, and taking responsibility for one’s own learning.

Central to the core mathematics sequence is the concept of problem solving in the modeling sense. Courses are designed around various applications of specific concepts to motivate the mathematical skills and deepen critical thinking to build confident and competent problem solvers. This iterative process, visualized as the mathematics modeling triangle below, allows students to:

1. **TRANSFORM** the Real World phenomena into a Mathematical Model.
2. Use appropriate mathematical tools and techniques to **SOLVE** the problem.
3. **INTERPRET** the solution in terms of the original problem and based off of assumptions iterate to make the model better.

This process bears similarity to other problem solving processes used at USMA and the broader academic community. Coursework explicitly covers each step of this process - although special emphasis is given to the higher-order skills of transforming and interpreting results.
The goal is for students to establish a foundation from which to address future problems. Therefore, the sequence is designed with applications to contemporary issues as well. This approach provides students with a broad appreciation and practice of mathematics. Examples of problem domains that are considered include: the fair distribution of resources among nations; scheduling transportation resources; network design; financial models; population models; statistical inference; motion in space; optimization models; position, location, and geometric models; accumulation models; growth and change models; long-term behavior of systems; algorithm analysis, numerical techniques; linear and non-linear systems, and heuristic techniques. Some problem domains are revisited several times at more sophisticated levels during the 3-semester sequence as students solidify fundamentals and develop into more competent problem solvers.

The sequence will include contrasting approaches to problem solving such as: Continuous and Discrete; Linear and Non-linear; Deterministic and Stochastic; Deductive and Inductive; Exact and Approximate; Local and Global; Quantitative and Qualitative; Science (what is) and Engineering (what can be). For example, mathematics education has the responsibility to develop a broad range of reasoning skills. Exposure to the art of reasoning will come through the process of induction and deduction. We will create learning opportunities where students move from the puzzling data to a suggested meaning (i.e., induction). Just as importantly, we will require students to move from the suggested meaning back to the data (i.e., deduction). This process of reasoning, along with other threads (e.g. approximation with error analysis, data analysis, discovery), runs throughout the core mathematics sequence.
Core Mathematics Requirements

The core mathematics program is a three-course sequence for all cadets consisting of:

1. one semester of a mathematical modeling course (MA103 or MA153),
2. one semester of a calculus course (MA104, MA205, or MA255), and
3. one semester of probability and statistics (MA206 or MA256).

Cadets are placed in the appropriate course based on previous academic preparation and aptitude for mathematics. For example, cadets who have completed single variable calculus in high school and show a high level of proficiency on validation or AP exams are placed in the advanced mathematics program. Cadets who have completed single variable calculus in high school but have demonstrated only average proficiency will remain in the standard mathematics program, but may take multivariable calculus instead of single variable calculus to meet their calculus requirement. Cadets who have taken calculus at the college level and demonstrated mastery of the material may be able to validate the calculus requirement. Cadets may also validate the modeling course and/or the probability and statistics course by taking a validation exam.

Potential paths through the core mathematics sequence are:

1. Standard core mathematics sequence (with little or no previous calculus experience):

   MA103 → MA104 → MA206

2. Standard core mathematics sequence (with previous calculus experience):

   MA103 → MA205 → MA206

3. Advanced core mathematics sequence (with previous calculus experience and high level of aptitude):

   MA153 → MA255 → MA256

Cadets may switch between sequences if they demonstrate higher or lower levels of proficiency than indicated by initial placement. Cadets who have shown proficiency in single- and multi-variable calculus, differential equations, as well as probability and statistics may be considered for alternate pathways on a case-by-case evaluation.

Cadets who demonstrate a deficiency in algebra or trigonometry may be placed into MA100 before entering the standard core mathematics sequence. Additionally, some cadets identified with deficiencies in algebra or trigonometry in MA103 may be placed into MA100 prior to enrolling in MA104. Enrollment in MA100 requires cadets to take a minimum of four mathematics courses resulting in an overloaded semester or enrollment in a Summer Term Academic Program (STAP) to make up this additional course.
Support of USMA Academic Program Goals

**USMA Academic Program Goal:** The USMA academic program is designed to accomplish the overarching USMA Academic Program Goal.

**USMA Academic Program Goal**
Graduates integrate knowledge and skills from a variety of disciplines to anticipate and respond appropriately to opportunities and challenges in a changing world.

**Academic Program Goals:** There are seven Academic Program Goals which support the above overarching USMA Academic Goal; of these, four are particularly pertinent to the core mathematics sequence, while key aspects of the core mathematics curriculum support a fifth goal.

**Science, Technology, Engineering, and Mathematics (STEM):** Graduates apply STEM concepts and processes to solve complex problems.

**Lifelong Learning:** Graduates demonstrate the capability and desire to pursue progressive and continued intellectual development.

**Communication:** Graduates communicate effectively with all audiences.

**Critical Thinking and Creativity:** Graduates think critically and creatively.

**Disciplinary Depth:** Graduates integrate and apply knowledge and methodological approaches gained through in-depth study of an academic discipline.

The core mathematics sequence is uniquely designed to enable accomplishment of the Science, Technology, Engineering, and Mathematics Goals. The Lifelong Learning, Communication, and Critical Thinking and Creativity goals are addressed in a successive and progressive manner by both the core mathematics sequence as well as other USMA programs. However, by virtue of its position at the beginning of the student’s academic program and the large amount of consecutive contact time, the core mathematics sequence has a large responsibility for initial growth in these three areas. Over the last decade, concerted efforts have led to interdisciplinary pedagogical approaches in the core mathematics sequence contributing to the development of cadets’ ability to collaboratively engage challenges that span multiple disciplines, which is
a sub-goal of Disciplinary Depth.

Science, Technology, Engineering, and Mathematics (STEM) Academic Program Goal: The focus of the STEM Program Goal is on helping students develop various modes of thought in a disciplined manner, utilize a formal problem-solving process to solve complex and ill-defined problems in science and engineering, develop scientific literacy sufficient to be able to understand and deal with the issues of society and our profession, and effectively apply technology to enhance their problem-solving abilities. The Math-Science Goal Team (outlined in Educating Army Leaders) deems students to have demonstrated evidence of achieving these goals when they can do the following:

What Graduates Can Do:
- Apply mathematics, science, and computing to model devices, systems, processes or behaviors.
- Apply the scientific method.
- Collect and analyze data in support of decision making.
- Apply an engineering design process to create effective and adaptable solutions.
- Explain and apply computing and information technology concepts and practices in the context of the cyber domain.

The two items in bold above are particularly pertinent to the core mathematics sequence.

Lifelong Learning Academic Program Goal: The core mathematics sequence plays an important role in inspiring and motivating independent learning. The three-course connected curriculum provides a gradual transition from detailed to minimal direction. Each course provides the cadets with opportunities to take responsibility for their own learning.

Communication Academic Program Goal: Throughout the core mathematics sequence, cadets are expected to communicate their problem-solving process both orally and in writing. They are introduced to technical writing and are evaluated according to the substance, organization, style, and correctness of their report. Each course progressively develops cadets’ ability to read and interpret technical material through increased reliance on the textbook and independent learning exercises, while also developing their ability to actively listen and participate in class instruction. Technical communication is formally assessed in MA206 through the formal “Writing in the Core” rubric.

Critical Thinking and Creativity Academic Program Goal: Cadets completing the core mathematics sequence will have developed a degree of proficiency in several modes of thought and habits of mind. Cadets learn to solve problems through deductive and inductive reasoning, algorithmically, by analogy, and with the ability to capture abstractions in models. Interdisciplinary projects and activities are introduced throughout the core mathematics sequence developing our students’ ability to transfer learning across disciplines. The critical thinking skills gained solving complex, ill-defined problems in the core mathematics sequence are a vital element in developing graduates that can think and act creatively.

Disciplinary Depth: The core mathematics sequence facilitates the Disciplinary Depth goal in several ways. Much of the content is an integral part of the ABET accreditation process for several of the engineering related majors. The core mathematics sequence provides students majoring in STEM disciplines as well as in Economics a broad overview of several fundamental tools and methods of inquiry. More generally, all cadets who complete the core mathematics sequence get introduced to some of the basic strengths and limitations of mathematical modeling and reasoning processes. Additionally, all cadets participate in several interdisciplinary applications throughout the three-course sequence, and thus gain exposure to the importance of collaborative efforts that span multiple disciplines to make progress on some challenging and complex problems.
Core Mathematics Goals

The goals of the core mathematics sequence support the USMA Academic Program goals. The six primary goals of the core mathematics sequence are provided and explained in further detail below.

**Acquire a Body of Knowledge**: Acquiring a body of knowledge is the foundation of the core mathematics sequence. This body of knowledge includes the fundamental skills requisite for entry at USMA as well as the incorporation of new skills fundamental to the understanding of modeling, calculus, and statistics.

**Communicate Effectively**: Students learn mathematics when they construct their own mathematical understanding. Successful problem solvers must be able to clearly articulate their problem-solving process to others. Throughout the core mathematics sequence, students will learn to:

- Actively listen to a presentation by a student or instructor and be able to synthesize information.
- Read a mathematics textbook and either synthesize the material or frame appropriate questions to clarify what they do not understand.
- Articulate mathematical ideas in writing to include development of a mathematical model, methods used to solve the model, and interpretation of the results.
- Appropriately document all assignments according to the procedures in Documentation of Written Work.
- Articulate mathematical ideas orally in the form of board briefings, in-class discussions, and project presentations.
Apply Technology: Along with increased visualization, computing power has opened up a new world of applications and solution techniques. Our students can solve meaningful real-world problems by leveraging computing power appropriately. They will develop competence in the following skills through the core mathematics sequence:

- Students will use Excel, Mathematica, and R to visualize, solve, analyze, and experiment with a myriad of mathematical functions (discrete, continuous, linear, non-linear, deterministic and stochastic). Specific skills are listed in the Required Skills for Scientific Computing section of this document.

- Students can interpret and correctly apply results obtained from Excel, Mathematica, or R.

- Students recognize the advantages and disadvantages of using technology, and can make wise choices regarding when to use technology to assist with a problem.

- Students are competent in using resources available to provide help in using technology, and are capable of detecting errors and fixing faulty code.

Following the core sequence, we expect students to be confident problem-solvers who appropriately leverage technology to solve problems.

Build Competent and Confident Problem Solvers: The ultimate goal of the core mathematics sequence is the development of competent and confident problem solvers. Students need to apply mathematical reasoning and recognize relationships, similarities, and differences among mathematical concepts in order to solve problems. Given a problem, students will learn to:

- Identify information relevant to the problem and determine what they are trying to find.

- Make reasonable and necessary assumptions to simplify the problem, and recognize the impact of those assumptions.

- Transform the problem into a mathematical model that can be solved using quantitative techniques.

- Choose appropriate techniques (numerical, qualitative, analytical, graphical, etc.) to solve the problem.

- Interpret and apply the solution.

- Perform sensitivity analysis to evaluate the impact of the assumptions and parameter values.

- If necessary, iterate through the problem-solving process again to refine the model and explore alternative solution techniques.

Provide Experience in Interdisciplinary Problem Solving: Problems that cadets will face require the ability to consider a variety of perspectives. Mathematical analysis and results should not be accepted without understanding the social, economic, ethical and other concerns associated with the problem. The core mathematics sequence seeks to:

- Expose and equip cadets to solve problems with an interdisciplinary scope. The goal is for cadets to consider what they have learned in other disciplines when faced with a problem requiring mathematical analysis.

- Equip cadets to appropriately apply mathematical concepts to support problems faced in other disciplines.
Develop Habits of Mind: Learning is an inherently inefficient process. Learning how to teach oneself is a skill that requires maturity, discipline, and perseverance. In studying mathematics, students learn good scholarly habits for progressive intellectual development. The core mathematics sequence seeks to improve each cadet’s habits of mind in areas to include:

- **Reasoning and Critical Thinking:** Students can identify relevant information, ask questions to clarify purpose or intent, make reasonable assumptions and recognize their affects, apply induction and deduction, develop a plan, and critique their own work.

- **Creativity:** Students can extend knowledge to new situations, draw upon previous experiences, develop illustrations to clarify concepts, establish connections between concepts, and take responsible risks.

- **Work Ethic:** Students strive for accuracy and precision, persist in the face of difficulty, attempt various methods without giving up, and remain focused on developing a solution strategy and implementing it.

- **Thinking Interdependently:** Students recognize potential contributions of team members, gather data from all sources, paraphrase other’s ideas, understand the diverse perspectives of others, and act responsibly in fulfilling group commitments.

- **Life Long Learning and Curiosity:** Students recognize the value of continuous learning, develop the ability to learn independently, learn to formulate questions to fill gaps between known and unknown, actively seek knowledge, and think about their own thinking (metacognition).
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<th>Core Mathematics Goals and Objectives</th>
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| **G1:** Students display effective habits of mind in their intellectual process.  
  **O1:** Demonstrate curiosity toward learning new mathematics.  
  **O2:** Reason and think critically through complex and challenging problems.  
  **O3:** Demonstrate creativity and a willingness to take risks in their approach to solving new problems.  
  **O4:** Display a sound work ethic, striving for accuracy and precision while maintaining strong resolve to complete problems in their entirety (e.g., persistence).  
  **O5:** Think interdependently when working in groups. |
| **G2:** Students demonstrate problem solving skills.  
  **O6:** Transform problems into mathematical models that can be solved using quantitative techniques.  
  **O7:** Select and apply appropriate mathematical models as well as algorithmic and other computational techniques in the course of solving problems.  
  **O8:** Interpret a mathematical solution to ensure that it makes sense in the context of the problem. |
| **G3:** When appropriate, students apply technology to solve mathematical problems.  
  **O9:** Use technology to visualize problems and, when appropriate, estimate solutions.  
  **O10:** Use technology to explore possible solutions to mathematical problems and to conduct a sensitivity analysis.  
  **O11:** Confidently use existing and new technologies to leverage their skills. |
| **G4:** Students demonstrate an ability to communicate mathematics effectively.  
  **O12:** Write to strengthen their understanding of mathematics and integrate their ideas.  
  **O13:** Discuss their knowledge of mathematics to strengthen their understanding of mathematics and integrate their ideas.  
  **O14:** Understand and interpret written material (e.g., the textbook).  
  **O15:** Listen actively to the instructor and to student presentations. |
| **G5:** Students demonstrate knowledge of mathematics.  
  **O16:** Demonstrate competency of the fundamental skills required for entry into USMA.  
  **O17:** Demonstrate competency of the fundamental skills (mathematical recall knowledge) required for each of the core mathematics courses.  
  **O18:** Demonstrate competency on major mathematical concepts for discrete mathematics, linear algebra, calculus, and probability and statistics covered in the core sequence. |
| **G6:** Students demonstrate an ability to consider problems from the perspective of multiple disciplines including mathematics.  
  **O19:** Recognize the connections between disciplines when facing complex problems.  
  **O20:** Consider the implications of issues from other disciplines when formulating and using mathematical models in problem solving.  
  **O21:** Include analytical approaches and mathematical models to help the problem solving process when faced with a problem from another discipline. |
The Role of Technology in the Core Mathematics sequence

“Mathematical sciences departments should employ technology in ways that foster teaching and learning, increase the students’ understanding of mathematical concepts, and prepare students for the use of technology in their careers or their graduate study. Where appropriate, courses offered by the department should integrate current technology. The availability of new technological tools and their pervasive use in the workplace have the potential for changing both the curriculum and the way that the mathematical sciences can and should be taught.”
–Guidelines for Programs and Departments in Undergraduate Mathematical Sciences, The Mathematical Association of America, February 2003.

“(T)he primary goals for using technology are to deepen student understanding of mathematics and statistics and to increase student motivation and engagement. We believe the informed and intelligent use of technology helps meet these goals.”
–2015 Curriculum Guide, Committee on the Undergraduate Program in Mathematics.

The core Mathematics sequence at USMA incorporates technology into the classroom to enhance student learning and understanding of mathematics, and to produce graduates who are competent and confident with evolving technologies. Students use technology in the core mathematics sequence with the following purposes:

1. Exploration. Technology enhances students’ ability to explore mathematical ideas. For example, using technology, students can experiment by manipulating parameters of functions to see the effects, analyzing large data sets to look for patterns, fitting functions to data for use in predictive analysis, and developing new visual and graphical representations of relationships among variables. By removing time-consuming manual computations, students can focus on making conjectures, using technology to investigate and refine those conjectures, and better understand the underlying mathematics.

2. Visualization. Technology provides an easy way for students to visualize and gain understanding of previously intractable concepts. Graphs of higher dimensional functions, histograms or bar charts of large data sets, phase portraits of differential equations, and graphs of network systems are examples of visual tools that technology provides to the learner to help them gain better understanding.

3. Scientific Computing. The computing capability provided by technology frees students from unnecessarily tedious computations and allows students to solve complex problems that cannot reasonably be completed manually. The appropriate use of technology helps students differentiate the problem solving skills that should be performed by humans (thinking, modeling, analyzing, developing algorithms) versus those that can be performed by machines (computing, iterating, executing algorithms.)

4. Conceptual understanding. The use of technology can significantly contribute to students’ conceptual understanding of mathematics. Technology enables students to use multiple problem solving strategies – graphical, numerical, algebraic – on a given problem and compare/contrast results. Using technology for computational efforts frees up time for students to focus on understanding the mathematical ideas instead of just focusing on rote processes. The ability to explore and visualize gives them deeper insight. In addition, the need to be “intelligent consumers of technology,” i.e. interpreting their results to determine if they make sense in the context of the problem, will help provide them with a better understanding of mathematics and problem solving.
Use of Technology in Core Courses

Associate program directors and instructors of core mathematics courses will use the following guidelines when incorporating technology into a particular lesson:

- Determine in advance the goal of using technology, and design the experience appropriately. Ensure the use of technology is pedagogy-based, (i.e. use it when it enhances teaching/learning). The focus should not be on learning the specific technological tool but on the mathematics that students can learn as a result.

- Focus on what the student needs from the technology and not on what the technology can do. Students should not be overwhelmed with complicated code and technology requirements. Rather, they should be able to focus on the mathematics instead of being distracted by the difficulty of syntax or procedure.

- Be clear about expectations on the use of technology. If technology is used in class for a particular type of problem, students should be able to use it on an exam for that type of problem.

Since the appropriate application of technology is one of the goals of the core mathematics sequence, program directors will also assess the students’ ability to use technology. Part of this assessment will involve the students’ ability to perform specific tasks or skills; however, the following areas should also be considered:

1. Do students recognize when it is appropriate to use technology to aid in problem solving?

2. Can students look at the results obtained through technology and interpret them appropriately? Can they reflect on the meaning of the results and evaluate whether the results make sense?

3. Can students use available resources, such as software-specific help menus, student guides, or on-line search engines, to perform tasks while using technology, including tasks that may or may not have been demonstrated in the classroom or by an instructor? When students encounter errors in using technology, can they interpret the meaning of those errors and troubleshoot to fix them?
Course Objectives

MA100 – Precalculus Mathematics

MA100 prepares cadets with background deficiencies in algebra and trigonometry for the core mathematics sequence. The course develops fundamental skills in algebra, trigonometry, functions, and systems of equations through an introduction to mathematical modeling and problem solving, providing the mathematical foundation for an introductory calculus course. Upon completion of MA100, cadets will enter MA103 to begin the standard core mathematics sequence.

The Course Learning Objectives for MA100 are:

1. Body of Knowledge: Students develop an understanding of algebra, trigonometry, functions, and systems of equations applications.

2. Problem Solving: Students model, solve, and interpret applied problems that can be solved using fundamental concepts. Applied problems cover multiple disciplines including chemistry, physics, economics, and decision sciences.

3. Technology: Students can leverage technology when appropriate to solve, gain insight, and/or visualize complex problems.

4. Communication: Students develop the ability to communicate mathematics both verbally and in writing.

5. Habits of Mind: Students improve their scholarly habits for progressive intellectual development in areas such as persistence, problem solving in a methodical manner, valuing accuracy and forming good questions to deepen understanding.

The course is broken down into four blocks of instruction. The major topics of these blocks are Precalculus (real numbers, basic arithmetic/algebraic operations, equations and inequalities, and Cartesian coordinate system), Functions, Trigonometry, and Systems of Equations. To successfully complete MA100 a student must satisfactorily complete course work and assessments from the four blocks, and demonstrate proficiency in a ‘Certification’ process. Certification requires near-perfect performance in a subset of the most foundational embedded skills. Certification is a semester long endeavor which involves solving select problems using an in-person assessment and technical writing in a journal.
Examples of Embedded Skills in MA100 which Support Course Objectives

1. Know the definition and understand the properties of real numbers, negative numbers, fractions, sets and intervals, and absolute values.

2. Be able to write radical expressions using exponents and exponential expressions using radicals.

3. Be able to use special product formulas to find products of (expand) and factor algebraic expressions.

4. Be able to simplify, add, subtract, multiply and divide fractional expressions.

5. Know the definition of a linear equation, and be able to solve linear equations.

6. Given an equation, be able to solve for one variable in terms of others.

7. Be able to solve linear inequalities, nonlinear inequalities, inequalities involving a quotient, and inequalities involving absolute values.

8. Be able to sketch a graph by plotting points, and find the $x$ and $y$ intercepts of the graph of equations.

9. Understand the definition of function.

10. Be able to evaluate a function (to include piecewise functions) at any value, numeric or symbolic.

11. Be able to find the domain and range of a function.

12. Understand average rate of change, how it is calculated and be able to apply the concept to problem solving.

13. Given the graph of $y = f(x)$, know and understand the transformation that shifts it vertically and horizontally and be able to sketch it.

14. Be able to combine functions and form new ones through algebraic operations on functions.

15. Be able to determine the end behavior of polynomials.

16. Be able to define complex numbers.

17. Understand the definition of exponential and logarithmic functions.

18. Understand the definition of the natural logarithm and its properties and be able to evaluate them.

19. Given a situation for circular motion, be able to find linear and angular speed.

20. Understand the definition of trigonometric functions.

21. Be able to express one trigonometric function in terms of another.

22. Be able to “solve a triangle” by determining all three angles and the lengths of all three sides.

23. Know the Laws of Sines and Cosines.

24. Be able to graph trigonometric functions and their transformations.

25. Be able to use the substitution, elimination, and graphical methods to solve both linear and nonlinear systems of equations.

26. Be able to determine the number of solutions of a linear system in two variables.

27. Understand the 3 possible number of solutions that a system of equations can have. For a linear system of equations with 2 variables, understand what the solutions mean graphically.

28. Write a linear system as a matrix equation.
MA103 – Mathematical Modeling and Introduction to Calculus

MA103 is the first course of the mathematics core curriculum. It emphasizes applied mathematics through modeling. Students develop effective strategies to solve complex and often, ill-defined problems. The course exercises a wide array of mathematical concepts while nurturing creativity, critical thinking, and learning through activities performed in disciplinary and interdisciplinary settings. The course introduces calculus using continuous and discrete mathematics while analyzing change in applied problems. Students employ a variety of technological tools to enhance the ability to visualize concepts, to explore ideas through experimentation and iteration, to complete complex and time-consuming computations, and to develop numerical, graphical, and analytical solutions that enhance understanding.

The **Course Learning Objectives** for MA103 are:

1. Understand the fundamental math skills as a foundation to develop complex models.
2. Construct models using discrete systems and difference equations to solve problems involving dynamic change.
3. Construct models using continuous functions to explain and analyze data.
4. Use matrix algebra to solve systems of equations.
5. Use discrete and continuous models to develop forecasts and make predictions.
6. Explain the concepts of limits, average rates of change, and instantaneous rates of change, and use these concepts to describe the long-term behavior of mathematical models.
7. Use technology, especially when developing and using mathematical models, to visualize concepts, to explore potential solutions, and to execute complex and time-consuming computations when solving applied problems.
8. Interpret results of mathematical models to explain conclusions about the underlying problems.
9. Identify the assumptions required to construct a mathematical model of a given applied problem, and justify the reasonableness and necessity of those assumptions to use the model.
10. Apply concepts from discrete mathematics, matrix algebra, and calculus to model and solve interdisciplinary problems.
11. Use a mathematics textbook, via independent reading and study, to learn, examine and explain mathematical concepts.
12. Explain assumptions, models, and results using technical language in written and oral presentation formats that demonstrate understanding of math and model in terms of substance, organization, style, and correctness.

The course is broken down into three blocks of instruction, all focusing on connections to calculus. The blocks are: Modeling with Discrete Dynamical Systems (Discrete Differential Equations), Modeling with Matrices and Systems of Recursion Equations, and Introduction to Calculus and Modeling with Continuous Functions.
Examples of Embedded Skills in MA103 which Support Course Objectives

1. Given an initial condition, iterate a difference equation to find the value of interest in a sequence generated by the underlying recursion equation.

2. Determine equilibrium values for recursion equations algebraically, numerically, and graphically.

3. Determine the long-term behavior of a discrete dynamical system using its analytic solution.


5. Algebraically verify an analytic solution to an initial value problem.

6. Find the sum and the difference of two vectors.

7. Multiply vectors by a scalar or a matrix.

8. Determine the dot product of two vectors.

9. Determine the parametric (vector) equations of a line and of a plane.

10. Multiply a matrix by a scalar, matrix, or vector.

11. Calculate the determinant of a matrix.

12. Solve (by hand) systems of equations (2 × 2) using substitution, row reduction, and the inverse of a matrix.

13. Use technology to solve systems of equations of virtually any size using row reduction or the inverse matrix method.

14. Explain the graphical interpretation of an eigenvector and eigenvalue and how to compute each for a 2 × 2 matrix.

15. Use matrix algebra to model and solve network flow and social network problems.


17. Using the definition of a function, determine which relationships are functions and which are not.

18. Describe the four different representations of functions.

19. Compute average rates of change to include slopes of secant lines.

20. Explain how parameters affect the shape of linear, exponential, power, and trigonometric functions.

21. Evaluate a model’s utility and make comparisons by computing its sum of squared errors (SSE) and its coefficient of determination ($r^2$).

22. Use the concepts of rates of change to characterize the stability of the equilibrium values of non-linear difference equations.

23. Apply modeling concepts to develop a solution to a complex problem of an interdisciplinary nature.
MA103 makes important contributions toward achieving the goals of the overall academic program. In addition to reinforcing fundamental concepts, introducing students to interdisciplinary problem-solving, facilitating the integration of technology into problem solving processes, and introducing students to problem-solving strategies for addressing ill-defined problems, MA103 establishes a foundation for many critical concepts that are reinforced and used throughout the core mathematics sequence and beyond. The following table lists some of the math concepts introduced in MA103 and indicates where they connect with the core mathematics sequence and beyond. (*Denotes critical introductory calculus material currently not covered in MA104.)

<table>
<thead>
<tr>
<th>MA103 Lesson Title</th>
<th>Supported Course or Topics</th>
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<tbody>
<tr>
<td>Introduction to Math Modeling</td>
<td>Mathematical Models and Math Modeling Process (Stewart Calculus 1.2)*</td>
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<tr>
<td>Sequences</td>
<td>Arithmetic and Geometric Sequences (Stewart Calculus 11.1)*</td>
</tr>
<tr>
<td>Long-Term Behavior and Limits</td>
<td>MA205 (Infinite series, long term behavior of solutions of diff eqs)</td>
</tr>
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<td>MA364 (Fourier series solutions to PDEs)</td>
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<tr>
<td>Analytic Solutions for Linear Discrete Differential</td>
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<tr>
<td>Equations</td>
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<tr>
<td>Vectors and the 3D Coordinate System</td>
<td>MA104 (3D coord system, vectors, dot products)</td>
</tr>
<tr>
<td></td>
<td>MA205 (Vector integration, projectile motion)</td>
</tr>
<tr>
<td></td>
<td>Core Physics (Vectors, kinematics, free body diagrams, work, torque)</td>
</tr>
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<td></td>
<td>MA364 (Vector calculus)</td>
</tr>
<tr>
<td>Matrix Algebra and Solving Systems of Linear Equations</td>
<td>Introduction to matrices and matrix algebra: inverses; row reduction, determinants, identity matrices. These topics are required material for the Fundamentals of Engineering Exam and are not covered elsewhere in the core sequence.</td>
</tr>
<tr>
<td>using Matrices</td>
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<tr>
<td>Systems of Recursion Equations (Probability)</td>
<td>MA206 (probability)</td>
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<tr>
<td>Eigenvalues and Eigenvectors and Analytic Solutions</td>
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<tr>
<td>to Systems of Recursion Equations</td>
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<tr>
<td>Properties of Functions</td>
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<td>Transformation of functions (Stewart Calculus 1.3)*</td>
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<td>Limits</td>
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<td>MA205 (improper integrals)</td>
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<td>Average and Instantaneous Rates of Change</td>
<td>MA104 (Tangent and velocity problems, and rates of change)</td>
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<td>Core Physics (velocity problems)</td>
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<tr>
<td>Linear Functions (Parameter Estimation, Modeling)</td>
<td>Math Models: Linear Functions (Stewart Calculus 1.2)*</td>
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<tr>
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<tr>
<td>Introduction to Power Models</td>
<td>Math Models: Polynomials and Power Functions (Stewart Calculus 1.2)*</td>
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<tr>
<td>Power Functions (Parameter Estimation, Modeling)</td>
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<tr>
<td>Introduction to Exponential Models &amp; Exponential</td>
<td>MA205 (Newton’s Law of Cooling, Exponential Growth Models)</td>
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<td>Functions</td>
<td>SS201 (Time Value of Money)</td>
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<tr>
<td>Introduction to Trig Functions and Models</td>
<td>Math Models: Trig Function (Stewart Calculus 1.3)*</td>
</tr>
<tr>
<td>Model Evaluation</td>
<td>MA206 (linear regression, estimating model parameters, and assessing model adequacy)</td>
</tr>
<tr>
<td>Parametric and Vector Equations of Lines, Planes and</td>
<td>MA104 (Instantaneous rates of change and projectile motion, Stewart Calculus 10.1)</td>
</tr>
<tr>
<td>Curves</td>
<td>MA205 (Vector functions, motion in space Stewart Calculus 13.1)</td>
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</table>
MA104 – Calculus I

MA104 is the second semester of the mathematics core curriculum capitalizing on MA103’s introduction to calculus. This Single Variable Calculus course emphasizes the conceptual understanding of differentiation and integration in one variable using modeling and problem solving techniques. Applications such as optimization, related rates of change, work, and probability are used to motivate the study of calculus. The course provides the basic mathematical foundation for further studies in mathematics, the physical sciences, the social sciences, and engineering.

The **Course Learning Objectives** for MA104 are:

1. **Body of Knowledge:** Develop students’ knowledge and understanding of the single variable derivative as an instantaneous rate of change and the single-variable integral as an accumulation. Further, develop students’ understanding of vector operations and applications. Within this context, MA104 is organized into two blocks of material:
   - Block I: Single Variable Differentiation with Applications
   - Block II: Single Variable Integration with Applications

2. **Problem Solving:** Develop students’ abilities to model, solve, and interpret applied problems that can be solved using differential and integral calculus techniques. Applied problems cover multiple disciplines including chemistry, physics, economics, and decision sciences.

3. **Technology:** Develop students’ abilities to leverage technology when appropriate to solve, gain insight, and/or visualize complex problems. This is accomplished through multiple mechanisms including classroom applications, a technology tutorial, projects based on modeling and inquiry, and select block homework assignments.

4. **Communication:** Develop students’ written and verbal communication skills. This is accomplished through developmental experiences including classroom applications and projects where students analyze, interpret and articulate the results for applied single variable calculus problems.

5. **Habits of the Mind:** Develop students’ scholarly habits for progressive intellectual development in areas such as work ethic, creative and critical thinking, lifelong learning and curiosity. This is accomplished through classroom applications, daily homework, concept based exams, modeling and inquiry based exams, and block homework assignments.
Examples of Embedded Skills in MA104 which Support Course Objectives

1. Approximate instantaneous rates of change using average rates of change and graphically approximate the slope of the tangent line at a point on a given curve using the slopes of secant lines.

2. Understand what it means for a function to have a limit and approximate the limit of a function using graphical and numerical means.

3. Understand the mathematical and geometric definitions of continuity as well as the three different types of discontinuities at a point. Apply this understanding to determine if a function is continuous, both at a point and on an interval.

4. Understand the relationship between average rate of change and instantaneous rate of change graphically, numerically, and algebraically.

5. Interpret the physical meaning of the rate of change in the context of a given application.

6. Know and understand the definition of the derivative and apply the definition to find derivatives of basic polynomial functions.

7. Find the derivative of polynomial, exponential, logarithmic, and trigonometric functions, utilizing the properties of the derivative (constant multiple, sum, and difference rules), power rule, product rule, quotient rule, and chain rule.

8. With an understanding of implicitly versus explicitly defined functions, model problems involving quantities that change over time (i.e., “related rates” problems) and solve them using implicit differentiation.

9. Determine the critical number(s) of a differentiable single-variable function and classify them as local (or relative) and absolute (or global) extreme values using an appropriate test (Closed Interval Method, First Derivative Test, Second Derivative Test, or a graphical depiction of the function).

10. Model and solve single variable optimization problems.

11. Approximate net change using Riemann sums.

12. Find the anti-derivative of basic polynomial, exponential, logarithmic, trigonometric, rational, and power functions.

13. Understand the Fundamental Theorem of Calculus and how it establishes the relationship between the derivative and the antiderivative.

14. Use the Fundamental Theorem of Calculus to evaluate definite integrals of basic functions.

15. Evaluate definite and indefinite integrals using substitution.

16. Evaluate Type I improper integrals using an appropriate integration strategy.

17. Apply integration to solve net change problems involving probability, variable force work, and distance.

18. Apply integration to solve volumes of revolution using disks, washers, or cylindrical shells.
MA206 – Probability and Statistics

This is the final course in the mathematics core curriculum. The course develops cadets’ ability to structure their reasoning under conditions of uncertainty and presents fundamental probability and statistical concepts that support the USMA core curriculum. The course is built around a statistical investigative process where students answer research questions using data. They think critically about statistical results that are presented in scientific literature and the media, with the goal of developing a statistically “literate” graduate. Topical coverage includes data analysis, probabilistic models, independence, simulation, random variables and their distributions, hypothesis testing, confidence intervals, and linear regression. The course also introduces conceptual and engineering applications of probability and statistics. Applied problems build skills and motivate concepts, and technology enhances understanding, problem solving, and communication.

The **Course Learning Objectives** for MA206 are:

1. Use the investigative process in statistics to answer research questions of interest and be able to communicate results to general audiences.

2. Produce and interpret graphical displays and numerical summaries of data.

3. Recognize and explain the central role of variability and randomness in designing studies and drawing conclusions.

4. Employ appropriate single and multivariate statistical models and interpret and draw conclusions from standard output of statistical software packages.

5. Understand and use basic ideas of statistical inference (hypothesis tests and confidence intervals) in a variety of settings.

6. Be critical consumers of statistically-based results and recognize whether reported results reasonably follow from the study and analysis conducted.

7. Understand fundamentals of probability including axioms, probability of events, conditional probability and Bayes’ Rule, and modeling with probability distributions.

**Examples of Embedded Skills in MA206 which Support Course Objectives**

1. Use the investigative process to answer research questions of interest with data.

2. Understand, construct, and interpret visual representations of data, measures of location, and measures of variability.

3. Compare and contrast discrete and continuous random variables.

4. Create a probability mass function (PMF) in tabular or graphical format.

5. Given a probability mass function (PMF), calculate the cumulative distribution function (CDF) – and vice versa.

6. Understand the definition of conditional probability and apply Bayes’ rule.

7. Recognize and understand the assumptions underlying a binomial experiment.

8. Calculate probabilities and percentiles from the uniform and normal distributions.

9. Understand why the Central Limit Theorem is important to statistical inference. Be able to apply the Central Limit Theorem to solve probability problems.
10. Calculate and interpret confidence intervals for the population mean based on parameter values, the underlying population distribution, and sample data.

11. Understand hypothesis testing in general terms and be able to construct the proper null and alternative hypotheses for a given scenario.

12. Conduct a hypothesis test, understand the relationship between the test statistic and the \( p \)-value, and properly interpret the \( p \)-value.

13. Conduct a 2-sample hypothesis test and construct a 2-sample confidence interval to compare the means of two different populations.

14. Understand the difference between predictor (independent) variables and response (dependent) variables.

15. Given a bivariate data set, construct a simple linear regression model.

16. Be able to interpret and draw conclusions from the standard output of statistical software packages.

17. Understand the calculation and interpretation of the coefficient of determination (R-squared).

18. Use the slope parameter from a linear regression model to understand the relationship between the variables of interest.

19. Perform inference on regression coefficients.

20. Create a graph of the residuals of a linear regression model.

21. Construct and interpret a multiple linear regression model.
MA205 – Calculus II

This course provides a foundation for the continued study of mathematics and for the subsequent study of the physical sciences, the social sciences, and engineering. MA205 covers topics in multivariable differential and integral calculus, ordinary differential equations, and infinite series representations of functions. It is taught as a service course to many majors as a science depth, or complementary service course. Mathematical models motivate the study of topics such as optimization, accumulation, change in several variables, differential equations, and other topics from the natural sciences, the social sciences, and the decision sciences. The computer program MATLAB is introduced in this course to provide a basis for future STEM study in multiple engineering departments.

The **Course Learning Objectives** for MA205 are:

1. Students perform multivariable differentiation.
2. Students model and solve problems using optimization of multivariable functions.
3. Students integrate multivariable functions.
5. Students produce the solution equation for a variety of ODEs.

**Examples of Embedded Skills in MA205 which Support Course Objectives**

**Perform Multivariable Differentiation**

1. Understand the definition of a function of two variables and how it can be represented algebraically, numerically, graphically, and verbally.
2. Given a function of several variables, calculate its partial derivatives.
3. Understand the relationship and differences between average rate of change and instantaneous rate of change of a multivariable function using partial derivatives.
4. Given a function of two variables and a point, find the equation of the plane tangent to the surface at the given point.
5. Approximate the value of a multivariable function using a linear equation.
6. Given a multivariable function and parameterizations for the function, use the chain rule to solve a rates of change problem.
7. Understand what a directional derivative is in terms of a rate of change graphically, algebraically, and numerically.
8. Given a function \( f(x, y) \) of two variables, find the directional derivative of \( f(x, y) \) at a given point in any direction.
9. Given a function of two variables, find the gradient vector at a specified point. Understand that the gradient vector gives the direction of the greatest increase in functional value at a given point for a differentiable function.
10. Understand that the magnitude of the gradient vector gives the maximum rate of change of a differentiable function at a given point.
11. Understand geometrically how the gradient vector is related to level curves.
12. Employ technology to calculate, visualize, and understand the directional derivative and gradient of a multivariable function.

**Model and Solve Problems Through Multivariable Optimization**

13. Understand the definition of a critical point and be able to find the critical points of a function of two variables.

14. Use the Second Derivatives Test to classify a critical point as a local maximum, local minimum, or saddle point.

15. Find the absolute maximum and minimum of a given continuous function $f(x, y)$ on a closed and bounded set using the Closed Interval Method for functions of two variables.


17. Solve optimization problems for functions of several variables subject to a single constraint using the Method of Lagrange Multipliers.

18. Solve optimization problems for functions of several variables subject to two constraints using the Method of Lagrange Multipliers.

**Integrate Multivariable Functions**

19. Given a function, $f(x,y)$, use a double Riemann sum to estimate volume under a surface.

20. Understand the concept of volume under a surface as the limit of the sums of volumes of rectangular columns.

21. Use a table of data to estimate the volume under a surface.

22. Use level curves to estimate the volume under a surface.

23. Estimate the average value of a function of two variables.

24. Evaluate an iterated integral over a rectangular region.

25. For any region in the $xy$-plane, sketch the region, label the boundaries, and label the points of intersection.

26. Find the volume of any solid whose base is a specified region in the $xy$-plane and is bounded above by a given function, $f(x, y)$.

27. Understand and use the polar coordinate system to solve iterated integrals.

28. Find the mass, moments, and center of mass of any lamina whose area is a specified region in the $xy$-plane and whose density is a given function $\rho(x, y)$.

29. Employ technology to find the mass, moments, and center of mass of any lamina whose area is a specified region in the $xy$ plane and whose density is a given function $\rho(x, y)$.

30. Understand that a joint density function is a probability density function of multiple continuous random variables.

31. Given a joint density function of a pair of random variables $X$ and $Y$, determine the probability that $(X, Y)$ lies in a region $D$.

32. Verify if multivariable functions are valid joint density functions.
33. Evaluate a triple integral over a box.

34. Evaluate a triple integral over a general region.

35. Understand and use the cylindrical coordinate system to solve iterated integrals.

36. Understand and use the spherical coordinate system to solve iterated integrals.

**Model Functions Using Infinite Series**

37. Determine whether an infinite sequence diverges or converges.

38. Understand how an infinite series relates to an infinite sequence.

39. Apply the appropriate convergence test for a given series.
   
   (a) Apply the Test for Divergence to identify a divergent infinite series.
   (b) Recognize the geometric series, identify when it is convergent or divergent, and compute its sum when convergent.
   (c) Apply the Integral Test to determine if an infinite series converges.
   (d) Recognize the $p$-series and when it converges.
   (e) Apply the Ratio Test to determine if an infinite series converges.

40. Understand what a power series represents.

41. Find the interval and radius of convergence for a power series and understand what it represents.

42. Approximate a function using a Taylor Series.

43. Recognize the Maclaurin Series as a special case of the Taylor Series.

44. Understand that the accuracy of a Taylor Series is dependent upon the number of terms computed and the proximity of an $x$ value of interest to the center $a$.

**Produce Solution Equations for Standard ODEs**

45. Classify a differential equation.
   
   (a) Identify independent and dependent variables.
   (b) Determine the order of a differential equation.
   (c) Determine the separability of a differential equation.
   (d) Determine the linearity of a differential equation.
   (e) Identify a differential equation as homogeneous or non-homogeneous.
   (f) Identify autonomous differential equations.

46. Understand the analytic, graphical, and numerical solution techniques and determine when each is appropriate.

47. Determine general and particular solutions of differential equations.


49. Verify that a given function is a solution to a differential equation and/or initial value problem.

50. Create a slope field for a given first-order differential equation.
51. Given an initial condition, approximate a solution to a first-order ODE using a slope field.

52. Evaluate the long-term behavior and equilibrium solution of a first-order differential equation based on the slope field.

53. Approximate the values of the solution to a first-order differential equation using Euler’s (pronounced “Oiler”) Method.

54. Understand how Euler’s Method uses the concept of slope in order to solve first-order differential equations.

55. Determine a general and particular solution to a differential equation using the separation of variables technique.

56. Understand the standard form for a differential equation which may be solved with integrating factors.

57. Use integrating factors to solve first-order linear differential equations.

58. Given a second-order, linear, homogeneous differential equation, determine the characteristic or auxiliary equation.

59. Determine a general solution to a given second-order, linear, homogeneous differential equation with constant coefficients.

60. Find the particular solution to a second-order, linear, homogeneous initial-value problem.

61. Model and solve population growth problems using the natural and logistic growth models.

62. Identify equilibrium solutions and describe the long-term behavior of natural and logistic growth models.

63. Use a differential equation to model the temperature of an object with Newton’s Law of Cooling.

64. Use a differential equation to model a mixing problem.

65. Use a differential equation to model a spring-mass system, determine whether the system is undamped, critically damped, under damped, or over damped, and describe the long-term behavior.


67. Verify a solution to a system of differential equations

68. Model practical applications using systems of differential equations.

69. Apply Euler’s Method to numerically solve a system of first-order differential equations.

70. Use technology to plot the phase portrait and particular solution of a nonlinear system of two, first-order differential equations.

71. Find the equilibrium solutions to a system of first-order differential equations and use the graphical solution to help predict if those equilibria are stable or unstable.

72. Discuss the long-term behavior of the system of differential equations based on the graphical solution.
MA153 – Mathematical Modeling and Introduction to Differential Equations

MA153 is the first course of a two-semester advanced mathematics sequence for selected cadets who have validated single variable calculus and demonstrated strength in the mathematical sciences. It is designed to provide a foundation for the continued study of mathematics, sciences, and engineering. This course emphasizes the interaction between mathematics and the physical sciences and humanities through modeling with differential equations. An understanding of course material is enhanced through the use of Mathematica (computer algebra system) and Excel (spreadsheet).

The **Course Learning Objectives** for MA153 are:

1. Students learn good scholarly habits for progressive intellectual development and transition to the USMA curriculum.

2. Students build critical thinking skills to solve problems through learning complex mathematical concepts in differential equations.

3. Students develop a knowledge, curiosity and appreciation for the application of mathematics in different disciplines.

4. Students can design, solve, and interpret the results of mathematical models using ordinary differential equations (ODE) to address complex and interdisciplinary problems.

5. Students improve the quality of both their written and verbal communication of mathematics.

6. Students use technology appropriately to visualize ODE concepts, graphically and analytically explore solutions to ODE models and numerically approximate solutions to complex ODE models.

7. Students understand and can apply analytical, numerical and qualitative methods to solve first- and second-order differential equations.

8. Students understand and can apply analytical, numerical and qualitative methods to solve systems of linear and nonlinear first-order differential equations.
Examples of Embedded Skills in MA153 which Support Course Objectives

1. Derive differential equations that mathematically model simple problems.

2. Graph a direction field for a first-order ODE and sketch approximate solutions.

3. Understand what an initial value problem is, and how to show a given function is a solution to one.

4. Classify differential equations with respect to order, linearity, and homogeneity.

5. Compute the integrating factor and general solution for a linear differential equation.

6. Determine the general solution for a separable equation.

7. Understand the three identifiable stages that are always present in the mathematical modeling process: construction of the model (transform), analysis of the model (solve), comparison with experiment or observation (interpret).

8. Identify the logistic equation, determine the equilibrium solutions, and classify them as asymptotically stable, semistable, or unstable.

9. Understand when and how to apply Euler’s method and the improved Euler method to solve first-order initial value problems (by hand and with software).

10. Understand the components of an initial value problem for a second-order linear equation.

11. Understand the implications of the Existence and Uniqueness Theorem and the Principle of Superposition.

12. Understand the relationship between a fundamental set of solutions, linear independence, and a non-zero Wronskian.

13. Use the roots of the characteristic equation to find the general solution of a homogeneous equation with constant coefficients.

14. Use initial conditions to find the particular solution of a second-order differential equation with constant coefficients.

15. Use the method of undetermined coefficients to solve for the particular solution of a nonhomogeneous equation.

16. Determine the period, natural frequency, amplitude, and phase of undamped and unforced vibrations.

17. Determine the quasi-frequency and quasi-period of damped vibrations.

18. Understand the three different kinds of damping: overdamped, underdamped, and critically damped.

19. Understand the difference between forced and unforced vibrations.

20. Understand the definition and properties of the Laplace transform.

21. Determine the inverse Laplace transform of a function using a table of elementary Laplace transforms.

22. Determine the solution to an initial value problem for a second-order linear ODE using Laplace transforms.

23. Understand the properties and sketch the graphs of discontinuous functions (step functions).
24. Determine the solution to an initial value problem for a second-order, linear ODE with a discontinuous forcing function using Laplace transforms.

25. Transform a second-order ODE into a system of first-order ODEs.

26. Understand how to use row reduction to solve systems of equations; perform this process by hand for systems of three equations.

27. Understand and apply the properties of matrices and matrix algebra.

28. Find the eigenvalues and eigenvectors of a matrix.

29. Understand the definition of a fundamental set of solutions.

30. Find the general solution of a system of linear ODEs with real and different or complex eigenvalues.

31. Sketch the phase portrait for a system of two linear ODEs.

32. Classify the critical point of a system of two linear differential equations with respect to type and stability. Determine the phase plane and phase portraits of this system.

33. Understand stability as related to nonlinear systems.

34. Sketch the phase portrait of a nonlinear system.

35. Determine whether a system of ODEs is locally linear and classify the critical point of a locally linear system with respect to type and stability.

36. Use phase plane methods to investigate solutions to competing species and predator prey models.
MA255 – Advanced Multivariable Calculus

MA255 is the second course of a two-semester advanced mathematics sequence for selected cadets who have validated single variable calculus and demonstrated strength in the mathematical sciences. It is designed to provide a foundation for the continued study of mathematics, sciences, and engineering. This course consists of an advanced coverage of topics in multivariable calculus. An understanding of course material is enhanced through the use of Mathematica (computer algebra system).

The Course Learning Objectives for MA255 are:

1. Students learn good scholarly habits for progressive intellectual development.
2. Students build critical thinking skills to solve problems through learning complex mathematical concepts in multivariable and vector calculus.
3. Students develop a knowledge, curiosity and appreciation for the application of mathematics in different disciplines.
4. Students can design, solve, and interpret the results of mathematical models using multivariable calculus to address complex and interdisciplinary problems.
5. Students improve the quality of both their written and verbal communication of mathematics.
6. Students use technology appropriately to visualize complex geometries, facilitate their understanding of concepts, and solve complex, applied calculus problems.
7. Students can use vectors and vector functions to model motion and surfaces in three dimensional space.
8. Students understand the concepts of multivariable differential calculus and can apply it to solve unconstrained and constrained optimization problems.
9. Students understand the concepts of multivariable integral calculus and can apply it solve problems involving accumulation.
10. Students understand and apply the concepts of vector calculus.
Examples of Embedded Skills in MA255 which Support Course Objectives

1. Understand the basic definitions of a sequences and series.
2. Understand the meanings of convergence and limit of a sequence.
3. Understand the notion of recursive sequences.
4. Know the geometric and harmonic series.
5. Understand the properties of convergent series.
6. Know how to apply the Test for Divergence, Alternating Series Test, and Ratio Test.
7. Understand absolute convergence and conditional convergence.
8. Understand the definition of a power series and the importance of the Ratio Test in determining the radius and interval of convergence of a power series.
10. Determine the vector between two points.
11. Determine the length (magnitude) of a vector.
12. Perform arithmetic operations on vectors.
13. Determine the unit vector in a given direction.
14. Determine the resultant force of several forces using vector operations.
15. Determine the dot (scalar) and cross (vector) products of two vectors.
16. Determine the angle between vectors, lines, and planes.
17. Determine the scalar projection and vector projection of one vector onto another.
18. Use vectors to compute work.
19. Use vectors to parameterize a line segment between two points.
20. Determine if two vectors are parallel or orthogonal.
21. Use vectors to compute the equations of lines and planes.
22. Determine the point of intersection of a line and a plane.
23. Determine the equation for the line of intersection of two planes.
24. Determine the distance between a point to a plane and two parallel planes.
25. Determine limits, derivatives, and integrals of vector functions.
26. Determine the tangent line to a space curve at any point.
27. Determine arc length and curvature of space curves.
28. Determine the normal and binormal vectors of a space curve.
29. Determine velocity from acceleration and position from velocity.
30. Determine domain, range, limits, and continuity of a function of two or more variables.
31. Compute the partial derivatives of a function of two or more variables.
32. Determine the tangent plane to a surface.
33. Determine the linear approximation to a function of two variables.
34. Determine the total differential of a function of two variables.
35. Use the chain rule to differentiate a function of several variables.
36. Determine the directional derivative and gradient vector of a function of two or more variables.
37. Determine the critical points of a multivariable function.
38. Determine the absolute maximum and minimum values of a function of two variables.
39. Use the method of Lagrange multipliers to determine the absolute maximum and minimum values of a function with one or two constraints.
40. Approximate a double integral using double Riemann sums.
41. Evaluate double and triple integrals using iterated integrals.
42. Evaluate a double integral by changing to polar coordinates.
43. Evaluate a triple integral by changing to cylindrical or spherical coordinates.
44. Use triple integrals to determine the density, mass, and center of mass of a solid of variable density.
45. Be able to interpret two- and three-dimensional vector fields.
46. Determine the line integral of a multivariable function and a vector field.
47. Understand how line integrals relate to work.
48. Understand the Fundamental Theorem for Line Integrals.
49. Use Green’s Theorem to evaluate a line integral.
50. Compute the curl and divergence of a vector field and understand the physical significance of curl and divergence.
51. Determine the parametric representation of a surface.
52. Compute the surface integral of scalar and vector functions over a surface.
53. Develop an understanding of flux (surface integrals of vector fields).
54. Use surface integrals to determine the electric flux, net charge, and heat flow across a solid region.
55. Use Stokes’ Theorem to evaluate a surface integral.
56. Understand and apply the Divergence Theorem.
MA364 – Engineering Mathematics

MA364 is an applied mathematics course designed for engineers and scientists. It provides additional mathematical techniques and deepens the understanding of concepts in mathematics to support continued study in mechanical, electrical, and nuclear engineering as well as select students of the applied sciences. A goal of the course is to provide students a solid foundation in mathematical reasoning and problem solving. Emphasis is placed upon using mathematics to gain insight into natural and man-made phenomena that give rise to problems in vector calculus and differential equations. Vector calculus topics include vector functions and fields; curl and divergence; line and surface integrals; and Green’s Theorem, Stokes’ Theorem and the divergence theorem. Topics in ordinary differential equations include modeling with second-order differential equations, the method of undetermined coefficients and Laplace transforms. Additionally, an introduction to Fourier series and classical partial differential equations is included in the course.

Objectives at the topical level are:

1. Vector Calculus
   (a) Review of curve and surface parameterization, polar coordinates, integration techniques, the gradient and directional derivatives, dot and cross products of vectors, calculating double integrals, and determining the normal vector to a surface at a point.
   (b) Understand vector fields and how to calculate the divergence, understand divergence as a measure of source/sinks or expansion/compression.
   (c) Calculate the curl of a vector field and understand it as a measure of the local rotation at a point in the field.
   (d) Solve line integrals of curves through both scalar and vector fields. Understand a line integral through a vector field is a calculation of work.
   (e) Understand the difference between conservative and non-conservative vector fields, determine and understand path independence of a line integral.
   (f) Understand and apply the Fundamental Theorem of Line Integrals. Understand the relationship between the vector field and the potential function, find the potential function using the vector field equation.
   (g) Understand and apply Green’s Theorem to determine work around a closed path through a non-conservative vector field.
   (h) Understand and apply Stokes’ Theorem to determine work around a closed path through a non-conservative vector field. Understand the relationship between Stokes’ and Green’s Theorems
   (i) Find surface area and calculate surface integrals.
   (j) Understand flux as a representation of flow through a surface, calculate the flux through a surface.
   (k) Find the volume of a solid through integration.
   (l) Determine flux through a closed surface using the Divergence Theorem.

2. Ordinary Differential Equations
   (a) Understand the geometry of the complex plane and the relationship between rectangular and polar representations of complex numbers.
   (b) Perform arithmetic with complex numbers, including multiplication, division, powers, roots, and exponentiation.
   (c) Classify ordinary differential equations by order, linearity, homogeneity, and coefficient type.
(d) Model physical systems, such as spring/mass systems and LRC circuits, with ordinary differential equations.

(e) Solve second-order linear constant coefficient homogeneous differential equations using the characteristic equation. Understand how complex roots of the characteristic equation lead to real-valued solutions to the differential equation.

(f) Solve second-order linear constant coefficient homogeneous differential equations using the method of undetermined coefficients.

(g) Know the three conditions for and the behavior of the three types of damping: under, critical, and over, for a spring/mass system.

(h) Understand and apply the unit step (Heaviside) and Dirac Delta functions in modeling physical systems with discontinuous inputs.

(i) Solve second-order ordinary differential equations with continuous and discontinuous inputs using Laplace transforms.

(j) Understand and apply both translation theorems as they apply to Heaviside and Dirac delta functions.

(k) Review Taylor and Maclaurin Series

3. Fourier Series and Partial Differential Equations

(a) Find the inner product between functions, define orthogonality between functions and determine if two functions are orthogonal, determine if a set of functions is an orthogonal set.

(b) Use a Fourier series to represent a periodic function, understand how orthogonal functions allows us to solve for the coefficients of the series.

(c) Define an even or odd function, determine the Fourier sine or Fourier cosine series for even and odd functions, generate the half range expansion of a function.

(d) Classify PDEs by type: linear / nonlinear, homogeneous / nonhomogeneous, and hyperbolic / parabolic / elliptic.

(e) Understand the three types of boundary conditions: Dirichlet, Neumann, and Robin.

(f) Understand the construct of solutions pertaining to the one-dimensional heat equation using separation of variables and Fourier series methods. Solve the one-dimensional heat equation with various boundary conditions.

(g) Understand the construct of solutions pertaining to the one-dimensional wave equation using separation of variables and Fourier series methods. Solve the one-dimensional wave equation with various boundary conditions.
Interdisciplinary Goal

The Department of Mathematical Sciences believes that cadets learn mathematics by doing mathematics. Traditionally, this has meant daily work that enables cadets to practice and explain their problem-solving skills, which enables instructors to gauge the cadet’s understanding. While this practice continues, the advent of calculators, computers, and associated software now provides opportunities for cadets and teachers to create and explore more sophisticated problems. In developing these types of problems instructors have naturally turned to applications in nature, science, engineering, economics, and other fields including physical education and psychology. As a result, the mathematics faculty is often engaged in conversations dealing with how to best prepare cadets to wield these new weapons of discovery and learning.

Interdisciplinary projects are valuable as they:

- excite cadets with the power of mathematics as a tool in describing, investigating, and solving applied problems.
- excite cadets about the fields that become accessible to them as they master more mathematics.
- acquaint downstream departments with their future customers, allow these departments to demonstrate the utility of mathematics within their discipline, and involve these departments in exciting these cadets at an early stage of their development.
- enhance inter-departmental cooperation and make the four-year cadet experience more cohesive.

Usually, each core math course presents from one to three problems or projects during the course of the semester that demonstrate realistic applications of the mathematics being studied. A sampling of recent interdisciplinary projects used in the core mathematics courses include the following:

**MA103:**
Modeling Population Dynamics – GENE – 15-1  
Battery Power Planning for the 25th ID – 16-1  
Military Retirement: Legacy vs. Blended – 17-1  
The Syrian Refugee Crisis – 18-1  
Modeling the Opioid Epidemic – 19-1

**MA104:**
Optimizing DCS Tailgate Profits via Sales and Advertising – 12-2  
Sensor Placement in Afghanistan – 15-2  
“Deflate Gate” (Ideal Gas Law) – 16-2  
Space Colonization – 17-2  
Base Camp Planning – COP Keating – 18-2

**MA205:**
Modeling Planetary Motion – Physics – 18-1  
Modeling Cyber Attacks with Differential Equations – EECS – 18-2  
AEROSTAT Volume Estimation and Sensor Placement – CME/Physics/Systems – 19-1  
Machine Learning and Anomaly Detection – EECS – 19-2
The interdisciplinary goal embodies the belief that early mastery of mathematics skills produces in cadets the realization that they can indeed formulate and analyze interesting problems arising in engineering, science, business, and many other fields. Our experience is that this realization increases cadet motivation. We have seen benefits of this increased motivation as cadets move from core mathematics into their disciplines.
Liaison Professors

Most departments at USMA rely on at least some portion of the core mathematics sequence to prepare cadets for studies in their Department. Additionally, the Department of Mathematical Sciences has found that the study of mathematical principles and methods is almost always made easier by considering problems in applied settings with realistic scenarios. In order to facilitate success in both of these areas, the Department of Mathematical Sciences has instituted a liaison program in which a tenured faculty member is designated as the Liaison Professor primarily responsible for coordination with a particular client Department. The designations for this academic year are as follows:

<table>
<thead>
<tr>
<th>DEPARTMENT</th>
<th>LIAISON PROFESSOR</th>
<th>PHONE</th>
<th>OFFICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS&amp;L (Human Factors)</td>
<td>Dr. Diana Thomas</td>
<td>x5619</td>
<td>TH254</td>
</tr>
<tr>
<td>Chemistry &amp; Life Science</td>
<td>LTC Andy Lee</td>
<td>x5627</td>
<td>TH227B</td>
</tr>
<tr>
<td>C&amp;ME</td>
<td>LTC Jamie Blumen</td>
<td>x7461</td>
<td>TH249</td>
</tr>
<tr>
<td>EE&amp;CS</td>
<td>COL Joe Lindquist</td>
<td>x7436</td>
<td>TH225</td>
</tr>
<tr>
<td>GEnE</td>
<td>COL Jake LaPorte</td>
<td>x5988</td>
<td>TH246B</td>
</tr>
<tr>
<td>Physics</td>
<td>Fr. Gabe Costa</td>
<td>x5625</td>
<td>TH235B</td>
</tr>
<tr>
<td>Soc Sci (Econ)</td>
<td>COL Mike Yankovich</td>
<td>x2276</td>
<td>TH239A</td>
</tr>
<tr>
<td>Systems Eng</td>
<td>Dr. Bill Pulleyblank</td>
<td>x3200</td>
<td>TH246A</td>
</tr>
</tbody>
</table>

COL Mike Scioletti has primary responsibility for oversight of the Liaison Professor program.

The major focus of this program is to achieve a more integrated student MSE experience by promoting coordination and collaboration between the Department of Mathematical Sciences and the other MSE Departments. The role of the Liaison Professor is to serve as the principal point of contact for members of the client department. The Liaison Professor fields questions, accepts suggestions, and works issues from the client department with respect to course material, procedures, timing, and any other matters of mutual interest. Additionally, the Liaison Professor is a first point of contact for course directors in the Department of Mathematical Sciences who are looking for examples and applications appropriate to their course and need referrals.

This program is not intended to preclude closer coordination (for instance, at the course director or instructor level), but rather is intended to provide a continuing source of information and input at the senior faculty level who can ensure that inter-departmental cooperation is accomplished across several courses in a consistent fashion, as required.
The following pages provide a brief description of the fundamental mathematics skills with which you should be proficient prior to your arrival at the United States Military Academy. To be successful in the science, technology, engineering and mathematics (STEM) courses at USMA, cadets must arrive with knowledge of certain mathematical skills and concepts. USMA deems the items below to be fundamental skills and concepts that all entering cadets must possess. Early in Cadet Basic Training, the Department of Mathematical Sciences administers a “Gateway” exam to assess fundamental skills. Therefore, we encourage incoming cadets to evaluate their mathematics skills and work at remediation of any deficiencies prior to arriving at USMA.

Failure to demonstrate proficiency in the initial “Gateway” exam during Cadet Basic Training may result in placement in MA100 (Pre-Calculus) during the first semester rather than the standard placement into MA103 (Mathematical Modeling and Introduction to Calculus). Cadets enrolled in MA100 still have to complete the rest of the standard core mathematics requirements in order to satisfy graduation requirements, which means they would likely have to overload in future semesters or enroll in STAP. Failure to demonstrate proficiency in these fundamental mathematical skills by the end of their first year may be separated from USMA.

The Department of Mathematical Sciences administers two additional “Gateway” exams during the first semester in MA103 (Discrete Dynamical Systems and Introduction to Calculus). Failure to pass at least one “Gateway” exam at the proficiency level (≥ 80%) will require remediation which may result in the cadet having enroll in MA100 in the spring semester instead of continuing into MA104 (Single Variable Calculus). This would likely require course overloading in future semesters or enrollment in the summer academic term.

Almost any high school algebra book is a suitable reference for these fundamental skills. Other references and opportunities for self-remediation are available at the Department of Mathematical Sciences’ “Prospective Students” webpage.

Note: All calculations must be done without the use of technology (i.e., calculator). Some examples of skills are provided in parentheses.

1. Algebra and Real Numbers

   (a) Use symbols and operators to represent ideas and objects and the relationships existing between them.

   (b) Understand the relationship between measures of the physical world. (Velocity, distance and time: On a 40 mile car trip to Middletown, NY, you drive the first twenty miles at 40 mph and the last twenty miles at 60 mph. What is your average speed during the trip?)

   (c) Know and apply the following algebraic properties of the real number system: identity, associative, commutative, inverse, and distributive.

   (d) Express numbers using scientific notation. (Express 0.004312 in scientific notation.)
2. Radicals and Exponents
   (a) Convert between radical and rational exponent form.
       (Transform \( \frac{1}{\sqrt{x+2}} \) to the rational exponent form \((x + 2)^{-\frac{1}{2}}\)).
   (b) Manipulate algebraic expressions that contain integer and rational exponents.
       (Simplify \(4^{-\frac{3}{2}} \cdot 27^{-\frac{2}{3}}\)).

3. Algebraic Expressions
   (a) Add, subtract, multiply, and divide expressions.
       (Find the remainder when \(x^3 - 7x^2 + 9x\) is divided by \(x - 2\)).
   (b) Simplify algebraic expressions. (Expand and simplify \((x - 3)(x - 2)(x - 1)\)).

4. Factoring / Prime Numbers
   (a) Write a number as the product of factors. (Write 42 as the product of prime factors.)
   (b) Solve for the roots of a polynomial by factoring. (Find the roots of \(x^2 - 5x + 6 = 0\)).

5. Linear Equations, Inequalities and Absolute Values
   (a) Solve 2 simultaneous linear equations by graphing and by substitution. (Use a graph to estimate
       the point of intersection of the lines \(2x + 3y = 7\) and \(-x + y = 4\). Verify your result using back
       substitution.)
   (b) Solve linear equations and inequalities (graphically and algebraically).
       (Solve \(5(3 - x) > 2(3 - 2x)\) for \(x\)).
   (c) Solve linear equations and inequalities with absolute values. (Solve \(|x - 4| \geq 3\) for \(x\)).

6. Polynomials and Rational Inequalities
   (a) Solve simple polynomial inequalities. (Solve \(x^2 + 3x + 6 > x - 4\) for \(x\)).
   (b) Solve simple rational inequalities. (Solve \(\frac{x - 3}{x + 1} < 2\) for \(x\)).

7. Straight Lines
   (a) Determine the equation of a line.
       (Find the equation of a straight line passing through the points \((2, 1)\) and \((5, 4)\)).
   (b) Determine the equation of a line that is parallel or perpendicular to a given line.
       (Find the equation of a line parallel to \(2y - 3x = 7\) and passing through the point \((1, 2)\)).
8. Functions

(a) Identify the independent and dependent variables of a function.

(b) Determine the domain and range of a real-valued function.
   (Find the domain and range of the real valued function \( g(x) = \frac{1}{x^2 - 2} \).)

(c) Evaluate a function at a point. (Given \( f(x) = 3x^2 - 2x + 4 \), find \( f(2a) \).)

(d) Evaluate composite functions. (Given \( h(r) = 3r^2 \) and \( g(s) = 2s \), find \( h(a + 2) - h(g(2a)) \).)

9. Quadratic Equations and Systems

(a) Solve for real and complex roots using the quadratic formula. (Find the roots of \( 3x^2 + 2x = -1 \).)

(b) Solve a system of quadratic equations in 2 variables by substitution.
   (Solve the system \( y = 3 - x^2 \) and \( y = 4 + 2x^2 - 2x \).)

10. Trigonometric Functions

(a) Define each of the 6 trigonometric functions (\( \sin \theta, \cos \theta, \tan \theta, \cot \theta, \sec \theta, \csc \theta \)) in terms of the sides of a right triangle. (\( \cos \theta = \frac{x}{r} \) where \( x \) is the adjacent side and \( r \) is the hypotenuse.)

(b) Define each of the 6 trigonometric functions in terms of \( \sin \theta \) and \( \cos \theta \). (\( \tan \theta = \frac{\sin \theta}{\cos \theta} \).)

(c) Know the domain and ranges for the sine, cosine, and tangent functions.

(d) Convert angle measures between degrees and radians. (Write 120 degrees as a radian measure.)

(e) Memorize and use the 30/60/90 and 45/45/90 degree reference triangles.

(f) Know and apply the trigonometric identity \( \sin^2 \theta + \cos^2 \theta = 1 \).
   (Simplify the expression \( 2 \cos^2 \theta + \sin^2 \theta - 1 \).)

11. Logarithmic and Exponential Functions

(a) Know the relationship between logarithm and exponential functions (\( y = \log_a x \) is the inverse of the function \( y = a^x \) for \( a > 0, a \neq 1; \log_a x = y \Rightarrow a^y = x \)). (Evaluate \( \log_3 27 \).)

(b) Know the properties of the logarithmic and exponential functions and use them to simplify logarithmic expressions. (Express as a single logarithm: \( 0.5 \log_{10} x - \log_{10} y \).)

(c) Solve simple logarithmic and exponential equations. (Solve the equation \( 3^{x+4} = 4 \) for \( x \).)
12. **Graphs and Graphing**

(a) Graph equations and inequalities.
   (Sketch a graph of the function \( f(x) = 3x^2 - 2x + 7 \) for \( 1 < x < 5 \).)

(b) Properly label a graph (axes, intercepts, asymptotes, and roots).

(c) Know the general characteristics and shapes of the graphs of polynomial, logarithm, exponential and trigonometric functions.

(d) Transform the graph of a known function.
   (From the graph of \( f(x) \), graph the function \( g(x) = 2f(x) - 3 \).)

13. **Analytic Geometry**

(a) Know and apply the distance formula between 2 points.
   (Find the distance between the two points \( A(1, 2) \) and \( B(-5, -3) \).)

(b) Know and apply the circumference and area formulas for circles, triangles, and rectangles.
   (If you double the radius of a circle, what happens to its circumference?)

(c) Know and apply the surface and volume formulas for cylinders, spheres, and rectangular solids.

(d) Know the relationship between similar triangles. (A rectangle with base \( x \) and height 5 is inscribed in an isosceles triangle with base 10 and height 20. Determine \( x \).)

(e) Know and apply the Pythagorean Theorem to simple geometric problems.
   (Given a rectangle that is 4 ft by 7 ft determine the length of the diagonal.)
Sample Fundamental Concept Exam Questions

1. Completely simplify $\frac{1}{x(1-\frac{1}{3})}$.

2. Solve $7^x = \frac{1}{40}$ for $x$.

3. Expand the product $(3x - 4)(5 - 2x)$ and combine like terms.

4. Find all roots of $x^2 - 10x + 23 = 0$.

5. Find the distance between the points $(1, 6)$ and $(6, 18)$.

6. Solve $x^2 + 3x \leq 4$ for $x$.

7. Find an equation of the line that passes through the points $(-1, 0)$ and $(2, 6)$.

8. If the independent variable of $w(\theta) = 2\theta^2$ is restricted to values in the interval $[2, 5]$, what is the interval of all possible values of the dependent variable?

9. Completely simplify $2 \cot(x) \sec(x) \sin(x)$.

10. Completely simplify $(\frac{3a^2}{b})^{-2}$.

11. Solve the system of equations $3x = 2 - 7y$ and $y = x - 5$ for $x$ and $y$.

12. Given $g(t) = 2 + 3t$ and $h(s) = 4s^2$, evaluate $g(3) - h(2)$.

13. Graph $y = \sin(x) - 2$ on the domain $[0, 2\pi]$. Label the axes.

14. If you double the length of the sides of a cube, by how much is the volume affected?

15. Completely simplify $\log_4 \frac{3}{8+\log_4 2}$.

16. Find an equation of the line that is parallel to the line $-4x + 8y = 12$ and passes through the point $(1, 1)$.

17. Solve $x^2 + 7x - 8 = 0$ for $x$.


19. Solve $3 + x > -(x - 2)$ for $x$.

20. Write 42 as a product of its prime factors.

21. Write $\sqrt[3]{x^2}$ without using radical notation.

22. If Joe runs the first mile of the APFT in 5 minutes 40 seconds, and he wants to finish the 2-mile course in, at most, 12 minutes 40 seconds, at what speed must he run the remaining mile?

23. Solve $|x - 5| = 7$ for $x$.

24. Find the equation of the line that passes through the points $(-2, 1)$ and $(-3, 1)$.

25. What is the range of the function $h(x) = -x^2$?

26. If the hypotenuse of a 30-60-90 right triangle measures 6 inches, how long is the side of the triangle that is opposite from the 30 degree angle?

27. Solve $4 = \log_3 x$ for $x$.  

45
28. Given that \( f(x) = \sin x \), write the function that shifts \( f(x) \) to the right by 3 units.

29. Write \( 6.371 \times 10^{-5} \) in decimal notation.

30. Expand the product \((x - 5)(5 - x)\).

31. Solve for the roots of \( g(x) = x^2 - 3x - 10 \).

32. Solve \( x^2 - 5x + 3 > 5 - 4x \) for \( x \).

33. Find the equation of the line that passes through the point \((1, 2)\) and that is parallel to the line \( y = -3x + 4 \).

34. Evaluate \( j(x) = \sqrt{x^2 + 5} \) at \( x = a \).

35. What is the domain of the function \( m(\theta) = \sin(3\theta) \)?

36. Completely simplify \( \log_3 a + \log_3 b \).

37. Sketch a graph of \( y = x^3 + 3 \). Label 3 points on your sketch.

38. How long is the diagonal of a 3” by 5” rectangle?
Mathematical Recall Knowledge

The following constitutes a basic mathematical vocabulary that will be built upon during each cadet’s three-semester core mathematics experience and in his or her future science/technology/engineering/mathematics (STEM) courses. Once each of these basic ideas has been covered in class, each cadet can be required to reproduce, upon demand in any future lesson of any STEM course, that idea exactly as shown here. Annotated beside each heading or item is the course number in which the cadet is responsible for each item. These items are recall knowledge - cadets are also required to be proficient in the more conceptual, less-verbatim ideas and skills reflected in each core mathematics course’s Course Objectives section of this document. The items listed in the sections from Algebra through (and including) Properties of Functions are the mathematical skills represented on the Fundamental Concepts Exam (FCE) and are knowledge required of entering cadets.

ALGEBRA (MA100 / MA103)

1. \(ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)

2. \(a^b \cdot a^c = a^{b+c}\)

3. \((ab)^c = a^c b^c\)

4. \(\frac{a^b}{a^c} = a^{b-c}\)

5. \(y = \log_b x \Rightarrow x = b^y\)

6. \(\log_b (b^x) = b^{\log_b (x)} = x\)

7. \(\log_b x^a = a \log_b x\)

8. \(\log_b ac = \log_b a + \log_b c\)

9. \(\log_b \frac{a}{c} = \log_b a - \log_b c\)

10. \(\log_b a = \frac{\log_c a}{\log_c b}\)

ANALYTIC GEOMETRY (MA100 / MA103)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>(lw)</td>
<td>(2l + 2w)</td>
</tr>
<tr>
<td>Circle</td>
<td>(\pi r^2)</td>
<td>(2\pi r)</td>
</tr>
<tr>
<td>Rectangular Solid</td>
<td>(lwh)</td>
<td>(2lw + 2lh + 2hw)</td>
</tr>
<tr>
<td>Cylinder</td>
<td>(\pi r^2l)</td>
<td>(2\pi r^2 + 2\pi rl)</td>
</tr>
<tr>
<td>Sphere</td>
<td>(\frac{4}{3} \pi r^3)</td>
<td>(4\pi r^2)</td>
</tr>
</tbody>
</table>

Distance between \((x_1, y_1)\) and \((x_2, y_2)\) = \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)

TRIGONOMETRY (MA100 / MA103)

With reference to the right triangle:

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
\tan \theta &= \frac{y}{x} \\
\csc \theta &= \frac{1}{\sin \theta} \\
\sec \theta &= \frac{1}{\cos \theta} \\
\cot \theta &= \frac{1}{\tan \theta} \\
\tan \theta &= \frac{\sin \theta}{\cos \theta} \\
x^2 + y^2 &= r^2 \\
\sin^2 \theta + \cos^2 \theta &= 1
\end{align*}
\]

\[2\pi \text{ radians} = 360 \text{ degrees}\]
RELATIONSHIPS (MA100 / MA103)

1. Corresponding sides of similar triangles are proportional
2. Distance = average rate × time

DIFFERENTIATION (MA104)

1. \( \frac{d}{dx} (a) = 0 \)
2. \( \frac{d}{dx} (x) = 1 \)
3. \( \frac{d}{dx} (au) = a \frac{du}{dx} \)
4. \( \frac{d}{dx} (u + v) = \frac{du}{dx} + \frac{dv}{dx} \)
5. \( \frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx} \) (Product Rule)
6. \( \frac{d}{dx} (\frac{u}{v}) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \) (Quotient Rule)
7. \( \frac{d}{dx} (u^n) = nu^{n-1} \frac{du}{dx} \) (Power Rule)
8. \( \frac{d}{dx} [f(u)] = \frac{d}{du} [f(u)] \frac{du}{dx} \) (Chain Rule)
9. \( \frac{d}{dx} (\sin u) = \cos u \frac{du}{dx} \)
10. \( \frac{d}{dx} (\cos u) = -\sin u \frac{du}{dx} \)
11. \( \frac{d}{dx} (e^u) = e^u \frac{du}{dx} \)
12. \( \frac{d}{dx} (\ln u) = \frac{1}{u} \frac{du}{dx} \)

INTEGRATION (MA104)

13. \( \int adx = ax + C \)
14. \( \int (u + v) dx = \int u dx + \int v dx \)
15. \( \int x^n dx = \frac{x^{n+1}}{n+1} + C (n \neq -1) \)
16. \( \int e^{ax} dx = \frac{e^{ax}}{a} + C \)
17. Understand and be able to apply the Substitution Rule
18. \( \int \frac{du}{u} = \ln |u| + C \)
19. \( \int \sin (ax) dx = -\frac{1}{a} \cos (ax) + C \)
20. \( \int \cos (ax) dx = \frac{1}{a} \sin (ax) + C \)

Fundamental Theorem of Calculus:
21. If \( f \) is integrable on \([a, b] \), then \( \int_a^b f(x) dx = F(b) - F(a) \) where \( \frac{dF}{dx} = f(x) \)

VECTOR CALCULUS (MA103/MA104/MA205)

22. \( |\vec{A}| = \sqrt{a_i^2 + a_j^2 + a_k^2} \)
23. \( \vec{A} \cdot \vec{B} = a_i b_i + a_j b_j + a_k b_k = |\vec{A}| |\vec{B}| \cos \theta \)
24. \( |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta \)
25. \( \nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} \)
26. \( \vec{A} \times \vec{B} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix} = (a_j b_k - a_k b_j) \hat{i} - (a_i b_k - a_k b_i) \hat{j} + (a_i b_j - a_j b_i) \hat{k} \)
27. PMFs, PDFs and CDFs:

Discrete: PMF: \( P(X = x) = p(x) \)
CDF: \( P(X \leq x) = \sum_{y \leq x} p(y) \)

Continuous: PDF: \( f(x) \) is used to find probabilities of the form \( P(a \leq X \leq b) = \int_a^b f(x) \, dx \)
CDF: \( F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y) \, dy \)

28. The total accumulation of a probability distribution and probability mass function is 1.

Discrete: \( \sum p(x) = 1 \) 
Continuous: \( \int_{-\infty}^{\infty} f(x) \, dx = 1 \)

29. Calculate and interpret the expected value (mean) of a random variable.

Discrete: \( E(X) = \sum x \cdot p(x) \) 
Continuous: \( E(X) = \int_{-\infty}^{\infty} x \cdot f(x) \, dx \)

30. Calculate and interpret the variance of a random variable.

The variance is an expected value: \( V(X) = E[(X - \mu)^2] = E(X^2) - \mu^2 \)

31. Percentiles of random variables including the median (50\textsuperscript{th} percentile).

32. Central Limit Theorem: Let \( X_1, X_2, \ldots, X_n \) be a random sample from any population with a finite mean and variance. Then, if \( n \) is sufficiently large, \( \bar{X} = \sum_i \frac{X_i}{n} \) and \( T_0 = \sum X_i \) are \textit{approximately} normally distributed.
United States Military Academy Required Skills in Scientific Computing

The following list constitutes the basic, required scientific computing skills that Cadets learn and use during their three-semester core mathematics experience and in your future math/science/engineering courses. Once each of the listed topics is covered in class, students are expected to execute this skill upon demand in any future lesson or course. Reference books, for the respective computing system, are the chief source of information for learning these skills. Annotated before each item is the course, which denotes the exact point in time students are responsible for that particular item (e.g. MA103 indicates responsibility for that skill following MA103.

**LAPTOP SKILLS (MATHEMATICA, MS EXCEL, and R)**

The following are all laptop (or desktop) computer skills. Most can be accomplished in either Mathematica, Excel, or R, but some are easier in one package. As you become proficient in each skill, it is your job to decide which tool is the right one for the task at hand.

1. (MA103) Enter a recursive relationship and evaluate analytically, numerically and graphically. For example:

    Given the difference equation \( p_{n+1} = 3p_n + n^2 \):

    (a) Find \( p_{100} \).

    (b) Display a table of \( n \) and \( p_n \) values.

    (c) Graph the time-series plot of \( p_n \).

2. (MA103) Perform basic matrix and vector operations:

    (a) Given 2 matrices, compute (if defined) their sum and product.

    (b) Find the inverse of a given square matrix (if it exists).

    (c) Find the eigenvalues and eigenvectors of a given matrix.

    (d) Find the magnitude of a given vector.

    (e) Given 2 vectors, compute their dot product.

3. (MA103) Perform matrix operations on linear systems of equations in two and three variables. For example:

    \[
    \begin{bmatrix}
    2 & 5 & 6 \\
    7 & 8 & 4 \\
    3 & 2 & 5
    \end{bmatrix}
    \begin{bmatrix}
    x \\
    y \\
    z
    \end{bmatrix}
    =
    \begin{bmatrix}
    4 \\
    7 \\
    4
    \end{bmatrix}
    \]

    (a) Solve the system of equations.

    (b) Perform row reduction operations on the augmented matrix.
4. (MA103) Use the Solve or FindRoot commands in Mathematica to find the roots of a polynomial or transcendental function (both real and complex) or to solve simultaneous equations. For example:

(a) Find the roots of the function \( r^3 - 7r^2 - 14r + 8 = 0 \).
(b) Find the roots of the functions \( e^x - 2 = 0 \).
(c) Solve to find the point of intersection between \( 2x + 3y = 7 \) and \( -x + y = 4 \).

5. (MA103) Define and evaluate a function at a specific point in Mathematica and Excel.

6. (MA103) Plot a function or several functions over a specified domain.

7. (MA103) Use the Excel Solver to minimize the sum of the squared error (SSE) when fitting models to data.

8. (MA104) Evaluate limits. For example:

(a) \( \lim_{x \to 5} \frac{x^2 - 5}{x^2 - 25} \).
(b) \( \lim_{x \to \infty} \frac{2\sqrt{x} + x^{-1}}{3x^2 - 1} \).

9. Evaluate derivatives. For example:

(a) (MA104) Find the derivative of \( y = \frac{1}{x} + \sin(2x^3 - x) \).
(b) (MA205 / MA255) Find the partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) of \( f(x, y) = y^2 + \cos(2x^3 - y) \).
(c) (MA205 / MA255) Find the directional derivative of \( f(x, y) = x^2y + xy^2 \) at the point \((2, 3)\) in the direction of \( \vec{v} = \langle 2, 4 \rangle \).

10. (MA104) Calculate Riemann sums to approximate net change.

11. Evaluate definite and indefinite integrals. For example:

(a) (MA104) Evaluate \( \int (x \ln(x) - e^x) \) \( dx \).
(b) (MA104) Evaluate \( \int_0^2 (x^2 + \cos(2x)) \) \( dx \).
(c) (MA205 / MA255) Evaluate \( \int_0^1 \int_0^y (3y^3 + e^{xy}) \) \( dx \) \( dy \).

12. (MA205) Plot regions of integration.

13. (MA104) Calculate distance traveled along a curve using the concept of arc length.

14. (MA205 / MA255) Plot 3-dimensional objects (surfaces, space curves, and level curves). For example:

(a) Plot the space curve \( \vec{r}(t) = \langle t^2, 2t, \sin(t) \rangle \) over the domain \( t \in [0, \pi] \).
(b) Plot the surface \( z = \frac{x^2}{4} + \frac{y^2}{16} \).
(c) Plot the level curves for \( x^2 + y^2 = k \).

15. (MA205 / MA255) Solve problems involving differential equations (DE). For example: Given the differential equation \( \frac{dy}{dx} = \cos(\pi x) + e^{-x} - 2 \),

(a) Plot the direction (slope) field of the DE.
(b) Graphically approximate a solution curve to the DE for \( y(0) = 2 \).
(c) Use Euler’s Method to find a numerical approximation for \( y(3) \) given \( y(0) = 2 \).
(d) Verify a given solution to the DE.
16. (MA206) Fit a linear regression model to data using R.

17. (MA206) Compute sample statistics from a data set.

18. (MA206) Plot a histogram and boxplot.


20. (MA206) Integrate probability density functions to calculate probabilities.

21. (MA206) Calculate expected value and population variance using integration.

22. (MA206) Calculate the inverse CDF using FindRoot or Solve commands.

23. Seek on-line HELP to learn new commands, uses, and methods, as well as to troubleshoot one’s own efforts.

**MS-WORD, with Graphics Objects from other Packages**

(MA103) Create a formal document in electronic form that includes textual, graphical, numerical, and analytical modeling and analysis.
History of the Department of Mathematical Sciences at USMA

A teacher affects eternity; he can never tell where his influence stops. – Henry Adams

The Department of Mathematical Sciences, USMA, has a rich history of contributing to the education of cadets as confident problem solvers and of developing its faculty as effective teachers, leaders, and researchers. The story of mathematical education at West Point is full of interest: faculty curriculum developments, teaching methods and tools, and technological equipment. Many of the Department’s advances have been exported outside the Academy to be employed by other civilian and military educational institutions.

EARLY BEGINNINGS: The actual teaching of mathematics at West Point dates from even before the Academy was established. In 1801, George Baron taught a few Cadets of Artillery and Engineers some of the fundamentals and applications of algebra. The Academy at West Point was instituted by act of Congress and signed into law by President Thomas Jefferson on 16 March 1802. The first acting Professors of Mathematics were Captains Jared Mansfield and William Barron. They taught the first few cadets topics in algebra, geometry, and surveying.

CONTRIBUTIONS TO THE NATION: Since the Academy was the first scientific and technical school in America, the early mathematics professors at USMA had the opportunity to make significant contributions not only to the Academy, but also to other American colleges. Perhaps the most prominent contributors were the early 19th century department heads Charles Davies and Albert E. Church. The work of these two professors had a significant impact on elementary schools, high schools, and colleges across the country. Davies became the Professor of Mathematics in 1823. He was one of the most prolific textbook authors of his day, writing over 30 books from elementary arithmetic to advanced college mathematics. His books were used in schools throughout the country from grade school to college. He had tremendous influence on the educational system of America throughout the 19th century. Albert Church succeeded Davies in 1837, and served as Department Head for the next 41 years. Another influential author, he published seven college mathematics textbooks.

PRODUCING LEADERS FOR THE NATION: Faculty from the Department have been notable military leaders for the country. Robert E. Lee was a standout cadet-instructor in the Department, Omar Bradley served as an Instructor for four years, Harris Jones and William Bessell were Deans of the Academic Board at USMA for a total of 15 years, and Department Heads Harris Jones, William Bessell, Charles Nicholas, John Dick, and Jack Pollin served impressively during two world wars.

The unique technical curriculum in place at the Academy during the middle of the 19th century produced many successful mathematicians and scientists for the country at large. West Point graduates Horace Webster, Edward Courtenay, Alexander Bache, James Clark, Francis Smith, Richard Smith, Henry Lockwood, Henry Eustis, Alexander Stewart, and William Peck filled positions as professors of mathematics or college presidents at other schools such as the U. S. Naval Academy, Geneva College, University of Virginia, University of Pennsylvania, University of Mississippi, Yale, Brown, Harvard, Columbia, Virginia Military Institute, Cooper Institute, and Brooklyn Polytechnic Institute. Two mathematics department heads became college presidents after leaving USMA; Alden Partridge founded and became the first president of Norwich University, and David Douglass served as president of Kenyon College in Ohio for four years. Jared Mansfield was appointed surveyor-general of the Northwest Territory, and Ferdinand Hassler became superintendent of the United States Coastal Survey. Capable individuals such as these exported the West Point model of undergraduate mathematics education throughout the nation.
While the faculty at USMA has been primarily military, the Department has benefited from civilian visiting professors since 1976. As part of the goal for civilianization of 25% of the faculty by 2002, begun in 1992, the Department established in 1994 a Center for Faculty Development in Mathematics. This Center establishes faculty development models and curricula and provides for the development of the "Davies Fellows", who serve as rotating civilian faculty members.

**HISTORICAL AND LIBRARY HOLDINGS:** Sylvanus Thayer’s first task before assuming the Superintendent in 1817 was to tour the technical institutions of Europe and assess what features USMA could use to advantage. One of Thayer’s many accomplishments was to obtain numerous mathematics and science books from Europe. Thayer’s book collection included many of the finest books available at that time. His books provided a solid foundation for the USMA library to build upon. Today, the West Point Library has one of the finest collections of pre-20th century mathematics books in the world. Also during the middle of the 19th century, the Academy instructors used elaborate physical models made by Theodore Oliver to explain the structures and concepts of geometry. This magnificent collection of string models is still in the Department today.

**CURRICULAR DEVELOPMENT:** After Thayer studied the military and educational systems of Europe, he reorganized the Academy according to the French system of the Ecole Polytechnic. The Department of Mathematics faculty (which included as Professor the distinguished scientist and surveyor Andrew Ellicott, and the famous French mathematician Claude Crozet whom Thayer recruited during his European trip to bring to USMA and America his expertise in Descriptive Geometry, advanced mathematics, and fortifications engineering) combined the French theoretical mathematics program with the practical methods of the English to establish a new model for America’s program of undergraduate mathematics. This program of instruction in Mathematics grew over several decades and was emulated by many other schools in the country. The initial purpose of the Military Academy was to educate and train military engineers. Sylvanus Thayer, the "Father of the Military Academy" and Superintendent from 1817-1833, instituted a four-year curriculum with supporting pedagogy to fulfill this purpose. Thayer’s curriculum was very heavy in mathematics; from Thayer’s time to the late 1800’s, cadets took the equivalent of 54 credit hours of mathematics courses. The topics covered in these courses were algebra, trigonometry, geometry, descriptive geometry (engineering drawing), analytic geometry, and calculus. Over the years, the entering cadets became better prepared and fewer of the elementary subjects were needed. During Davies’ tenure (1823-37), calculus was introduced as a requirement for all cadets, and was used in the development of science and engineering courses. The time allotted for the mathematics curriculum decreased to 48 credit hours by 1940, and to 30 credit hours by 1950. During the 1940’s, courses in probability and statistics and in differential equations were introduced into the core curriculum and a limited electives program was started for advanced students. In the 1960’s, department head Charles Nicholas (previously one of the organizers of the Central Intelligence Agency) wrote a rigorous and comprehensive mathematics textbook (the "Green Death") that cadets used during their entire core mathematics sequence. With this text, he was able to adapt the mathematics program to keep up with the increasing demands of modern science and engineering. In the 1970’s, Academy-wide curricular changes provided opportunities for cadets to major in mathematics.

During the 1980’s, a mathematical sciences consulting element was established that allowed faculty members and cadets to support the research needs of the Army. This type of research activity continues today in the Army Research Laboratory (ARL) Mathematical Sciences Center of Excellence and in the Operations Research Center (ORCEN). In 1990 the Department introduced a new core mathematics curriculum that included a course in discrete dynamical systems, with embedded linear algebra. In that same year, the department changed its name to the Department of Mathematical Sciences to reflect broader interests in applied mathematics, operations research, and computation.
TECHNOLOGICAL DEVELOPMENT: USMA has a long history of technological innovation in the classroom. It was Crozet and other professors at USMA in the 1820’s who were the first professors in the nation to use the blackboard as the primary tool of instruction. In 1944, the slide rule was issued to all cadets and was used in all plebe mathematics classes. During William Bessell’s tenure (1947-1959), the mathematics classrooms in Thayer Hall (a converted riding stable) were modernized with overhead projectors and mechanical computers. Bessell was also instrumental in establishing a computer center at West Point. The hand held calculator was issued to all cadets beginning in 1975, and pre-configured computers were issued to all cadets beginning in 1986.

NATIONAL COMPETITION: In the spring of 1933 West Point entered an interesting competition in mathematics. After USMA defeated Harvard at a football game the previous fall, a chance remark from President Lowell of Harvard to Superintendent Jones of USMA led the two schools to arrange a mathematics challenge match between the two schools with the two competing teams each composed of 12 second-year students. Army was the home team, so the Harvard competitors traveled by train to West Point. All the competitors took a test written by the president of the Mathematics Association of America. The New York Times, which had promoted the event with a series of articles in its sports section, reported the results. The West Point “mathletes” defeated Harvard in the competition that was the precursor to the national Putnam Competition. Since its inception in 1984, the Academy has entered teams in the International Mathematics Competition in Modeling. USMA won the top prize in 1988, 1993, 2000-2004, 2006, 2009-2010, 2016-2017, and 2019.

During the first half of the 1990’s, the Department of Mathematical Sciences at USMA became recognized as one of the more progressive mathematics programs in the country. The Department developed a strong “7 into 4” program that combined the seven topics of Differential Calculus, Integral Calculus, Multi-variable Calculus, Differential Equations, Linear Algebra, Probability and Statistics, Discrete Math into the four allotted semesters of core mathematics. Throughout the 1990’s and into the 21st century, the Department of Mathematical Sciences has led the nation in curricular reform and in nurturing interdisciplinary cooperation and collaboration between and among partner disciplines. Project INTERMATH, a National Science Foundation grant to develop integrated curriculum, assisted in supporting these efforts. The USMA mathematics program has had great influence on mathematics education in America throughout its history, and strives to continue contributing to the improvement of mathematical education in America.

For a more extensive exposition on the history of the Department, refer to The Department of Mathematical Sciences website.