Missile Technology Basics

David Wright
Senior Scientist and Co-Director, Global Security Program
Union of Concerned Scientists

June 19, 2014
• Trajectory Basics:
  • Phases of flight
  • Speed, angle, range

• Propulsion:
  • Thrust
  • Rocket equation and structural mass
  • Staging

• Guidance and Control (G&C)
  • Guidance system
  • Steering

• Reentry and Heating

• Accuracy
  • G&C errors
  • Reentry errors
Physics View of a Ballistic Missile

- Warhead
- Guidance & control
- Airframe
- Propulsion
Phases of Missile Flight

• Boost Phase: when the engines are burning
  • Typically lasts a few minutes

• Midcourse Phase: for ranges > ~500 km, that part of the trajectory where atmospheric drag is negligible
  • Above ~100 km altitude
  • Can last 20-30 minutes for long ranges

• Re-entry Phase: when atmospheric effects are large at the end of flight
  • Lasts a couple minutes
  • Leads to intense heating and inaccuracy
Missile Range Designations

- **Short** < 1,000km
- **Medium** 1,000-3,000 km
- **Intermediate** 3,000-5,500 km
- **ICBM** > 5,500 km

Range is not an intrinsic characteristic of a missile, since it depends on the payload.

**Example:** U.S. Trident SLBM

- Fully loaded: 8 warheads (1,500 kg) → Range = 7,500 km
- Half loaded: 4 warheads (750 kg) → Range = 11,000 km
Variation in Range with Burnout Angle

**Lofted**

**Depressed**

(maximum range)

Powered flight
Optimal Burnout Speed vs. Range
## Approximate Burnout Parameters

<table>
<thead>
<tr>
<th>Range (km)</th>
<th>Burntime (s)</th>
<th>Burnout altitude (km)</th>
<th>Apogee (km)</th>
<th>Burnout speed (km/s)</th>
<th>Burnout angle (deg)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>62</td>
<td>25</td>
<td>85</td>
<td>1.6</td>
<td>43</td>
<td>Scud-B</td>
</tr>
<tr>
<td>500</td>
<td>80</td>
<td>35</td>
<td>120</td>
<td>2.0</td>
<td>42</td>
<td>Stretched Scud</td>
</tr>
<tr>
<td>1,000</td>
<td>100</td>
<td>50</td>
<td>250</td>
<td>3.0</td>
<td>41</td>
<td>Nodong</td>
</tr>
<tr>
<td>2,000</td>
<td>170</td>
<td>140</td>
<td>500</td>
<td>4.0</td>
<td>38</td>
<td>Nodong+ Scud</td>
</tr>
<tr>
<td>3,000</td>
<td>140</td>
<td>100</td>
<td>700</td>
<td>4.5</td>
<td>38</td>
<td>Chinese DF-3</td>
</tr>
<tr>
<td>5,000</td>
<td>250</td>
<td>330</td>
<td>1,200</td>
<td>5.4</td>
<td>33</td>
<td>Chinese DF-4</td>
</tr>
</tbody>
</table>
“½ Rule”

A missile that can launch a payload to a maximum range of $R$ can launch *that same payload* vertically to an altitude of roughly $R/2$

- exact at short distances
- holds approximately even for long ranges
Structure of Liquid Missiles

German V-2

Control vanes (air and jet vanes)  Rocket motor  Oxidizer (liquid oxygen)  Fuel (ethanol-water mixture)  Guidance System  Warhead

Soviet Scud-B
Rocket Propulsion

Conservation of momentum:

\[ M \frac{dV}{dt} = -V_e \frac{dM}{dt} \]

Then:

\[ \text{Thrust} = M \frac{dV}{dt} = -V_e \frac{dM}{dt} \]

Exhaust velocity

Mass flow rate out of the engine
\[ Thrust = V_e \frac{dM}{dt} \]

- **Mass flow rate:**
  - Determined largely by size and design of engine
  - Turbo pumps, etc.

- **Exhaust velocity:**
  - Determined largely by type of propellant
  - For chemical propellants: \( V_e = 2 \text{ - } 3 \text{ km/s} \)

- “Specific Impulse”:
  \[ I_{sp} = \frac{V_e}{g_o} \]
  \[ g_o = gravity = 9.8 \text{ m/s}^2 \]
North Korean Missile Development

- **Nodong**: 1,200 km, 0.7t
- **Scuds**: 300-500 km, 1t
- **Musudan**: 3,000 km, 0.75t
- **TD-1**: (untested)
- **TD-2/Unha-3**: (untested)
\[ \text{Thrust} = V_e \frac{dM}{dt} \]

Evolution of North Korean missiles

<table>
<thead>
<tr>
<th>Missile</th>
<th>( V_e ) (km/s)</th>
<th>Mass flow rate (kg/s)</th>
<th>Thrust (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scud-B</td>
<td>2.3</td>
<td>58</td>
<td>130</td>
</tr>
<tr>
<td>Nodong</td>
<td>2.3</td>
<td>130</td>
<td>290</td>
</tr>
<tr>
<td>TD-2 (stage 1)</td>
<td>2.3</td>
<td>520</td>
<td>1200</td>
</tr>
<tr>
<td>Musudan (est)</td>
<td>2.7</td>
<td>96</td>
<td>254</td>
</tr>
<tr>
<td>Titan II (stage 1)</td>
<td>2.9</td>
<td>803</td>
<td>2090</td>
</tr>
</tbody>
</table>

Changing \( V_e \):

Scud, Nodong, and TD-2 use:
- fuel: kerosene
- oxidizer: IRFNA (nitric acid)

Musudan is thought to be a version of the Soviet SS-N6 missile
- fuel: UDMH
- oxidizer: NTO (nitrogen tetraoxide)
$$Thrust = V_e \frac{dM}{dt}$$

Evolution of North Korean missiles

<table>
<thead>
<tr>
<th>Missile</th>
<th>$V_e$ (km/s)</th>
<th>Mass flow rate (kg/s)</th>
<th>Thrust (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scud-B</td>
<td>2.3</td>
<td>58</td>
<td>130</td>
</tr>
<tr>
<td>Nodong</td>
<td>2.3</td>
<td>130</td>
<td>290</td>
</tr>
<tr>
<td>TD-2 (stage 1)</td>
<td>2.3</td>
<td>520</td>
<td>1200</td>
</tr>
<tr>
<td>Musudan (est)</td>
<td>2.7</td>
<td>96</td>
<td>254</td>
</tr>
<tr>
<td>Titan II (stage 1)</td>
<td>2.9</td>
<td>803</td>
<td>2090</td>
</tr>
</tbody>
</table>

Increasing mass flow:

Nodong engine is essentially a scaled-up Scud engine.

© Brügge 2010-11

Evolution of North Korean missiles
\[ \text{Thrust} = V_e \frac{dM}{dt} \]

Evolution of North Korean missiles

<table>
<thead>
<tr>
<th>Missile</th>
<th>( V_e ) (km/s)</th>
<th>Mass flow rate (kg/s)</th>
<th>Thrust (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scud-B</td>
<td>2.3</td>
<td>58</td>
<td>130</td>
</tr>
<tr>
<td>Nodong</td>
<td>2.3</td>
<td>130</td>
<td>290</td>
</tr>
<tr>
<td>TD-2 (stage 1)</td>
<td>2.3</td>
<td>520</td>
<td>1200</td>
</tr>
<tr>
<td>Musudan (est)</td>
<td>2.7</td>
<td>96</td>
<td>254</td>
</tr>
<tr>
<td>Titan II (stage 1)</td>
<td>2.9</td>
<td>803</td>
<td>2090</td>
</tr>
</tbody>
</table>

Increasing mass flow:

TD-2 first stage uses a cluster of 4 Nodong engines
"Rocket Equation"

\[
MdV = -V_e \, dM \quad \text{or} \quad dV = -V_e \frac{dM}{M}
\]

Conservation of momentum gives:

\[
\Delta V \equiv \int_{V_i}^{V_f} dV = V_f - V_i = -V_e \int_{M_i}^{M_f} \frac{dM}{M} = -V_e \left( \ln M_f - \ln M_i \right)
\]

\[
\Delta V = V_e \ln \left( \frac{M_i}{M_f} \right)
\]

\(M_i = \text{initial mass}\)

\(M_f = \text{final mass}\)
Rocket Equation

- Ignore the mass of the payload, and gravity
- Initial mass \( M_i = \text{mass of structure} + \text{propellant mass} = M_S + M_P \)
- Final mass \( M_f = \text{mass of structure} = M_S \)

\[
\Delta V = V_e \ln \left( \frac{M_i}{M_f} \right) = V_e \ln \left( \frac{M_S + M_P}{M_S} \right) = V_e \ln \left( 1 + \frac{M_P}{M_S} \right)
\]

- \( \Delta V \) depends on the ratio \( M_S/M_P \)
- One way to increase range is to add propellant. That makes the missile heavier and the forces greater.
- If you scale both the propellant and structure up by the same factor, you don’t gain any velocity.
Reducing Structural Mass

Evolution of North Korean missiles

<table>
<thead>
<tr>
<th>Missile</th>
<th>$V_e$ (km/s)</th>
<th>Mass flow rate (kg/s)</th>
<th>Thrust (kN)</th>
<th>Total Mass (tons) (no payload)</th>
<th>Propellant Mass (tons)</th>
<th>$\frac{M_s}{M_p}$</th>
<th>Range/payload (km &amp; kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scud-B</td>
<td>2.3</td>
<td>58</td>
<td>130</td>
<td>4.9</td>
<td>3.8</td>
<td>0.29</td>
<td>300/1,000</td>
</tr>
<tr>
<td>Nodong</td>
<td>2.3</td>
<td>130</td>
<td>290</td>
<td>14.3</td>
<td>12.4</td>
<td>0.15</td>
<td>950/1,000</td>
</tr>
<tr>
<td>TD-2 (stage 1)</td>
<td>2.3</td>
<td>520</td>
<td>1200</td>
<td>71.3</td>
<td>64.0</td>
<td>0.11</td>
<td>2,000/1,000</td>
</tr>
<tr>
<td>Musudan (est)</td>
<td>2.7</td>
<td>96</td>
<td>254</td>
<td>18.6</td>
<td>17.1</td>
<td>0.088</td>
<td>2,700/1,000</td>
</tr>
<tr>
<td>Titan II (stage 1)</td>
<td>2.9</td>
<td>803</td>
<td>2090</td>
<td>122</td>
<td>118</td>
<td>0.034</td>
<td></td>
</tr>
</tbody>
</table>
The Effect of Structural Mass

The graph shows the relationship between structural mass ratio $\Delta V/V_e$ and mass ratio $M_S/M_P$ for different missiles. The graph includes data points for Titan II, Musudan, TD-2, Nodong, and Scud. The trend suggests a negative correlation, indicating that as the structural mass ratio increases, the mass ratio also increases.
Staging: An Example

1 Stage:

Total mass = 111 t   Propellant mass = 100 t

\[ M_P = 100 \text{ t} \quad M_S = 10 \text{ t} \]

\[ \Delta V = V_e \ln \left( \frac{(100+10+1)}{(10+1)} \right) = 2.3V_e \]

Payload = 1 t
Staging: An Example

1 Stage: Total mass = 111 t Propellant mass = 100 t

\[ M_P = 100 \text{ t} \quad M_S = 10 \text{ t} \]

\[ \Delta V = 2.3 V_e \]

\[ \Delta V = V_e \ln \left[ \frac{111}{8 + 20 + 2 + 1} \right] + V_e \ln \left[ \frac{20 + 2 + 1}{2 + 1} \right] = 3.3 V_e \]

→ Staging leads to 43% increase in velocity
Chinese Liquid Rockets

DF-3: similar in size but more capable than TD-2 first stage

CZ-1: China’s first satellite launcher - similar in size but more capable than Unha

DF-5: China’s first ICBM - much larger than Unha
Guidance and Control

• Guided missile requires:
  
  – A way to control the direction of thrust during boost phase
  
  – A way to know the missile’s location and velocity during boost
  
  – A computer to know when it has reached the velocity, angle, and altitude to reach its intended target
  
  – A way to terminate thrust at that point
Steering: Jet Vanes on a Scud and DF-2
Steering Engines on Unha

Unha first stage (rear view)
Guidance Antennas on a DF-2a
Reentry

- Warhead encounters increasing atmosphere below ~ 100km altitude
- Atmospheric drag slows the warhead
- Kinetic energy of the warhead is transformed into heat

- Controlled by “weight-to-drag ratio” or “ballistic coefficient”

\[ \beta = \frac{gM}{C_D A} \]

- \( g \) = gravity  \( M \) = mass
- \( C_D \) = drag coefficient
- \( A \) = cross-sectional area
How $\beta$ Affects Reentry Speed

$\beta = 150$ vs $2500 \text{ lb/ft}^2$

$150 \text{ lb/ft}^2 = 7.5 \text{ kN/m}^2$

$2500 \text{ lb/ft}^2 = 125 \text{ kN/m}^2$
Drag $\sim (\rho V^2)/\beta$ For lower $\beta$, RV slows at higher altitude, where $\rho$ is small

Heating rate $\sim \rho V^3$
Reentry Vehicles

Low $\beta$: Mercury capsule

Medium $\beta$: Titan reentry vehicle

High $\beta$: MM-III reentry vehicle
Sources of Inaccuracy

• Uncertainties in location of launch point and target
• Errors in calculating trajectory due to gravity variations, etc.

• Guidance and Control errors
  • Errors in accelerometers
  • Errors in computing speed and location
  • Errors in thrust termination

• Reentry errors
  • Unpredictable lateral forces due to:
    – Local winds, density variations
    – Non-zero angle of attack
    – Corkscrewing or tumbling
    – Asymmetries of reentry vehicle
CEP: Circular Error Probable

Statistical measure of accuracy

Radius of a circle, centered on the mean impact point (P), that includes half of the impact point

Sometimes called “Circle of Equal Probability”

Distance between P and aim point is a measure of systematic inaccuracy, called “bias”
Typical CEPs

- Scud-B: CEP ~ 1 km
- Nodong: CEP ~ 4 km
- DF-5 ICBM: 1-3 km
- Trident II SLBM: CEP ~ 50-100 m
Liquid vs. Solid Propellant

Solid Propellant Motor

Liquid Propellant Engine
Chinese Missiles

DF-5: Liquid, 183 tons

DF-31A: solid, ~63 tons mobile
Difficulties of Solid Propellant

- Very difficult to manufacture large solid motors

- China
  - Developed solid 300 km M11 in 1970s (0.86 m diameter)
  - Deployed DF-31 ICBM in 2006 (2.25 m diameter)

- France
  - Deployed solid 2,500 km M1 in 1971 (1.5 m diameter)
  - Deployed M51 ICBM in 2010 (2.3 m diameter)

- Iran
  - Sajjil: 2,000 km range (1.25 m diameter)

- North Korea
  - KN-02: 100 km range (0.65 m diameter)
Iranian Missile Development

- Shahab-1 (Scud)
- Shahab-2 (Nodong)
- Shahab-3
- Shahab-3M
- Safir
- Simorgh (similar to Unha) (untested)