I. Introduction

1. The Manuscript

The present edition comprises twelve leaves from a papyrus codex (hereafter P.Math.), datable from its contents as well as on paleographic grounds to the fourth century CE, and containing texts of three kinds: mathematical problems, metrological texts, and model documents. Nine of the leaves (designated by the letters A through I, with E and F constituting an intact bifolium) were purchased by the anonymous owner of the “Archimedes Palimpsest” in 2001; the other three (designated M through O), around the same time, by the late Lloyd Cotsen, who donated them to the Cotsen Children’s Library, Princeton University (CTSN Q 87167). Previous to these acquisitions, all the leaves had been in the hands of the antiquities dealer Bruce Ferrini.

The first knowledge of P.Math. in the scholarly community dates from the early 1980s.1 In 1982 the Vatican Library received photographs (of very poor quality) of the bifolium and of leaves of a Greek papyrus codex of Exodus.2 In the same year, Ludwig Koenen received photographs (not traceable, but likely the same ones) of P.Math. and the Exodus codex as well as of the Coptic codex now known as Codex Tchacos (containing the Letter of Peter to Philip, the so-called First Revelation of James, the Gospel of Judas, and the Book of Allogenes), and in May, 1983 Koenen, David Noel Freedman, and Stephen Emmel were allowed a brief examination at a hotel in Geneva of these three codices and a fourth, a Coptic manuscript of the letters of Paul.3 The manuscripts were in extremely brittle condition, wrapped in newspaper and stored in three oblong boxes. The only available report of this examination, by Emmel, provides no details con-

1. For a more circumstantial account of the modern history of the group of four codices to which the mathematical codex belongs, see Krosney 2006. Rodolphe Kasser’s account of Codex Tchacos’s history in Kasser, Meyer, Wurst, and Gaudard 2007, 1–12 omits any mention of the other three codices.
2. Minutoli and Pintaudi 2011, 194–195 and plate 24 (photograph of E verso). We thank Rosario Pintaudi for providing us with a scan of a second photograph showing F recto.
cerning \textit{P.Math}. They were again seen briefly in March, 1984 in New York by one of the present editors (RSB) under conditions that prevented the recording of detailed observations.\textsuperscript{4}

Following Frieda Tchacos Nussberger’s purchase of the four codices in 2000, they were held for several months at the Beinecke Library, Yale University. A report by Robert Babcock, then the Beinecke’s Curator of Early Books and Manuscripts, supplies the earliest description of \textit{P.Math}.\textsuperscript{5}

At least 17 substantially complete leaves (including one complete bifolium) of a mathematical text, dealing with geometry (the measuring of triangles and liquid volume, among other things), and hundreds of small fragments. There are numerous drawings, some mathematical and related to the text, others appear to be purely decorative (crosses). Extensive searching indicates that the text cannot be identified with any known extant mathematical treatise from antiquity. The script suggests a fourth or fifth century date. No trace of the original binding is present, but the bifolium has sewing holes that show that the book was originally stab sewn. … The large leaves of this manuscript were placed in the front of the volume of Pauline letters…, with which they have no relationship. They do not belong to that binding, as the leaves are larger than that binding.

It thus appears, not only that Ferrini, who purchased \textit{P.Math.} from Tchacos Nussberger later that year, was responsible for its partition into separate lots of leaves, but that at least five leaves and many smaller fragments that still existed in 2000 are unaccounted for, perhaps sold by Ferrini to other collectors.\textsuperscript{6}

Unverifiable reports, relying on pseudonymous informants interviewed long after the claimed discovery, allege that the four codices were found in a tomb in the Jebel Qarara.\textsuperscript{7} Internal evidence in \textit{P.Math.} supports a general provenance in the Oxyrhynchite nome: one of the model documents, h3, is an agreement between (fictitious) residents of Oxyrhynchos and of the 6th pagus, and features of the model documents, including year numbers, are characteristic of documents from the Oxyrhynchite nome (see especially the commentary to text h3). That the codices were found together is credible, given the circumstances under which they were preserved when seen in the early 1980s; and the roughly contemporary Chester Beatty Codex AC.1390 (see below, section 3) even has mathematical problems in Greek and Christian literature (part of the Gospel of John) in Coptic in a single codex. The claim that the site was a tomb inspires less confidence but is not inherently unreasonable. Though the inclusion of a manu-

\textsuperscript{4} Krosney 2006, 148–149.
\textsuperscript{5} We thank Robert Babcock and Herb Krosney for providing us with this part of the report, in slightly variant versions.
\textsuperscript{6} Babcock (personal communication) recalls that all the substantial fragments were together at the front of the Pauline manuscript, and that there was a shoebox-size plastic container half filled with numerous small (“thumbnail or half-dollar size”) bits of all four codices.
\textsuperscript{7} Krosney 2006, 9–27. For scepticism see Nongbri 2018, 95–96.
script of more or less practical mathematics in a funerary ensemble might seem odd to present-day expectations, it can be paralleled with the findspots of other ancient mathematical papyri such as P.Cair. cat. 10758 (late 4th or 5th century, also known as the “Akhmim Mathematical Papyrus”), reportedly found in the necropolis of Panopolis, and the Middle Kingdom hieratic Moscow Mathematical Papyrus (Pushkin State Museum), reportedly from the necropolis of Dra’ Abu el-Naga. It is also not too far-fetched to bring into comparison recent discoveries of elite tombs in China, dating from the Warring States period through the Han Dynasty, in which were deposited collections of manuscripts of highly disparate genres, including mathematics.

2. The Leaves and Their Sequence

The larger portion of P.Math. in the collection of the owner of the Archimedes Palimpsest consists of nine pieces of papyrus, of which seven are single leaves (A–D and G–I), one intact bifolium (E+F), and one smaller fragment (X) which joins along the top of I and, for the purposes of this edition, is treated as part of I. The portion in the Cotsen Children’s Library consists of three single leaves (M–O). We estimate the original sheet dimensions as about 32 cm wide by approaching 30 cm. This would place the codex in Turner’s Group 6, with 16 × 28 as the approximate leaf dimensions but with several representatives between 29 and 30 cm in height. Peripheral damage to the leaves is most severe at the top: no significant upper margins survive, and many pages lack the top line of writing. Surviving binding holes show that the sheets were stab-bound.

Each recto or verso contains one or more texts, usually separated by decorative borders or by the diagrams that come at the end of many mathematical problems. We have assigned each text a reference name consisting of the letter of the relevant leaf, in lower case, followed by the ordinal number of the text starting from the top of the (known or presumed) recto side. In only two instances (a5 and d4) does a text continue from the verso of one leaf to the recto of another; only once (b3) does a text run over from recto to verso; and in each case the overflow is small, as if the writer had miscalculated the line spacing needed to fit the text on one page. On other pages, when a text ends well above the bottom, the remaining space is partly filled with drawings of ankh and other decorations. An index of the texts is provided at the end of this introduction.

In the following descriptions of the individual leaves, measurements are in centimeters, to a precision of half a centimeter except for the dimensions of the leaves themselves. The identifications of rectos and versos will be explained later in this section.

8. Baillet 1892, 2 (we reject Baillet’s dating of the codex to the 6th century); Struve and Turajeff 1930, vii.
9. For example Zhiangjiashan tomb no. 247, Jiangling County, Hubei, excavated in 1983–1984, apparently belonged to a low-level official who was buried with manuscripts containing medical, legal, philosophical, and mathematical texts; see Morgan and Chemla 2018, 153–154.
11. Such borders also appear, along with separators, in Bl. Add. MS 33569 (information from Todd Hickey).
A. 14.9 × 25.2. Margins: recto (↓) bottom 4.5, containing decorations, inner (2.0); verso (→) bottom 3.5, inner 1.5. Kollesis 5.5 from verso left edge. Remains of a binding hole 2.5 from bottom.

B. 14.4 × 26.2. Margins: recto (↓) bottom 2.5, inner 1.5; verso (→) bottom 4.0, inner 1.0. Top line of text on recto partially preserved. Kollesis 9.5 from verso left edge. Remains of a binding hole 2.5 from bottom, and binding hole 5 from bottom.

C. 15.2 × 27.1. Margins: recto (↓) bottom 2.0, inner 2.0; verso (→) bottom 1.0, inner 1.5. Apparent top line of text on both recto and verso partially preserved. Kollesis 13.5 from verso left edge.

D. 15.0 × 26.8. Margins: recto (↓) bottom 5.5, inner 1.5; verso (→) bottom 2.5, inner 0.5. No kollesis on verso.

E+F. 31.2 × 26.7. Margins: E recto (↓) bottom 1.5, inner 1.0 to binding fold, outer 0.5; E verso (→) bottom 1.0, no inner (writing runs across binding fold), outer 0.5; F recto (→) bottom 1.0, inner 1.0 to binding fold, outer 1.0; F verso (↓) bottom 1.0, inner 0.5 to binding fold, outer 1.0. Top lines of E recto and verso partially preserved. Binding holes on both leaves 2.0 and 4.5 from bottom. Kollesis 13.5 and 31 from E verso left edge.

G. 13.3 × 26.5. Margins: recto (→) bottom negligible, inner 1.5; verso (↓) bottom 3.5, containing decorations, inner 1.5. Top line of recto and apparent top line of verso partially preserved. Kollesis 12.5 from recto left edge. Binding holes 2.0 and 4.5 from bottom.

H. 11.3 × 25.7. Margins: recto (→) bottom 3.5; verso (↓) bottom 10.0, containing decorations. Top lines of both recto and verso partially preserved. Kollesis 1.5 from recto left edge.

I (incorporating X). 10.0 × 26.7. Margins: recto (→) bottom 1.0; verso (↓) bottom 3.0. No kollesis on recto.

M. 16.2 × 25.9. Margins: recto (→) bottom 8.0, inner 1.5; verso (↓) bottom 6.5, containing decorations, inner 1.0. Kollesis 8–9 from recto left edge. Binding holes 2.5 and 5 from bottom.

N. 14.8 × 25.1. Margins: recto (→) bottom 5.0, outer negligible; verso (↓) bottom 1.5, outer 1.5. Kollesis 2.5 from recto right edge.
We are not aware of any documentation of the order in which the extant leaves were arranged when they were discovered. Moreover, any page numbers that may once have existed were lost with the top margins.

As a working hypothesis, we suppose that the codex when complete comprised a single quire having the structure typical for fourth-century single-quire papyrus codices, namely that all sheets had their vertical-fiber sides outwards.\textsuperscript{12} We know that in 2000 there still existed at least 17 leaves, so that the quire would have had at least 9 sheets. The extant bifolium E+F was the middle sheet, since text lines on E\hspace{1pt}↓ (which is obviously E verso) run across the binding fold. Continuity of text from A→ to B\hspace{1pt}↓ and from D→ to E\hspace{1pt}↓ shows that these pairs of pages were consecutive, with their vertical-fiber sides as rectos. Binding holes (in some cases incompletely preserved as indentations along the broken edge of the leaf) establish or confirm that A\hspace{1pt}↓, B\hspace{1pt}↓, G→, M→, and O→ are rectos.

Continuity of horizontal fibers can be seen from C→ to G→, from G→ to B→, from B→ to M→, and from M→ to A→. Hence C+G and B+M constitute bifolia (in the case of C+G there is even a bit of common edge extant), and these bifolia and the one to which A belonged were originally adjacent pieces of the papyrus roll from which they were cut, making it very probable that B and C were consecutive leaves. Hence we have the following provisional sequence accounting for the eight leaves A–G and M:

First half of quire (↓ recto): \textit{m (≥ 0) leaves}, A, B, C, \textit{n (≥ 0) leaves}, D, E
Second half of quire (→ recto): F, \textit{n+1 leaves}, G, M, \textit{m+1 leaves}

Confirmation of this ordering of leaves A through E comes from consideration of their broken outlines (Fig. 1). A and B are both missing a roughly rectangular area at the top left of their vertical-fiber sides, in addition to other similarities of damage. C has a similar but slightly smaller loss in the same corner, with a less regular outline, and again in D there is a still smaller “bite” there, while E has what appears to be a small vestige of the same damage. Although the evidence considered above allows for one or more leaves between C and D, none of the extant leaves has an outline that would naturally fit into the progression in that place.

\textsuperscript{12.} Turner 1977, 64–65.
The bifolia E+F, C+G, and B+M preserve two kolleseis, separated respectively by 17.5, 14.5, and 13.5 cm, so that a rough average for the width of a kollema would be 15 cm. In Fig. 2, we reconstruct the portion of the original papyrus roll from which the five bifolia to which leaves A–G belonged were cut. Since from left to right the kolleseis show a gradual leftward trend against the pages, we can infer that a typical kollema was slightly less than half the breadth of a sheet, hence our estimate of 32 cm for the original sheet width.

Turning to the remaining four leaves, it is probable that M, N, and O were originally together, since this would explain why these particular leaves were separated by Ferrini from the rest of the codex; moreover, the last mathematical problem on N↓ and the first on O→ (known to be O verso) are of exactly the same special type, namely determination of the capacity of a vaulted granary, and solved in the same incorrect manner, suggesting that these were consecutive pages. Hence we have the sequence of leaves F (missing leaf) G M (possible missing leaves) N O for the second half of the quire. H and I seem to offer no evidence for identifying which side was recto, but from the comparatively extreme damage to these leaves we may conjecture that they were either towards the beginning of the quire before A (preferably in the order I H) or towards the end after O (preferably H I). An argument favoring placing them at the front is that this would bring together the three model documents (our texts a1, h3, and i3). Fig. 3 shows a possible reconstruction of the part of the original roll from which the four leaves came, which
would have been to the right of the part shown in Fig. 2. Another version would make N the conjugate of A and O the conjugate of either H or I.

On the other hand, we have not been able to show fiber continuity between either N or O and any of A, H, or I, such as we might expect if H and I were from the quire's front half. (Since only a narrow strip of papyrus survives left of the kollesis on N and O, however, it is difficult to rule out any continuities categorically.) With H and I at the front, seven sheets of the presumed minimum nine would be partly or entirely accounted for by the extant twelve leaves (Fig. 4); with them at the back, eight leaves would be accounted for (Fig. 5). In the light of the uncertainties about where H and I were, our edition presents them at the end, following the ten leaves whose order is better grounded.
3. Dating

The model contracts provide several elements pointing to a date after the middle of the fourth century. The first is the use of the solidus in the loan of money (a1). Solidi are essentially absent from the documentary papyri before the Constantian monetary reform of 351–353, and as the commentary to a1 notes, we do not in fact have any published loans of solidi dated before 364. The use of myriads (restored, but inescapably) in i3.6–7 points in the same direction; myriads become a commonplace descriptor of sums of money only after the Constantian reform.

For this reason, the editor’s date for the mathematical portions of Chester Beatty Codex AC. 1390, third–fourth century, with a preference for a date before 350, is almost certainly wrong. The Chester Beatty codex also has myriads of denarii. He also sought to use the supralinear curl as an indicator of amounts in the thousands as an indication of such a date (p. 35), but this is also a feature of our codex. It is not an indicator of sufficient chronological precision to bear the weight that Brashear put on it. This codex seems likely to date from roughly the same period as our codex, i.e., the third quarter of the fourth century.

The second indicator is the use of a year 10 in h3.4–5. As the commentary there points out, of the two plausible dates in the fourth century, the earlier (342/3) would not be consistent with the monetary indications of the model contracts. The later would be the Oxyrhynchite era year 41=10, or 364/5. Mu (40) is also the most likely reading of the first digit of the year number in i3.

It is, of course, possible for the writer to have used Oxyrhynchite era years referring to a year earlier than that in which he was writing, but it would be unlikely that he would refer to any year later than the year following that in which he was writing; any more ambitious futuristic reference is excluded by the monetary changes, which could not have been foreseen, and the use of the era year itself, which would not antedate Julian’s accession as Caesar in 355. Year 41=10 would in fact be only the second year in which the use of the regnal years of Constantius II and Julian can be properly seen as an era, following the death of Julian, so that neither year any longer refers to the reign of a living emperor. It seems on balance likely that 364/5 can be seen as the “dramatic date” of the composition of the model contracts, even if actual composition could have been slightly later.

4. Paleography

The first question that arises in looking at the writing of the codex is whether all of it was written by the same hand. It is, of course, easy to look simply at A recto and see that a1 and a2 reflect different styles of writing. The style of a1, like that of h3 and i3, is more rapid or “cursive” than that of a2 and the other mathematical and metrological portions of the codex. But neither does it look to us quite typical of the documentary hands found in contracts or official documents

13 Brashear, Funk, Robinson, and Smith 1990, 35. The date is given in DCLP, and thus Trismegistos, as 275–350.
of the fourth century written by professional bureaucrats or business agents. It is easy to get a synoptic view of these by looking at texts of a span of years in PapPal (pappl.info). Rather, the impression it conveys is of an attempt to write the kind of handwriting that we find in the mathematical texts, only in a more cursive fashion. That is a subjective judgment, to be sure, and the model contracts do at any rate resemble contemporary documentary handwriting far more than any literary hands. We can see no reason not to ascribe all three model contracts to the same writer.

It is not difficult to see considerable variation in the handwriting of the mathematical parts of the codex. Even inside a single page (one may take Or as an example), multiple forms of the same letter may be found (beta or epsilon, for example). But it is our impression that these variations do not exceed what may be found in the writing of a single individual, particularly over a period of time. For this reason, we take the codex to be a paleographical unit, on the view that an experienced writer could also have written in a similar but more cursive style appropriate to the different content of the model contracts.

Characterizing the hand of the mathematical sections is not simple. It is certainly not a professional book hand, although its general rightward slant is commonly found in literary texts as much as in documentary. The general character may be paralleled in some documentary texts; of those that appear on the PapPal gallery for this period there are similarities with P.Oxy. 14.1716 (333), P.Oxy. 66.4534 (335), P.Oxy. 63.4370 (354), P.Oxy. 48.3393 (365), and P.NYU 1.24 (373). But our writer, in the mathematical sections, is less professionally cursive than these. His script also bears some resemblance to teachers’ models of the sort described by Raffaella Cribiore. Her catalogue numbers 296 (Pl. XXXVI) and 389 (Pl. LXXIII) are the closest parallels of which she has illustrations. These are naturally without any indication of date; they have been assigned to the third to fourth centuries by editors, although Cribiore suggests that no. 389 may be later (a judgment with which we agree). It should also be said that the handwriting of the Chester Beatty Codex AC. 1390 mentioned earlier is, although more cursive than that of our codex, similar in many letter shapes.

5. Language

The Greek of the codex is far from flawless. The majority of the deviations from the norms of koine Greek are phonetic interchanges, mostly of a character well known from the documentary papyri of the Roman and Byzantine periods. These are listed below, with references in square brackets to the place in Gignac’s Grammar (Gignac 1976–1981) where they are treated. Pure omissions of letters, signaled in the text with angle brackets, are not included. Similarly, but mainly in the mathematical problems, we find a number of morphologically non-standard forms. There are also some more complicated errors, which provide useful clues to the process of the creation of the codex. Because the model contracts present a somewhat different picture

from the metrological and mathematical texts, the phonetic and morphological phenomena in
them are presented separately. The model contracts have an error rate of 25 percent (about 40
of 160 forms are nonstandard), almost all of them phonetic interchanges except for a few case
or number errors, all in I verso. These phonetic errors are virtually all well paralleled in docu-
mentary papyri and require no special comment. It is common for the same word to be spelled
both correctly and incorrectly on the same page.

But not all errors are plausibly explained as phonetic interchanges, as a look through the
list below will indicate. It is more difficult to see what process could have produced them. The
numerous other errors, including failure to use the correct case endings, are also readily paral-
leled in papyri written in this period. Such errors are generally explained as representing the
difficulty that an Egyptian, whose native language did not decline nouns and adjectives, would
have had in fully grasping the Greek system of accidence. Some other errors, entirely in the
mathematical problems, seem to reflect mistakes in understanding the substance of the prob-
lems. These need to be considered in the context of the substantive errors of procedure made by
the writer (or his source) in many of these problems.

**Phonology**

*Model contracts.*

**Vowels.**
\[
\begin{align*}
\varepsilon & \rightarrow \iota (G. 1.189–90): H ↓12, I ↓11 \\
\iota & \rightarrow \varepsilon (G. 1.190–1): H ↓4, H ↓12, H ↓15 \\
\varepsilon & \rightarrow \varepsilon (G. 1.257–9): H ↓5 \\
\omicron & \rightarrow \upsilon (G. 1.197–8): Ar3, Ar7, Ar10, Mr3 \\
\omicron & \rightarrow \iota (G. 1.272): H ↓12 \\
\eta & \rightarrow \upsilon (G. 1.264): H ↓12, H ↓15, I ↓9 \\
\alpha & \rightarrow \varepsilon (G. 1.192–3): Ar4, Ar8 (2x), H ↓10 \\
\varepsilon & \rightarrow \alpha (G. 1.193): Ar11, H ↓3, I ↓10, I ↓11 \\
\omicron & \rightarrow \omicron (G. 1.277): Ar11 (2x), H ↓7, H ↓9, H ↓14 \\
\upsilon & \rightarrow \omicron (G. 1.276–7): Ar3, Ar7, Ar8, Ar12 \\
\omicron & \rightarrow \alpha (G. 1.287–8): Ar3 \\
\omicron \upsilon & \rightarrow \upsilon (G. 1.208): H ↓3, H ↓6, I ↓8 \\
\omicron & \rightarrow \omicron (G. 1.212–13): I ↓8
\end{align*}
\]

**Consonants.**
\[
\begin{align*}
\delta & \rightarrow \tau (G. 1.81): Ar4 \\
\tau & \rightarrow \delta (G. 1.80–1): Ar4, H ↓7 \\
\kappa & \rightarrow \gamma (G. 1.77): Ar5 \\
\rho & \rightarrow \rho (G. 1.156): H ↓6 \\
\sigma & \rightarrow \sigma (G. 1.158–9): H ↓7
\end{align*}
\]
Introduction

Metrology and mathematics.

Vowels.

\[\alpha > o \text{ (G.1.286–7): Cr19}\]
\[\varepsilon > \eta \text{ (G. 1.244–5): Dr13, Dr19, Er13, Ov14}\]
\[\varepsilon > \iota \text{ (G.1.249–51): Mv1}\]
\[\varepsilon > o \text{ (G. 1.290–92): Fr2, Fr2, Fr9}\]
\[\varepsilon > \alpha \iota \text{ (G.1.193): Cr14, Mr13, Nv2}\]
\[\eta > \iota \text{ (G. 1.235–7): Av11, Br5, Cv1 (also wrong gender)}\]
\[\eta > \upsilon \text{ (G. 1.264): Bv11, Cr14, Cr19, Ev17, Mr11, Mr12, Or6, Ov7}\]
\[\eta > o \text{ (not phonetic): Mr17}\]
\[\eta > \alpha \iota \text{ (G 1.239–40): Br21}\]
\[\eta > o \iota \text{ (G.1.266): Br11, Fr23, Mr6, Mv11, Nv8}\]
\[\iota > \alpha \text{ (not phonetic): Dr15, Dr19}\]
\[\iota > \eta \text{ (G. 1.237–9): Bv12, Cv3, Ev10, Ev12, Ev16, Nv4}\]
\[\iota > \upsilon \text{ (G.1.269–71): Dr4, Ev21}\]
\[\iota > \alpha \iota \text{ (G. 1.190–1): Dv5, Er11, Fr2, Gr3, Gr4, Gr6, Gr13, Gr19, I \rightarrow 8, Nv15, Ov7}\]
\[\iota > o \iota \text{ (G. 1.272): I \downarrow 13}\]
\[o > \omega \text{ (G. 1.277): Cr1, Cr20, Cv15, Cv16, Dr4, Dr18, Dv3, Dv24 (2x), Er5, Er6, Ev14 (2x), Fr3, Fv3, Fv18, Fv21, Gr8, Mr10 (3x), Mr16–17, Mv4, Mv6, Mv7, Mv9, Mv10, Mv11–12, Nv1, Nv2, Or24, Ov4}\]
\[o > o \omega \text{ (G. 1.212–13): Dv19, Er2, Er11, Ev19, Ev22, Gr24}\]
\[\upsilon > \alpha \text{ (not phonetic): Cv13}\]
\[\upsilon > o \text{ (G.1.273–4): Dv3, Nr5}\]
\[\upsilon > \eta \text{ (G. 1.262–3): Ar17, Av2, Av4, Av6, Av13, Bv10 (2x), Bv11, Bv16, Cv15, Ev5, Ev3, Ev4, Ev17, Gr2, Gr3, Gr5, Gr11, Mr13, Mv4, Nv13}\]
\[\upsilon > o \text{ (G. 1.293): Br10}\]
\[\upsilon > o \text{ (G.1.294): I \rightarrow 2}\]
\[\upsilon > \alpha \iota \text{ (G.1.272): Ev7, Ev16, Gr15, Gr16, Gr18, Gr19, Mr12}\]
\[\upsilon > o \iota \text{ (G. 1.198–9.): Cr17, Cr10, Dv21, Er6, Er14, Fr1, Fv14, Gr6, Gr16, I \rightarrow 20, Mr15 (2x), Mr18, Mv7, Mv8, Mv10, Mv11, Mv12, Or11, Or24}\]
\[\upsilon > o \omega \text{ (G. 1.215): Gr6 (by correction)}\]
\[\omega > o \text{ (G. 1.276–7): Br11, Cv6, Cv14, Dr18 (with \omega \text{ then also written), Er12, Ev3, Fr16, Fv21, Gr1, Gr7, I \rightarrow 12, Nv2, Or3, Or16, Ov17}\]
\[\omega > o \omega \text{ (G. 1.209–11): Ov3}\]
\[\alpha > \varepsilon \text{ (G. 1.192–3): Av14, Av17, Dr18, Er6, Ev8, Ev9, Ev10, Fr1, Fr19, Gr3, Gr9, Gr12, Gr14 (2x), Gr15, H \rightarrow 2, H \rightarrow 4, H \rightarrow 6, H \rightarrow 8, H \rightarrow 10, H \rightarrow 11, H \rightarrow 13, H \rightarrow 14, H \rightarrow 18, Mr5, Mv7, Mv8, Mv9, Mv11, Mv12, Ov17}\]
\[\alpha > \eta \text{ (G. 1.244–5): Gr10, Gr11, Gr16 (2x), Gr18, Gr19, H \rightarrow 15, I \rightarrow 1}\]
\[\alpha > \iota \text{ (G. 1.249): Gr10}\]
\( \alpha \omega > \alpha \) (G. 1.226–8): Er14
\( \alpha \omega > \omega \omega \) (cf. G. 1.230–31): Mr9
\( \varepsilon \eta \ > \varepsilon \) (G. 1.257–9): Fr7, Fr8, Fr9, Or3
\( \varepsilon \eta \ > \eta \) (G. 1.240–42): Er18, Mr12
\( \varepsilon \eta \ > \iota \) (G. 1.189–90): Av7, Bv14, Cr10, Cr15, Cr16 (2x), Cr17 (2x), Cr20, Dv4, Dv5, Dv9, Dv14, Dv15, Er17, Ev17, Fr1, Fr7, Fr10, Fv11, Fv12, Fv15, Fv17, Fv20, Gr6, Gr9, Gr14, Gr16, Gr24, Gr24–25, Mr12, Nv5, Nv12, Nv18, Or6, Ov7
\( \varepsilon \eta \ > \omicron \) (not phonetic): Gv17 (?)
\( \omicron \iota \ > \varepsilon \) (G. 1.274–5): Cv1, Dr3, Mv2
\( \omicron \iota \ > \eta \) (G. 1.265–6): Av5, Av13, Bv14, Gv9, Ov14 (?)
\( \omicron \iota \ > \iota \) (G. 1.272): Dr12
\( \omicron \iota \ > \omicron \) (G. 1.200–01): Ev18, Gv5
\( \omicron \iota \ > \omicron \omicron \) (G. 1.197–8): Cr10, Gv2, Mv5
\( \omicron \omicron \ > \omicron \omicron \) (G. 1.215: rare): Bv15
\( \omicron \omicron \ > \omicron \omicron \omicron \) (G. 1.208): I →3
\( \omicron \omicron \ > \omicron \) (G. 1.211–12): Ar20, Br14, Bv14, Dr18, I →8 (2x), I →9
\( \omicron \omicron \ > \omicron \) (G. 1.208): Ar15, Av15, Fr14

**Consonants.**
\( \gamma \sigma \) (G. 1.78): Dv3, Ev23, Gv12, Mr13 (2x), Or22
\( \delta \sigma \) (G. 1.81): Av1, Av4, Av6, Br12, Bv9 (2x), Bv14, Cr15, Cr19, Dr18 (2x), Er12, Ev9, Ev11, Fr2, Fr3, Fr8 (2x), Fr14, Gr12, H →11, H →15, Mr6, Mr11, Mr15, Mv8, Nv10
\( \zeta \sigma \) (G. 1.123): Nv7 (?)
\( \zeta \sigma \delta \) : Bv8
\( \kappa \sigma \) (G. 1.77): Dr18, Mv2
\( \sigma \delta \) : Cr19, Nv2, Nv13, Nv15, Nv17
\( \tau \delta \) (G. 1.80–1): Av4, Av19 (2x), Cr15, Dr13, Dr19, Ev9, I →5, I ↓13, Nr15
\( \gamma \gamma \sigma \) (G. 1.116): Gr5
\( \gamma \gamma \sigma \kappa \) (G. 1.116): Dv19
\( \gamma \gamma \sigma \nu \kappa \) (G. 170–71): Er2, Er11, Gr22, Gv10
\( \gamma \kappa \sigma \) (G. 116): I →8, I →9
\( \gamma \kappa \sigma \nu \kappa \) (G. 1.168–9): Gr9, I →8
\( \mu \beta \sigma \nu \beta \) (G. 1.168–9): Av19, Av20, Br8, Bv4, Cv6, Dv9, Dv11, Ev7, Ev19, Ev21, Ev22, Ev24, Gv6, Gv8, Mr7, Or19
\( \mu \kappa \sigma \nu \pi \) (G. 1.168–9): Mv12
\( \rho \rho \sigma \rho \) (G. 1.156): Ov4
\( \sigma \pi \sigma \psi \) (G. 1.154 on inversion; no examples of this type): Gr2, Gr11, H →1, H →4, H →8, H →11, H →12, H →13, H →18
σσ > σ (G. 1.158–9): I ↓13, Nv9
φ > π (G. 1.93): Er13, I ↓18
Omission of nu before consonant (G1.116–7): Br18, Dr4
Omission of final nu (G1.111–12): Dr6
Omission of final sigma (G. 1.124–6): Ev23, Fv7, I ↓18
Omission of word termination: Fr11, Or22
Insertion of nasal before stop (G. 1.118: normal in future of λαμβάνω and compounds): Mv8, Mv9, Mv11, Mv12, Ov15

Mistaken word: Fr11 (2x), Fr12

**Morphology (all texts)**

Masculine formation in place of neuter: Gr6
Masculine formation in place of feminine: Or2–3
Incorrect declension: Fv3
α instead of η as dative participial ending: Dv15
α instead of η as vowel in definite article: Or24, Or25
Formation on r-stem, ending in –ας: I →11
Third-declension accusative in –αν (G. 2.45–46): Cr19, Ov6
Uncontracted form (βορέας for βορρᾶς): Ov4
Thematic forms of –μι verb (προστίθομεν, συντίθω, συντίθομεν): passim (see index)

By-formation of present from aorist stem (έλω compound ύφελ- from root εἷλον, aorist of αἰρέω):

Av13 ῥή ἀπὸ τῆς κηνῆς βάσεως ἐφήλω τὴν κορη-
Av14 φίν, ἀπὸ τῶν ῥ γεφέλομεν γ. λοιπὲ κ.δ.
Dr6 γί(νεται) ῥη. ἀπὸ τῶν [ἴ]ις ὑφέλομαι θ. λοιπαί χ.
Dr17 γί(νεται) ῥη. ἀπὸ τῶν ἄ τοις ἱκάρομαι [ὁ] ἱκάρω τ.δ. λοίπας [χοβ.]
Nv5 ἐρ' ἐστια. γί(νεται) ἁγος. τούτων ἐφέλω [ὁ] ἱκάρω τ.δ. λοίπας [χοβ.]
Er5 τ. γί(νεται) σνς. τούτων ἱκάρομαι το τέταρτον;
Er6 ζ. ἐπ᾽ [π]ὶ τῶν σνς ὑφέλομεν ζ. λοιπὲ [ρ' ρ' οβ' .]
Fv14 γί(νεται) ζ. ἀπὸ τῶν σπὸ ὑφέλομεν ζ. λοίπας σκλ. ὄν
Mv4 [ὁ. ἐπὶ] τόν ἡ. γί(νεται) ῥκ. ἀπὸ τῶν τ ἱκάρω σ.μεν [ῥκ.]
Or11 παρὰ τόν ὅ. γί(νεται) ις. ἀπὸ τῶν ις ὑφέλομεν ις. [γί(νεται) ιβ.]
Or24 τόν μίαν. γί(νεται) πα. ὄν πλευρὰ θ. ὑφέλομεν [ἐν]
Gv5 [ἀπὸ τὸ] ν ἤ ἱκάρομαι σνς. λοιπαὶ ρμ. ὄν πλευρὰ ιβ.

For the formation of thematic forms from τίθημι and its compounds, see Gignac 1981, 380–381 and Mandilaras 1973, 86. Active forms are much rarer than middle, and in documentary papyri participles are more common than finite forms. Forms of ύφαινω
Mathematics, Metrology, and Model Contracts

occur in Chester Beatty AC.1390, but they are abbreviated after ὑφελ, leaving the mood uncertain. In this respect it is very similar to the Akhmim codex, where again the form is abbreviated. Brashear, like Baillet, resolved these as aorist imperatives, which is plausible in light of the use of imperatives in these manuscripts, a respect in which they differ from P.Math., where the first person is consistently used (sometimes singular, sometimes plural). In the Akhmim codex, in some places there is no indication of whether indicative or imperative would be more correct. In PSI 3.186v, as reedited in Shelton 1981b, υφελω occurs. Shelton took this as a future and accented accordingly. But it is in light of P.Math. more likely that this is in fact a present indicative. The absence of such presents outside mathematical texts may lead one to see these forms as instances of the confusion of tenses (on which see Mandilaras 1973, 57) rather than a true birth of an alternative present.

Syntax

Model contracts.
Accusative for nominative: Ev19, Ev22
Genitive for dative: H ↓11
Accusative for dative: H ↓10
Plural for singular: H ↓10
Singular for plural: H ↓9

Metrological and mathematical texts.
Nominative for genitive: Cv4, Er4, Fv12, Mr7
Nominative for accusative: Br7, Bv3, Dv14, Fv2
Nominative for dative: Dv15
Genitive for nominative: Av12
Genitive for accusative: Br6, Fv19, Fv22, Ov6
Dative for nominative: Gr15, Nv11, Or20
Accusative for nominative: Cr17, Dv12–13
Accusative for genitive: Mv5 (2x), Nv19, Ov13
Accusative for dative: Dr15, Dr19
Singular for plural: Fr7, Gr11, H →3, H →4, H →6, I →7, I →18
Plural for singular: Gr9, H →7, I →10, Ov13
Participle for finite verb: Mr4
Omission of augment: Cr16
Second person singular for third person plural: Cr16
Neuter for masculine: Mr15
Masculine for neuter: Dv22
Three of the texts in the codex, of which only one is fully preserved, fall into this category, a1, h3, and i3. What do we mean with the term “model contract”? Model contracts are to be distinguished from actually executed contracts, which should have contained official dating formulas (by the consuls, at the period to which we assign our codex) and the subscription of the acknowledging party or parties and of their hypographeus in the case of an illiterate party. Actual contracts also stand on separate sheets of papyrus and are not simply portions of pages of a codex. Because we have the endings of all three of the model contracts, we can be certain that they lack the concluding elements needed in actual contracts.

Somewhat more complex is the distinction between a model contract and a draft of a contract. An example of a draft contract, in this case a rider to a contract, has recently been published as PBagnall 47, with a detailed commentary by David Ratzan. Like our model contracts, this lacks the elements of execution, including the subscriptions, that would be needed in actual contracts, but unlike them it does not attempt to include the names of parties at the start, starting baldly with χαίρειν. It is thus pure draft legal language. In contrast, we have the names of the parties, even if imperfectly preserved, in h3, and it is clear that they also stood in a1.

The question of distinguishing actual from model contracts is not always simple, however, particularly with the presence of elements like the names of parties. This issue was confronted by Rosario Pintaudi and P. J. Sijpesteijn in the introduction to the volume of wooden and wax tablets in various collections (T.Varie, p. 8): “Alcuni dubbi si hanno a proposito di alcune tavole della Biblioteca Apostolica Vaticana (1, 2, 3, 8, 10, 11): si hanno testi di tipo documentario (contratti), scritti da scribi con una certa esperienza grafica, per quanto l’ortografia in molti casi risulti orribile; le parti coinvolte sono menzionate con l’indicazione delle località di provenienza, che spesso risultano attestate qui per la prima volta. Tutti elementi questi, che ci porterebbero a considerare i testi come veri e propri atti giuridicamen
ti validi. A questo si oppone il fatto che nessuno di questi contratti risulta però completo, anche quando lo spazio a disposizione nella tavoletta ne avrebbe permesso la conclusione regolare (formule finali, sottoscrizioni e sim.), senza presupporre il seguito su altre tavolette ora perseute! Non si può quindi escludere che anche quei testi più manifestamente documentari siano di provenienza scolastica, cioè scritti dal maestro perché gli studenti li potessero copiare ed acquisire confidenza con formule e situazioni quanto più vicine alla realtà.”

When those words were written thirty years ago, the corpus of comparable texts was small (see PRain.Unterricht, where only no. 108 is really similar), but the analysis remains sound. With the benefit of the material provided by P.Math., we can add that the combination of model contracts with texts of other types, including mathematical, in a codex appears as a diagnostic element for the scholastic use of the texts and thus their model character. This is particularly striking in the case of the tablets if one takes into account the probability that the tablets in that volume and others cited by the editors on pp. 7–8 belong to a single find, a large part of which was school texts of one sort or another, many of them mathematical, including tables and
problems. Because that tablet find belongs to a much later period than \textit{P.Math.} (the sixth and seventh centuries), the publication of \textit{P.Math.} adds importantly to our knowledge of the use of model contracts in the school curriculum.\textsuperscript{15} It is significant to note that only h3 concerns an area of land, a type of business transaction in which one might suppose a direct relationship with the geometric knowledge that the problems of \textit{P.Math.} tried to convey.

The corpus of published model contracts will be enlarged in the near future by the publication of several other texts. We cannot incorporate their information fully into the present edition, but, thanks to their editors, we can briefly mention them and some of their chief characteristics. Most extensive is BL Add. MS 33369, a codex of ten wooden tablets from the Panopolite nome and to be dated to the fifth century. Each leaf is broken vertically, with a loss of about a quarter to a third of the width, but a substantial amount survives.\textsuperscript{16} It has points of similarity with our codex, but also differences; see the commentary to a1 below for more detail.

Also on tablets, but apparently separate items rather than forming part of a codex, is a series of school texts in the Yale collection, which are now in press and will appear as \textit{P.Yale} 4.184–191, edited by Ruth Duttenhöfer. These date to the seventh century and include a model receipt for \textit{annona civica}, loans for repayment in kind, Coptic letter formula, and Greek contract formulas, along with multiplication and fraction tables.

7. The Metrological Texts

\textit{P.Math.} contains five metrological texts: e3, g1, h1, i1, and m1. Like the model contracts, they all start at the top of a page, on the recto side except for e3. Texts e3 and m1 occupy an entire page; the rest are followed by mathematical problems. Perhaps this reflects how lessons were organized, with mathematical exercises taking up any time remaining after other material.

The first four texts all have extensive word-for-word parallels in other papyri, and parts of m1 have less exact parallels in e3:

\begin{itemize}
\item e3: \textit{P.Oxy.} 4.669 (late 3rd or early 4th century).
\item g1: \textit{P.Oxy.} 4.669.
\item h1: \textit{P.Lond.} 5.1718 (late 6th century), \textit{P.Ryl.} 2.64 (4th or 5th century).
\item i1: \textit{P.Oxy.} 1.9 + 49.3456 (3rd or 4th century),\textsuperscript{17} \textit{P.Oxy.} 49.3457 (1st or 2nd century), 3458 (3rd century), 3459 (3rd century), and 3460 (2nd or 3rd century).
\end{itemize}

\textsuperscript{15} We are indebted to Jean-Luc Fournet for the information that model contracts also occur in a codex of tablets that he is to publish together with Todd Hickey.

\textsuperscript{16} The catalogue record may be found at https://urldefense.proofpoint.com/v2/url?u=https-3A__tinyurl.com_yxrvt4ug&d=DwIfaQ&c=slrrB7dE8n7gBjbeOog-IQ&r=a1GzvfjAQ4FOT67EmNqRIAkSU OmrC2-_N3pEK_8DkHo&m=TLnpq-YcNRech5Y7m-YFuehiSwy46bulYkF8iaLlAc&=EixQi4BL2WDKr- _60UolOIC74WrB-JVwnVMHP-De&=*, derived from M. Richard, \textit{Inventaire des manuscrits grecs du British Museum I, Fonds Sloane, Additional, Egerton, Cottonian et Stowe}, Paris 1952, p. 56. It is to be published by Jean-Luc Fournet, Todd Hickey, Yasmine Amory, and Valérie Schram.

\textsuperscript{17} We are indebted to a referee for the information that a full edition of these tables will appear in the forthcoming Festschrift for G. Messeri, arguing for a third-century date.
Thus the teaching of systems of units involved the copying—and memorization?—of a few standard texts that circulated in some cases for several centuries, with some allowance for variation through reordering, omission, or insertion of material.

The first part of g1 (G recto 1–6) represents an initial approach to learning units, through lists of their names in order of magnitude, in this instance increasing and beginning with units of length followed by units of area. The main object of the metrological texts, however, was to teach how the various units were quantitatively related, so that one could perform unit conversions such as we encounter in many of the mathematical problems.

The relations expressed between units fall into two kinds, depending on whether the units in question measure the same kind or different kinds of magnitude. A relation between two units of the same kind, say two units of length, is expressed as how many of one unit are contained in one of the other. In the metrological texts in P.Math., relations between units of different kinds are limited to definitions of units of area or volume in terms of units of length applied to the two or three dimensions; we do not find, for example, a definition of a unit of weight in terms of a specified volume of some material. A unit appropriate for a particular kind of object or commodity may, however, be related to a less particularized unit of the same general kind, as in e3 (E verso 16–18) and m1 (M recto 12–13) where equivalents are given of the solid cubit, a general unit of volume, in volume units suitable for grain (artabas) and for liquids (metretai).

The texts make much use of syllogism-like statements on the pattern, “Unit A contains \(l\) unit Bs, and unit B contains \(m\) unit Cs, so that unit A contains \(n\) unit Cs” (where \(n = l \times m\)). This formula may have been preferred for ease of memorization, since it connects three or more units of the same kind and provides an arithmetical relation that can be verified. Text i1 consists entirely of quasi-syllogistic statements, which do not appear to follow a larger-scale plan, since the kinds of unit in each statement have no obvious connection with the ones that precede or follow.

Text h1, which does not employ quasi-syllogistic statements, is the most systematically organized. It provides relations among eleven units of length, devoting to each, in order of diminishing magnitude, a sentence listing the equivalents in each of the smaller units, again in order of diminishing magnitude, so that by the end a direct relation has been given for each possible pair of units. (For the extraneous last sentence of the text, H \(\rightarrow\)16–19, see the commentary.)

8. Relations in the Metrological Texts

This section summarizes the relations between units in the five metrological texts e3, g1, h1, i1, and m1. In the tables, the quantities in the cells represent the number of the unit named at the top of the column contained in the unit at the left end of the row. Square brackets and angle brackets have the same meaning as in the transcriptions, while italics indicate quantities that we have included as supplements for the user’s convenience. When a table merges data from more than one text, quantities or digits of quantities that are preserved in at least one text are not bracketed. We have not incorporated data from other papyri. Since a large part of the met-
logical system is reducible to defined relations to the cubit, orders of magnitude in terms of modern units can be estimated from the approximate equivalent of one Egyptian cubit to 0.53 meters; see also Bagnall 2009b.

\[
\text{length (} g_1, h_1, i_1 \text{)}.
\]

<table>
<thead>
<tr>
<th>surveyor's schoinion</th>
<th>hamma</th>
<th>reed</th>
<th>xylon</th>
<th>pace</th>
<th>cubit</th>
<th>foot</th>
<th>spithame</th>
<th>lichas</th>
<th>palm</th>
<th>finger</th>
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<tbody>
<tr>
<td></td>
<td>[8]</td>
<td>[16]</td>
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<td>144</td>
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<td>12</td>
<td>[18]</td>
<td>24</td>
<td>36</td>
<td>72</td>
<td>[288]</td>
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</tr>
<tr>
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<td>3</td>
<td>6</td>
<td>[9]</td>
<td>12</td>
<td>18</td>
<td>36</td>
<td>[144]</td>
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<td>4½</td>
<td>6</td>
<td>9</td>
<td>18</td>
<td>[72]</td>
<td></td>
<td></td>
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<td>pace</td>
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<td>3</td>
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</tr>
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<td>4</td>
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<td></td>
<td></td>
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\[
\text{length (} e_3 \text{)}
\]

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\[
\text{length (} e_3 \text{)}
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<td>18</td>
<td>[7]2</td>
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</tr>
<tr>
<td>private xylon</td>
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<td>64</td>
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### Introduction

#### Length (g1)

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<tr>
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<th>Ptolemaic (Egyptian) foot</th>
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<th>builder's foot</th>
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<td>13½</td>
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#### Length (g1)

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<th>nilometric cubit</th>
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<th>?</th>
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#### Length (m1)

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<th>stade</th>
<th>surveyor's schoinon</th>
<th>cubit</th>
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#### Area (m1)

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<td>laura</td>
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#### Area (e3)

building-site cubit = [100] area cubits
Mathematics, Metrology, and Model Contracts

**area \( (e^3, m^1) \)**

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</thead>
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<tr>
<td>country aroura</td>
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<td>9216</td>
</tr>
<tr>
<td>urban aroura (aroura for building sites)</td>
<td>50</td>
<td>10,000</td>
</tr>
<tr>
<td>(country) bikos</td>
<td>192 (text: 992)</td>
<td>200</td>
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</table>

**volume \( (e^3, m^1) \)**

1 (public) naubion = 1 xylon x 1 xylon x 1 xylon = 27 solid cubits

1 private naubion = \(18 + \frac{1}{2} + \frac{1}{3} + \frac{1}{9} + \frac{1}{54}\) (i.e. \(\frac{18}{27}\)) solid cubits
(i.e. 1 private naubion = 1 private xylon x 1 private xylon x 1 private xylon)

1 solid cubit = 3 + \(\frac{1}{4} + \frac{1}{8}\) artabas (i.e. \(\frac{3}{8}\)) = 3 metretai
(i.e. 1 artaba = 1 foot x 1 foot x 1 foot)

**volume, liquid capacity \( (i^1) \)**

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<tr>
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<th>chous</th>
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<tr>
<td>metretes</td>
<td>12</td>
<td>144</td>
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<td>chous</td>
<td>[12]</td>
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**volume? \( (h^1) \)**

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<th>hamma</th>
<th>reed</th>
<th>xylon</th>
<th>pace</th>
<th>cubit</th>
<th>foot</th>
<th>spithame</th>
<th>lichas</th>
<th>palm</th>
<th>finger</th>
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(Note: this seems to be a non-standard scale of units, combining the familiar relation 1 naubion = 27 solid cubits with the relations of linear cubits to other length units.)

**weight \( (i^1) \)**

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<th>thermos</th>
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<td>tetarte</td>
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<td>thermos</td>
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9. The Mathematical Problems

Most Greek mathematical papyri contain problems, arithmetical tables such as multiplication tables and tables of fractions, metrological texts, or a combination of these genres. The normal format for presenting problems is as a series of statements and solutions, with no intervening commentary (see Appendix A). Successive problems are often thematically related (e.g., MPER NS 1.1 consists of problems concerning volumes of three-dimensional figures such as parallelepipeds and pyramids, and PChic. 3 of problems concerning areas of polygonal fields). 44 problems are preserved in P.Math., most of them complete or with enough surviving so that we can recover what is to be solved, the method of solution, and the results. This is not quite the largest known collection of problems in a Greek papyrus—P.Cair. cat. 10758 has 50—but it is the most varied collection, and especially rich in geometrical problems. There are occasional instances of thematic grouping (e.g., d4, e1, e2); but for the most part each problem seems to have little relation to the preceding and following problems, and in the leaves A–G, whose order is well established, we see little sign of a progression from more elementary to more advanced problems.

In common with the problem texts in other Greek and Demotic mathematical papyri as well as in those from many ancient and medieval societies, most problems of P.Math. are about how to find unknown quantities in scenarios defined at the outset in terms of given quantities.18 The most common pattern begins with the specification, as a nominal phrase, of an object un-

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18. Beyond the broad characterization of the problem texts delineated above, the Greek and Demotic problem texts have surprisingly little in common, either with respect to their scenarios or the algorithms employed to solve them, the exceptions including a few that were practically universal to the genre across cultures, such as the Diagonal Rule (“Theorem of Pythagoras”), Quadrilateral Area Algorithm (“Surveyor’s Formula”), and algorithms for a circle’s area that assume the approximation π = 3. For the corpus of Demotic mathematical papyri see Parker 1972 and 1975; note that in the former volume Parker followed the unusual practice of numbering the mathematical problems continuously through all the papyri.
der consideration, which may be simply a “real-world” entity such as “a breach in a dike” (a2), or such an entity with an added indication of its shape, such as “a circular excavation” (d4), or a mathematical abstraction such as “an isosceles triangle” (c3). Then a participial or relative construction may introduce the given dimensions or other quantities, usually expressed as a number of units. (Sometimes the units are not named at this point but become explicit later in the text.)

The solution, optionally introduced by a phrase such as οὕτως ποιοῦμαι (“I proceed as follows”), consists of a series of arithmetical operations and their results, culminating in a result that is the solution of the problem. The writer of P.Math. is fond of concluding a problem text with οὕτως ἔχει or οὕτως ἔχει ὁμοίως, a phrase that we interpret as an assurance that the same procedure is applicable to similar problems with different data (“This way for similar cases,” cf. the alternative phrase καὶ ἐπὶ τῶν ὁμοίων, “And (so) for similar cases,” at the conclusion of n3, N verso 6).19

The operations may be expressed using first-person (singular or plural) indicative verbs or by verbless prepositions (e.g., ἐπί for multiplication, παρά for division); imperatives are not used in P.Math., though they are common in other mathematical papyri. While the arithmetical operations are typically not explained, exceptions are frequent, such as the following instance where the numerical operation is preceded by both an explanation that this is a unit conversion and by the specific relation of units (d4, D verso 21–23):

I convert the naubia to cubits. One naubion contains 27 cubits. 21 1/3 times 27. The result is 576.

Explanatory supplements are not limited to metrology, but can also express a step of a computational algorithm in generalized terms relating to the scenario before repeating the step using the numbers in the particular problem. In the following passage of a5 (A verso 12–15), we have not only this kind of functional explanation preceding an arithmetical operation, but also, following an intermediate result, a statement of what that result means in terms of the geometrical configuration:

Since from the common base I subtract the top (dimension), we subtract 6 from 30. The remainder is 24. Half of this is 12. This will be the base of the right-angled (triangle).

Such intermittent explanations suggest that the teaching of algorithms in the classroom made more use of abstraction and generalization than was expected in the written solutions, and that

19. The same phrase, however, concludes the metrological texts ε3 (E verso 24) and h1 (H →19, curtailed to merely οὕτως), where such an appeal to generalization makes no sense. Cf. also P.Gen. 3.124 i.9–10: ὁμοίως δὲ καὶ ἐκ ἄλλων ἀριθμῶν εὑρήσομεν, “We will find similarly for other numbers.”
the student was expected to have some comprehension of the meaning (in terms of the scenario) of intermediate results.20

Alternatively, an operation may be followed by a question and explanation, as if a teacher is responding to a student’s oral question (or conversely, as if the teacher is quizzing the student), as in the following example (n1, N recto 6–8):

The result is 77,760. I divide this by 27. Why? (Because) one naubion [contains] 27 cubits. The result is 2880.

Or the teacher may, as it were, ask the student an arithmetical question (f2, F recto 10–13):

- 200 contains how many hundreds? 2.
- Three hundred contains how many hundreds? [3.]
- 400 contains how many hundreds? 4.
- I add 2 and 3 and [4.] The result is 9.

A few of the problem texts in P.Math. (f2, m2, and perhaps d3) conclude with a verification that the result obtained at the end correctly satisfies the conditions of the problem.

The solutions in problem texts in Greek mathematical papyri such as P.Math. have the appearance of being models possessed by the teacher in advance of presenting the problems to students. Even the question-and-answer passages in P.Math. were likely scripted, not genuine transcripts of interchanges that occurred in the classroom; they can be paralleled in other mathematical papyri.21 The most crucial decisions that the solver of a problem must make were never written down. The student was expected to know a repertoire of algorithms applicable to a variety of general scenarios, such as algorithms for finding the area of a rectangle or of any quadrilateral figure given the sides (see next section); so the first questions to be addressed in the face of a new problem were, which algorithm applies, and how are the given data to be matched up with the givens assumed in the algorithm? In some cases, a complete solution may have called for one algorithm applied to part of the data, and another applied to the result of that algorithm together with the rest of the data. Algorithms sometimes also had to be transformed so that what was originally the result becomes a given while one of the original givens

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20. See also P.Lond. 5.1718, 71–77 for statements of geometrical problems, first in generalized terms and then with specific numbers.

21. P.Cair. cat. 10758 has numerous examples of the interrogatory formulas ἐν ποίᾳ ψήφῳ (asking what division of whole numbers would result in a given expression of unit-fractions), τί ἐπὶ τί (asking for whole-number factorizations of a given number), and διὰ N πώς (asking what a given fraction of N amounts to). Chester Beatty Codex AC. 1390 also has ἐν ποίῳ ψήφῳ and τί ἐπὶ τί questions (1.24–25, 3.4, 3.20–21). In the same manuscript are several διὰ τί (“why?”) questions concerning metrological conversions (1.10–11, 2.25–27, 3.11–12, cf. 2.23 for a metrological question not using the διὰ τί formula) and an arithmetical operation (3.28–4.1). Another metrological διὰ τί question is in PSI 3.186.6–8, for which see Shelton 1981b, 100, but we propose restoring thus: γί(νεται) ‘Αυμ. ‘Αυμ εἰς τὸν γδη. διὰ τί ἐπὶ τὸν | γδη; ὅτι ὁ στερεός πῆχ(υς) χωρήσει ξηροῦ | (ὑποστήριγμα) γδη. δὲρα ὁ τετραπόδος (?) χωρήσει ξηροῦ ὑποστήριγμα ‘Δωξ. Shelton, p. 100, cites an instance of a διὰ τί (concerning an arithmetical operation) in pseudo-Heron, Stereometrica, ed. Heiberg p. 140.2.
becomes the result; for example an algorithm for finding the area of a plane figure from its linear dimensions may be turned into an algorithm for finding one of the linear dimensions from the others together with a known area. The artificiality of such inverted problems shows that the goal of problem-solving was not just preparedness for routine and repetitive practical situations but a certain degree of mathematical versatility. The calculations involved in the arithmetical operations, even when involving large numbers and fractions, are omitted in the texts, and it is not clear whether these were actually performed in the didactic setting or whether the results were taken for granted.

Taken as a whole, the mathematical problems in P.Math. are riddled with mistakes. These are of diverse kinds, and offer clues about the processes underlying the composition of the manuscript and hence the way that mathematical skills were taught in fourth-century Egypt. First of all, there are problems that presume mathematically impossible scenarios. In b3, a rectangular wall of constant thickness is presupposed, such that the outer and inner perimeters and the thickness are treated as independent data, whereas in reality once one has chosen any two of these, the third is determined—and the particular given quantities in b3 are grossly inconsistent. Problem o2 is of the artificial, inverted algorithm kind, presupposing a quadrilateral with three sides and the area known and requiring the fourth side to be found. A figure with the prescribed dimensions, however, turns out to be impossible, and the only available algorithm that a student could apply, which is by its nature approximate, leads to the absurd result that the quadrilateral degenerates to a straight line with zero area. These are blunders that one would hope would have been exposed as soon as the solutions were attempted in the classroom.

Next, we have instances in which the problem-solver has chosen an algorithm that is inappropriate for the problem’s scenario, indeed for any plausible scenario. Problem b4 is set out as a very simple demand to find the area of a rectangular field with given dimensions (“side” and “base”). Instead of just multiplying the two numbers, the solution begins by squaring the “side,” and then multiplies the product by the “base,” as if the object was to find the volume of a prism with a square cross-section. The solution is rescued by a final division by the “side,” but one can only wonder what triggered this diversion into three dimensions. Still stranger are two forays into the fourth dimension, n4 and o1, in which the scenario asks for the volume of what is apparently a box-like shape with a vaulted roof, so that dimensions are given for the length, width, and depth of the box part and (we suppose) for the height of the roof. The intended algorithm would perhaps have consisted of calculating the volume of the box, and separately the volume of a rooftlike figure having the given vault height and the rectangle formed by the given length and width as base. Instead, all four quantities are multiplied together, and the product is considered to be the volume.

The solver also occasionally introduced spurious steps in an algorithm, spoiling the result. In g2, for example, an intermediate calculation of a circle’s area from its known circumference, a division of the circumference by 2 is performed that has no place in the algorithm, causing the problem’s solution to be a quarter what it should be. In n2, where the algorithm requires division by 2, the solver divides instead by 4, so that the solution is too small by half.
If the solver’s grasp of the algorithms was sometimes shaky, he nevertheless appears to have been well grounded in basic arithmetic; there seem to be no errors of calculation throughout the problems. Multiplications that involve fractions and that result in products on the order of hundreds of thousands are correctly executed (e.g., b3, i2, and n4). Divisions usually lead to whole-number results. Square roots also always turn out to be whole numbers, never exceeding 30, such as could have been found in a table of squares.

Lastly, we have copying errors. A few isolated missing or miscopied digits in the numerals (e.g. 32 for 12 in d2, D recto 13, and missing unit-fractions in o5, O verso 6 and 8) just show that we are reading a transcript. More revealing are passages in the solutions as they appear in *P.Math.* that are clearly distortions or misunderstandings of an underlying correct version. Thus in f5, the area of a field has been found as 1176 area cubits, and this is to be divided by 9216 to get the area in arourai. These numbers are correctly written in the manuscript, and the original solver of the problem certainly performed the division correctly, obtaining $1/8 + 1/384$. In the manuscript, however, the first fraction has somehow become the symbol representing 4000, so that the figure 4384 is presented (twice!) as the number of arourai in the field, a result whose absurdity should have been obvious both from the relative sizes of the numbers involved in the division and from a common-sense realization that area of a field whose longest side is just 90 cubits can scarcely exceed one aroura, let alone several thousand. Again in f2, the passage quoted above with its three questions asking how many hundreds are contained in 200, 300, and 400 is turned into nonsense through repeated confusion of the word for “hundred” (ἕκατον) with that for “each” (ἕκαστος) or for “hundredths” (ἑκατοστή). Such a series of errors are symptomatic of incomprehension, not mere inattention, and could hardly have been made by the original solver.

The diagrams in *P.Math.*, which were presumably drawn and certainly labelled by the same hand that wrote the texts, are among the manuscript’s odder features. Diagrams were a common, though not indispensable, accompaniment to problem texts in mathematical papyri, usually drawn below, but sometimes preceding, the text. Unlike the diagrams of Greek deductive geometry, they were not integral to the logic of the texts, but served to visualize the scenario, data, and results in a highly schematic manner. For example, if a problem hypothesizes a quadrilateral field with given lengths of its north, south, east, and west sides and required one to find the area, the diagram could consist of a crudely drawn quadrilateral with the four sides labelled with the direction names and the numerical lengths, and with the numerical area inscribed inside the figure.

Problems e1 and e2, dealing with cylindrical pits, have diagrams that are reasonably normal for the genre. In e1, the givens are the “upper diameter,” that is, the diameter of the circular top face of the cylinder (16 cubits), and the depth (3 cubits), and one is required to find the volume (21 1/3 naubia). The diagram consists of a crudely drawn circle with a diameter drawn across it from top to bottom; there is no attempt to represent the pit as a three-dimensional object. Written outside the circle are the numerals 16 (near the upper end of the diameter), 3 (near the diameter’s lower end), and 21 1/3 (to the circle’s right). There is no text other than these numbers,
and only from the text can one know which number represents which element in the problem. In e2, we are given the pit’s depth (9 cubits) and its volume (16 naubia), and we are asked to find the circumference (24 cubits). The diagram is just a crude circle with the numerals 24, 16, and 5 written outside it in roughly the 12 o’clock, 3 o’clock, and 6 o’clock positions. Thus the numbers for the depth and volume occupy the same position as in e1’s diagram, so there is at least some consistency of layout.

Problem a5 has a more complicated geometrical scenario, a trapezoidal figure that can be partitioned into a central rectangle flanked by two similar right-angled triangles. The givens are the four sides, namely 6 schoinia for the “top” (i.e., the shorter parallel side), 30 schoinia for the “common base” (i.e., the longer parallel side), and 15 schoinia for the “legs” (i.e., the two sloping sides). In the solution part of the text, various intermediate results are obtained, including the base of the right-angled triangles (12 schoinia), the square of this base (144, units not specified but actually around), the square of the triangles’ leg (225), the altitude of the trapezoid (9 schoinia, for some reason called the “base of the rectangle”), and the areas of the triangles and the rectangle (in each case 54). The expected final result, namely the total area (162 around), is omitted. The diagram shows the trapezoid partitioned by vertical lines into the two triangles and the rectangle, as expected (compare our own diagram in the commentary to this problem).

Again there is no text, only numerals. Of the givens, the 15 for the “legs” is written next to the right sloping side, but also above the upper parallel side where we might have expected this side’s length, 6. In fact neither 6 nor 30 appears anywhere. 12, the base of the triangles, is written in a central position below the rectangle, and 9, the altitude, is written just above the 12 inside the rectangle. The square numbers 144 and 225 are written near the sides of the right-side triangle to which they pertain, and 54 is written inside both triangles (but not inside the rectangle). All in all, the selection and placement of the numerical data in this diagram are so wayward that it would surely have been a source of confusion to anyone using it as an aid to following the text.

Elsewhere it is not merely the placement of the numbers in the diagrams that seems disconnected from the problems as written, but the very way that a geometrical scenario is imagined. In g4 the statement of the problem admittedly invites trouble. Though the beginning is mutilated, it is evident that the (three-dimensional) object in question is described as “circular” (τροχικόλοιν, scil. τροχογύλον), which in other problem texts means a solid shape having a circular cross-section, namely a cylinder, cone, or conical frustrum; however, the givens are “length,” “width,” and “thickness,” which would be appropriate for a parallelepiped. The solver who wrote the solution ignored the alleged circularity, and applied an algorithm appropriate for a parallelepiped. The diagram, however, consists of two horizontal parallel lines joining two small circles (the left one is incompletely preserved), evidently representing a cylinder—though grossly out of proportion with the rather thin slab implied by a figure whose thickness is in fingers while the other dimensions are in cubits.

In other cases, however, it is hard to imagine how the person who drew the diagram imagined that it represented the problem’s scenario, even though we usually find at least some of the data from the problem scattered around the drawing. Thus c5, a problem about a rectangular
Introduction

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parallelepiped, is accompanied by a drawing looking a bit like the reverse side of a letter envelope, such as might be meant for a triangular prism. On the same page, problem c3 concerns finding the area of a plot of land explicitly described as an isosceles triangle, but the diagram shows a more or less right-angled triangle with an additional line crossing it parallel to its shorter leg. Nor is this a one-time accident, since two other problems about isosceles triangles, d1 and g3, have similar drawings. Problem o2 concerns a quadrilateral, yet its diagram consists of a single horizontal line with the lengths of three of the sides written above the line and the fourth below. As it happens, a quadrilateral with the specified dimensions is a geometrical impossibility, but this cannot be said for the quadrilateral in o5, which is illustrated by a horizontal line in exactly the same manner. Whoever created such diagrams must have had only a tenuous grasp of the spatial meaning of the geometrical terms in the problems, even if he usually knew which were the right algorithms to apply to them.

10. Algorithms Used in the Problems

As we have seen, the approach to solving problems in P.Math. is algorithmic, that is, the student was expected to learn a sequence of arithmetical operations (additions, subtractions, multiplications, divisions, and square roots) that was appropriate for finding the desired unknown quantity in each kind of scenario. The following survey includes both algorithms used in P.Math. and algorithms attested in other Greek mathematical papyri but that are related to algorithms in P.Math. (see also Appendix A). The repertoire ranges from the trivial (e.g., finding the area of a rectangle) to moderately complex procedures (e.g., finding an arithmetical sequence comprising a specified number of terms with given sum and increment). Although a respectable level of analytic reasoning underlay the more advanced algorithms, the texts give no sign that students were expected to understand why they worked.

Also characteristic of the ancient tradition of problem texts is the absence of any distinction drawn between mathematically exact and approximate algorithms. There are in fact several different ways in which the algorithms could be inexact. All algorithms that involved the relationship between the diameter and circumference of a circle (P9), or the relationship between either linear measure of a circle and its area (P10A–D), involve (as we would now say) an assumed approximation for $\pi$, which was usually just 3. Hence both circumferences and areas computed from given diameters are systematically too small, whereas areas computed from given circumferences are systematically too large, by a little under 5 percent. In the algorithms relating to pyramids or pyramidal frusta with equilateral triangular bases (S5A and S6A), the area of an equilateral triangle is found by multiplying the square of the triangle’s side by $1/3 + 1/10$, which is obviously an approximation (a good one) of $\sqrt{3}/4$. Algorithms for finding the volume or surface area of a conic frustum (i.e., a truncated right cone) required as givens the altitude of the frustum and either the diameters or the circumferences of the two circular faces, and thus had the same built-in inaccuracy arising from equating $\pi$ with 3; but several of the frustum algorithms (S3A, S3B, and S4) overlay this with another approximative procedure, finding the
average of the given diameters or areas and then calculating as if the object was a cylinder hav-
ing that dimension, on analogy with a mathematically exact algorithm (P3A) for trapezoids. The error resulting from this short-cut would typically not be very large. For example if we are given circumferences of 1 and 2 units for the two circular faces, algorithm S3B, which employs the averaging short-cut yields a volume of $27/144 \times c$ (where $c$ is the altitude) whereas S3C and S3D, which assume the correct relation between the diameters and the volume, yield $28/144 \times c$.

The algorithm commonly employed to find the area of a quadrilateral from the given lengths of its four sides (P4, often called the “Surveyor’s Formula”), also involves averaging, in this case by assuming that the area is the same as that of a rectangle whose two dimensions are the averages of each pair of opposite sides of the quadrilateral (say the average of the east and west sides times the average of the north and south sides). So long as the quadrilateral is reasonably close to being a rectangle, the algorithm yields a meaningful approximation of the true area, though for anything but a rectangle the result is always an overestimate. The error becomes extreme when the quadrilateral has very acute or obtuse internal angles. For an example of how disregard of this limitation can make nonsense of a problem, see o2, O recto 8–13.

The results of arithmetical operations are generally expressed as exact quantities, not approx-imations. Problems involving use of the Diagonal Rule (the so-called Theorem of Pythagoras) raise the potential of leading to square roots that could not be expressed as exact numbers, but in practice the people who devised the problems in the mathematical papyri showed a strong preference for “Pythagorean triangles,” that is, right-angled triangles having all three sides in whole-number ratios. The 3–4–5 and 5–12–13 triangles were most commonly used, but in f6 and in P.Bagnall 35.1–6 we find 8–15–17 triangles. Only P.Berl. 11529v 1–10 and 11–19 have right-angled triangles with irrational hypotenuses, respectively 8–10–2√41 (approximated fairly accurately as 12 2/3 1/15 1/26 1/30) and 7–9–√130 (approximated rather crudely as 11 1/2).

Plane figures.

P1. Diagonal rule.

Given a rectangle with sides $a$ and $b$ (Fig. 6, top) or a right-angled triangle with legs $a$ and $b$ (Fig. 6, bottom), to find the diagonal or hypotenuse $c$:

(i) Multiply $a$ by itself.
(ii) Multiply $b$ by itself.
(iii) Add the product obtained in (i) to the product obtained in (ii).
(iv) $c$ is the square root of the sum.

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22. For a proof see Pottage 1974, 302 note 4; and for further discussion of the accuracy of the formula, Tou 2014.

23. We follow recent historiography of mathematics in antiquity (e.g., Friberg 2007, 200 or Britton, Proust, and Shnider 2011, 521) in preferring the name “diagonal rule” to “theorem of Pythagoras,” partly to avoid anachronism but also to emphasize that, in the context of non-deductive mathematical problems, the relation is primarily an algorithmic means of obtaining numerical results.
c = √(a^2 + b^2).

Not used directly in *P.Math.*, but cf. f6.
Other papyri: *P.Berl.* 11529.1–10 and 11–19; *P.Gen.* 3.124.26–40.

**P1i. Inverse diagonal rule.**
Given a rectangle with one side *a* and diagonal *c* (Fig. 6, top) or a right-angled triangle with one leg *a* and hypotenuse *c* (Fig. 6, bottom), to find the remaining side or leg *b*:

(i) Multiply *c* by itself.
(ii) Multiply *a* by itself.
(iii) Subtract the product obtained in (ii) from the product obtained in (i).
(iv) *b* is the square root of the difference.

\[ b = \sqrt{(c^2 - a^2)}. \]

Other papyri: *P.Chic.* 3.3.16–20 (problem 5); *P.Bagnall* 35.2.23–37; *P.Gen.* 3.124.1–10 and 11–25.

**P2. Rectangle area algorithm.**
Given a rectangle with sides *a* and *b*, to find the area *A*:
(i) A is \(a\) multiplied by \(b\).

\[ A = a \times b. \]

Used in: \(b2, f7\).
Other papyri: P.Berl. 11529v.1.1–10; Chester Beatty Codex AC. 1390.2.14–27; MPER NS 1.1.13.1–4 (problem 30).

**P2i. Inverse rectangle area algorithm.**

Given a rectangle with side \(a\) and area \(A\), to find the other side \(b\):

(i) \(b\) is \(A\) divided by \(a\).

\[ b = A \div a. \]

Not used in \(P.Math\).
Other papyri: \(P.Mich. 3.151.3.6.5–8; SB 16.12680 verso 7–10 and 11–16\).

**P3A. Trapezoid area algorithm A.**

Given a trapezoid having parallel sides \(a\) and \(b\) and altitude \(c\) (Fig. 7), to find the area \(A\):

(i) Add \(a\) and \(b\).
(ii) Divide sum by 2.
(iii) \(A\) is the quotient multiplied by \(c\).

\[ A = \left(\frac{a + b}{2}\right) \times c. \]

Used in: \(a2, a3, b5\).

**P3B. Trapezoid area algorithm B.**

Given a trapezoid having parallel sides \(a\) and \(b\) (with \(a > b\)) and equal oblique sides \(d\) (Fig. 8), one considers the trapezoid to be subdivided by perpendiculars dropped from the endpoints of the shorter parallel side into two right triangles flanking a rectangle. To find the area \(A\):
(i) Subtract $b$ from $a$.
(ii) Divide the difference by 2. The quotient is the base $e$ of the two right triangles.
(iii) Subtract the result multiplied by itself from $d$ multiplied by itself. The difference is the altitude $c$.
(iv) Multiply $b$ by $c$. The product is the area of the rectangle.
(v) Divide the product by two. The quotient is the area of one of the triangles.
(vi) Multiply $e$ by $b$.
(vii) $A$ is the sum of the areas of the two triangles and the rectangle.

The algorithm embeds the inverse diagonal rule (P1i).

$$c = \sqrt{(d^2 - ((a - b)/2)^2)}$$

$$A = \left(\left((a - b)/2\right) \times c\right) + (b \times c).$$

Fig. 8. Diagram for algorithm P3B.

Used in: a5.
Other papyri: presumably P.Chic. 3.2.1–2 (problem 3).

P3C. Trapezoid area algorithm C.

Given a trapezoid having parallel sides $a$ and $b$ (with $a > b$) and oblique sides $d$ and $e$ (with $d > e$, Fig. 9), one considers the trapezoid to be subdivided by perpendiculars dropped from the endpoints of the shorter parallel side into two right triangles flanking a rectangle. To find the area $A$:

(i) Multiply $e$ by itself.
(ii) Multiply $d$ by itself.
(iii) Subtract the square of $e$ from the square of $d$.
(iv) Subtract $b$ from $a$.
(v) Divide the difference into the difference of squares found in (iii).
(vi) Subtract the quotient from the difference found in (iv).
(vii) Divide the difference by 2. This is the base of the smaller right-angled triangle.
(viii) Multiply the quotient by itself.
(ix) Subtract the product from the square of $e$. 
The altitude $c$ is the square root of the difference.

Multiply $c$ by the base of the smaller right-angled triangle found in (vii).

Divide the difference by 2. This is the area of the smaller right-angled triangle, $A_T$.

Multiply $c$ by $b$. This is the area of the rectangle, $A_r$.

Multiply $c$ by the quotient found in (v) and divide the product by 2. This is the area of the scalene triangle that is the difference between the larger and the smaller right-angled triangle, $A_S$.

$A$ is the sum of twice $A_T$, $A_r$, and $A_S$.

The algorithm embeds the inverse diagonal rule (P1i).

\[ \beta_S = \frac{(d^2 - e^2)}{(a - b)} \]
\[ \beta_T = \frac{[(a - b) - \beta_S]}{2} \]
\[ c = \sqrt{(e^2 - \beta_T^2)} \]
\[ A_T = \frac{(c \times \beta_T)}{2} \]
\[ A_R = (c \times b) \]
\[ A_S = \frac{(c \times \beta_S)}{2} \]
\[ A = 2A_T + A_R + A_S \]

Not used in *P.Math.*

Other papyri: *P.Chic.* 3.2.3–15 (problem 4); *P.Bagnall* 35.1.1–16 (variant).

**P3D. Trapezoid area algorithm D.**

Given an obtuse trapezoid having parallel sides $a$ and $b$ and oblique sides $d$ and $e$ (with $d > e$, Fig. 10), one considers the trapezoid to be subdivided by perpendiculares dropped from the endpoints of the shorter parallel side into two right triangles flanking a rectangle. To find the area $A$:

(i) Multiply $e$ by itself.
(ii) Multiply $d$ by itself.
(iii) Subtract the square of $e$ from the square of $d$.
(iv) Subtract $b$ from $a$. 
(v) Divide the difference into the difference of squares found in (iii).
(vi) Subtract the difference found in (iv) from the quotient.
(vii) Divide the difference by 2. This is the base of the smaller right-angled triangle.
(viii) Multiply the quotient by itself.
(ix) Subtract the product from the square of e.
(x) The altitude c is the square root of the difference.
(xi) Subtract the base of the smaller right-angled triangle obtained in (vii) from b. This is the base of the rectangle.
(xii) Subtract the difference from a. This is the base of the larger right-angled triangle.
(xiii) Multiply c by the base of the smaller right-angled triangle obtained in (vii).
(xiv) Divide the product by 2. This is the area of the smaller right-angled triangle, $A_{T_1}$.
(xv) Multiply c by the base of the rectangle obtained in (xi). This is the area of the rectangle, $A_r$.
(xvi) Multiply c by the base of the larger right-angled triangle obtained in (xii) and divide the product by 2. This is the area of the larger right-angled triangle, $A_{T_2}$.
(xvii) $A$ is the sum of $A_{T_1}$, $A_r$, and $A_{T_2}$.

The algorithm embeds the inverse diagonal rule (P1i).

\[ \beta_{T_1} = \frac{(e^2 - e^2)/(a - b) - (a - b))/2} \]
\[ c = \sqrt{e^2 - \beta_{T_1}^2} \]
\[ \beta_r = b - \beta_{T_1} \]
\[ \beta_{T_2} = a - \beta_r \]
\[ A_{T_1} = (c \times \beta_{T_1})/2 \]
\[ A_r = (c \times \beta_r) \]
\[ A_{T_2} = (c \times \beta_{T_2})/2 \]
\[ A = A_{T_1} + A_r + A_{T_2} \]

Not used in *P.Math*.
Other papyri: *PChic. 3.3.1–15; PBagnall 35.2.6–21 (variant).*
**P4. Quadrilateral area algorithm (approximate).**

Given a quadrilateral having pairs of sides \(a\) opposite \(b\) and \(c\) opposite \(d\) (Fig. 11), to find the area \(A\):

(i) Add \(a\) and \(b\).
(ii) Divide the sum by 2.
(iii) Add \(c\) and \(d\).
(iv) Divide the sum by 2.
(v) Multiply the quotient by the quotient obtained in (ii).

The algorithm is exact only in the trivial case when the quadrilateral is a rectangle.

Fig. 11. Diagram for algorithms P4, P4iA, and P4iB.

\[
A = \left(\frac{a + b}{2}\right) \times \left(\frac{c + d}{2}\right).
\]

Used in: c1, f5, o5.

Other papyri: P.Berl. 11529v.1.1–10 (for a rectangle!); P.Col. inv. 157a.A.1–11 and B.1–16 (for a rectangle!); Chester Beatty Codex AC. 1390.1.2–14 and 2.14–27 (for a rectangle!).


**P4iA. Inverse quadrilateral area algorithm A (approximate).**

Given a quadrilateral with side \(a\) opposite side \(b\), side \(c\), and area \(A\) (Fig. 11), to find the remaining side \(d\):

(i) Multiply \(A\) by 2.
(ii) Divide the product by half the sum of \(a\) and \(b\).
(iii) Subtract \(c\) from the quotient to obtain \(d\).

\[
d = \frac{2A}{(a + b)/2} - c.
\]

Used in: o2.
P4iB. Inverse quadrilateral area algorithm B (approximate).
Given a quadrilateral with area $A$ and side $a$ (Fig. 11), to find the remaining sides $b$ (opposite $a$), $c$, and $d$:

(i) Factorize $A$ into $e \times f$.
(ii) Multiply $e$ by 2.
(iii) Subtract $a$; the difference is $b$.
(iv) Let $c$ and $d$ both equal $f$.

Not used in $P.Math$.
Other papyri: Chester Beatty Codex AC. 1390.3.1–12.

P5. Hollow rectangle area algorithm.
Given a hollow rectangle having outer perimeter $a$, inner perimeter $b$, and constant perpendicular distance between outer and inner perimeter $c$ (Fig. 12), to find the area $A$:

(i) Add the outer and inner perimeters.
(ii) Divide the sum by 2.
(iii) $A$ is the product of the quotient and $c$.

This is algorithmically homologous to trapezoid area algorithm A (P3A), and may be derived from that algorithm by subdividing the hollow rectangle into four trapezia by the straight lines joining corresponding corners of the two perimeters.

![Fig. 12. Diagram for algorithm P5.](a)

$$A = (\frac{a + b}{2}) \times c.$$ 

Used in: b3.

P6. Triangle area algorithm.
Given a triangle with base $a$ and altitude $c$ (Fig. 13), to find the area $A$:

(i) Multiply $a$ by $c$.
(ii) $A$ is the product divided by 2.
A = \frac{1}{2} ac.

Used in: c3, d1, g3, n3?
Other papyri: P.Berl. 11529v.2.21–32; MPER NS 1.1.7.12–16 (problem 16).

**P7. Right-angled triangle area algorithm.**
Given a right-angled triangle with legs a and b (Fig. 14), to find the area A:
(i) Multiply a by b.
(ii) A is the product divided by 2.

A = \frac{1}{2} ab.

Used in: d3.
Other papyri: P.Berl. 11529v.1.11–19; PChic. 3.3.16–20 (problem 5).
**P8A. Isosceles triangle sides algorithm A.**
Given an isosceles triangle with base $a$ and altitude $c$ (Fig. 15), to find the equal sides $d$:

1. Multiply $c$ by itself.
2. Divide $a$ by 2.
3. Multiply the quotient by itself.
4. Add the product obtained in (1) to the product obtained in (3).
5. $d$ is the square root of the sum.

$$d = \sqrt{c^2 + \left(\frac{a}{2}\right)^2}.$$  

Used in: c3.

**P8B. Isosceles triangle vertical algorithm B.**
Given an isosceles triangle with base $a$ and equal sides $d$ (Fig. 15), to find the altitude $c$:

1. Divide $a$ by 2.
2. Multiply the quotient by itself.
3. Multiply $d$ by itself.
4. Subtract the product obtained in (2) from the product obtained in (3).
5. $c$ is the square root of the difference.

The algorithm embeds the inverse diagonal rule (P1i).

$$c = \sqrt{d^2 - \left(\frac{a}{2}\right)^2}.$$  

Used in: d1, g3.

Other papyri: P.Berl. 11529v.2.21–32.
Given a circle having diameter $d$ (Fig. 16), to find the circumference $c$:

(i) Multiply $d$ by 3. The product is the circumference.

$$c = 3 \times d.$$ 

The algorithm effectively assumes that $\pi = 3$.

Not used in *P.Math.*
Other papyri: BM Add. MS 41203A.r.1–6; *P.Oxy.* 3.470.31–46.

P9i. Circle inverse circumference algorithm (approximate).
Given a circle having circumference $c$ (Fig. 16), to find the diameter $d$:

(i) Divide $c$ by 3. The quotient is the diameter.

$$d = c / 3.$$ 

The algorithm effectively assumes that $\pi = 3$.

Not used in *P.Math.*
Other papyri: *MPER* NS 15.172–174.1–6.

P10A. Circle area algorithm A (approximate).
Given a circle having diameter $d$ (Fig. 16), to find the area $A$:

(i) Multiply $d$ by itself.
(ii) Subtract one quarter of the product from itself. The difference is $A$.

$$A = (d^2 - d^2/4) = 3(d^2)/4.$$
Since the exact formula is $A = \pi(d^2)/4$, the algorithm effectively assumes that $\pi = 3$.

Used in: e1.
Other papyri: MPER NS 1.1.9a.1–2 (problem 20), 9a.2–5 (problem 21), 9a.5–6 (problem 22), 9a.6–7 (problem 23); MPER NS 15.178.3.1–6; PSI 3.186v.1–8.

P10Ai. Inverse circle area algorithm A (approximate).
Given a circle having area $A$ (Fig. 16), to find the diameter $d$:
(i) Add to $A$ one-third of itself.
(ii) $d$ is the square root of the result.

$$d = \sqrt{(A + A/3)} = \sqrt{(4A/3)}.$$

Used in: d4.

P10B. Circle area algorithm B (approximate).
Given a circle having circumference $c$ (Fig. 16), to find the area $A$:
(i) Multiply $c$ by itself.
(ii) Divide the result by 12. The quotient is $A$.

$$A = (c^2)/12.$$

The exact formula is $A = (c^2)/4\pi$, so again the algorithm assumes $\pi = 3$.

Used in: g2 (incorrectly).
Other papyri: MPER NS 15.178.2.1–7; MPER NS 15.172–174.1–6 (14 instead of 12 in step ii is surely a textual error).

P10Bi. Inverse circle area algorithm B (approximate).
Given a circle having area $A$ (Fig. 16), to find the circumference $c$:
(i) Multiply $A$ by 12.
(ii) $c$ is the square root of the product.

$$c = \sqrt{(12A)}.$$

Used in: e2.

P10C. Circle area algorithm C (approximate).
Given a circle having diameter $d$ and perpendicular half-diameter $e$ (Fig. 16), to find the area $A$: 
(i) Add \(d\) and \(e\).
(ii) Multiply the sum by itself.
(ii) Divide by 3. The quotient is \(A\).

\[ A = \frac{(d + e)^2}{3}. \]

The algorithm effectively assumes that \(\pi = 3\).

Not used in \textit{P.Math}.
Other papyri: \textit{MPER NS} 15.178.4.1–9.

\textbf{P10D. Semicircle area algorithm D (approximate).}

Given a semicircle having diameter \(d\) and perpendicular half-diameter \(e\) (Fig. 16), to find the area \(A\):

(i) Add \(d\) and \(e\).
(ii) Divide by 2.
(ii) Multiply by \(e\). The quotient is \(A\).

\[ A = e(d + e) / 2. \]

The algorithm effectively assumes that \(\pi = 3\).

Not used in \textit{P.Math}.
Other papyri: \textit{MPER NS} 15.178.5.1–9.

\textit{Solids.}

\textbf{S1. Parallelepiped volume algorithm.}

Given a rectangular parallelepiped with linear dimensions \(a, b, c\) (Fig. 17), to find the volume \(V\):

(i) Multiply \(a\) by \(b\).
(ii) \(V\) is the product multiplied by \(c\).
$V = a \times b \times c$.

Used in: c5, g4, i2.

Other papyri: Chester Beatty Codex AC. 1390.2.1–6; *P.Cair. cat*. 10758.3v1.8–12 (problem 2); *P.Lond*. 5.1718.4.71; *T.Varie* 20.3–4 and 5–6; *MPER* 1.1.2.4–7 (problem 2), 2.8–13 (problem 3), 3.7–12 (problem 5), 3.12–15 (problem 6), and 4.3–8 (problem 8).

**S2. Prism volume algorithm.**

Given a prism (or cylinder) having cross section area $A$ and perpendicular dimension $h$ (Fig. 18), to find the volume $V$:

(i) $V$ is $A$ multiplied by $h$. 

Fig. 17. Diagram for algorithm S1.
$$V = A \times h.$$ 

Used in: a2, a3, b3, b5, c1, e1, g2, n3, n2.

Other papyri: Chester Beatty Codex AC. 1390.1.2–14; PSI 3.186v.1–8 (cylinder); MPER NS 1.1.7.12–16 (problem 16), 9a1–2 (problem 20), 9a2–5 (problem 21), 9a5–6 (problem 22), and 9a6–7 (problem 23).

**S2i. Inverse prism volume algorithm.**

Given a prism having volume $V$ and perpendicular dimension $h$ (Fig. 18), to find the area of the cross section $A$:

(i) $A$ is $V$ divided by $h$.

$$A = V/h.$$ 

Used in: d4, e2.

**S3A. Conic frustum volume algorithm A (approximate).**

Given a conic frustum having circular faces with diameters $a_1$ and $a_2$ and perpendicular dimension $c$ (Fig. 19), to find the volume $V$:

(i) Add $a_1$ and $a_2$.
(ii) Divide the sum by 2.
(iii) Multiply the sum by itself.
(iv) Subtract a quarter of the result from itself.
(v) $V$ is the difference multiplied by $c$. 

![Fig. 18. Diagram for algorithms S2 and S2i.](image)
V = (d^2 - d^2/4) \times c, \text{ where } d = (a_1 + a_2)/2.

The algorithm embeds Circle Algorithm A (P10A) with its implied value of 3 for \( \pi \), and moreover calculating the frustum’s volume as if of a cylinder with diameter equal to the average of \( a_1 \) and \( a_2 \) is only an approximation.

Used in: n1.
Other papyri: BM Add. MS 41203.11–16 (erroneously treating circumferences as diameters).

**S3B. Conic frustum volume algorithm B (approximate).**

Given a conic frustum having circular faces with circumferences \( p_1 \) and \( p_2 \) and perpendicular dimension \( c \) (Fig. 19), to find the volume \( V \):

(i) Add \( p_1 \) and \( p_2 \).
(ii) Divide the sum by 2.
(iii) Multiply the sum by itself.
(iv) Divide the product by 12.
(v) \( V \) is the difference multiplied by \( c \).

\[ V = d^2 \times c/12, \text{ where } d = (p_1 + p_2)/2. \]

Not used in *P.Math*.
Other papyri: Chester Beatty Codex AC. 1390.1.15–23; *P.Cair. cat.* 10758.3v1.1–7 (problem 1, but for the division by 36 instead of 12 in step iv cf. S3C).
S3C. Conic frustum volume algorithm C (approximate).

The application of this algorithm in Greek mathematical papyri is a conjecture based on the divisions by 36 cited below.\(^\text{24}\)

Given a conic frustum having circular faces with circumferences \(p_1\) and \(p_2\) and perpendicular dimension \(c\) (Fig. 19), to find the volume \(V\):

(i) Multiply \(p_1\) by itself.
(ii) Multiply \(p_2\) by itself.
(iii) Multiply \(p_1\) by \(p_2\).
(iv) Add the three products.
(v) Multiply the sum by \(c\).
(v) \(V\) is the product divided by 36.

\[
V = \left( p_1^2 + p_1 \times p_2 + p_2^2 \right) \times c / 36
\]

Not used in \textit{P.Math}.
Other papyri: Possibly \textit{P.Lond.} 5.1718.4.75–76, and cf. the division by 36 instead of 12 in \textit{P.Cair. cat.} 10758.3v1.1–7 (problem 1).

S3D. Conic frustum volume algorithm D (approximate).

Given a conic frustum having circular base and top with circumferences \(p_1\) and \(p_2\), and slope height (measured along the conical surface) \(s\) (Fig. 19), to find the volume \(V\):

(i) Divide \(p_1\) by 3. The quotient is the diameter of the base, \(a_1\).
(ii) Divide \(p_2\) by 3. The quotient is the diameter of the top, \(a_2\).
(iii) Subtract \(a_2\) from \(a_1\).
(iv) Divide the difference by 2.
(v) Multiply the quotient by itself.
(vi) Multiply \(s\) by itself.
(vii) Subtract the product obtained in step (v) from the product obtained in step (vi).
(viii) Find the square root of the difference. This is the altitude \(c\).
(ix) Add \(a_1\) and \(a_2\).
(x) Divide the sum by 2.
(xi) Multiply the quotient by itself.
(xii) Subtract a quarter of the result from itself.
(xiii) Subtract \(a_2\) from \(a_1\).
(xiv) Divide the difference by 2.
(xv) Multiply the quotient by itself.
(xvi) Divide the product by 4.
(xvii) Add the results obtained in steps (xii) and (xvi).
(xviii) \(V\) is the sum multiplied by \(c\).

\(^{24}\) Smyly 1920, 107; Kurt Vogel in MPER NS 1, 39 doubts that an exact algorithm taking this form existed in antiquity.
\[ a_1 = \frac{p_1}{3} \]
\[ a_2 = \frac{p_2}{3} \]
\[ c = \sqrt{s^2 - d_1^2} \text{ where } d_1 = \frac{(a_1 - a_2)}{2} \]
\[ V = \left( \frac{d_2}{2} - \frac{d_1}{2} \right) \times c, \text{ where } d_2 = \frac{(a_1 + a_2)}{2} \]

Not used in *P.Math*.
Other papyri: *MPER* NS 1.1.9b.1–9 (problem 24) and 10.7–16 (problem 25).

**S4. Conic frustum surface algorithm (approximate).**

Given a conic frustum having circular faces with diameters \( a_1 \) and \( a_2 \) and perpendicular dimension \( c \), to find the surface \( A \):

(i) Add \( a_1 \) and \( a_2 \).

(ii) Divide the sum by 2.

(iii) Multiply the sum by 3.

(iv) \( V \) is the product multiplied by \( c \).

\[ V = 3 \times \frac{(a_1 + a_2)}{2} \times c \]

Not used in *P.Math*.
Other papyri: BM Add. MS 41203A.r.1–6; *P.Oxy.* 3.470.31–46.

**S5A. Pyramid volume algorithm A.**

Given a pyramid having an equilateral triangle with side \( a \) as base and similar equilateral triangles with sloping side \( s \) as the other faces (Fig. 20), to find the volume \( V \):

(i) Multiply \( a \) by itself.

(ii) Divide the product by 3. The quotient is the *epipedos* (square of a line from the base’s center to a vertex of the base).

(iii) Multiply \( s \) by itself.

(iv) Subtract the *epipedos* from the product.

(v) Find the square root of the difference. The result is the vertical, \( c \).

(vi) Multiply the square of \( a \) (from step i) by 1/3 1/10. The product is the area of the base, \( A \).

(vii) Multiply \( A \) by \( c \).

(viii) \( V \) is the product divided by 3.
Given a pyramid having a square with side $a$ as base and similar equilateral triangles with sloping side $s$ as the other faces (Fig. 21), to find the volume $V$:

(i) Divide $a$ by 2.
(ii) Multiply the quotient by itself.
(iii) Add the product to itself. The sum is the *epipedos* (square of a line from the base’s center to a vertex of the base).
(iv) Multiply $s$ by itself.
(v) Subtract the *epipedos* from the product.
(vi) Find the square root of the difference. The result is the vertical, $c$.
(vii) Multiply $a$ by itself. The product is the area of the base, $A$.
(viii) $V$ is the product divided by 3.

Not used in *P.Math*.
Other papyri: *MPER NS 1.1.5.1–5* (problem 10), *5.19–6.3* (problem 12), and *11.10–18* (problem 27).

**S5B. Pyramid volume algorithm B.**

Given a pyramid having a square with side $a$ as base and similar equilateral triangles with sloping side $s$ as the other faces (Fig. 21), to find the volume $V$:

\[
c = \sqrt{s^2 - \frac{a^2}{3}}
\]

\[
A = \frac{13}{30} \times a^2
\]

\[
V = A \times \frac{c}{3}
\]

Fig. 20. Diagram for algorithm S5A.
\[ c = \sqrt{s^2 - \frac{a^2}{2}} \]
\[ A = a^2 \]
\[ V = A \times \frac{c}{3} \]

Not used in *P.Math*.
Other papyri: *MPER NS 1.1.8.1–12* (problem 18) and 12.8–13 (problem 29).

**S5B. Pyramid volume algorithm C.**
Given a pyramid having base with area \( A \) and vertical \( c \), to find the volume \( V \):

(i) Multiply \( A \) by \( c \).
(ii) \( V \) is the product divided by 3.

\[ V = A \times \frac{c}{3} \]

Not used in *P.Math*.
Other papyri: *MPER NS 1.1.7.9–12* (problem 15) and 13.1–4 (problem 30).

**S6A. Pyramidal frustum volume algorithm A.**
Given a pyramidal frustum having equilateral triangles with sides \( a_1 \) and \( a_2 \) as base and top, and similar equilateral triangles with sloping side \( s \) as the other faces (Fig. 22), to find the volume \( V \):

(i) Subtract \( a_2 \) from \( a_1 \).
(ii) Multiply the difference by itself.
(iii) Divide the product by 3.
(iv) Multiply \( s \) by itself.
(v) Subtract the quotient obtained in step (iii) from the product.
(vi) Find the square root of the difference. The result is the vertical, \( c \).
(vii) Add \( a_1 \) and \( a_2 \).
(viii) Divide the sum by 2.
(ix) Multiply the quotient by itself.
(x) Multiply the product by \(1/3 \ 1/10\).
(xi) Divide the difference of \( a_1 \) and \( a_2 \) (from step i) by 2.
(xii) Multiply the quotient by itself.
(xiii) Multiply the product by \(1/3 \ 1/10\).
(xiv) Divide the product by 3.
(xv) Add the results obtained in steps (x) and (xiv).
(xvi) \( V \) is the sum multiplied by \( c \).

\[
c = \sqrt{s^2 - (a_1 - a_2)^2/3}
\]

\[
V = 13/30 \times (((a_1 + a_2)/2)^2 + ((a_1 - a_2)/2)^2 / 3) \times c
\]

Not used in \( P.Math \).
Other papyri: \( MPER \ NS \ 1.1.5.6–18 \) (problem 11), 6.3–11 (problem 13), 7.1–8 (problem 14), and 11.1–10 (problem 26).

**S6B. Pyramidal frustum volume algorithm B.**

Given a pyramidal frustum having squares with sides \( a_1 \) and \( a_2 \) as base and top, and similar equilateral triangles with sloping side \( s \) as the other faces (Fig. 23), to find the volume \( V \):

(i) Subtract \( a_2 \) from \( a_1 \).
(ii) Multiply the difference by itself.
(iii) Divide the product by 2.
(iv) Multiply \( s \) by itself.
(v) Subtract the quotient obtained in step (iii) from the product.
(vi) Find the square root of the difference. The result is the vertical, \( c \).
(vii) Add \( a_1 \) and \( a_2 \).
(viii) Divide the sum by 2.
(ix) Multiply the quotient by itself.
(x) Divide the difference of $a_1$ and $a_2$ (from step i) by 2.
(xii) Multiply the quotient by itself.
(xiii) Divide the product by 3.
(xiv) Add the results obtained in steps (ix) and (xiii).
(xv) $V$ is the sum multiplied by $c$.

\[ c = \sqrt{s^2 - (a_1 - a_2)^2/2} \]
\[ V = \frac{13}{30} \times \left( \frac{(a_1 + a_2)/2}{2} + \frac{(a_1 - a_2)/2}{3} \right) c \]

Not used in *P.Math*.
Other papyri: *MPER NS 1.1.8.12–19* (problem 19) and 12.1–8 (problem 28).

**Numbers.**

**N1. Arithmetical sequence algorithm.**

It is required to find $n$ numbers $i_1 \ldots i_n$ increasing by constant differences $d$, and such that their sum is $s$. To find the smallest of the numbers $i_1$:

(i) Multiply $n$ by itself.
(ii) Add $n$ to the product.
(iii) Divide the sum by 2.
(iv) Multiply the quotient by $d$.
(v) Subtract the product from $s$.
(vi) Divide the difference by $n$.
(vi) Add $d$ to the quotient. The sum is $i_1$.

Successive additions of $d$ yield the other numbers.

\[ i_1 = \frac{(s - ((n^2 + n)/2) \times d)/n + d}{n} \]

Used in: m2.
11. Partitions into Unit-Fractions

Six of *P.Math.*’s mathematical problems (a4, c4, f3, f4, and the very poorly preserved h2 and i4) are of a special type, with elliptical statement of the problem λοιπὰ ρ, τὸ D ἐν n μόρια· μὴ πρόβα ρ (with R, D, and n standing for numerals), which we believe should be translated, “remainder R, the Dth part in n unit fractions; do not surpass 100.” This is immediately followed by the answer, taking the form ἔσται τὰ μόρια followed by a sequence of numerals to be read as unit-fractions, always in order of increasing denominators. The meaning of such a problem is as follows. It is to be supposed that a division of some number by D has been carried out, such that the remainder beyond a whole number of Ds is R. We are now required to divide R by D and express the resulting fractional quotient R÷D as a sum of n distinct unit-fractions, none of whose denominators should exceed 100. (What the original dividend or the integer part of the quotient were is immaterial to the problem.) In contrast to the other problems, no steps leading to the answer are provided.

*P.Yale* 4.187 consists of comparable fraction partitions in a didactic context, for example ις: ις, τὸ ξθ, ιβ κγ μϛ ρβ, to be interpreted as “11, the 69th part (is) 1/12 1/23 1/46 1/92.” The thirty-four such partitions do not constitute a reference table; notwithstanding a few groups of consecutive lines having the same dividend, including a sequence (ii 15–iii 22) dividing the same quantity by 10 through 17, there is no broad pattern in the numbers. The partitions can be into as few as two or as many as five unit-fractions, none of which has a denominator greater than 98, so it seems that there was an unstated restriction to denominators not exceeding 100.

Another manuscript that contains comparable problems is *P.Cair. cat.* 10758. In *P.Cair. cat.* 10758, problems 18, 21, 22, 23, 38, 39, and 40 are stated with an algorithm such as τῆς R τὸ D, “of R the Dth part,” and the solution (incomplete in 40) is typically given as ὡς εἶναι, “so that they are,” followed by a sequence of unit-fractions, with denominators never exceeding 99, that sum to R÷D. Moreover, *P.Cair. cat.* 10758 problems 16, 19, 20, and 50 are similar but also include a stipulation of the number of unit-fractions, e.g., χώρισον D εἰς n μόρια, “partition the Dth part into n unit-fractions,” such as we have in the *P.Math.* problems. In contrast to *P.Math.*, *P.Cair. cat.* 10758 provides a step-by-step solution of each problem. These solutions are made up of several recurring types of operations:

- Conversion of a division to lowest whole-number terms R÷D (i.e., R and D are whole numbers with no common factor).
- Scaling up of a division R÷D by a whole-number multiple k to obtain (kR)÷(kD).
- Peeling off of a unit-fraction from a division in whole number terms R/D, such that E×F = D, obtaining the unit-fraction 1/F plus the remainder (R–E)÷D.
- Peeling off of a unit-fraction from a division in whole-number terms R÷D, obtaining the unit-fraction 1/D plus the remainder (R–1)÷D. [This is a special case of the preceding operation.]
• Splitting of a division in whole-number terms $R\div D$ into two unit-fractions $1/(E\times[E+F]+R)$ and $1/(F\times[E+F]+R)$ such that $E\times F = D$ and $E+F$ is divisible by $R$.
• Splitting a single unit-fraction $1/D$ into two unit-fractions $1/(E\times[E+F])$ and $1/(F\times[E+F])$ such that $E\times F = D$. [This is a special case of the preceding operation.]

For example, *PCair. cat.* 10758 problem 19 requires us to express a quantity given as the sum of the three unit fractions $1/55 + 1/56 + 1/70$ as a sum of four unit-fractions. The solution is as follows:

i. Convert $1/55 + 1/56 + 1/70$ to a division in lowest whole number terms: $31\div 616$.
ii. Peel off the unit-fraction $1/88$ (i.e. $7\div 616$), leaving remainder $24\div 616$.
iii. Convert to lowest terms: $3\div 77$.
iv. Peel off the unit-fraction $1/77$, leaving remainder $2\div 77$.
v. Split into two unit-fractions $1/63$ and $1/99$ (using $77 = 7 \times 11$).

As this example illustrates, problems of this kind are puzzles with no immediate practical application. Why would one want to express a given quantity as a sum of a predetermined number of unit-fractions, especially when we know from the start that it can be expressed in fewer? In the foregoing problem we are given the quantity as a sum of three unit-fractions and asked to re-express it as a sum of four; in others, the given quantity already is, or can be simplified to, a single unit-fraction.\(^\text{25}\) Secondly, such problems could not be solved mechanically by applying a single memorized algorithm; the approach is exploratory, with arbitrary choices such as whether to scale up a division and if so by what multiple, and which of multiple factorizations to use in a splitting operation. For example a division $1\div 24$ can be split as $1/25 + 1/600$ or as $1/28 + 1/168$ or as $1/33 + 1/88$ or as $1/40 + 1/60$, depending on which factorization of 24 one chooses. The prohibition of denominators greater than 100 in *P.Math.*’s problems would rule out some of these possibilities, and its intention may have been to make the problems more challenging, in particular by eliminating the lazy option of splitting any unit-fraction $1/R$ into $1/(R+1) + 1/R(R+1)$. The partitions offered in both *PCair. cat.* 10758 and *P.Math.* tend to consist of unit-fractions whose denominators are of roughly the same order of magnitude.

Although the problem of partitioning a quantity into say four unit-fractions might be approached in multiple ways, for example by an initial splitting into two parts, each of which is then split into a pair of unit-fractions, the solutions in *PCair. cat.* 10758 exhibit a linear approach, comprising a sequence of peelings-off terminating in a splitting. The denominators of

\(^{25}\) In special circumstances there is a point in splitting a single unit-fraction into two. Specifically, splitting an odd-numbered unit-fraction into two even-numbered ones makes it easy to express double the fraction in distinct unit-fractions.
the unit-fractions resulting from a splitting have the property that their sum is either a square or a small multiple of a square. This fact provides a clue to reconstructing the routes by which the solutions in P.Math. were obtained. On the other hand, unit-fractions peeled off at the beginning tend to share factors with the given divisor; for example in PCair. cat. 10758 problem 19, 616 is $7 \times 88$, and the first unit-fraction peeled off is $1/88$.

The following conjectural reconstructions of P.Math.’s partition problems may not be exactly how the originators of these problems proceeded, but we suspect that their approaches must have been along similar lines. Problem f3 turns out to be fairly challenging.

*Conjectural reconstructions of the partition problems in P.Math.*

**a4.** $4 \div 80$, partition into four unit-fractions.

i. Convert to lowest terms: $1 \div 20$.

ii. Scale up by $7$: $7 \div 140$.

iii. Peel off $2 \div 140$, i.e. $1/70$, leaving remainder $5 \div 140$.

iv. Convert to lowest terms: $1 \div 28$.

v. Scale up by $3$: $3 \div 84$.

vi. Peel off $1/84$, leaving remainder $2 \div 84$.

vii. Convert to lowest terms: $1 \div 42$.

viii. Split into two unit-fractions $1/78$ and $1/91$ (using $42 = 6 \times 7$).

ix. Answer: $1/70 + 1/78 + 1/84 + 1/91$.

**c4.** $1 1/4 \div 7$, partition into seven unit-fractions.

i. Convert to lowest terms: $5 \div 28$.

ii. Peel off $1/28$, leaving remainder $4 \div 28$.

iii. Convert to lowest terms: $1 \div 7$.

iv. Scale up by $5$: $5 \div 35$.

v. Peel off $1/35$, leaving remainder $4 \div 35$.

vi. Scale up by $2$: $8 \div 70$.

vii. Peel off $1/70$, leaving remainder $7 \div 70$.

viii. Convert to lowest terms: $1 \div 10$.

ix. Scale up by $6$: $6 \div 60$.

x. Peel off $1/60$, leaving remainder $5 \div 60$.

xi. Convert to lowest terms: $1 \div 12$.

xii. Scale up by $2$: $2 \div 24$.

xiii. Peel off $1/24$, leaving remainder $1 \div 24$.

xiv. Scale up by $2$: $2 \div 48$.

xv. Split into $1/42$ and $1/56$ (using $48 = 6 \times 8$).

xvi. Answer: $1/24 + 1/28 + 1/35 + 1/42 + 1/56 + 1/60 + 1/70$. 
f3. 9 ÷ 119, partition into four unit-fractions.
   i. Scale up by 2: 18 ÷ 238.
   ii. Peel off 7 ÷ 238 (i.e. 1/34), leaving 11 ÷ 238.
   iii. Scale up by 3: 33 ÷ 714.
   iv. Peel off 14 ÷ 714 (i.e. 1/51), leaving remainder 19 ÷ 714.
   v. Scale up by 2: 38 ÷ 1428.
   vi. Split into 1/68 and 1/84 (using 1428 = 34 × 42, with 34 + 42 divisible by 38).
   vii. Answer: 1/34 + 1/51 + 1/68 + 1/84.

f4. 36 ÷ 228, partition into five unit-fractions.
   i. Peel off 19 ÷ 228 (i.e. 1/12), leaving remainder 17 ÷ 228.
   ii. Peel off 4 ÷ 228 (i.e. 1/57), leaving remainder 13 ÷ 228.
   iii. Peel off 3 ÷ 228 (i.e. 1/76), leaving remainder 10 ÷ 228.
   iv. Convert to lowest terms: 5 ÷ 114.
   v. Split into 1/30 and 1/95 (using 114 = 6 × 19, with 6 + 19 is divisible by 5).
   vi. Answer: 1/12 + 1/30 + 1/57 + 1/76 + 1/95.

12. Metrological Relations and Coefficients in the Mathematical Problems

Only a small subset of the units of the metrological texts appear in the problems. Among the numerous units of length, for example, only the cubit, finger, and schoinion occur frequently, and additionally we find the xylon, pace, and reed in one problem each (c5 and f5), where quantities in these units are to be converted to cubits. Again, areas are almost always expressed in arourai, just once in bikoi (o3).

Most relations assumed in problems can be found somewhere in the metrological texts. For example, f5 converts area cubits to arourai by dividing by 9216, a relation found in e3 (E verso 23–24), while in o3 area cubits are divided by 200 to obtain bikoi, as stated again in e3 (E verso 19). On the other hand, none of the metrological texts asserts the relation that one aroura is the area of a square whose side is one schoinion, which is frequently invoked in the problems; at best one might derive it roundabout by recognizing that the number of area cubits in an aroura, 9216, is the square of 96, the number of cubits in a schoinion. The liquid measure unit xestes (sextarius), in which i2 requires one to express the capacity of a vat, does not appear in the metrological texts at all, at least so far as they are preserved.

Four problems, a3, b5, c1, and g4, present a metrological enigma. Each of them concerns a long, sticklike shape, whose length is given in cubits while the dimensions defining the cross-section are given in fingers, and the assignment is to find the volume. As an intermediate step, the area of the cross-section (in square fingers) is multiplied by the length (in cubits) to obtain the volume in quasi-units corresponding to a rectangular parallelepiped one finger by one finger by one cubit. This product is then divided by either 192 (b5) or 288 (a3, c1, g4) to obtain a result in unnamed units, with any remainder in “fingers” that are one-twenty-fourth of the
unnamed unit, i.e., the same fraction that a regular linear finger is of a linear cubit, from which one might expect that the unnamed unit is supposed to be the volume cubit. The diagram of g4, G verso 17, makes this identification explicit, if the word πηχος prefixed there to the numerical result represents πῆχεις, “cubits.” The number of finger-by-finger-by-cubits in a volume cubit, however, is 576, so that dividing the interim result in these problems by 192 or 288 amounts to converting into units equivalent respectively to one-third or one-half of a solid cubit. The metretes is indeed one-third of a solid cubit, but it is a unit of liquid measure, whereas the problems in question are clearly about solid objects. The nonstandard system of apparently volumetric units at the end of text h1 (H → 16–19) seems to offer a volumetric spithame equal to half a solid cubit, and a volumetric lichas equal to a third of a solid cubit, but even if this strange system was not just a figment of the text’s writer’s imagination, its fingers are twenty-fourths specifically of the volumetric cubit, so not consistent with the volumetric fingers in the problems.

Problems b3 and f7 illustrate the use of standard coefficients, that is, constants required for the conversion of a magnitude expressed in general units into a number of concrete objects potentially constituting that magnitude, specifically the number of bricks (48) making up one solid cubit, and the number of area cubits of land (4) that would be devoted to a single vine. Like metrological relations, these coefficients were presumably memorized, and they are applied in the final steps of the relevant problems without any explanation of what they are.

13. The Nature of the Codex

What is this codex, and who was responsible for it? Despite all of the information that we have elicited and compiled, the answers to these questions are not immediately obvious. We initially thought that it might be a teacher’s compendium of material, but we have found this hypothesis more and more difficult to defend as we have looked carefully at its characteristics.

(1) As we have seen, we are missing part of the codex and in all likelihood are lacking the opening pages. If there were any indications of authorship or ownership in its original state—and there are often such indications in education-related texts, particularly tablets—they are lost. What remains does not have any obvious organization. If H and I were located at the front, the model documents may have been roughly grouped, but we have no clear evidence that they were, or of what might have preceded them. There is no progression in terms of subject or difficulty, either. This is certainly not a systematic compendium of any sort.

(2) The handwriting is fairly fluent and at times stylish. If the model contracts and the mathematical texts are all by the same hand but in different styles, this suggests someone with enough education to have acquired a reasonable level of skill in writing, but by no means a completely fluent documentary script. Moreover, there is a high level of inconsistency in the writing and the layout does not always make a good impression.

(3) The level of spelling errors is high, and there are many inconsistencies; they do not all trend in the same direction. Most are explicable as the normal phonetic variants found in documentary and epistolary papyri, where people are writing words down as they heard them.
There is no sign of the command of spelling that would have been acquired in a grammarian’s classroom.

(4) More damagingly, there are many errors that are not phonetic, that suggest a failure to understand the text at all. Some interchanges are unparalleled in the papyri, and these are not phonetic interchanges. Some might be failures to understand a character and thus to copy it correctly. Morphology is mediocre at best, and in many instances the case endings are wrong. Again, this is a sign of a lack of any literary education above the most elementary level.

(5) Arithmetic is unfailingly correct, but the skill in computation is often applied in the service of mistaken algorithmic procedures, to the point of nonsense in some cases, as where four dimensions seem to be envisaged. The diagrams attached to the problems are often wrong or even disconnected from the givens of the problems. The writer cannot be said to have made up for a weak literary grounding with a high level of expertise in geometry.

Taking all of these characteristics together, we believe that the most likely explanation is that the codex belonged to a student in a school devoted to training business agents and similar professionals. Much of it may have been copied by eye rather than from dictation, but there are elements of both processes at different points, and the exemplar may itself have been the result of things taken down by ear. It is perfectly conceivable that a student took down information on tablets in the classroom, then made an intended “fair copy” onto papyrus at a later time. Such a process could account for the mixture of types of errors in the codex. It would also be a plausible setting for the habit visible in many places of beginning a section with some ambition for elegant presentation but gradually becoming less and less in command of the writing as time went on and line succeeded line. Such initial ambition followed by tailing off can also be seen in many private letters.

A brief account of numerical literacy is given by Raffaella Cribiore in *Gymnastics of the Mind.* She notes the presence of some basic training in arithmetic in the lower schools, but points to the acquisition of deeper competence in multiplication and fractions in what she calls “specialized scribal schools.” She points in particular to the relatively skilled handwriting of these “apprentice scribes” as evidence that this teaching does not belong to the earliest stages of education. Our codex fleshes out this picture importantly, suggesting that model contracts, metrological information, geometry, and some types of fractional computation all were part of an education that was indeed separate from that of the grammarian and did not presuppose any time in a grammarian’s classroom. At the same time, it suggests that “scribal” is not the best term for an education that aimed to inculcate a number of skills relevant for the administration of land, rents, and taxes, skills that would have been equally relevant for the agents of liturgists.

27. For other apparently didactic manuscripts from late antiquity combining multiple genres such as mathematical problems, arithmetical tables, metrology, and model contracts, see BL Add. MS 41203 (of which only tablet A, containing mathematical problems and isopsephisms, has been published in Skeat 1936, but a complete edition of all four tablets by Julia Lougovaya and Rodney Ast is in preparation), BL Add. MS 33669 (forthcoming, see section 6 above), and P.Yale 4.184–187 (forthcoming). In Add. MS 33669, Todd Hickey observes that the handwriting points to an early stage of education.
who had to assess and collect taxes and the stewards of the estates of large landowners: quite possibly the same people at different times, in fact, as the landowners were the liturgists. The precise nature of this schooling has never been studied in detail, but our codex makes a substantial contribution to the investigation of this subject.

14. Index of Texts

a1 (Ar 1–12) model document: loan of money  
a2 (Ar 14–20) trapezoidal solid (breach in dike, διάκοπος ἐπὶ χώματος)  
a3 (Av 1–7) trapezoidal solid  
a4 (Av 8–9) partition into unit-fractions  
a5 (Av 11–12, Br 1–3) trapezoid  
b2 (Br 4–8) rectangle (field)  
b3 (Br 10–21, Bv 1) complex solid (tower, πύργος, bricks)  
b4 (Bv 2–7) rectangle  
b5 (Bv 8–16) trapezoidal solid  
c1 (Cr 1–12) quadrilateral prism (beam?, ξύλον)  
c2 (Cr 14–21) shipload of wheat  
c3 (Cv 1–9) isosceles triangle  
c4 (Cv 11–12) partition into unit-fractions  
c5 (Cv 13–20) rectangular solid (excavation of river, διῶρυξ ποτάμου)  
d1 (Dr 1–12) isosceles triangle  
d2 (Dr 13–19) speeds of runners  
d3 (Dv 1–17) right-angled triangle  
d4 (Dv 19–25) conical frustum (circular pit, ὄρυγμα στρογγύλον)  
e1 (Er 1–10) conical frustum (circular pit, ὄρυγμα στρογγύλον)  
e2 (Er 11–20) conical frustum (circular pit, ὄρυγμα στρογγύλον)  
e3 (Ev 1–24) metrology: units of length, area, volume  
f1 (Fr 1–5) sale of artabas of wheat  
f2 (Fr 7–18) three granaries  
f3 (Fr 19–21) partition into unit-fractions  
f4 (Fr 22–23) partition into unit-fractions  
f5 (Fv 1–9) quadrilateral (field)  
f6 (Fv 11–17) right-angled triangle  
f7 (Fv 18–22) rectangle (vineyard, χωρίον ἀμπέλιον, vines)  
g1 (Gr 1–20) metrology: units of length  
g2 (Gr 22–28) cylinder (circular naubion, ναύβιον στρογγύλον)  
g3 (Gv 1–9) isosceles triangle  
g4 (Gv 10–17) circular or rectangular solid (circular naubion?, [ναύβιον] γόνατον στρογγύλον)  
m1 (Mr 1–19) metrology: units of length, area, volume, liquid capacity
m2 (Mv 1–15) distribution of artabas of wheat
n1 (Nr 1–10) conical frustum (cistern?)
n2 (Nr 12–17) triangular solid (granary, θησαυρός τρίγωνος)
n3 (Nv 1–7) rectangular solid (granary, θησαυρός, artabas of grain)
n4 (Nv 8–20) complex solid (vaulted granary, θησαυρός καμαρωτός)
o1 (Or 1–7) complex solid (vaulted granary, θησαυρός καμαρωτός)
o2 (Or 8–13) square (field, σφραγίς)
o3 (Or 14–20) trapezoid? (vacant lot, ψιλός)
o4 (Or 22–26) triangle
o5 (Ov 1–11) quadrilateral (field?)
o6 (Ov 12–19) pay
h1 (H → 1–19) metrology: units of length
h2 (H → 21–22) partition into unit-fractions
h3 (H ↓ 1–15) model document: undertaking to lease arable land
i1 (I → 1–14) metrology: units of length, liquid capacity, weight
i2 (I → 16–23) rectangular solid (vat, ληνός)
i3 (I ↓ 1–12) model document: loan of money
i4 (I ↓ 13–14) partition into unit-fractions
i5 (I ↓ 16–18) unidentified

15. Index of Texts by Type

Model Documents.
a1 (Ar 1–12) loan of money
h3 (H ↓ 1–15) undertaking to lease arable land
i3 (I ↓ 1–12) loan of money

Metrology.
c3 (Ev 1–24) units of length, area, volume
g1 (Gr 1–20) units of length
m1 (Mr 1–19) units of length, area, volume, liquid capacity
h1 (H → 1–19) units of length
i1 (I → 1–14) units of length, liquid capacity, weight

Two-dimensional geometry.
a5 (Av 11–12, Br 1–3) trapezoid
b2 (Br 4–8) rectangle (field)
b4 (Bv 2–7) rectangle
c3 (Cv 1–9) isosceles triangle
d1 (Dr 1–12) isosceles triangle
d3 (Dv 1–17) right-angled triangle
f5 (Fv 1–9) quadrilateral (field)
f6 (Fv 11–17) right-angled triangle
f7 (Fv 18–22) rectangle (vineyard, χωρίον ἀμπέλιον, vines)
g3 (Gv 1–9) isosceles triangle
o2 (Or 8–13) square (field, σφραγίς)
o3 (Or 14–20) trapezoid? (vacant lot, ψιλός)
o4 (Or 22–26) triangle
o5 (Ov 1–11) quadrilateral (field?)

**Three-dimensional geometry.**
a2 (Ar 14–20) trapezoidal solid (breach in dike, διάκοπος ἐπὶ χώματος)
a3 (Av 1–7) trapezoidal solid
b3 (Br 10–21, Bv 1) complex solid (tower, πύργος, bricks)
b5 (Bv 8–16) trapezoidal solid
c1 (Cr 1–12) quadrilateral prism (beam?, ξύλον)
c5 ( Cv 13–20) rectangular solid (excavation of river, διώρυξ ποτάμου)
d4 (Dv 19–25) conical frustrum (circular pit, ὄρυγμα στρογγύλον)
e1 (Er 1–10) conical frustrum (circular pit, ὄρυγμα στρογγύλον)
e2 (Er 11–20) conical frustrum (circular pit, ὄρυγμα στρογγύλον)
g2 (Gr 22–28) cylinder (circular naubion, ναύβιον στρογγύλον)
g4 (Gv 10–17) circular or rectangular solid (circular naubion?, [ναύβιο]ν? στρογγύλον)
n1 (Nr 1–10) conical frustrum (cistern?)
n2 (Nr 12–17) triangular solid (granary, θησαυρὸς τρίγωνος)
n3 (Nv 1–7) rectangular solid (granary, θησαυρός, artabas of grain)
n4 (Nv 8–20) complex solid (vaulted granary, θησαυρός καμαρωτός)
o1 (Or 1–7) complex solid (vaulted granary, θησαυρός καμαρωτός)
i2 (I →16–23) rectangular solid (vat, ληνός)

**Distribution of shares.**
c2 (Cr 14–21) shipload of wheat
m2 (Mv 1–15) distribution of artabas of wheat

**Miscellaneous problems.**
d2 (Dr 13–19) speeds of runners
f1 (Fr 1–5) sale of artabas of wheat
f2 (Fr 7–18) three granaries
o6 (Ov 12–19) pay
i5 (I ↓16–18) unidentified
Introduction

Partition into unit-fractions.

a4 (Av 8–9)
c4 (Cv 11–12)
f3 (Fr 19–21)
f4 (Fr 22–23)
h2 (H → 21–22)
i4 (I ↓ 13–14)

16. Note on Editorial Procedure

Texts in this volume are presented according to the usual papyrological practices. The following signs have their usual meanings:

( ) Resolution of an abbreviation or a symbol
[ ] Lacuna in the text
< > Letters omitted by the scribe
{ } Letters erroneously written by the scribe
[ ′ ′ ] Letters written, then cancelled, by the scribe
 αβγδε Letters the reading of which is uncertain or would be uncertain outside the context
...... Letters remaining in part or in whole which have not been read
[ ± 5 ] Approximate number of letters lost in a lacuna and not restored
 \ Letters inserted above the line by the scribe

Italicized letters on E verso were read from the photograph referred to above, Introduction section 1 at note 2. In general, abbreviations are resolved in the text. Fractions printed are d = ¼ and s = ½. The line notes correct non-standard Greek. Errors of case and varieties of spelling are very numerous.