TO THE ILLUSTRIOUS JOHN SCHÖNER, as to his own revered father, G. Joachim Rheticus sends his greetings.

On May 14th I wrote you a letter from Posen in which I informed you that I had undertaken a journey to Prussia,[1] and I promised to declare, as soon as I could, whether the actuality answered to report and to my own expectation. However, I have been able to devote scarcely[2] ten weeks to mastering the astronomical work of the learned man to whom I have repaired; for I had a slight illness and, on the honorable invitation of the Most Reverend Tiedemann Giese, bishop of Kulm, I went with my teacher to Löbau and there rested from my studies for several weeks.[3] Nevertheless, to fulfill my promises at last and gratify your desires, I shall set forth, as briefly and clearly as I can, the opinions of my teacher on the topics which I have studied.

First of all I wish you to be convinced, most learned Schöner, that this man whose work I am now treating is in every field of knowledge and in mastery of astronomy not inferior to Regiomontanus. I rather compare him with Ptolemy, not because I consider Regiomontanus inferior to Ptolemy, but because my teacher shares with Ptolemy the good fortune of completing, with the aid of divine kindness, the reconstruction of astronomy which he began, while Regiomontanus—alas, cruel fate—departed this life before he had time to erect his columns.

My teacher has written a work of six books in which, in imitation of Ptolemy, he has embraced the whole of astronomy,

[1] The basic study for the biography of Rheticus will be found in *Vierteljahrsschrift für Geschichte und Landeskunde Vorarlbergs*, neue Folge, II(1918), 5-46. For subsequent work consult *Forschungen zur Geschichte Vorarlbergs und Liechtensteins*, I(1920), 128-30; *Schriften des Vereines für Geschichte des Bodensees*, LV(1927), 120-37; and Martin Bilgeri, *Das Vorarlberger Schrifttum* (Vienna, 1936), pp. 64-70.

[2] Reading *vix* (Th 447.8) instead of *viri* (PII, 295.7).

[3] In the light of this remark, we must regard as incorrect Prowe's statement (PI^2, 395) that the *Narratio prima* was written at Löbau. Prowe himself declares that Rheticus's trip to Löbau kept him from his studies (PI^2, 428).

stating and proving individual propositions mathematically and by the geometrical method.

The first book contains the general description of the universe and the foundations by which he undertakes to save the appearances and the observations of all ages. He adds as much of the doctrine of sines and plane and spherical triangles as he deemed necessary to the work.

The second book contains the doctrine of the first motion[4] and the statements about the fixed stars which he thought he should make in that place.

The third book treats of the motion of the sun. And because experience has taught him that the length of the year measured by the equinoxes depends, in part, on the motion of the fixed stars, he undertakes in the first portion of this book to examine by right reason and with truly divine ingenuity the motions of the fixed stars and the mutations of the solstitial and equinoctial points.

The fourth book treats of the motion of the moon and eclipses; the fifth, the motions of the remaining planets; the sixth, latitudes.

I have mastered the first three books, grasped the general idea of the fourth, and begun to conceive the hypotheses of the rest. So far as the first two books are concerned, I have thought it unnecessary to write anything to you, partly because I have a special plan,[5] partly because my teacher's doctrine of the first motion does not differ from the common and received opinion,[6] save that he has so constructed anew the tables of declinations, right ascensions, ascensional differences, and the other tables belonging to this branch of the science that they can be brought by the method of proportional parts into agree-

[4] The apparent daily rotation of the heavens; see p. 41, above.

[5] Rheticus doubtless refers to his plan for writing a "Second Account." For an explanation why this "Second Account" was never written see p. 10, above.

[6] But in the common and received opinion the first motion was real; in Copernicus's system, apparent. Rheticus ignores the distinction, for it involves the motion of the earth. Throughout the first third of this *Account* he withholds all reference to Copernicus's principal alteration of astronomical theory, the shift from a stationary to a moving earth, and from geocentrism to heliocentrism (cf. below, pp. 135-36, n. 115).

ment with the observations of all ages. Therefore I shall set forth clearly to you, God willing, the subjects treated in the third book together with the hypotheses of all the remaining motions, so far as at present with my meager mental attainments I have been able to understand them.

The Motions of the Fixed Stars

My teacher made observations with the utmost care at Bologna, where he was not so much the pupil as the assistant and witness of observations of the learned Dominicus Maria;[7] at Rome, where, about the year 1500, being twenty-seven years of age more or less, he lectured on mathematics before a large audience of students and a throng of great men and experts in this branch of knowledge; then here in Frauenburg,[8] when he had leisure for his studies. From his observations of the fixed stars he selected the one which he made of Spica Virginis in 1525. He determined its distance from the autumnal point[9] as about 17°21′, and its declination as not less than 8°40′ south of the equator. Then comparing all the observations of previous writers with his own, he found that a revolution of the anomaly or of the circle of inequality had been completed and that the second revolution extends from Timocharis to our own time. Thereby he geometrically determined the mean motion of the fixed stars and the equations of their unequal motion.

Timocharis's observation of Spica in the 36th year[10] of the first Callippic cycle, when compared with his observation in the 48th year of the same cycle, shows us that the stars moved 1° in 72 years in that era.[11] From Hipparchus to

[7] Concerning whom Lino Sighinolfi has assembled some material, chiefly biographical, in his article "Domenico Maria Novara e Nicolò Copernico" (*Studi e memorie per la storia dell' università di Bologna*, V [1920], 211-35).

[8] Cf. Th 193, note to line 9.

[9] The first point of Libra (cf. Th 161.24-25).

[10] 295/4 B. C. A Callippic cycle contained 76 years (HII, 25.16-17; Th 159.11). See F. K. Ginzel, *Handbuch der mathematischen und technischen Chronologie* (Leipzig, 1906-14), II, 409-19; and J. K. Fotheringham in *Monthly Notices of the Royal Astronomical Society*, LXXXIV(1924), 387-92.

[11] HII, 28.11-30.17.

Menelaus they regularly completed 1° in 100 years.[12] My teacher therefore concluded that Timocharis's observations fell in the last quadrant of the circle of inequality,[13] in which the motion appears mean-diminishing, and that between Hipparchus and Menelaus the motion of inequality was slowest. A comparison of Menelaus's observations with Ptolemy's shows that the stars then moved 1° in 86[14] years. Therefore Ptolemy's

[12] Ptolemy accepts this estimate as the approximate value for the entire period from Hipparchus to himself (HII, 23.11-16); and he regards the rate of precession as constant (HII, 34.11-17).

[13] Copernicus held that the rate of precession varied. To represent the variation he constructs a "circle of inequality," (Fig. 26) in which *a* is the point of slowest

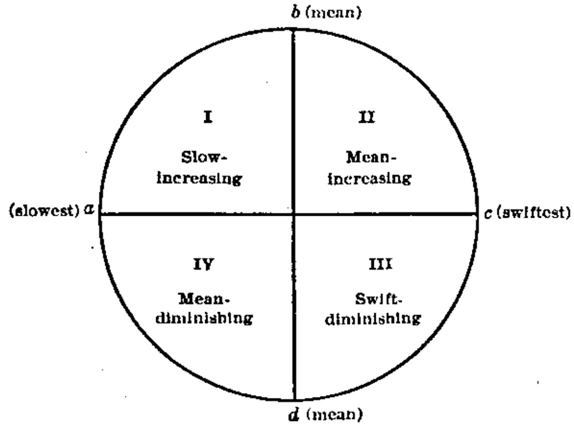

FIGURE 26

motion; *c*, the point of swiftest motion; *b* and *d*, the points of mean motion. The first quadrant *ab* is the quadrant of slow-increasing motion; the second quadrant *bc*, of mean-increasing motion; the third quadrant *cd*, swift-diminishing; the fourth quadrant *da*, mean-diminishing. See Th 169.25-170.4, and p. 100, above.

[14] This should be 96, as an examination of Menzzer's chart (p. 21 of his notes) shows. Menelaus determined the longitudinal distance of Spica from the summer solstice as 86° 15′, and of β Scorpii from the autumnal equinox as 35° 55′ (HII, 30.18-31.16, 33.3-24; Th 159.24-29). For Ptolemy, 40 years later, the corresponding values were 86° 40′ and 36° 20′ (HII, 103.16, 109.18; Th 159.29-160.4; *LXXXVI s.* in Th 160.1 is wrong, as can be seen in the *Letter against Werner* [cf. p. 104, above] and in the three editions of the *Syntaxis* available to Copernicus: 1515, p. 83r; 1528, p. 78r; 1538, p. 187; cf. Th 161.28-31). For both stars the motion is 25′ in 40 years, or 1° in 96 years. It is more likely that an X has fallen out of LXXXXVI than that Rheticus made an error in his computations.

observations were made when the motion of anomaly was in the first quadrant, and the stars then moved with a slow-increasing[15] or -augmenting motion. Further, from Ptolemy to Albategnius, 66 years correspond to 1°;[16] a comparison of our observations with those of Albategnius shows that the stars in their unequal motion again completed 1° in 70 years;[17] and a comparison of the observation which I mentioned above with the others which my teacher made in Italy shows that the fixed stars in their unequal motion are once more passing through 1° in 100 years. Therefore it is clearer than sunlight that between Ptolemy and Albategnius the motion of inequality passed the first boundary of mean motion and the entire quadrant of mean-increasing motion, and about the time of Albategnius was in the region of swiftest motion. Between Albategnius and ourselves the third quadrant of unequal motion was completed (during this time the stars moved with a swift-diminishing motion) and the other boundary of mean motion was passed. In our era the anomaly has again entered the fourth quadrant of mean-diminishing motion, and hence the unequal motion is once more approaching the point of slowest motion.

To reduce these calculations to a definite system in which they would agree with all the observations, my teacher computed that the unequal motion is completed in 1,717 Egyptian years,[18] the maximum correction is about 70′, the mean motion

[15] *addito* (Th 449.3) has dropped out of Prowe's text (PII, 298.14).

[16] Nallino, *Al-Battānī*, I, 124.32-33, 128.2-4. A translation of Albategnius's work into Latin was included in a book printed at Nuremberg in 1537 and was presumably available to Copernicus and Rheticus, as may be inferred from the latter's remark about Albategnius on p. 124, below. The volume opened with a treatise entitled in some copies *Rudimenta astronomica Alfragani*, and in others, *Brevis ac perutilis compilatio Alfragani*. I have been unable to consult this translation, but the relevant passage was excerpted by Menzzer (n. 81). Rheticus ignores the distinction made by Copernicus (Th 162.7-11) that the rate of precession was 1° in 66 years from Menelaus to Albategnius, and 1° in 65 years from Ptolemy to Albategnius. The distinction is based on the observations cited in Th 159.24–160.10.

[17] Copernicus states this rate as 1° in 71 years (Th 162.12-14); calculation from his data gives the fractional result 1° in 70¼ years.

[18] The changing rate of precession requires 1717 years to pass through the four quadrants of the circle of inequality.

of the stars in an Egyptian year is about 50″,[19] and the complete revolution of the mean motion will take 25,816 Egyptian years.[20]

General Consideration of the Tropical Year

This theory of the motions of the fixed stars is supported by the length of the year reckoned from the equinoctial points. It is quite clear why from Hipparchus[21] to Ptolemy there was a deficiency of 19/20 of a day;[22] from Ptolemy to Albategnius, of about 7 days;[23] and from Albategnius to the observations which my teacher made in 1515, of about 5 days.[24] These discrepancies are not at all caused by a defect in the instruments, as was heretofore believed, but occur according to a definite and completely self-consistent law. Hence equality of motion must be measured, not by the equinoxes, but by the fixed stars, as observations of the motions not merely of the sun and moon but of the other planets as well testify with a remarkable unanimity of all ages.

[19] The mean rate of precession is about 50″ a year, or 1° in about 72 years; and the greatest difference between the mean equinox and the true equinox is about 70′ (Th 179.4-7).

(*a*) The slowest rate of precession is 1° in 100 years, or 36″ a year. The difference between the slowest rate and the mean rate is 14″ a year, and in 300 years (the three centuries before Menelaus) the maximum difference of 70′ between mean and true equinox is attained.

(*b*) The swiftest rate is 1° in 66 years, or slightly more than 54½″ a year. The difference between the swiftest rate and the mean rate is about 4½″ a year, and in 743 years (between Ptolemy and Albategnius) about ⅚ of the maximum difference of 70′ is attained; the remaining ⅙ accumulates because the rate of precession during the 620 years between Albategnius and Copernicus's observations in Italy is slightly more rapid than the mean rate.

Copernicus's estimate of the mean precession, about 50″ a year, agrees quite closely with the determination accepted at present. His belief in the cyclic variation of the rate of precession is of course erroneous.

[20] The complete passage of the stars around the celestial sphere requires 25,816 years.

[21] Michael Mästlin, editor of the fourth (1596) and fifth (1621) editions of the *Narratio prima*, correctly substituted "Hipparchus" for the older reading "Timocharis." Unfortunately the incorrect reading was revived in Th 449.24-25.

[22] HI, 203.22–204.18. The tropical year (t) is less than 365¼ days; the "deficiency" from year x to year y is (y − x) (365¼ days − t).

[23] Nallino, *Al-Battānī*, I, 42.10-14.

[24] Th 193.20-21.

It is the accepted opinion that because from Timocharis to Ptolemy the stars moved very slowly the year was less than 365¼ days by only 1/300 of a day;[25] and from Ptolemy to Albategnius, because the stars moved rapidly, by 1/106 of a day.[26] If the observations of our age are compared with those of Albategnius, it is clear that the difference is 1/128 of a day.[27] Therefore a greater length of the tropical year apparently corresponds to a slow motion of the stars, a lesser length to a swift motion, and the lengthening of the year to a diminishing velocity; so that if the length of the tropical year in our era is accurately determined, it will again be almost the same as Ptolemy's value. Hence we must say that the equinoctial points, like the nodes of the moon,[28] move in precedence, and not that the stars move in consequence.[29]

We must accordingly imagine a mean equinox moving in precedence from the first star of Aries in the sphere of the fixed stars, and displacing them by its uniform motion. The true equinox deviates to either side of this mean equinox in an unequal and regular motion; but the radius of the distance between the true equinox and the mean equinox does not much exceed 70′. Thus a definite law governing the length of the tropical year has existed in all ages, and it can be ascertained today. It agrees very closely, moreover, and almost to the minute with the observations which all scholars have made of the fixed stars.

To offer you some taste of this matter, most learned Schöner, I have computed for you the true precession of the equinoxes at certain times of observation.

[25] HI, 205.9-14, 207.24–208.1.

[26] Albategnius's estimate of the length of the tropical year was $365^d14^m26^s$ (Nallino, *op. cit.*, I, 42.17). The difference between this value and 365¼ days ($= 365^d15^m$) is 34^s or 34/3600 of a day. Albategnius expresses this difference as $\frac{3\frac{2}{5}}{360}$ (*op. cit.*, I, 127.19-20). It is much closer to 1/106 than to 1/105 of a day, and Copernicus writes the more accurate fraction (Th 193.2-3). Hence I have followed Mästlin in changing our text from 1/105 to 1/106.

[27] Th 193.20-21, 194.4-5.

[28] See p. 73, above.

[29] Copernicus interprets precession as a motion of the equator.

Egyptian Year		*True Precession* °	′	*Period*
B.C.	293	2	24	Timocharis
	127	4	3	Hipparchus
A.D.	138	6	40	Ptolemy
	880	18	10	Albategnius
	1076	19	37[30]	Arzachel
	1525	27	21	present

Ptolemy's precession subtracted from the positions of the stars as given by Ptolemy leaves a remainder equal to the distance of the stars from the first star of Aries; then the addition of Albategnius's precession gives the true position of the observed star. A similar procedure is followed in all the other cases. The results thus obtained coincide to the utmost degree of exactness with the observations of all scholars, even where the minutes are noted, or are derived from recorded declinations or from the motion of the moon reduced to greater precision, as a comparison of our observations with those of the ancients shows us. For when the minutes are neglected, as you see, at least a part of a degree is cut off,[31] ½° or ⅓° or ¼°, etc. However, these results do not agree with the motions of the planetary apsides, and therefore an independent motion had to be assigned to them, as will be clear from solar theory.[32]

Realizing that equality of motion must be measured by the fixed stars, my teacher carefully investigated the sidereal year. He finds that it is 365 days, 15 minutes, and about 24 seconds[33] and that it has always been of this length from the time of

[30] Prowe states (PII, 300 n) that the first edition read incorrectly 12° 37′, and that the change to 19° 37′ was made by Mästlin. But the Basel edition of 1566 gave 19° 37′ (p. 198r); and that number is suspect, for it would make the rate of precession (*a*) between Albategnius and Arzachel too slow (1° 27′ in 196 years, or 1° in 135 years); and (*b*) between Arzachel and Copernicus too fast (7° 44′ in 449 years, or 1° in 58 years). To be consistent with the theory and the rest of the table, the true precession for Arzachel must be about 20° 57′.

[31] Reading *recidant* (Th 450.28) instead of *recitant* (PII, 301.9).

[32] See pp. 119-21, below.

[33] The minute of the text is a *minutum diei* = 1/60 of a day = 24^m; in like manner, the second = 1/60 of a *minutum diei* = 24^s. The length of the sidereal year, then, is given here as about $365^d6^h9^m36^s$. In *De rev.* it is given as $365^d6^h9^m40^s$; cf. p. 67, above.

the earliest observations. For the fact that the Babylonians, according to Albategnius, assign 3 seconds more,[34] and Thābit 1 second less,[35] can be safely ascribed to either the instruments and observations, which, as you know, cannot have been entirely accurate, or to the inequality in the motion of the sun, or even to the circumstance that the ancients, having no sure theory of eclipses, neglected to take account of the solar parallax in their observations. In any case, this discrepancy over the entire period from the Babylonians to ourselves cannot be compared with the discrepancy of 22 seconds between Ptolemy and Albategnius.[36] That there necessarily was a deficiency of $^{19}/_{20}$ of a day from Hipparchus to Ptolemy, and from Ptolemy to Albategnius of about 7 days, I have deduced, not without the greatest pleasure, most learned Schöner, from the foregoing theory of the motions of the stars and from my teacher's treatment of the motion of the sun, as you will see a little further on.[37]

The Change in the Obliquity of the Ecliptic

My teacher found that the cycle of maximum obliquity stands in the following relation: while the unequal motion of the fixed stars is once completed, half of the change in the obliquity occurs. He therefore concluded that the entire period of the change in the obliquity is 3,434 Egyptian years.[38]

[34] The Latin translation of Albategnius gave $365\frac{1}{4}^d + \frac{1}{131}^d = 365^d15^m27\frac{1}{2}^s$ (see above, p. 67, n. 25). However, the Arabic MS on which Nallino based his text reads (I, 40.28-29) $365\frac{1}{4}^d + \frac{1}{120}^d = 365^d15^m30^s$.

[35] Rheticus and Copernicus (Th 194.8-12) probably drew this information about Thābit from the *Epitome*, Bk. III, Prop. 2 (see above, p. 65, n. 19), which gave Thābit's value as $365^d6^h9^m12^s$ (= $365^d15^m23^s$). Proof that Rheticus used the *Epitome* is afforded by two references to it (p. 124, below) and by a quotation from it (pp. 133-34, below). For Thābit see George Sarton, *Introduction to the History of Science* (Baltimore, 1927–), I, 599-600.

[36] Rheticus is referring to the difference in their determinations of the length of the tropical year: Ptolemy $365^d14^m48^s$ (HI, 208.11-12)
Albategnius $365^d14^m26^s$ (See above, p. 115, n. 26)
22^s

[37] Pages 128-30, below.

[38] It was stated above (p. 113) that the period of the unequal motion of the fixed stars is 1,717 Egyptian years.

In the time of Timocharis, Aristarchus, and Ptolemy the change in the obliquity was very slow, so that they believed the maximum arc of declination to be invariable, always having the value of $^{11}/_{83}$ of a great circle.[39] After them, Albategnius announced the obliquity for his own era as about 23°35′;[40] Arzachel, about 190 years after him, 23°34′; and Prophatius Judaeus, 230 years later, 23°32′. In our own era it appears not greater than 23°28½′.[41] Accordingly it is clear that in the 400 years before Ptolemy the change in the obliquity was very slow. But from Ptolemy to Albategnius, a period of about 750 years, the obliquity decreased by 17′, and from Albategnius to ourselves, a period of 650 years, by only 7′. Hence it follows that the variation of the obliquity, like the deviation of the planets from the ecliptic,[42] is governed by a motion in libration or motion along a straight line. It is a property of such motion that in the middle the motion is quickest, and slowest at the ends. Then about the time of Albategnius the pole of the equator or of the ecliptic was approximately in the middle of this motion in libration, while at present it is near the second limit of slowest motion, where the poles approach each other most closely. But I stated above[43] that the motions of the fixed stars and the variation in the length of the tropical year are saved by the motion of the equator. Now the poles of the equator

[39] $^{11}/_{83} \times 360° = 47° \, 42' \, 40''$, which makes the obliquity 23° 51′ 20″ (HI, 68.4-6, 81.50).

[40] Nallino, *Al-Battānī*, I, 12.20-22; Menzzer, n. 87.

[41] Th 162.24-25, 171.31-172.4; cf. above, p. 64, n. 15. The foregoing statement about the history of the determinations of the obliquity is virtually identical with the scholion in Reinhold's 1542 edition of Peurbach's *Theoricae novae planetarum*, fol. e8r-v; cf. Boncompagni, *Bulletino di bibliografia e di storia delle scienze*, XX(1887), 594-95. Since the statement in our text is earlier than Reinhold's, but Reinhold's contains additional items, apparently they both drew from some common source. For Arzachel and Prophatius Judaeus see Sarton, *Introduction*, I, 758-59; II, 850-53. Copernicus believed that Prophatius obtained his value of 23° 32′ by a direct determination; but it was rather a calculation from Arzachel's tables, according to Duhem (*Le Système du monde*, III, 311). J. Millàs i Vallicrosa has published Prophatius's translation of Arzachel's *Saphea* in *Don Profeit Tibbon, Tractat de l'assafea d'Azarquiel* (Barcelona, 1933).

[42] Cf. pp. 80-81, 84-85, above and pp. 180, 182-85, below.

[43] See above, p. 115, n. 29.

are the prolongations of the earth's axis, and it is from them that the altitude of the pole is measured. Let me in passing call your attention, most learned Schöner, to the sort of hypotheses or theories of motion that the observations require; but you will hear clearer evidence.

Furthermore, my teacher assumes that the minimum obliquity will be 23°28′, and the difference between the minimum and the maximum, 24′. On this basis he geometrically constructed a table of[44] proportional minutes, from which the maximum obliquity of the ecliptic may be derived for all ages. Thus the proportional minutes were, in the time of Ptolemy 58, Albategnius 18, Arzachel 15, and in our own time 1.[45] If, using these figures, we take proportional parts of the 24′ difference between minimum and maximum, we shall have a sure rule for the change in the obliquity.[46]

The Eccentricity of the Sun and the Motion of the Solar Apogee

Since every difficulty in the motion of the sun is connected with the variable and unstable length of the year, I must first speak of the change in the apogee and eccentricity, in order that all the causes of the inequality of the year may be enumerated. However, by the assumption of theories suitable to the purpose, my teacher shows that these causes are regular and certain.

When Ptolemy declared that the apogee of the sun was fixed,[47] he preferred accepting the common opinion to trusting his own observations, which differed perhaps but little from the common opinion. But it can be definitely established from

[44] In his 1621 edition (p. 99), Mästlin inserted "sixty."

[45] Thus in the case of Arzachel, $\frac{15}{60} \times 24' = 6' + 23° 28' = 23° 34'$. For Albategnius, the editions of our text put the number of proportional minutes at 24; I have emended this obviously incorrect number to 18.

[46] For modern astronomy the change in the obliquity is a progressive diminution. The evidence available to Copernicus warranted only the same conclusion (Th 76.27-28). But he believed that after the obliquity had decreased to 23° 28′, it would increase to 23° 52′, completing a cycle which would then be repeated.

[47] HI, 232.18-233.16; cf. n. 13, pp. 62-63, above.

his own account that about the time of Hipparchus, that is, 200 years before his own time, the eccentricity was 417[48] of the units of which 10,000 constitute the radius of the eccentric, and in his own time 414.[49] In the time of Arzachel (in whom Regiomontanus had great faith) the eccentricity was about 346, according to the maximum inequality.[50] But in our own time it is 323, since my teacher states that he finds the maximum inequality not greater than $1°50\frac{1}{2}'$.[51]

Furthermore, carefully investigating the motions of the apsides of the sun and of the other planets, he learned, as you see from what has been said above,[52] that the apsides have independent motions in the sphere of the fixed stars. We are no more justified in attributing the apparent motions of the fixed stars and apsides, and the change in the obliquity to a single motion and a single cause than is one of your experts, who speak of the motions of the planets as self-moving, in attempting to produce the motions and appearances of each of the planets by one and the same device; or than anyone undertaking to defend the view that the foot, hand, and tongue exercise all their functions by means of the same muscle and by the same motive force. Therefore my teacher assigned two motions to the solar apogee, one mean and the other unequal, with which it moves in the eighth sphere. Moreover, since the true equinox moves with a regular unequal motion in the reverse

[48] HI, 233.5-8; Hipparchus determined the eccentricity as approximately $\frac{1}{24}$ of the radius of the eccentric: $\frac{1}{24} \times 10{,}000 = 416\frac{2}{3}$.

[49] HI, 236.15-18. Ptolemy's value for the eccentricity is slightly smaller than Hipparchus's; but since he believed the eccentricity to be constant (see above, p. 61, n. 9), he ignored the small difference between the two values, which he denotes as approximate (ἔγγιστα) in any case. Copernicus held that the eccentricity varies, and hence utilized the difference. Ptolemy's value would be more accurately expressed as 415 than as 414 (Th 209. n. to line 12).

[50] For the method of computing the eccentricity from the maximum inequality, see above, p. 61, n. 11. The information that Arzachel had put the maximum inequality at 1° 59′ 10″ (cf. Th 212.15-16) was obtained by Copernicus and Rheticus from the *Epitome*, Book III, Prop. 13. By the Table of Chords (Th 44.18-19), this inequality would correspond to an eccentricity of 346 (cf. Th 210.1-6).

[51] Th 211.16-19; 212.16; 224.8, 37.

[52] Page 116.

order of the signs, the apogees of the sun and of the other planets, like the fixed stars,[53] are displaced eastward. Consequently, to harmonize the observations of all ages in a consistent law, my teacher was compelled to distinguish these three motions.

To understand this analysis, assume a maximum eccentricity of 417 units, and a minimum of 321. Let the difference, 96, be the diameter of a small circle, on whose circumference the center of the eccentric moves from east to west. The distance from the center of the universe, then, to the center of the small circle will be 369 units. You will recall that 10,000 of these units constitute the radius of the eccentric. This is the device which my teacher derived from the three above-mentioned eccentricities, in a manner closely resembling the surely divine discovery by which the uniform motions of the moon are determined from three lunar eclipses.[54]

My teacher further established that the velocity with which the center of the eccentric revolves is the same as that with which each value of the changing obliquity recurs. This discovery is indeed worthy of the highest admiration, since it is achieved with such great and remarkable agreement.

The eccentricity was greatest about 60 B.C., when the declination of the sun was also at its maximum. The eccentricity has decreased, moreover, in accordance with this single law, similar to no other. This and other[55] like sports of nature often bring me great solace in the fluctuating vicissitudes of my fortunes, and gently soothe my troubled mind.

The Kingdoms of the World Change with the Motion of the Eccentric

I shall add a prediction. We see that all kingdoms have had their beginnings when the center of the eccentric was at some special point on the small circle. Thus, when the eccentricity of the sun was at its maximum, the Roman government be-

[53] See p. 115, above.
[54] HI, 265.16-19, 268.3-12; Th 236.15-17, 28-32; 246.3-8.
[55] Reading *alii* (Th 453.7) instead of *alibi* (PII, 305.1).

came a monarchy; as the eccentricity decreased, Rome too declined, as though aging, and then fell. When the eccentricity reached the boundary and quadrant of mean value, the Mohammedan faith was established; another great empire came into being and increased very rapidly, like the change in the eccentricity. A hundred years hence, when the eccentricity will be at its minimum, this empire too will complete its period. In our time it is at its pinnacle from which equally swiftly, God willing, it will fall with a mighty crash. We look forward to the coming of our Lord Jesus Christ when the center of the eccentric reaches the other boundary of mean value, for it was in that position at the creation of the world. This calculation does not differ much from the saying of Elijah, who prophesied under divine inspiration that the world would endure only 6,000 years,[56] during which time nearly two revolutions are completed. Thus it appears that this small circle is in very truth the Wheel of Fortune, by whose turning the kingdoms of the world have their beginnings and vicissitudes. For in this manner are the most significant changes in the entire history of the world revealed, as though inscribed upon this circle. Moreover, I shall soon, God willing, hear from your own lips how it may be inferred from important conjunctions and other learned prognostications, of what nature these empires were destined to be, whether governed by just or oppressive laws.[57]

[56] From Rheticus's language it appears that he attributed the dire prophecy to the prophet Elijah. But the Old Testament contains no such prediction by Elijah; however, the late Prof. Ralph Marcus kindly called my attention to the following passage in the *Babylonian Talmud:* "The Tanna debe Eliyyahu teaches: The world is to exist six thousand years" (*Babylonian Talmud*, English translation, ed. Isidore Epstein [London, 1935–], *Sanhedrin*, Vol. II [= *Nezikin*, Vol. VI], p. 657).

[57] Rheticus again displays his devotion to astrological superstition in the Preface to Werner's *De triangulis sphaericis.* He there declares: "The changes in empires depend upon celestial phenomena. Lands formerly distinguished for their culture, fertile soil, and possessions now lie barren and desolate, inhabited by barbarians, oppressed by tyranny . . . The fiercest nations become civilized, unproductive land is brought under cultivation, from heaven are sent down new forms of earth, culture, and physical type of man. And we see that at intervals of about three hundred and fifty years there always occurs some significant change in the sub-

Now while the center of the eccentric descends toward the center of the universe, the center of the small circle, it is clear, moves in the order of the signs about 25″ each Egyptian year. And starting from the point of its greatest distance from the center of the universe, the center of the eccentric moves in precedence. Hence the inequality arising from the motion of the anomaly for any specified time is subtracted from the mean motion, until a semicircle is completed; but in the other semicircle it is added, in order to obtain the true[58] motion of the apogee. Now the greatest difference between the true and mean apogee is deduced, in the proper geometrical manner, from the above-mentioned data as 7°24′;[59] the other differences are determined, in the customary way, from the position of the center of the eccentric on the small circle. The unequal motion is known, since three positions are given. With regard to the mean motion there is some doubt, since we do not have for these three positions the true place of the solar apogee on the ecliptic. The doubt arises from the disagreement between

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lunar world, corresponding to some important alteration in the motion of the sphere of the fixed stars" (*Abh. zur Gesch. d. math. Wiss.*, XXIV, 1, fol. a2v). Later in the same Preface he asserts: "As far as the stars are concerned, I have no doubt that for the Turkish empire there is impending disaster, momentous, sudden, and unforeseen, since the influence of the fiery Triangle is approaching, and the strength of the watery Triangle is declining. Moreover, the anomaly of the sphere of the fixed stars is nearing its third boundary. Whenever it reaches any such boundary, there always occur the most significant changes in the world and in the empires, as history makes clear" (*ibid.*, fol. a5r).

In a letter to Tycho Brahe, Christopher Rothmann censures Rheticus and asks: "How can the variation in the eccentricity of the sun produce a change of empires?" (*Tychonis Brahe opera omnia*, ed. Dreyer, VI, 160.28-29). I know of no evidence indicating that Copernicus shared the astrological views of Rheticus. Dreyer would perhaps not have advanced this suggestion (*Planetary Systems*, p. 333), had he been familiar with the aforementioned Preface by Rheticus.

Schöner's *Opera mathematica* appeared at Nuremberg in 1551, and again in 1561. The first paper is an introduction to judicial astrology (*Isagoge astrologiae iudiciariae*), and the second is a fearfully thorough essay in genethlialogy (*De iudiciis nativitatum*).

[58] Reading *verus* (Th 453.35) instead of *versus* (PII, 306.3).

[59] In *De rev.* Copernicus puts the greatest difference at about 7½° (Th 223.5-8); while an earlier passage gives 7° 28′ (Th 221.3-5), Rheticus has chosen to follow Copernicus's tables, which give 7° 24′ (Th 224.8-10).

Albategnius and Arzachel, pointed out by Regiomontanus in the *Epitome*, Book III, Proposition 13.[60]

Albategnius is too free in his treatment of the inner secrets of astronomy, as can be seen in many passages. Did he commit this fault in his determination of the solar apogee? Let us grant that he had the correct time of the equinox. Nevertheless, it is impossible, as Ptolemy states,[61] by means of instruments to determine with precision the times of the solstices. For a single minute of declination, which of course easily escapes the eye, may deceive us in this matter by about 4°, to which four days correspond. How was Albategnius able to determine the position of the solar apogee? If he used the method of intermediate positions on the ecliptic, explained by Regiomontanus in the *Epitome*, Book III, Proposition 14, he failed to employ a more trustworthy procedure. He is therefore himself to blame for going astray, since he selected eclipses occurring not near the apogee, but near the middle longitudes of the eccentric of the sun, where the solar apogee, even if mistakenly located 6° from its true position, could produce no noticeable error in eclipses.

According to Regiomontanus,[62] Arzachel boasts that he made 402 observations, and determined from them the position of the apogee. We grant that by this diligence he found the true eccentricity. But since it is not clear that he took into account lunar eclipses occurring near the apsides of the sun, it is apparent that we must no more accept his[63] determination of the higher apse than that of Albategnius.

[60] "Albategnius determined the eccentricity as 2° 4′ 45″, and the arc BH as 7° 43′. Arzachel, however . . . found the same eccentricity as Albategnius, but his value for the arc BH was 12° 10′. This is certainly surprising, since Arzachel lived after Albategnius." The arc BH is the distance from the apogee to the summer solstice.

[61] HI, 196.21–197.11.

[62] *Epitome*, Book III, Prop. 13: "Arzachel, 193 years after Albategnius, made 402 observations [*considerationes*] of the four points midway between the equinoctial and solstitial points, and found BH to be 12° 10′." It should be noted, with reference to the equivalence of *consideratio* and *observatio* (see above, p. 99, n. 28), that in citing this passage Rheticus altered *considerationes* to *observationes*.

[63] Reading *ei* (Th 454.20) instead of *eis* (PII, 306.32).

Now you see what great effort my teacher had to put forth to determine the mean motion of the apogee. For nearly 40 years in Italy and here in Frauenburg, he observed eclipses and the motion of the sun. He selected the observation by which he established that in A.D. 1515 the solar apogee was at 6⅔° of Cancer.[64] Then examining all the eclipses in Ptolemy and comparing them with his own very careful observations, he concluded that the mean annual[65] motion of the apogee with reference to the fixed stars was about 25″,[66] and with reference to the mean equinox about 1′15″.[67] Through this result it is established, by applying the true precession to both the mean and the unequal motions, that the true[68] position of the apogee was in the time of Hipparchus 63° from the true equinox, Ptolemy 64½°, Albategnius 76½°, and Arzachel 82°, while in our time all the calculations agree with experience. These figures are surely more satisfactory than the Alfonsine, which put the solar apogee at 12° of Gemini in the time of Ptolemy, and at the beginning of Cancer in our time.[69] We are 2° closer than the Alfonsine Tables to the estimate of Arzachel.[70] Albategnius's computation of the position of the apogee exceeds the Alfonsine by 1°, while we, for a good reason, fall short of his figure by 6°.[71] For my teacher cannot depart from Ptolemy and from his own observations, not only because he made and

[64] Th 210.10–211.26.

[65] Reading *annuum* (Th 454.26) instead of *annum* (PII, 307.8).

[66] Th 221.32–222.3.

[67] The mean annual motion of the equinox (mean precession) is about 50″ (see pp. 113-14, above; cf. Th 172.14-17), and it is retrograde (see p. 115, above). The motion of the apogee is direct (see p. 123, above). Hence, to obtain the motion of the apogee relative to the equinox, the two mean annual rates must be added: 25″ + 50″ = 1′ 15″.

[68] Reading *verus* (Th 454.29) instead of *versus* (PII, 307.12).

[69] That is, at 72° for Ptolemy's time, and at 90° for Copernicus's time.

[70] As we saw above (notes 60 and 62 on p. 124), the *Epitome* stated that Arzachel found the apogee 12° 10′ from the summer solstice = 77° 50′ from the equinox; cf. Th 210.5-8.

[71] Albategnius located the apogee 7° 43′ from the summer solstice = 82° 17′ from the equinox; cf. above, p. 124, n. 60 and Nallino, *Al-Battānī*, I, 44.29-33. The version of the Alfonsine Tables to which Rheticus refers evidently contained the following values: for Ptolemy's time, 72°; Albategnius's, 81°; Arzachel's, 72° (or 84°?); Copernicus's, 90°.

noted his own observations with his own eyes, but also because he knows that Ptolemy, working with the utmost care and making use of eclipses, accurately investigated the motions of the sun and the moon and established them correctly, so far as he could. We are compelled, nevertheless, to differ from him by about 1°,[72] as the motion of the apogee has made clear to us. For Ptolemy regarded the apogee as fixed and therefore showed little care in his treatment of this topic.

You have the opinion of my teacher regarding the motion of the sun. He has accordingly drawn up tables in which he collects for any specified time the true position of the solar apogee, the true eccentricity, the true inequalities, the uniform motions of the sun with reference to the fixed stars and to the mean equinoxes, and hence the true position of the sun corresponding to the observations of all ages. Clearly the tables of Hipparchus, Ptolemy, Theon, Albategnius, and Arzachel, and the Alfonsine Tables, which are to some extent a composite of the others, are temporary only and can endure at most 200 years, until, that is, the discrepancy in the length of the year, eccentricity, inequality, etc., becomes evident, a thing which occurs in the motions and appearances of the other planets for a similar definite reason. Not undeservedly, therefore, could the astronomy of my teacher be called perpetual, as the observations of all ages testify, and the observations of posterity will doubtless confirm. But he calculates his motions and the positions of the apsides from the first star of Aries,[73] since equality of motion is measured by the fixed stars. Then by adding the true precession, he computes and determines the distance in each age of the true positions of the planets from the true equinox.

If such an account of the celestial phenomena had existed a

[72] Hipparchus found the apogee 24½° from the summer solstice (HI, 233.8-10) = 65½° from the equinox, and Ptolemy accepted his determination (HI, 237.6-11).

[73] γ Arietis (Th 130.6-7), not α Arietis (Th 130.22) as Berry thought (*A Short History of Astronomy*, p. 110 n); cf. above, p. 63, n. 13; Rudolf Wolf, *Geschichte der Astronomie* (Munich, 1877), p. 240; Dreyer, *Planetary Systems*, p. 330; Armitage, *Copernicus*, pp. 105-6.

little before our time, Pico would have had no opportunity, in his eighth and ninth books,[74] of impugning not merely astrology but also astronomy. For we see daily how markedly common calculation departs from the truth.

Special Consideration of the Length of the Tropical Year

In improving the calendar most scholars enumerate various lengths of the year as computed by writers. But they do this in a confused way and come to no conclusion—surely a remarkable procedure for such great mathematicians.

From what has been said above, however, most learned Schöner, you see the four causes of the unequal motion of the sun as measured by the equinoxes: the inequality of the precession of the equinoxes, the inequality of the motion of the sun in the ecliptic, the decrease of the eccentricity, and last, the motion of the apogee for a twofold reason. By virtue of the same causes, the tropical year cannot be equal.

We may readily pardon Ptolemy for measuring equality of motion by the equinoxes,[75] since he held that the fixed stars move in consequence,[76] the position of the apogee is fixed,[77] and the eccentricity of the sun does not decrease.[78] How others would excuse themselves, I do not know. Let us even grant them that the stars and the solar apogee have the same motion in consequence; that therefore time measured by the true equinox in reality does not change; and that the entire inequality (though to assert this in our time would be most absurd) is caused by the defect in the instruments, since the motion of the solar apogee produces only a slight change in the length of the year. Nevertheless, it will not therefore follow that the sun regularly returns to the true equinox always in equal times, as we say that the moon regularly increases its distance from the mean apogee of the epicycle, and returns to the same position in equal times—a statement quoted by the

[74] Pico della Mirandola, *Disputationes adversus astrologos*, Books VIII–IX (pp. 457-82 in the Venice, 1498, edition of Pico's *Opera omnia*).

[75] See above, p. 65, n. 18.

[76] HI, 193.14-16; cf. above, p. 63, n. 13.

[77] See p. 119, above.

[78] See above, p. 120, n. 49.

learned Marcus Beneventanus[79] from the Alfonsine Tables. For since we surely cannot deny that the eccentricity of the sun changes, how can they assert that the variation of the angle of anomaly from the mean motion does not alter the length of the tropical year? I heartily congratulate the state and all scholars, whom the work of my teacher will advantage, that we have a sure understanding of the inequality of the year.

But that you may the more readily grasp all these ideas, most learned Schöner, I set them forth numerically before your eyes, in order that I may at length fulfill the pledge I made above.[80]

Let the sun be at the mean vernal equinoctial point, which was 3°29′ west of the first star of Aries at the time of the observation of the autumnal equinox made by Hipparchus in 147 B.C.[81] Let the sun move from this point in the eighth sphere and return to it in a sidereal year (365 days, 15 minutes, and about 24 seconds).[82] However, because the mean equinox in a sidereal year moves about 50″ in the direction opposite to that of the sun, the result[83] is that the sun reaches the new position of the mean vernal equinoctial point before it reaches the starting point, where the sun and the mean equinox had occupied the same position on the ecliptic. Therefore the year as measured by the mean equinox is shorter than the sidereal year,[84] and is computed to be, on the basis of our hypotheses, 365 days, 14 minutes, and about 34 seconds.[85] Now if, for the year measured by the mean equinox, we inquire what the excess[86] in days and fractions of days amounted to in the

[79] For a brief account of the life and work of Marco da Benevento see L. Birkenmajer in *Bulletin international de l'académie des sciences de Cracovie, classe des sciences math. et naturelles* (1901), pp. 63-71; and A. Birkenmajer in *Philosophisches Jahrbuch*, XXXVIII (1925), 336-44.

[80] Page 117.

[81] HI, 195.17-20, 204.1-6.

[82] See above, p. 116, n. 33.

[83] Reading *fit* (Th 456.17) instead of *sit* (PII, 310.11).

[84] See p. 46, above.

[85] That is, $365^d5^h49^m36^s$. Newcomb's determination (1900) is $365^d.24219879 = 365^d5^h48^m46^s$ (*American Ephemeris for 1940*, p. xx).

[86] That is, the length of time by which the tropical year exceeds the Egyptian year of exactly 365 days.