Introduction
1. **Title:** "Graphing Quadratic Functions"

2. **Grade Level:** 10 – 11

3. **Target Group** Mainstream Class with Integrated ELL students

4. **Source of Written Reading Materials** Primarily “McDougal Littell Algebra II”, 2001 (Textbook and Note-Taking Guide)

5. **Source of Lessons** Rick Padro – Math Teacher

6. **3-4 Learning Goals** I want students to know:

   - How to graph a basic quadratic function, using an x-y table and resulting in a parabola.
   - How to identify the parts of a specific parabola, including its vertex, axis of symmetry, x and y-intercepts.
   - How to graph a specific quadratic function (in standard form) which models a real-life situation.
   - How to infer from the graph of a quadratic function what the maximum or minimum value of the function would be.

7. **Modification Note:** This unit will contain lessons and materials which have been modified for students with varying levels of English Language Proficiency to help make content comprehensible while developing vital academic language skills. At the end of each lesson, the Appendix will be followed by the original lesson with original text.
Lesson 1
### Performance Standards: Graphing Quadratic Functions – Lesson 1, “Basic Quadratic Function”

<table>
<thead>
<tr>
<th>Content Objectives</th>
<th>Language Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Analyze the terms of a function and determine whether or not the function is quadratic.</td>
<td></td>
</tr>
<tr>
<td>2. a. Graph a basic quadratic function by completing an x-y table and plotting points.</td>
<td></td>
</tr>
<tr>
<td>b. Identify the steps in graphing a basic quadratic function.</td>
<td></td>
</tr>
<tr>
<td>3. Identify the structure of a parabola, including vertex.</td>
<td></td>
</tr>
<tr>
<td>1. In small groups and pairs, students will orally identify the terms of a function and determine whether or not the function is quadratic.</td>
<td></td>
</tr>
<tr>
<td>2. b. In small groups and pairs, students will orally list the steps involved in graphing a basic quadratic function.</td>
<td></td>
</tr>
<tr>
<td>3. In small groups and pairs, students will orally describe the structure of a parabola, including vertex.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain/Topic</th>
<th>Fluent Bridging Level 5</th>
<th>Expanding Fluency Level 4</th>
<th>Speech Emerging Level 3</th>
<th>Early Production Level 2</th>
<th>Production Level 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speaking and Listening:</td>
<td>In pairs, students will orally describe how to identify a quadratic function, using complete sentences, and they will create unique quadratic functions.</td>
<td>In pairs, students will orally describe how to identify a quadratic function, using complete sentences.</td>
<td>In small groups, students will recite the terms of various functions, and -using language prompts- state why they are quadratic or not, using 2-3 phases from a phrase bank.</td>
<td>In small groups, students will recite only the lead term of various functions and state orally whether they are quadratic or not, using one or two words from word bank.</td>
<td>In small groups students label the lead term of various functions and state orally whether they are quadratic or not, using one or two words from word bank.</td>
</tr>
<tr>
<td>Identify Quadratic Function in Standard Form.</td>
<td>In pairs, students will discuss and describe the 5 steps for graphing a basic quadratic function, using complete sentences, and they will demonstrate the process with a specific function.</td>
<td>In pairs, students will discuss and describe the 5 steps for graphing a basic quadratic function, using complete sentences and a word bank.</td>
<td>In small groups, students will state orally the 5 steps for graphing a basic quadratic function, using language prompts and a word bank.</td>
<td>In small groups, students will describe orally list the 5 steps for graphing basic quadratic function by repeating short phrases, using language prompts and a word bank.</td>
<td>In small groups, students will state the 5 steps for graphing basic quadratic function by repeating short phrases, using language prompts and a word bank.</td>
</tr>
<tr>
<td>Sequence math processes in correct order.</td>
<td>Students will describe the structure of a parabola in paragraph form, listing its components and comparing it to a real-life object.</td>
<td>Students will describe the structure of a parabola in paragraph form, listing its components and similarities to a real-life object.</td>
<td>Working in small groups, students will describe the structure of a parabola by writing 3-4 complete sentences, and they will compare to visual objects using 2-3 phases from a phrase bank.</td>
<td>Working in small groups, students will label the components of a parabola and compare to visual object using graphics, sentence starters and a word bank.</td>
<td>Working in small groups, students will label the components of a parabola and compare to visual object using graphics and a word bank.</td>
</tr>
<tr>
<td>Writing:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>Situation</td>
<td>Expression</td>
<td>Words/Phrases</td>
<td>Grammar</td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>----------------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>-------------------------------</td>
<td></td>
</tr>
<tr>
<td>Identify</td>
<td>Algebraic terms in quadratic functions.</td>
<td>y = 6x + 3 can be read as: “y (1)________ six (2)________ “x” (2)________ three.”</td>
<td>1. Equals Is equal to</td>
<td>Mathematical operations and expressions</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>y = 3x^2 - 5 can be read as: “y (1)________ three (2)________ “x” (3)<strong><strong><strong><strong>, (2)</strong></strong></strong></strong> five.”</td>
<td>2. Times Plus Minus</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3. Squared Cubed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sequence</td>
<td>Five steps to graphing a basic quadratic function.</td>
<td>(1)<strong><strong><strong><strong>, you need to (2)</strong></strong></strong></strong> “x” values.</td>
<td>1. First Next Then</td>
<td>Transitional Words</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)<strong><strong><strong><strong>, you should (3)</strong></strong></strong></strong> the y value also known as the __________.</td>
<td>2. Input Enter Plot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For the (5)<strong><strong><strong><strong>, write-out each x-y (6)</strong></strong></strong></strong> as (6)________.</td>
<td>3. Calculate Evaluate</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For the (5)<strong><strong><strong><strong>, (2)</strong></strong></strong></strong> each (6)________ on the coordinate plane.</td>
<td>4. Input Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5. First step Second Step Third Step Fourth Step Fifth Step Next Step Final Step</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6. Pair Coordinates Point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare</td>
<td>Visual objects to the shape of a parabola.</td>
<td>It (1)________ a parabola ____________...</td>
<td>1. Looks like Is shaped like Does not look like Is not shaped like Because...</td>
<td>Verb: To Have Expression:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>It (2)________ curved sides.</td>
<td>2. Has Does not have</td>
<td>To Look Like Be Shaped Like</td>
<td></td>
</tr>
</tbody>
</table>
LESSON 1
(70 minutes)

Warm-Up Activity 1:

Students will have been given a Graphic Organizer (p.1) the night before to prep them for the lesson. The teacher will activate background knowledge by writing a simple linear function \((y = 3x + 7)\) on the board and asking students to recite it aloud. Next, the teacher will write a similar function but adding an exponent of 2 to the variable \((y = 3x^2 + 7)\). What does the “2” mean? The teacher will asked students to try and recite the function with the aid of the graphic organizer. The teacher will repeat and model slowly. The concept of “x squared” will be introduced by the teacher drawing a square on the board, and labeling one of the sides “x.” What would the area formula be? \((A = x^2)\). This is an example of a quadratic function. “And any function which ‘Begins’ (write ‘Lead Term’ on board) with an “x-squared term,” is a quadratic function. The teacher will show additional examples of functions and ask students whether or not they are quadratic and why. The teacher will repeat often “x-squared signals a quadratic function.” Thus, the teacher will guide students make the connection between the lead variable (squared) and a quadratic function. The teacher will break the class into small groups according to language proficiency, and they will complete the oral warm-up exercise “Group Dicussion” (p. 6-10) with the first Proficiency Levels using the modified hand-outs (p. 8-10). (20 min)

Teacher Talk & Scaffolding:

The teacher will begin by writing two question on the board: “How do you think squaring the variable will affect the graph of the function?” “Will it still make a simple line?” The teacher will hand out and review with students the Five Steps to Graphing a Basic Quadratic Function modified text sheets according to Language Proficiency Level (LP) (p. 11-13). The teacher will read aloud while highlighting Key vocabulary which students should be familiar with from the previous unit on linear functions (such as input, output, variable, negative, plug-in, evaluate, plot, points, coordinates). Students should follow along. (10 min)

Activity 2:
In small groups, students will discuss and list orally the five steps to graphing a basic quadratic function. LP levels 1-3 will use language guides (p. 14-16). (5 minutes)

Activity 3:

The teacher will hand out the Graphing with x-y table worksheet (p. 17) to all students. The teacher will model the process of inputting an x value and calculating an output value.

Then, students will pair up and break up into small groups. In their groups, they will complete the x-y table, write out coordinates, plot points and produce their graphs on the given x-y plane. The teacher will go around and monitor progress and discussion,
amplifying key academic words where appropriate. “So when the input value is 3, the output is 9...” or “When you evaluate the function at negative two, the output value becomes 4.” (10 min)

Follow-up & Realia:

After students have finished their graphs, the teacher will return to the previous questions and ask: “What shape is the graph of a quadratic function?” “Is it a simple line?” After allowing students to answer with descriptive words or even similes, the teacher will give them the “Basic Parabola” listening guide (p. 18) and explain that the graph of a quadratic function is called a “parabola.”

The teacher will then begin a realia presentation by showing students a real-life horseshoe. “Does it look similar? Is it a parabola?” The teacher will repeat several times? The teacher will point out the “curved sides” and write the words on the board. The teacher will show a banana which also has a “curve.” Students may recall other objects which are curved such as a road. Next, the teacher will talk about the point where the two curved sides meet and write the word “vertex” on the board. The teacher will show a big letter “V” which also has a vertex but not the curved sides.

Finally, the teacher will talk about the imaginary line which runs down the middle of the graph and write on the board: “axis of symmetry.” The teacher will show an open book as well as a cardboard butterfly-objects which fold down the middle to reveal an “axis of symmetry.” The teacher will guide students as they label the “Upside Down” parabola on the Listening Guide (p.19). And the teacher will explain that this parabola had a lead term which was negative. That is why the parabola turned upside down. (10 min)

Activity 3:

The teacher will hand out the parabola picture gallery hand-out sheet (p. 20) and instruct students to work in small groups to compare each of the objects to a parabola, using the 3 main terms (vertex, curved sides and axis of symmetry) which had been emphasized with the realia, written on the board and included in the listening guide (p. 18). A question sheet “Is it shaped like a Parabola?” (p. 21) will be handed out along with Modification hand-outs (p.22-27) according to LP level. As the teacher goes around to monitor work, the teacher will ask higher-order questions such as: “Is it an upside-down parabola?” “Is the object’s vertex a maximum or a minimum?” “...so would the lead term of the function be negative or positive?” (15 min)

Homework/Assessment:

Have students graph on graph paper the relatively basic quadratic function $y = 2x^2$ by making an x-y table and plotting points.
Quadratic Functions - Graphic Organizer

What is it?
- A formula
- A rule
- A relationship \((x - y)\)

<table>
<thead>
<tr>
<th>What does it look like?</th>
<th>What does it sound like?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = x^2)</td>
<td>“(y) equals (x) squared.”</td>
</tr>
<tr>
<td></td>
<td>“(y) equals (x) times itself.”</td>
</tr>
<tr>
<td>(y = 3x^2)</td>
<td>“(y) equals three times (x)-squared.”</td>
</tr>
<tr>
<td>(y = -7x^2 + 6x)</td>
<td>“(y) equals negative seven times (x)-squared plus six times (x)”</td>
</tr>
</tbody>
</table>

Not Quadratic:
Famous Example:
“Area of a Square = Side Squared”
\(A = s^2\)

<table>
<thead>
<tr>
<th>Function</th>
<th>Because...</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = 7x + 1)</td>
<td>The lead variable (in 7(x)) is not squared</td>
</tr>
<tr>
<td>(y = x^3)</td>
<td>The lead variable (in (x^3)) is not squared</td>
</tr>
<tr>
<td>(y = 2x^5 + 8x)</td>
<td>The lead variable (in 2(x^5)) is not squared</td>
</tr>
</tbody>
</table>

How does a quadratic function work?

```
INPUT VALUE X  GETS squared! (x\cdot x) to help make OUTPUT VALUE Y
```

The result is an ordered pair: \(x, y\)
...also known as a coordinate \((x, y)\)
...which can be plotted on an x-y graph.

How is it related to a smile?
Group Discussion

Warm-up

- Recite each function below, including the individual terms.
- Check if the lead variable is squared.
- State in full sentences whether or not it is a quadratic function and explain why.

<table>
<thead>
<tr>
<th>Circle the lead term</th>
<th>Recite the function</th>
<th>Is it a quadratic function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 9x - 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = -x^2 + 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = 5x^3 + x^2 - 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = 8x^2 - 7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Group Discussion

Warm-up

- Recite each function, including the individual terms.
- Check if the lead variable is squared.
- State in full sentences whether or not it is a quadratic function and explain why using the sentence starters and phrase bank.

<table>
<thead>
<tr>
<th>Circle the lead term</th>
<th>Recite the function</th>
<th>Is it a quadratic function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 9x - 1$</td>
<td>It ________________ because _________________.</td>
<td></td>
</tr>
<tr>
<td>$y = -x^2 + 4$</td>
<td>It ________________ because _________________.</td>
<td></td>
</tr>
<tr>
<td>$y = 5x^3 + x^2 - 6$</td>
<td>It ________________ because _________________.</td>
<td></td>
</tr>
<tr>
<td>$y = 8x^2 - 7$</td>
<td>It ________________ because _________________.</td>
<td></td>
</tr>
</tbody>
</table>

Phrase Bank

- $y$ equals ____ times $x$ (minus plus) ______
- $y$ equals ____ times $x$-squared (minus plus) ______
- $y$ equals ____ times $x$-cubed (minus plus) ______

Is a quadratic function Is not a quadratic function

The lead variable is squared The lead variable is not squared
**Group Discussion**

**Warm-up**

- Recite the **lead term** of each function.
- Check if the **lead variable** is squared.
- State whether or not it is a quadratic function and explain why using sentence starters and phrase bank.

<table>
<thead>
<tr>
<th>Circle the <strong>lead term</strong></th>
<th>Recite the <strong>lead term</strong> (choose correct form)</th>
<th>Is it a <strong>quadratic function</strong>? - Use the phrase bank below to choose “1” or “2” phrases.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>y = 9x − 1</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
  "y equals 9 times x minus 1"  
  "y equals 9 times x-squared minus 1"  |
| It (1)__________________  
  because (2)____________.  |
| **y = -x^2 + 4**         | 
  "y equals negative x-squared plus four."  
  "y equals two times x plus four."  |
| It (1)__________________  
  because (2)____________.  |
| **y = 5x^3 + x^2 - 6**   | 
  "y equals five times x-squared plus x-squared."  
  "y equals five times x-cubed plus x-squared minus six"  |
| It (1)__________________  
  because (2)____________.  |
| **y = 8x^2 - 7**         | 
  "y equals eight times x-squared plus seven."  
  "y equals eight times x-squared minus seven."  |
| It (1)__________________  
  because (2)____________.  |

**Phrase Bank**

1. Is a quadratic function  
2. The lead variable is squared  
1. Is not a quadratic function  
2. Lead variable is not squared
Group Discussion

Warm-up  LP 1

- Recite the lead term of each function.
- Check if the lead variable is squared.
- State whether or not it is a quadratic function.

<table>
<thead>
<tr>
<th>Circle the lead term</th>
<th>Recite the lead term</th>
<th>Is it a quadratic function?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>“Yes it is a quadratic function.” or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>“No, it is not a quadratic function.”</td>
</tr>
<tr>
<td>$y = 9x - 1$</td>
<td>Nine x</td>
<td></td>
</tr>
<tr>
<td>$y = -x^2 + 4$</td>
<td>Negative x squared</td>
<td></td>
</tr>
<tr>
<td>$y = 5x^3 + x^2 - 6$</td>
<td>Five x cubed</td>
<td></td>
</tr>
<tr>
<td>$y = 8x^2 - 7$</td>
<td>Eight x squared</td>
<td></td>
</tr>
</tbody>
</table>
Five Steps to Graphing a Basic Quadratic Function ($y = x^2$)

Text Modification/Enhancement

(McDougal Littell p. 249)

LP 4 & 5

Step 1: In x-y table, enter domain values (x values or input values), including both positive and negative numbers.

Step 2: For each x value, calculate the y value (output value) by plugging the x value into the function at each x variable and evaluating or solving for y.

Step 3: Write out each x-y ordered pair in the table as coordinates.

Step 4: Plot each coordinate (point) on the coordinate plane. Remember, the x value always comes first and represents the horizontal movement left or right. The y value represents the vertical position up or down.

Step 5: Connect the dots, revealing vertex with two curves sides, and place arrow tips at the ends of each curve to show that the graph will continue on indefinitely (to infinity).
Five Steps to Graphing a Basic Quadratic Function \( y = x^2 \)

Text Modification/Enhancement
(McDougal Littell p. 249)

**Step 1:** In x-y table, enter positive and negative input values.

\[ y = x^2 \]  
**input values** (x): -3, -2, -1, 0, 1, 2, 3

**Step 2:** Plug the input value into the x variable, and calculate the output value.

<table>
<thead>
<tr>
<th>( y = (x)^2 )</th>
<th>Output (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = (-3)^2 )</td>
<td>9</td>
</tr>
<tr>
<td>( y = (-2)^2 )</td>
<td>4</td>
</tr>
<tr>
<td>( y = (-1)^2 )</td>
<td>1</td>
</tr>
<tr>
<td>( y = (0)^2 )</td>
<td>0</td>
</tr>
<tr>
<td>( y = (1)^2 )</td>
<td>1</td>
</tr>
<tr>
<td>( y = (2)^2 )</td>
<td>4</td>
</tr>
<tr>
<td>( y = (3)^2 )</td>
<td>9</td>
</tr>
</tbody>
</table>

**Step 3:** Write each x-y ordered pair as coordinates.

\((x, y) = \)  
(-3, 9)  
(-2, 4)  
(-1, 1)  
(0, 0)  
(1, 1)  
(2, 4)  
(3, 9)

**Step 4:** Plot each coordinate (point) on the graph.

**Step 5:** Connect the dots, revealing vertex with two curves sides.
Five Steps to Graphing a Basic Quadratic Function ($y = x^2$)

Text Modification/Enhancement
(McDougal Littell p. 249)

**Step 1:** Enter input values (positive and negative).

\[ y = x^2 \quad \text{input values (x): -3, -2, -1, 0, 1, 2, 3} \]

**Step 2:** Plug into x variable, and calculate the output value.

\[
\begin{array}{c|c}
\text{Input (x)} & \text{Output (y)} \\
\hline
(-3)^2 & 9 \\
(-2)^2 & 4 \\
(-1)^2 & 1 \\
0^2 & 0 \\
1^2 & 1 \\
2^2 & 4 \\
3^2 & 9 \\
\end{array}
\]

**Step 3:** Write each x-y pair as coordinates.

\[
(x, y) = (-3, 9) \\
(-2, 4) \\
(-1, 1) \\
(0, 0) \\
(1, 1) \\
(2, 4) \\
(3, 9)
\]

**Step 4:** Plot each coordinate on the graph.

**Step 5:** Connect the dots.
What are the five steps to graphing a basic quadratic function?

Discuss each step by finishing each sentences below, using your Five Steps Guide with Word bank.

1. First, ____________________________.

2. The second step is ____________________________.

3. Next, ____________________________.

4. Then, ____________________________.

5. Finally, ____________________________.
What are the five steps to graphing a basic quadratic function?

Discuss each step by finishing each sentence below, using 2-3 words or phrases from your Five Steps Guide with Word bank.

1. First, ________________.

2. The second step is ________________.

3. Next, ________________.

4. Then, ________________.

5. Finally, ________________.
What are the five steps to graphing a basic quadratic function?

Discuss each step by reciting 2-3 key words from the Five Steps Guide with Word bank.

Step 1: **Enter input** values.
Step 2: **Plug into x variable**, and **calculate** the **output** value.
Step 3: **Write** as **coordinates**.
Step 4: **Plot** each **coordinate**.
Step 5: **Connect** the dots.

1. First, ____________________________.

2. The second step is ____________________________.

3. Next, ____________________________.

4. Then, ____________________________.

5. Finally, ____________________________.
Graph The Basic Quadratic Function \( y = x^2 \)
by following the Five Steps

Use the domain (input values) of: \{-3, -2, -1, 0, 1, 2, 3\} to calculate the outputs.

**X - Y TABLE**

<table>
<thead>
<tr>
<th>Input (x value)</th>
<th>Evaluate Expression</th>
<th>Output (y value)</th>
<th>Coordinates (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( y = (-3)^2 )</td>
<td>9</td>
<td>(-3, 9)</td>
</tr>
</tbody>
</table>

Plot Points and Connect Dots:
Basic Parabola
Graphical Representation of
Quadratic Function \( y = x^2 \)

"Up-Facing" Parabola:
Vertex is the "Minimum"

"Down-Facing" Parabola:
Vertex is the "Maximum"

Key Characteristics of a Parabola

✓ Two curved sides
✓ Vertex – Point where sides meet
✓ Axis of Symmetry
Basic Parabola
Graphical Representation of Quadratic Function \( y = x^2 \)

“Up-Facing” Parabola:
Vertex is the “Minimum”

“Down-Facing” Parabola:
Vertex is the “Maximum”

Key Characteristics of a Parabola
- Two curved sides
- Vertex – Point where sides meet
- Axis of Symmetry

...And when lead term is negative

Vertex (Minimum)
1. Horseshoe
2. Ice Cream Cone
3. Open Book
4. Mountain
5. Bowl of Soup
6. Smile
7. Broken Egg
8. Nike Symbol
9. McDonald’s Sign
Is it shaped like a Parabola? Compare.
LP 4 & 5

Using the Parabola Picture Gallery and your Basic Parabola Listening Guide, compare each picture to a parabola. Explain in a written paragraph whether or not the object has a parabolic shape and why? Remember the three main components of a parabolic structure.
Is it shaped like a Parabola? Compare.

LP 3

Write in the correct response below with all reasons that apply, using the words below and the Basic Parabola Listening Guide:

<table>
<thead>
<tr>
<th>Is</th>
<th>Is not</th>
<th>It has</th>
<th>does not have</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axis of symmetry</td>
<td>Vertex</td>
<td>Curved sides</td>
<td></td>
</tr>
</tbody>
</table>

1. Horseshoe

The ____________________________ because

2. Ice Cream Cone

The ____________________________ because
3. Open Book

The ______________________ because

4. Mountain

The ______________________ because

5. Bowl of Soup

The ______________________ because
**Is it shaped like a Parabola? Compare.**

**LP 2**

Write in the correct response below with all reasons that apply, using the following word/phrases and referring to the Basic Parabola Listening Guide:

<table>
<thead>
<tr>
<th>Is</th>
<th>Is not</th>
<th>It has</th>
<th>does not have</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axis of symmetry</td>
<td>Vertex</td>
<td>Curved sides</td>
<td></td>
</tr>
</tbody>
</table>

2. **Horseshoe**

The _______ _______ shaped like a _______ because _______

a _______ , two _______ and an _______.

2. **Ice Cream Cone**

The _______ _______ shaped like a _______ because _______

a _______ , two _______ and an _______.
3. Open Book

The _______ _______ shaped like a _________ because _________

a ______________ , two _______________ and an ___________________.

4. Mountain

The _______ _______ shaped like a _________ because _________

a ______________ , two _______________ and an ___________________.
Is it shaped like a Parabola? Compare.

LP 1

Write in the correct response below with all reasons that apply.

1. Horseshoe

The _______ _______ shaped like a ________ because _______
   (is / is not) (it has / it does not have)

   a _______ , two _______ and an ________.
   (vertex) (curved sides) (axis of symmetry)

2. Ice Cream Cone

The _______ _______ shaped like a ________ because _______
   (is / is not) (it has / it does not have)

   a _______ , two _______ and an ________.
   (vertex) (curved sides) (axis of symmetry)
3. Open Book

The __________ __________ shaped like a __________ because __________
(is / is not) (it has / it does not have)

a ______________, two ______________ and an ________________.
(week) (curved sides) (axis of symmetry)

4. Mountain

The __________ __________ shaped like a __________ because __________
(is / is not) (it has / it does not have)

a ______________, two ______________ and an ________________.
(week) (curved sides) (axis of symmetry)
Narrative – Lesson 1

This unit on quadratic functions offers students their first glimpse of an nonlinear relationship between two variables. Since students have already become familiar with a line graph (slope and y-intercept), the original text from the textbook assumes that students will automatically grasp the idea that the graph of a quadratic function is simply different: namely a parabola. Yet, in my teaching experience, I have found that students benefit from discovering this relationship of the visual representation to the original function by using an x-y table.

While many of the terms associated with graphing may be familiar, they should be reviewed and amplified with all students, especially ELLs. New content vocabulary such as “squared,” “vertex” and “axis of symmetry” should be introduced via deliberately-designed teaching methods (gesturing, visuals, repetition, realia…) and engaging activities in which students may have many opportunities to negotiate meaning and develop their academic language.

I believe the lesson starts out effectively when students are given the original graphic organizer which offers the standard depictions of basic quadratic functions along with visuals and ends with the question: “How is a smile related?” Many opportunities are given to students to discuss new concepts (and listen) while using hand-outs with key vocabulary and other modifications according to LP level. The various activities also give them chances to negotiate meaning while using the language assistance materials needed to perform and grow linguistically. I believe that the graphic organizers and listening guides in conjunction with peer interaction will prove most beneficial to student progress in this unit.
Lesson 2
## Performance Standards: Graphing Quadratic Functions – Lesson 2, “Standard Form Functions”

<table>
<thead>
<tr>
<th><strong>Content Objectives</strong></th>
<th><strong>Language Objectives</strong></th>
</tr>
</thead>
</table>
| 1. SWBAT identify “a,” “b” and “c” values of a standard form quadratic function.  
2. SWBAT graph the parabola of a standard form quadratic function, using the formula \( x = -\frac{b}{2a} \).  
3. SWBAT identify and describe the max, min, axis of symmetry and y-intercept of a standard form quadratic function: \( y = ax^2 + bx + c \). | 1. In small groups, students will take part in an identification activity. They will orally describe items using determining factors and reach consensus in the group discussion.  
2. In small groups, students will take part in a graphing activity. They will orally list and verify in the small group the steps to graphing a parabola.  
3. In small groups, students will identify and describe in writing the key components of a specific function’s parabola. |

<table>
<thead>
<tr>
<th><strong>Domain/Topic</strong></th>
<th><strong>Fluent Bridging Level 5</strong></th>
<th><strong>Expanding Fluency Level 4</strong></th>
<th><strong>Speech Emerging Level 3</strong></th>
<th><strong>Early Production Level 2</strong></th>
<th><strong>Preproduction Level 1</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Listening and Speaking: Identify “a,” “b” and “c” values of quadratic functions</td>
<td>Students will take a lead role in small group identification activity, orally describing traits of “a,b,c” values and modeling pronunciation of math terms.</td>
<td>Students will take part in small group identification activity, orally describing traits of “a, b, c” values and modeling pronunciation of math terms.</td>
<td>Students will take part in small group activity, orally describing traits of “a,b,c” values, using simple phrases, using graphic organizer and functional lang. chart.</td>
<td>In small groups, students will identify the “a,b,c” values by orally discussing and describing traits with single words in graphic organizer &amp; funct. lang chart. Use L1 if needed.</td>
<td>In small groups, students will identify the “a,b,c” values by orally discussing, pointing to and repeating single words in graphic organizer &amp; funct. lang chart. Use L1 if needed.</td>
</tr>
<tr>
<td>Speaking and Writing: 5-step process to graphing standard form</td>
<td>Students will work in pairs on a graphing activity, orally describing the 5-step process w complete sentences and answering questions posed by teacher.</td>
<td>Students will work in pairs on a graphing activity, orally describing the 5-step process w sentences and answering questions posed by teacher.</td>
<td>Students will work in small groups on a graphing activity, orally describing the 5-step process, using simple sentences, FL chart &amp; modified text.</td>
<td>Students will work in small groups on a graphing activity, orally describing the 5-step process, using single words, pointing to modified text, reciting and using FL chart &amp; L1.</td>
<td>Students will work in small groups on a graphing activity, orally describing the 5-step process, using single words, pointing to modified text, reciting and using FL chart &amp; L1.</td>
</tr>
<tr>
<td>Writing: List and describe 5 key components of a graphed function.</td>
<td>Working in pairs, students will describe in writing 5 components of a graphed quadratic function, using complete sentences &amp; word bank.</td>
<td>Working in pairs, students will describe in writing 5 components of a graphed quadratic function, using complete sentences &amp; word bank.</td>
<td>Working in small groups, students will describe in writing 5 components, using sentence starters and word bank.</td>
<td>Working in small groups, students will describe in writing 5 components, by choosing words to complete sentences &amp; using graphic organizer.</td>
<td>Working in small groups, students will describe in writing 5 components, by rewriting words to complete sentences &amp; using graphic organizer.</td>
</tr>
</tbody>
</table>
LESSON 2
(70 minutes)

Warm-Up (10 minutes): The teacher will write on the board several quadratic functions, including the basic quadratic function \( y = x^2 \), which students should now be familiar with. Students will take turns reciting the quadratic functions and stating which term is the Lead Term or first term, which is the second term and which is the third.

Front-loading key vocabulary: The teacher will write out the words coefficient and constant on the board and discuss their meaning with students.

Coefficient is the number connected to a variable by multiplication, such as “7” in \( 7x \) or \( 7x^2 \). The teacher may connect the word Coefficient to other “co” words such as “coexist”, “contact,” or even “connect.”

Constant is a number which stands alone without a variable. The teacher may show a picture of a tree or a person “standing” alone and connect the “stant” to “stand.”

The teacher will point to each term of the various functions and ask whether the number in that term is a coefficient or constant. Students should justify their answers by saying “stands alone” or “connected to a variable.”

The teacher will hand out the a, b, c graphic organizer (p. 33) and review with students, while discussing how to identify the a, b and c values of a quadratic function in standard form. The teacher will inform students that the a, b and c values shall be an important part of using formulas related to quadratic functions.

Activity 1 (10 minutes): The students will break up into small groups of three according to language proficiency; however, at least one higher-level (4-5) language student will work within each group to help model language functions and pronunciation of math terms. Each group will be given a small response board, and level 1-3 students will also be given the Functional Language chart (p. 30) to use.

The teacher will write a quadratic function on the board and ask students to discuss what is the “a” value of the function and why? What is the “b” value of the function and why. What is the “c” value of the function and why? Groups should discuss, come to a consensus and write their answer on the response board. For each question, a group representative may present their answer to the class. The teacher will repeat this activity with another function, making sure to amplify key terms: Coefficient, constant, x-squared, lead term, first term, second term, third term, variable, etc.

Teacher Talk (10 minutes): The teacher will ask the students to think about the last time they graphed the basic quadratic function, and students will take out their Basic Parabola Graphic Organizer (p. 19) The teacher will ask: What was the most important point on the graph? Why? Is it possible to graph a single curve if you don’t know where the vertex of a parabola is? Of course! The teacher will explain that you may input dozens of x values but still not get a parabola
if you do not know where the vertex is! The teacher will explain that there is a very good formula for helping find the exact vertex of a parabola, and the teacher will introduce the X-of-the-Vertex Formula \( x = -\frac{b}{2a} \) by writing it out on the board. The teacher will explain that this formula can help find the x value of the vertex. The teacher will show a horsehoe again and draw it out on the board or overhead with an arrow pointing to the vertex. The teacher will draw a large question mark near the vertex and write out “Coordinates?” What is the x value? What is the y value? The teacher should write out \((x, y) ???\) on the board. Next, the teacher will say: “We can find the x value. We have a formula: \( \text{The x-of-the-vertex Formula } x = -\frac{b}{2a} \). The teacher will write this formula underneath the vertex and make an arrow pointing to the vertex and make an arrow pointing to the x value.

Next, students will be asked to take out the original hand-out on How to Graph a Quadratic Function in Standard Form (p. 34-35), while Levels 1-3 will be given modified text (p. 36) with hand-written samples on the sides.

The teacher will read through the steps with the class, pausing, asking questions, repeating key words (a value, b value, input, calculate, output, plot) and modeling each step with a sample quadratic function.

**Whole Class Activity** (15 minutes): The teacher will hand out Listening Guides (p. 36 a-d) and the new Graphic Organizer “Let’s Together Graph” (p. 37) and put a copy on the overhead projector. Together, the teacher will call up different students to the board to complete each step of the process, while another student recites the key parts of the step. For example: “Calculate the x of the vertex.” The teacher will paraphrase student comments and correct pronunciation while amplifying key words. At the end, the key components on the graphed parabola will be labeled, and the teacher will also emphasize the \( y \)-intercept – the value of \( y \) when \( x \) is 0 by evaluating the function at \( x = 0 \). Students should be familiar with this term from the unit on linear functions. Students will fill out their graphic organizers (p. 37), so they resemble the completed graphic organizer (p. 38) (15 minutes).

**Group Activity & Review** (25 minutes): Students will break up into small groups according to language proficiency. The teacher will hand out the Graphing Activity Worksheet (p. 39) along with a Key Components of the Parabola worksheet (p. 40-43) according to language proficiency. Students will discuss within their groups the steps to graphing the given quadratic function \( y = 2x^2 - 4x + 3 \) with levels 1-3 using their modified text and Functional Language charts (p. 36 & 30). They will proceed to graph the function and label key points, using their graphic organizers (p. 38) Finally, students will complete in writing the Key Components of the Parabola worksheet (p. 40-43).

The teacher will monitor discussion, repeat key words and language functions, correct pronunciation and ask questions justification and higher-order questions such as: “Is the Axis of Symmetry a real or imaginary line? What is the difference? How is the x-of-the-vertex and related to the axis of symmetry?” The teacher will review and discuss the results of this activity with the whole class, sharing samples of work.

Graphic Organizer
Identifying a, b & c values

Sample Quadratic Function (in Standard Form): $y = 2x^2 - 8x + 5$

- **a**
  - Location: 1st Term or Lead Term ($2x^2$)
  - Identity: Coefficient
  - Connected to: $x^2$
  - So: $a = 2$

- **b**
  - Location: 2nd Term (-8x)
  - Identity: Coefficient
  - Connected to: $x$
  - So: $b = -8$

- **c**
  - Location: Last Term (5)
  - Identity: Constant
  - Connected to: Nothing (Stands Alone)
  - So: $c = 5$
How to Graph a Quadratic Function (Standard Form)

Step 1: Verify that you indeed have a quadratic function... in standard form.

a. Remember, a function begins with \( y = \)

b. The first term should have a variable raised to the 2\(^{\text{nd}}\) power.
   Example: \( y = 5x^2 + 7x - 10 \) ... or the function may simply be: \( y = x^2 \)

c. **Standard form** is a method of representing a quadratic function without parentheses.

   Example: \( y = -9x^2 + 6x + 5 \)
   .... as opposed to \( y = (x - 3)^2 + 1 \) (not standard form!)

   Standard form may also be known as the "ABC" form in which the coefficient of the first term is \( a \), the coefficient of the second term is \( b \), and the coefficient of the 3\(^{\text{rd}}\) term is \( c \).

   **Standard Form Model:** \( y = ax^2 + bx + c \)
   Thus, for the example above: \( a = -9 \quad b = 6 \quad \text{and} \quad c = 5 \)

Step 2: Identify the \( a, b \) and \( c \) values of the function and apply the \( x \) of the vertex formula \(- b \div (2a)\) in order to calculate the \( x \) coordinate of the vertex of the parabola.

Example: If \( y = 3x^2 + 12x + 1 \), then \( a = 3 \quad b = 12 \quad c = 1 \)

So \(- b \div (2a)\) would mean \(-12 \div (2 \cdot 3)\) and this equals -2

Thus, the \( x \) of the vertex is -2.
Step 3: Calculate the y coordinate of the vertex by plugging in the x value of the vertex at each variable and evaluating the quadratic function with that input. Thus, in the example above:

Since \( x = -2 \), \( y = 3(-2)^2 + 12(-2) + 1 \)
Thus, the y value of the vertex is -11.

Step 4: After plotting the vertex on the coordinate plane, find two x values to the left of the vertex (lesser values) and two x values to the right of the vertex (greater values) and evaluate the quadratic function at each of those inputs in order to find the corresponding outputs- or y values.

Plot each of the 4 new ordered pairs. There should be 5 total points plotted on the coordinate plane: The vertex, 2 points to the left of the vertex, and 2 points to the right of the vertex.

Step 5: Connect the dots, and don’t forget to add arrow tips at the ends of the parabola to indicate a continuation of the graph to infinity.
Modified Text

How to Graph a Quadratic Function
In Standard Form
\[ y = ax^2 + bx + c \]

Step 1: Verify Make sure the function is in Standard Form with a lead term of \( ax^2 \), such as \( x^2 \) or \( 5x^2 \).

\[
\begin{align*}
y &= -2x + x^2 + 3 & \text{... Not Standard Form! Does not begin w} & x^2 \text{ term} \\
Y &= x^2 - 2x + 3 & \text{... Yes, Standard Form! Begins w} & \text{an} & \ x^2 \text{ term}
\end{align*}
\]

Step 2:

a. Identify Find the \( a, b \) & \( c \) values.

If \( Y = x^2 - 2x + 3 \), then...

\[
\begin{align*}
a &= 1 \\
b &= -2 \\
c &= 3
\end{align*}
\]

b. Calculate the "x" of the vertex, using the formula \( x = -b/2a \)

\[
x = -(-2)/(2 \cdot 1) \quad \ldots \text{which equals} \ 1
\]

So, \( x = 1 \)

Step 3:

a. Calculate the "y" of the vertex by inputting the \( x \) value into the function and finding the output.

If \( x = 1 \) and \( Y = x^2 - 2x + 3 \), then...

\[
Y = (1)^2 - 2(1) + 3 \quad \ldots \text{which equals} \ 2
\]

So, \( y = 2 \)
Step 3: b. Plot the vertex \((x, y)\)
\[(1, 2)\]

Step 4: Calculate and Plot 4 more points, using 2 inputs to the left of the vertex and 2 inputs to the right.

<table>
<thead>
<tr>
<th>Left side</th>
<th>Vertex</th>
<th>Right side</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\[y = x^2 - 2x + 3\]

<table>
<thead>
<tr>
<th>Input</th>
<th>(y = (_)^2 - 2(_)+3)</th>
<th>Output</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>((-1)^2 - 2(-1)+3)</td>
<td>6</td>
<td>((-1, 6))</td>
</tr>
<tr>
<td>0</td>
<td>((0)^2 - 2(0)+3)</td>
<td>3</td>
<td>((0, 3))</td>
</tr>
<tr>
<td>2</td>
<td>((2)^2 - 2(2)+3)</td>
<td>3</td>
<td>((2, 3))</td>
</tr>
<tr>
<td>3</td>
<td>((3)^2 - 2(3)+3)</td>
<td>6</td>
<td>((3, 6))</td>
</tr>
</tbody>
</table>

Step 5: Connect the dots, and add arrows to each end of the graph.

Higher-Order Question: Do you notice the symmetry in the graph?
Can you predict that the graph will be symmetric just by looking at the input-output table?
Listening Guide (to be used during discussion):  

\[ y = ax^2 + bx + c \]
\[ y = 3x^2 + 12x + 1 \]

\[ x = \frac{-b}{2a} = \frac{-12}{2(3)} = -2 \]

\[ y = 3(-2)^2 + 12(-2) + 1 = -11 \]

---

**Left Side of Vertex**
- Input: -3, Output: -8 point: (3, -8)
- Input: -4, Output: 1 point: (-4, 1)

**Right Side of Vertex**
- Input: -1, Output: -8 point: (-1, -8)
- Input: 0, Output: 1 point: (0, 1)

---

Fill in the blanks above with the correct words or phrases in **bold**:

1. The **standard form** of a quadratic function is \( y = ax^2 + bx + c \).
2. The **x of the vertex** is found with the formula: \( x = \frac{-b}{2a} \)
3. The **y of the vertex** is found by plugging the x value into the function.
4. **Plot 4 extra points** using other (left-right) input values.
5. **Connect the dots**
Listening Guide (to be used during discussion): Level 2

\[ y = ax^2 + bx + c \]
\[ y = 3x^2 + 12x + 1 \]

\[ x = \frac{-b}{2a} = \frac{-12}{2(3)} = -2 \]

\[ y = 3(-2)^2 + 12(-2) + 1 = -11 \]

---

Left Side of Vertex
Input: -3, Output: -8 point: (3, -8)
Input: -4, Output: 1 point: (-4, 1)

Right Side of Vertex
Input: -1, Output: -8 point: (-1, -8)
Input: 0, Output: 1 point: (0, 1)

---

Complete each sentence and label the above blanks with words in **bold**:

1. The **standard form** of a quadratic function is ________________.
2. The **x of the vertex** is found with the formula: ________________.
3. The **y of the vertex** is found by ________________.
4. **Plot 4 extra points** using ________________.
5. ________________ the dots.

(Answer in Bold)
Label the blanks using Phrase below & describe the process shown.

X of Vertex formula \((x = -b/2a)\)
Evaluate 4 extra points
Evaluate \(y\) of vertex

Standard Form \((y = ax^2 + bx + c)\)
Connect the Dots
Listening Guide (to be used during discussion): Levels 4 & 5

\[ y = 3x^2 + 12x + 1 \]

\[ x = \frac{-12}{2(3)} = -2 \]

\[ y = 3(-2)^2 + 12(-2) + 1 = -11 \]

**Left Side of Vertex**
- Input: -3, Output: -8 point: (3, -8)
- Input: -4, Output: 1 point: (-4, 1)

**Right Side of Vertex**
- Input: -1, Output: -8 point: (-1, -8)
- Input: 0, Output: 1 point: (0, 1)

Label the blanks above and describe each process shown.
Graphic Organizer
Let’s Together Graph: \[ y = -x^2 + 6x - 10 \]
and identify key components of the parabola.
(H.O./Prediction: Will it be an up-facing or down-facing parabola?)

<table>
<thead>
<tr>
<th>a =</th>
<th>b =</th>
<th>c =</th>
<th>x of vertex (-\frac{b}{2a}) =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**X – Y Table**

<table>
<thead>
<tr>
<th>Input (x value)</th>
<th>Evaluate Expression</th>
<th>Output (y value)</th>
<th>Coordinates (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

**Plot Points and Connect Dots:**

[Graph with a grid showing a parabola and labeled axes]
Graphic Organizer

Let’s Together Graph: \( y = -x^2 + 6x - 10 \)

and identify key components of the parabola.

(H.O./Prediction: Will it be an up-facing or down-facing parabola?)

\[
\begin{align*}
a &= -1 \\
b &= 6 \\
c &= -10 \\
x \text{ of vertex } (-b/2a) &= \frac{-6}{2(-1)} = \frac{-6}{-2} = 3
\end{align*}
\]

<table>
<thead>
<tr>
<th>Input (x value)</th>
<th>Evaluate Expression</th>
<th>Output (y value)</th>
<th>Coordinates (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( y = -(3)^2 + 6(3) - 10 )</td>
<td>-1</td>
<td>(3, -1)</td>
</tr>
<tr>
<td>1</td>
<td>( y = -(1)^2 + 6(1) - 10 )</td>
<td>-5</td>
<td>(1, -5)</td>
</tr>
<tr>
<td>2</td>
<td>( y = -(2)^2 + 6(2) - 10 )</td>
<td>-2</td>
<td>(2, -2)</td>
</tr>
<tr>
<td>4</td>
<td>( y = -(4)^2 + 6(4) - 10 )</td>
<td>-2</td>
<td>(4, -2)</td>
</tr>
<tr>
<td>5</td>
<td>( y = -(5)^2 + 6(5) - 10 )</td>
<td>-5</td>
<td>(5, -5)</td>
</tr>
</tbody>
</table>

Plot Points and Connect Dots:

- **Vertex**
- **2 Left-side Points**
- **2 Right-side Points**
- **2 Curved Sides**
- **Value of y when x is 0** \( y = -(0)^2 + 6(0) - 10 = -10 \)
- **Axis of Symmetry** Imaginary Vertical Line
Graphing Activity- Graph The Standard Form Quadratic Function \( y = 2x^2 - 4x + 3 \) by following the **Five Steps** and identify in writing at least 5 key components of the parabola. (H.O./Prediction: Will it be an up-facing or down-facing parabola?)

\[ a = \quad b = \quad c = \quad x \text{ of vertex } \left(-\frac{b}{2a}\right) = \]

<table>
<thead>
<tr>
<th>Input (x value)</th>
<th>Evaluate Expression</th>
<th>Output (y value)</th>
<th>Coordinates (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample, ( x = 5 )</td>
<td>( y = 2(5)^2 - 4(5) + 3 )</td>
<td>33</td>
<td>(5, 33)</td>
</tr>
</tbody>
</table>

**X - Y TABLE**

Plot Points and **Connect** Dots:

![Graph](image)
Key Components of the Parabola/
Graph of Function:  \( y = 2x^2 - 4x + 3 \)

**Level 1**
Use Your Graph & The Word Bank Below

1. The ___________ of the parabola is at ( , , )
   (vertex)

2. It is the point where the 2 ___________ sides meet.
   (cloudy, curved)

3. This vertex is also the ___________ point _______ graph.
   (maximum, minimum) (of the)

4. The ___________ is \( x = \) _________
   (axis of symmetry)
   It is a _________ line which divides the parabola in ______.
   (vertical) (half)

5. The value of \( y \) when \( x \) is zero is _________. This is also called the ___________.
   (y – intercept)

6. A point on the ___________ side _______ vertex is ( , , ).
   (left) (of the)

7. A point on the ___________ side _______ vertex is ( , , ).
   (right) (of the)
Key Components of the Parabola/
Graph of Function: \[ y = 2x^2 - 4x + 3 \]

Level 2
Use Your Graph & Write in the
correct words & Phrases Below

1. The \( \underline{\text{vortex, vertex}} \) of the parabola is \( (\underline{\text{, }}) \).

   It is the point where \( \underline{\text{the 2 curved sides meet, the arrow ends}} \).

2. This vertex is also known as the \( \underline{\text{maximum point of the graph, minimum point of the graph}} \).

3. The \( \underline{\text{sharp axe, axis of symmetry}} \) is \( x = \underline{\text{}} \).

   It is the \( \underline{\text{horizontal, vertical}} \) line which divides the parabola in \( \underline{\text{half, thirds}} \).

4. The value of \( y \) when \( x \) is zero is \( \underline{\text{}} \). It is also called the \( \underline{\text{y – international, y – intercept}} \).

5. A point on the \( \underline{\text{left, live}} \) side \( \underline{\text{on the, of the}} \) vertex is \( (\underline{\text{, }}) \).

6. A point on the \( \underline{\text{write, right}} \) side \( \underline{\text{of the, in the}} \) vertex is \( (\underline{\text{, }}) \).
Key Components of the Parabola/
Graph of Function: \( y = 2x^2 - 4x + 3 \)

Level 3
Use Your Graph & The Word - Phrase Bank Below to complete the sentence starters.

1. The ____________ of the parabola is ( , , ).
   It is defined as ____________________________.

2. The vertex of this parabola is also known as the ________________

3. The ________________ is \( x = \) ____________.
   It is defined as the ________________ line which ____________________________.

4. The value of \( y \) when \( x \) is zero is ____________. It is also called the ____________________________.

5. A point on __________________ of the vertex is ( , , ).

6. A point ____________________________ is ( , , ).

\( y \) – intercept \hspace{1cm} \text{Vertex} \hspace{1cm} \text{Axis of Symmetry}

The point where the 2 curved sides meet.
Maximum point of the graph \hspace{1cm} Minimum point of the graph
the left side \hspace{1cm} on the right side of the vertex
divides the parabola in half
horizontal \hspace{1cm} vertical
Key Components of the Parabola/
Graph of Function:  \( y = 2x^2 - 4x + 3 \)

**Levels 4 & 5**
Use the Word-Phrase bank below to list and describe at least 5 key components. Write in complete sentence.

1. 

2. 

3. 

4. 

5. 

6. 

\( y - \) intercept  
Vertex  
Axis of Symmetry  
The point where the 2 curved sides meet.  
Maximum point of the graph  
Minimum point of the graph  
the left side  
on the right side of the vertex  
divides the parabola in half  
horizontal  
vertical
Narrative – Lesson 2

The Standard Form Quadratic Function is empowering for students, since the numerical components (a, b, and c) can be used with key formulas such as the **x-of-vertex formula** and the **quadratic formula** to answer important questions about quadratic relationships and find key points: the maximum, the minimum and the zeros.

Many students have difficulty grasping the language of coefficients and separating the numbers from their respective variables, so I felt it was important in this lesson to have students collaborate and discuss strategies for identifying the a, b and c values. Coefficient –vs- constant could be treated as antonyms with skillful teaching, so students may recognize the difference and see that the constant stands alone as the c value.

During the first lesson in this unit, students were deliberately directed to enter negative inputs when graphing a basic quadratic function. But in the case of a Standard Form Functions, negative inputs alone will not necessarily lead to a complete graph with vertex, therefore, this lesson asks students to consider the important aspects of a quadratic graph or parabola. Graphic organizers are provided to allow ELLs to instantly view and check their labels to see the important components of a parabola, so they may participate in the discussion and progress of the lesson. Once it has been established that the vertex is key to graphing, figuring out how to find the vertex with given numerical clues (a, b, c) and an ingenious formula ($x = -b/2a$) becomes the central objective of this lesson. As in the first lesson, a graphic organizer which lists the steps in simple language and provides visuals/examples along with each step becomes a vital tool for ELLs.

The lesson, however, is not complete until students are given an opportunity to analyze and describe their graphs with the key components. ELLs at varying levels are given modifications to complete this task. Not only does this exercise strengthen their vocabulary and content language skills, but it gives them more ownership of the amplified terms and concepts. They will need this increasingly solid foundation to enter the next lesson which leaps into real-life word problems and applications of quadratic functions and graphs.
Lesson 3

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<thead>
<tr>
<th>Content Objectives</th>
<th>Language Objectives</th>
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<tr>
<td>1. SWBAT calculate the maximum or minimum value of a given real-life quadratic function and interpret the results.</td>
<td></td>
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<tr>
<td>2. SWBAT use technology to graph and verify results.</td>
<td>1a. In small groups, students will describe orally the relationship between two real-life variables given verbally and students will connect verbal variables to math variables.</td>
</tr>
<tr>
<td></td>
<td>1b. In small groups, students will interpret in writing the answer to a quadratic word problem.</td>
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<td>Reading and Speaking: Describe the variables in a word problem.</td>
<td>Students will take part in a small group summarizing activity, reading word problems aloud and determining which 2 variables could be represented by x and y variables. Students will justify answers, using complete sentences and key terms.</td>
<td>Students will take part in a small group summarizing activity, reading word problems aloud and determining which 2 variables could be represented by x and y variables. Students will justify answers, using key terms.</td>
<td>Students will take part in a small group summarizing activity, reading modified problems aloud and determining which 2 variables could be represented by x and y variables. Students will select “either-or” words to complete sentences. They will use a functional language chart and L1 where needed.</td>
<td>Students will take part in a small group summarizing activity, reading modified word problems aloud and determining which 2 variables could be represented by x and y variables. Students will select from words that have clues to complete sentences. They will use a functional language chart and L1 where needed.</td>
<td>Students will take part in a small group summarizing activity, reading modified word problems aloud and determining which 2 variables could be represented by x and y variables. Students will select from words that have clues to complete sentences. They will use a functional language chart and L1 where needed.</td>
</tr>
<tr>
<td>Writing: Interpret the answer to a quadratic problem</td>
<td>Students will independently write out the results of a quadratic problem using original verbal terms.</td>
<td>Students will independently write out the results of a quadratic problem using original verbal terms.</td>
<td>Students will work in small groups to write out results of a quadratic problem by completing sentences with the use of a word bank, a functional language chart and L1 where needed.</td>
<td>Students will work in small groups to write out results of a quadratic problem by selecting “either-or” words to complete sentences. They will use a functional language chart and L1 where needed.</td>
<td>Students will work in small groups to write out results of a quadratic problem by selecting “either-or” words to complete sentences. They will use a functional language chart and L1 where needed.</td>
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<tr>
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<td>Situation</td>
<td>Expression</td>
<td>Words/Phrases</td>
<td>Grammar</td>
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<td>-----------------------------------------------</td>
<td>-------------------------------------------------</td>
<td>-------------------------------</td>
<td>---------------</td>
<td></td>
</tr>
</tbody>
</table>
| Describe, Connect and Justify | Connect verbal variables in a quadratic word problem to math variables. | The variable (t) which (1)__________
Time is the (2)______________ variable. (3)______________ it can also be represented by the variable ________.
The variable (h) which (1)______________
Height is the (2)______________ variable. | 1. Represents
Stands for
Shows
2. Independent
Dependent
3. Therefore
Thus
So
4. “x”
“y” | Vocabulary, Conjunctions |
| Identify Explain      | The real-life meaning of an x-y answer.       | The (1)______________ height (2)______________
by the (3)______________ of the vertex. The ball reaches a (1)______________ height of 25 feet. The ball reaches this height after 2 seconds. This (2)______________ by the x value of the vertex. | 1. Maximum
Minimum
2. Is represented
Is shown
3. The x value
The y value | Vocabulary |
LESSON 3
(50 minutes)

Warm-Up Activity: The teacher will ask students if they have ever played soccer or kicked a ball. The teacher will also throw a small ball up in the air while asking: How does it behave? It goes up...it comes down. How high do you think it went? How long did it stay in the air? What is it’s position if it goes so high? The teacher will write out the word “Height” on the board. How can we measure that? The teacher will write out the word feet and show a ruler. How about the Time when it is at this height? The teacher will write out the word Time, and the class may discuss why we would measure in seconds rather than say, minutes or hours. What could we call the highest point that the ball reaches? The teacher will write out the word Maximum Height and draw a picture of an arcing ball on the board.

The teacher will ask: How could we measure the Maximum height of a ball that is kicked by a strong soccer player? Well let’s first see how such a ball might behave... The teacher will hand out the warm-up sheet depicting the soccer player kicking a ball (p. 48). Students will break up into pairs, discuss the clues given and then put the behavior events of the ball in correct order. While reviewing students’ answers, the teacher will ask questions based on the Discussion Questions sheet (p. 49). The teacher will amplify key words such as Independent and Dependent variables. (15 minutes)

Overhead Projector Graphing Activity: The teacher will hand out TI-84 Graphing calculators to the students and will use a TI-84 Presenter which allows a graph to be projected on the big screen, using the overhead projector. The teacher will hand out the “Graphing on the Graphing Calculator GUIDE” (p. 50) and introduce students to the actual function which depicts the height of the soccer ball over time. The teacher will model the steps to graphing a function on the TI-84 calculator while the students follow along with their own calculators. A key part of the lesson will be to convert the real-life variables into x and y variables! The teacher must emphasize this point and repeat often “x is the independent variable and time is the independent variable, so t can be replaced by x.” The teacher should stress these points on the board.

A visual depiction of the function will be presented on the screen. The class will discuss the graph using the “Discussion Guide” (P. 51) And all students will be given a hand-written graphic organizer (p. 52) which clearly illustrates how each point along the graph corresponds to a time and corresponding location of the ball’s height. In other words, a coordinate... an x-y relationship. Students should be brought back to recall the original variables as often as possible throughout the discussion/discovery activity. (20 minutes)

Small Group Activity: Students will be broken up into small groups and handed the Group Activity sheet (p. 53) in which they will calculate the maximum height of fireworks. Levels 1-3 will also be given accommodation sheets (p. 54-56) and The Functional Language chart (p. 46) to give them the language to help answer question #4. The teacher will monitor discussion and work while asking higher-order questions such as: “Would it make sense to have negative x values? Why or why not?” (No. time cannot be negative). “What was the original height of the fireworks? What do we call that starting point? Hint: The value of y when x is zero. (answer: y-intercept)” (15 minutes)
Warm-up Activity - How will the ball behave?

A soccer ball is kicked straight up in the air from 3 feet off the ground.

What happens next? Put a number next to the event in the correct order.

<table>
<thead>
<tr>
<th>Event #</th>
<th>Event</th>
<th>Time</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>_____</td>
<td>The soccer ball begins to fall</td>
<td>1.7 seconds</td>
<td>41.8 feet</td>
</tr>
<tr>
<td>_____</td>
<td>The soccer ball goes higher</td>
<td>.5 seconds</td>
<td>24 feet</td>
</tr>
<tr>
<td>_____</td>
<td>The soccer ball falls further</td>
<td>2.5 seconds</td>
<td>28 feet</td>
</tr>
<tr>
<td>_____</td>
<td>The soccer ball slows down</td>
<td>1 second</td>
<td>37 feet</td>
</tr>
<tr>
<td>_____</td>
<td>The soccer ball stops in mid-air</td>
<td>1.6 seconds</td>
<td>42 feet</td>
</tr>
<tr>
<td>_____</td>
<td>The ball hits the ground</td>
<td>3.2 seconds</td>
<td>0 feet</td>
</tr>
<tr>
<td>_______</td>
<td>The ball is at 3 feet</td>
<td>0 seconds</td>
<td>3 feet</td>
</tr>
</tbody>
</table>
Discussion Questions

1. What strategies did you use to number the events?

2. What happened when the ball “stopped in mid-air?”

3. Could you have correctly numbered the events based on the verbal clues alone?

4. What are the 2 important variables that give numerical clues about the ball’s behavior?

5. Could you have correctly numbered the events based on the ball’s height alone?

6. Could you have correctly numbered the events based on time alone?

7. Would you say that Height affects Time or that Time affects Height?

8. Which variable do you think is Independent?

9. Which variable is Dependent?
Graphing on the Graphing Calculator- GUIDE

(Overhead Projector)

The actual formula for the ball’s Height as a function of Time \( t \):

\[
\text{Height} = -16t^2 + 50t + 3
\]

\[
H = -16t^2 + 50t + 3
\]

Replace Independent Variable \( t \) with \( x \)

\[
H = -16x^2 + 50x + 3
\]

Replace Dependent Variable \( H \) with \( y \)

\[
y = -16x^2 + 50x + 3
\]

Is this a quadratic function?

Can you predict the shape of the graph?

Graph in the “\( y = \)” option of the calculator

For Input-Output \( (x, y) \) Values, Enter 2\textsuperscript{nd} Table
Discussion Questions

Re function: \( y = 16x^2 + 50x + 3 \)

Looking at the graph of the function on the overhead projector:

1. Does it look familiar?

2. What shape is it?

3. Point to the vertex.

4. What does the vertex represent in terms of the soccer ball that was kicked? ...Is it the height after 1 second? Is it when the ball lands? Is it the minimum height? Is it the maximum height?

5. Can you calculate the x and y values of the vertex?

6. The vertex is at (1.6, 42). What does this mean in terms of the soccer ball?

   Input (x) is 1.6   Output (y) is 42

7. Discuss these numbers in terms of the soccer ball and the Time and Height variables.
FUNCTION:

\[ H = -16t^2 + 50t + 3 \]
\[ y = -16x^2 + 50x + 3 \]

GRAPHIC ORGANIZER

WHAT DOES IT MEAN?

(1.37, 37) (After 1 second, ball is 37 feet high)

(1.6, 42) MAXIMUM

(2, 39) (After 2 seconds, ball is 39 feet high)

(3, 9) (After 3 seconds, 9 feet)

(3.1, 0) (Ball has landed)

Ball Height (feet) = y

STARTING POINT (0, 3) (0 seconds, 3 feet)

Time (seconds) = x
Group Activity – Assessment

Some fireworks are shot straight up in the air, and they explode when they reach their maximum height.

The Height over time can be Represented by the function:

\[ H = -16t^2 + 224t + 5 \]

Where \( H \) is Height (in feet) and \( t \) is time (in seconds)

1. Rewrite the function in terms of \( x \) and \( y \) variables.

2. Graph on the Graphing Calculator. Is the vertex a maximum or minimum?

3. What is the vertex? Calculate using the \( x \) of the vertex formula, and verify your answer with the Graphing Calculator.

4. Interpret your answer (vertex values) in terms of the fireworks, answering the following questions:

   a. What does the \( x \) value mean?
   b. What does the \( y \) value mean?
   c. Which is the independent variable? The Dependent?
   d. How high do the fireworks go?
   e. When do they explode?
Choose the correct answer, and write in the blank to complete the sentences.

- The $x$ variable represents the real variable. Height Time

- The $y$ variable represents the real variable. Height Time

- Time is the variable while Height is the Independent Dependent variable. Independent Dependent

- The fireworks reach a height of Maximum Minimum feet as shown in the $x$ value $y$ value of the vertex.

- The fireworks explode after seconds as shown in the $x$ value $y$ value of the vertex.
Choose the correct answer, and write in the blank to complete the sentences.

-The x variable ____________ the real
  Represents/ Reads

variable ____________________________.
  Height    Time

-The y variable ____________ the real
  Represents/ Erases

variable ____________________________.
  Height    Time

-Time is the ____________ variable while Height is the
  Independent    Dependent

__________________________ variable.
  Independent    Dependent

-The fireworks reach a __________________________ height of
  Maximum    Minimum

_____________________ feet as shown in the ____________ of the vertex.
  x value    y value

-The fireworks explode after ________ seconds as shown in the

_____________________ of the vertex.
  X value    y value
Complete the sentence starters using the word bank below.

The x variable ________________________________.

- The y variable ________________________________.

- Time is the ________________________________.
- Height is the ________________________________.

- The fireworks reach a ________________________________.

I know this because ________________________________.

- The fireworks explode after ________________________________.

I know this because ________________________________.

**Word-Phrase Bank**

<table>
<thead>
<tr>
<th>Represents</th>
<th>Height</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum height of...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum height of...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Independent Variable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

... it is shown in the x value of the vertex.
... it is shown in the y value of the vertex.
This lesson on quadratic functions should be highly relevant to students and their interests just as a mystery should have a strong ending which puts all of the pieces into place. The lesson brings back the vertex (maximum/minimum) concept as a tool ready to be used by the student to calculate the actual height that a real-life object will attain. Since readily-available forms of measurement may not be practical, students soon realize that quadratic functions have real power. Yet, they immediately face another language barrier: The real-life word problems give variables other than \( x \) and \( y \). The task in this lesson is to teach students how to recognize which variable man be replaced by \( x \) and which one may be replaced by \( y \). The reason for this is two-fold: Firstly, students have become accustomed to the language of \( x \) as in “input” and \( y \) as an “output,” so it makes comprehending the function and carrying out calculations much easier. Secondly, students will increasingly use technology (namely, the TI-84 Graphing Calculator) to graph functions and determine key values; however, the graphing calculator only used \( x \) and \( y \) variables.

The lesson begins with a real-life, relevant problem of a person kicking a soccer ball, and this leads students to question and predict the behavior of the ball. The simple verbal and numerical clues which are given to all students are also vital to ELLs and quite sufficient to allow their participation when used in conjunction with the Functional Language chart. Once students have discovered a parabolic relationship (up, then down) between time and the height of the ball, the lesson becomes extremely visual with the teacher graphing on the overhead projector and students following along on their own Graphing calculators.

With the help of a graphic organizer, ELLs may see how the (x,y) coordinates occur along the parabola and relate back to the original function. Discussion, repetition and amplification of key terms and concepts allow students to grasp exactly how the \( x \) and \( y \) variables relate back to the original word problems.

The final activity allows students to practice finding and interpreting key points with the language of the original problem. In a sense, all students are second-language learners, since they must move back and forth from the language of mathematical representations and symbols (variables) to the real-world verbal language of actual relationships between two values.
Checklists
FLA 518: Sheltered ELL Strategies Checklist

Write the page numbers and any other identifying features to identify those parts of your lessons that employ the following strategies.

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Original Lessons
Original Lesson Plan and Text on Following Pages
# Graphing Quadratic Functions

**Name of Lesson:** Discovering the Parabola

**Grade Level:** 10-11  
**Subject:** Algebra II  
**Prepared By:** R. Padro

## Overview & Purpose
Introduction to quadratic functions and the graphical representations of quadratic functions.

## Educational Standards
Common Core State Standards F.IF.7  
Analyze Functions using different representations.

## Objectives- SWBA
(Students will be able to...)

- Identify a quadratic function and the shape of its graph.

## Information Required Skills
- PEMDAS  
- Evaluating Algebraic Expressions  
- Plotting Coordinates

## Warm-up
Discuss how linear functions lead to input-output (values). How would output values change if variable was squared?

- Check for prior knowledge.  
- Amplify “input”-“output” relationship and process.

## Activity
Have students work in groups to produce table of values (ordered pairs) and plot points for parent function $y = x^2$

- Parent Function-“Simplest quadratic Function possible”
- Can add values to transform graph

## Vocabulary
- Quadratic Function  
- Parabola  
- Vertex  
- Axis of Symmetry  
- Maxima  
- Minima

## Teacher Guide
- Launch from linear functions  
- Function as a formula  
- Input... Evaluate for Output  
- x-y table

## Student Guide
- “Quadratic” –  
- “Quad” (4 tires)  
- “Quadrilateral” (4 sides)  
- Square, Rectangle  
- Leads to Area  
- Formula for Area of square: $A = s^2$

## Materials Needed
- Recall: x-y table leads to ordered pairs... coordinates.  
- Order of Operations  
- Quadrants of Coordinate Plane

## Other Resources
- Note-Taking Guide  
- Graph Paper  
- TI-84 Graphing Calculator

- Predict  
- Visualize  
- Connect to formula

- How does graph behave?  
- Is there only one line or curve?  
- Did you include negative inputs? What happens?

- Which words mean the same thing?  
- What other objects have axis of symmetry?
How to Graph a Quadratic Function (Standard Form)

Step 1: Verify that you indeed have a quadratic function...in standard form.

a. Remember, a function begins with $y =$

b. The first term should have a variable raised to the $2^{nd}$ power.
   Example: $y = 5x^2 + 7x - 10$ ...or the function may simply be: $y = x^2$

c. **Standard form** is a method of representing a quadratic function without parentheses.
   Example: $y = -9x^2 + 6x + 5$
   ....as opposed to $y = (x - 3)^2 + 1$ (not standard form!)

Standard form may also be known as the “ABC” form in which the coefficient of the first term is $a$, the coefficient of the second term is $b$, and the coefficient of the $3^{rd}$ term is $c$.

Standard Form Model: $y = ax^2 + bx + c$
Thus, for the example above: $a = -9 \quad b = 6 \quad and \quad c = 5$

Step 2: Identify the $a$, $b$ and $c$ values of the function and apply the $x$ of the vertex formula $-b \div (2a)$ in order to calculate the $x$ coordinate of the vertex of the parabola.

Example: if $y = 3x^2 + 12x + 1$, then $a=3 \quad b=12 \quad c=1$
So $-b \div (2a)$ would mean $-12 \div (2 \cdot 3)$ and this equals $-2$
Thus, the $x$ of the vertex is $-2$.  

**Unit:** Graphing Quadratic Functions  
**Name of Lesson:** Standard Form Functions  
**Grade Level:** 10-11  
**Subject:** Algebra II  
**Prepared By:** R. Padro  
**Educational Standards**  
Common Core State Standards F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.

<table>
<thead>
<tr>
<th>Teacher Guide</th>
<th>Student Guide</th>
<th>Materials Needed</th>
</tr>
</thead>
</table>
| **Objectives- SWBA**  
(Students will be able to…)  
Graph any Standard Form Quadratic Function by Applying the x-of vertex formula | • Coefficients are key  
• Why is the vertex so important?  
• Is it enough to calculate the x of the vertex? | “How To” Hand-Out  
5 Steps to Graphing Quadratic Functions (in Standard Form) |
| **Information/ Required Skills**  
Cognitive connection: Need “x” input before being able to calculate “y” output. | • Label a, b, c  
• Evaluate for x of vertex  
• Evaluate for y  
• Next: sides  
• Order of Operations  
• Do exponents come before multiplication?  
• double negative? | Order of Operations  
Match w template  
y = ax² + bx + c |
| **Warm-up**  
Game  
Students use coefficients to decode letters and words  
Create like functions | • Order of terms-x² comes first  
• What if no coefficient is shown?  
• Match w template  
y = ax² + bx + c | Other Resources  
Warm-up worksheet |
| **Activity**  
Teacher models procedure for graphing a quadratic function, then students graph a function, finding 5 pts including vertex. Use “How-To” guide. | • Check to see that students are labeling values  
Work in Group. Discuss and apply “How-To” Guide.  
Will the function match if graphed on the TI-84 calculator? | “How To” Hand-Out  
5 Steps to Graphing Quadratic Functions (in Standard Form) |
| **Vocabulary**  
Coefficient Plug in Evaluate Up-facing Down-facing | • Discuss turning point  
• Peak  
• Lowest point  
• y-intercept  
Recall how vertex relates to maximum or minimum | McDougal Littell Textbook and Note-Taking Guide |


Warm-up Activity- Lesson 2

Alphabet Code

<table>
<thead>
<tr>
<th>1-A</th>
<th>2-B</th>
<th>3-C</th>
<th>4-D</th>
<th>5-E</th>
<th>6-F</th>
<th>7-G</th>
<th>8-H</th>
<th>9-I</th>
<th>10-J</th>
<th>11-K</th>
<th>12-L</th>
<th>13-M</th>
</tr>
</thead>
</table>

List the coefficient values (a, b, c) of each function on the left and then rearrange according to the ABC order (in middle column) to match with a word on the right according to the alphabet code above.

<table>
<thead>
<tr>
<th>Function</th>
<th>a-b-c Order</th>
<th>Word</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 19x^2 + 14x + 21$</td>
<td>a-c-b</td>
<td></td>
</tr>
<tr>
<td>$a= 19 \quad b=14 \quad c = 21$</td>
<td>$a= S \quad b = N \quad c = U$</td>
<td>S-U-N</td>
</tr>
<tr>
<td>$y = 13x^2 + x + 8$</td>
<td>c-b-a</td>
<td>rod</td>
</tr>
<tr>
<td>$y = 14x^2 + 21x + 14$</td>
<td>a-b-c</td>
<td>bat</td>
</tr>
<tr>
<td>$y = 5x^2 + 12x + 4$</td>
<td>b-a-c</td>
<td>ham</td>
</tr>
<tr>
<td>$y = 4x^2 + 18x + 15$</td>
<td>b-c-a</td>
<td>nun</td>
</tr>
</tbody>
</table>
### Overview & Purpose
Connect graph representations with real-life input-output variables.

### Educational Standards
Common Core State Standards F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.

### Objectives- SWBA
(Students will be able to...)
Interpret a quadratic function graph and determine key values such as maximum or minimum outputs.

### Information Required Skills
Graph standard form quadratic function using coefficients, locate vertex (x,y)

### Warm-up
Discuss gravity, motion and how this “negative” force is modeled w quadratic function:
\[ H = -16t^2 + (\text{initial veloc}) + (\text{initial height}) \]

### Activity
Model vertical motion problem/function (p.252) on TI-84 overhead. Discuss. Students work in groups to complete Vertical Motion sheets.

### Teacher Guide
- Quadratic functions relate to real-life variables such as time and height
- If real-life function has a maximum, will parabola face up or down?
- Minimum?
- Why do you think the “negative” downward force (-16t^2) prevails over init. vel.: (80t)?

### Student Guide
- Samples of real-life quadratics?
- Which is dependent, independent variable in real situation?
- If input is time, does t variable behave same as x?
- \[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
- What does the vertex really mean?
- What is the output when the input is zero?
- The max function on TI-84 gives x & y values. How do I interpret in words?

### Materials Needed
- TI-84 calculator with overhead presenter.
- Vertical Motion Handout Sheets.
- "How to Graph” hand-out.
- Other Resources
  - If available: Student interest sheet related to motion scenarios (sports, kicking ball, jumping..)
  - TI-84 calculators
  - Graph Paper
  - Vertical Motion handout sheets w sample problems.
5.1 Graphing Quadratic Functions

**GOAL 1** Graphing a Quadratic Function

A quadratic function has the form $y = ax^2 + bx + c$ where $a \neq 0$. The graph of a quadratic function is U-shaped and is called a parabola.

For instance, the graphs of $y = x^2$ and $y = -x^2$ are shown at the right. The origin is the lowest point on the graph of $y = x^2$ and the highest point on the graph of $y = -x^2$. The lowest or highest point on the graph of a quadratic function is called the vertex.

The graphs of $y = x^2$ and $y = -x^2$ are symmetric about the $y$-axis, called the axis of symmetry. In general, the axis of symmetry for the graph of a quadratic function is the vertical line through the vertex.

**ACTIVITY**

Investigating Parabolas

1. Use a graphing calculator to graph each of these functions in the same viewing window: $y = \frac{1}{2}x^2$, $y = x^2$, $y = 2x^2$, and $y = 3x^2$.

2. Repeat Step 1 for these functions: $y = -\frac{1}{2}x^2$, $y = -x^2$, $y = -2x^2$, and $y = -3x^2$.

3. What are the vertex and axis of symmetry of the graph of $y = ax^2$?

4. Describe the effect of $a$ on the graph of $y = ax^2$.

In the activity you examined the graph of the simple quadratic function $y = ax^2$.

The graph of the more general function $y = ax^2 + bx + c$ is described below.

**CONCEPT SUMMARY**

The graph of $y = ax^2 + bx + c$ is a parabola with these characteristics:

- The parabola opens up if $a > 0$ and opens down if $a < 0$. The parabola is wider than the graph of $y = x^2$ if $|a| < 1$ and narrower than the graph of $y = x^2$ if $|a| > 1$.

- The $x$-coordinate of the vertex is $-\frac{b}{2a}$.

- The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.
Graph \( y = 2x^2 - 8x + 6 \).

**SOLUTION**

*Note* that the coefficients for this function are \( a = 2 \), \( b = -8 \), and \( c = 6 \). Since \( a > 0 \), the parabola opens up.

*Find* and plot the vertex. The \( x \)-coordinate is:

\[
x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2
\]

The \( y \)-coordinate is:

\[
y = 2(2)^2 - 8(2) + 6 = -2
\]

So, the vertex is \((2, -2)\).

*Draw* the axis of symmetry \( x = 2 \).

*Plot* two points on one side of the axis of symmetry, such as \((1, 0)\) and \((0, 6)\). Use symmetry to plot two more points, such as \((3, 0)\) and \((4, 6)\).

*Draw* a parabola through the plotted points.

The quadratic function \( y = ax^2 + bx + c \) is written in *standard form*. Two other useful forms for quadratic functions are given below.

### VERTEX AND INTERCEPT FORMS OF A QUADRATIC FUNCTION

<table>
<thead>
<tr>
<th>FORM OF QUADRATIC FUNCTION</th>
<th>CHARACTERISTICS OF GRAPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex form: ( y = a(x - h)^2 + k )</td>
<td>The vertex is ((h, k)).</td>
</tr>
<tr>
<td></td>
<td>The axis of symmetry is ( x = h ).</td>
</tr>
<tr>
<td>Intercept form: ( y = a(x - p)(x - q) )</td>
<td>The ( x )-intercepts are ( p ) and ( q ).</td>
</tr>
<tr>
<td></td>
<td>The axis of symmetry is halfway between ((p, 0)) and ((q, 0)).</td>
</tr>
</tbody>
</table>

For both forms, the graph opens up if \( a > 0 \) and opens down if \( a < 0 \).

**EXAMPLE 2**  

*Graphing a Quadratic Function in Vertex Form*

Graph \( y = -\frac{1}{2}(x + 3)^2 + 4 \).

**SOLUTION**

The function is in vertex form \( y = a(x - h)^2 + k \)

where \( a = -\frac{1}{2} \), \( h = -3 \), and \( k = 4 \). Since \( a < 0 \),

the parabola opens down. To graph the function, first plot

the vertex \((h, k) = (-3, 4)\). Draw the axis of symmetry

\( x = -3 \) and plot two points on one side of it, such as

\((-1, 2)\) and \((1, -4)\). Use symmetry to complete the graph.
**EXAMPLE 3**  **Graphing a Quadratic Function in Intercept Form**

Graph \( y = -(x + 2)(x - 4) \).

**SOLUTION**

The quadratic function is in intercept form
\( y = a(x - p)(x - q) \) where \( a = -1, p = -2, \) and \( q = 4 \). The \( x \)-intercepts occur at \((-2, 0)\) and \((4, 0)\).
The axis of symmetry lies halfway between these points, at \( x = 1 \). So, the \( x \)-coordinate of the vertex
is \( x = 1 \) and the \( y \)-coordinate of the vertex is:

\[
y = -(1 + 2)(1 - 4) = 9
\]

The graph of the function is shown.

...............

You can change quadratic functions from intercept form or vertex form to standard form by multiplying algebraic expressions. One method for multiplying expressions containing two terms is **FOIL**. Using this method, you add the products of the **First** terms, the **Outer** terms, the **Inner** terms, and the **Last** terms. Here is an example:

\[
(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15
\]

Methods for changing from standard form to intercept form or vertex form will be discussed in Lessons 5.2 and 5.5.

**EXAMPLE 4**  **Writing Quadratic Functions in Standard Form**

Write the quadratic function in standard form.

**a.** \( y = -(x + 4)(x - 9) \)

**b.** \( y = 3(x - 1)^2 + 8 \)

**SOLUTION**

**a.** \( y = -(x + 4)(x - 9) \)

\[
= -(x^2 - 9x + 4x - 36)
= -(x^2 - 5x - 36)
= -x^2 + 5x + 36
\]

**b.** \( y = 3(x - 1)^2 + 8 \)

\[
= 3(x - 1)(x - 1) + 8
= 3(x^2 - x - x + 1) + 8
= 3(x^2 - 2x + 1) + 8
= 3x^2 - 6x + 3 + 8
= 3x^2 - 6x + 11
\]
GOAL 2 USING QUADRATIC FUNCTIONS IN REAL LIFE

EXAMPLE 6 Using a Quadratic Model in Standard Form

Researchers conducted an experiment to determine temperatures at which people felt comfortable. The percent $y$ of test subjects who felt comfortable at temperature $x$ (in degrees Fahrenheit) can be modeled by:

$$y = -3.678x^2 + 527.3x - 18,807$$

What temperature made the greatest percent of test subjects comfortable? At that temperature, what percent felt comfortable? ▶ Source: Design with Climate

SOLUTION

Since $a = -3.678$ is negative, the graph of the quadratic function opens down and the function has a maximum value. The maximum value occurs at:

$$x = -\frac{b}{2a} = -\frac{527.3}{2(-3.678)} = 72$$

The corresponding value of $y$ is:

$$y = -3.678(72)^2 + 527.3(72) - 18,807 = 92$$

▶ The temperature that made the greatest percent of test subjects comfortable was about 72°F. At that temperature about 92% of the subjects felt comfortable.

EXAMPLE 6 Using a Quadratic Model in Vertex Form

CIVIL ENGINEERING The Golden Gate Bridge in San Francisco has two towers that rise 500 feet above the road and are connected by suspension cables as shown. Each cable forms a parabola with equation

$$y = \frac{1}{8960} (x - 2100)^2 + 8$$

where $x$ and $y$ are measured in feet. ▶ Source: Golden Gate Bridge, Highway and Transportation District

a. What is the distance $d$ between the two towers?

b. What is the height $h$ above the road of a cable at its lowest point?

SOLUTION

a. The vertex of the parabola is $(2100, 8)$, so a cable's lowest point is 2100 feet from the left tower shown above. Since the heights of the two towers are the same, the symmetry of the parabola implies that the vertex is also 2100 feet from the right tower. Therefore, the towers are $d = 2(2100) = 4200$ feet apart.

b. The height $h$ above the road of a cable at its lowest point is the $y$-coordinate of the vertex. Since the vertex is $(2100, 8)$, this height is $h = 8$ feet.
GUIDED PRACTICE

Vocabulary Check ✓
1. Complete this statement: The graph of a quadratic function is called a(n) ___.

Concept Check ✓
2. Does the graph of \( y = 3x^2 - x - 2 \) open up or down? Explain.

3. Is \( y = -2(x - 5)(x - 8) \) in standard form, vertex form, or intercept form?

Skill Check ✓

Graph the quadratic function. Label the vertex and axis of symmetry.

4. \( y = x^2 - 4x + 7 \)
5. \( y = 2(x + 1)^2 - 4 \)
6. \( y = -(x + 2)(x - 1) \)

7. \( y = -\frac{1}{3}x^2 - 2x - 3 \)
8. \( y = -\frac{3}{5}(x - 4)^2 + 6 \)
9. \( y = \frac{5}{2}x(x - 3) \)

Write the quadratic function in standard form.

10. \( y = (x + 1)(x + 2) \)
11. \( y = -2(x + 4)(x - 3) \)
12. \( y = 4(x - 1)^2 + 5 \)

13. \( y = -(x + 2)^2 - 7 \)
14. \( y = -\frac{1}{2}(x - 6)(x - 8) \)
15. \( y = \frac{2}{3}(x - 9)^2 - 4 \)

16. **SCIENCE CONNECTION** The equation given in Example 5 is based on temperature preferences of both male and female test subjects. Researchers also analyzed data for males and females separately and obtained the equations below.

   **Males:** \( y = -4.290x^2 + 612.6x - 21,773 \)
   **Females:** \( y = -6.224x^2 + 908.9x - 33,092 \)

   What was the most comfortable temperature for the males? for the females?

PRACTICE AND APPLICATIONS

MATCHING GRAPHS Match the quadratic function with its graph.

17. \( y = (x + 2)(x - 3) \)
18. \( y = -(x - 3)^2 + 2 \)
19. \( y = x^2 - 6x + 11 \)

![Graphs A, B, C](image)

GRAPHING WITH STANDARD FORM Graph the quadratic function. Label the vertex and axis of symmetry.

20. \( y = x^2 - 2x - 1 \)
21. \( y = 2x^2 - 12x + 19 \)
22. \( y = -x^2 + 4x - 2 \)

23. \( y = -3x^2 + 5 \)
24. \( y = \frac{1}{2}x^2 + 4x + 5 \)
25. \( y = -\frac{1}{6}x^2 - x - 3 \)

GRAPHING WITH VERTEX FORM Graph the quadratic function. Label the vertex and axis of symmetry.

26. \( y = (x - 1)^2 + 2 \)
27. \( y = -(x - 2)^2 - 1 \)
28. \( y = -2(x + 3)^2 - 4 \)

29. \( y = 3(x + 4)^2 + 5 \)
30. \( y = -\frac{1}{3}(x + 1)^2 + 3 \)
31. \( y = \frac{5}{4}(x - 3)^2 \)
Vertical Motion

- Vertical motion model is an equation \[ h = -16t^2 + vt + s \] such as the following which units are ft/s

- Where \( t = \) time \( v = \) initial velocity \( s = \) starting height

An athlete throws a shot put with an initial vertical velocity of 40 ft per second. Write an equation that models the height in feet of the shot put as a function of time \( t \) after it is thrown.

\[ h = -16t^2 + 40t + 6.5 \]

What is the maximum height the shot put reaches?

What is a vertex? How do we find it?

\[ t = \frac{-b}{2a} \]

Gives us the \( t \) value
Example continued

- \( t = \frac{-40}{2(-16)} = 1.25 \) seconds

This is the time the shot will reach the maximum height but we need to calculate the height.

\[
h = -16t^2 + 40t + 6.5
\]
\[
h = -16(1.25)^2 + 40(1.25) + 6.5
\]
\[
h = -25 + 50 + 6.5
\]
\[
h = 31.5 \text{ ft}
\]

The shot put reaches a height of 31.5 ft
You try!

Ex. 3) The polynomial $-16t^2 + 140t$ gives the height, in feet, reached in $t$ seconds. If the fireworks explode 4 seconds after launch, at what height do they explode? At what time will they reach a maximum height?

$$h = -16(4)^2 + 140(4)$$

$$h = 304 \text{ feet}$$

$$t = \frac{-b}{2a} = \frac{-140}{2(-16)} = 4.375 \text{ Seconds}$$
How would we find the maximum height the fireworks reached?

Correct! Substitute 4.375 seconds in for the time......

\[ h = -16(4.375)^2 + 140(4.375) \]

\[ h = 306.25 \text{ feet} \]

You can substitute any time you want to find out the height of the fireworks...

It is all about the math!

What would happen if they exploded after 5 seconds? 6 seconds? 9 seconds?