Pile Supported Foundation Design

By

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Table of Contents

I. Introduction to Elements of Pile Supported Foundation Design
II. Allowable Soil Pressures
III. Brief Discussion on Commonly Used Load Bearing Pilings
IV. Pile Cap Design
   (A) Pile Load Calculation
   (B) Flexural Reinforcement
   (C) The equations for Punching Shear for Column or Piles
   (D) Direct Shear
   (E) Deep Beam Shear
   (G) Dowels Required Between Cast-in-place Column or Pedestal and Foundation
V. Pile Cap Support Conditions
VI. Critical Section & Pile Location
VII. Uplift on Piles
VIII. Horizontal Forces on Pile Caps
I. Introduction to Elements of Pile Supported Foundation Design

The various types of loads produced from buildings, bridges, or any other structure must be transmitted to the soil through foundations. Because soil bearing pressures are significantly lower than the compressive stresses of steel, concrete, masonry columns or walls, foundations must be used to reduce the pressure applied directly to the soil by spreading the column or wall load over an area large enough such that the soil bearing pressure is not exceeded. When the area required to support the load using a shallow foundation is found to be inadequate an alternative which is frequently used is a deep foundation or pile supported foundation. A pile supported foundation is designed with the presumption that the bottom surface of the pile cap receives no support from the underling soil. A comprehensive foundation design involves both a geotechnical study of the soil conditions to determine the most suitable type of deep foundation and a structural design to determine the proportions of the piling and foundation elements. Pilings provide resistance to downward vertical loads through either bearing pressure at the base or through “skin” friction along the perimeter of the length or both (See Figure 1 below). Pilings provide resistance to upward vertical loads through “skin” friction along the perimeter of the length (See Figure 2 below). Lateral load resistance by the pilings are provided by the width or diameter on one side of the piling from the upper portion and the opposite side from the lower portion of the length of the pile (See Figure 3 below). There are dozens of types of piles (See Figure 4 next page), each having advantages and disadvantages for a given set of conditions.
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Fig. 4
II. Allowable Soil Pressures

Values of allowable soil pressures for various types of soils are usually specified in building codes. The table below is from the Florida 2010 Building Code.

**TABLE R401.4.1**

<table>
<thead>
<tr>
<th>CLASS OF MATERIAL</th>
<th>LOAD-BEARING PRESSURE (pounds per sq. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystalline bedrock</td>
<td>12,000</td>
</tr>
<tr>
<td>Sedimentary and foliated rock</td>
<td>4,000</td>
</tr>
<tr>
<td>Sandy gravel and/or gravel (GW and GP)</td>
<td>3,000</td>
</tr>
<tr>
<td>Sand, silty sand, clayey sand, silty gravel and clayey gravel (SW, SP, SM, SC, GM and GC)</td>
<td>2,000</td>
</tr>
<tr>
<td>Clay, sandy clay, silty clay, clayey silt, silt and sandy silt (CL, ML, MH and CH)</td>
<td>1,500</td>
</tr>
</tbody>
</table>
III. Brief Discussion on Commonly Used Load Bearing Pilings

a) Timber Piles

Timber pilings are the oldest known pilings. The Neolithic tribes in what is now Switzerland placed logs vertically into soft or unsuitable soil for structural support around 6,000 years ago. Timber piles are still widely used today for industrial structures, commercial structures, bridges and marine structures, just to name a few of the applications. Timber piles can resist vertical downward loads from skin friction and bearing as well as uplift loads and lateral loads. The natural taper angle (typically about 1 in. change in diameter per 10 feet of length) of a timber pile increases the friction reaction and is recognized in the design formula. Timber piles lengths typically range from 40 feet to 70 feet in length and have been successfully load tested in excess of 200 Kips.

**Advantages:**
- Low Cost
- Tapered Section provides higher resistance in granular soils than uniform piles
- Easy to drive
- Can be used as friction or end bearing pile

**Disadvantages:**
- Difficult to splice
- Low axial capacity

For deep foundation evaluations the services of a geotechnical engineer are almost always required as soil borings are necessary to determine shear strength and bearing capacity of the soil. The table shown above provides some values for bearing capacity for different types of soils. The USDA classifies soil types according to a soil texture triangle chart which gives names to various combinations of clay, sand, and silt.
b) Composite Piles

Composite Piles are piles of two different materials which are driven one over the other, so as to enable them to act as a single pile. In such a combination, advantage is taken of the good qualities of both the materials. These prove economical as they permit the utilization of the great corrosion resistance property of one material with the cheapness or strength of the other.

**Advantages:** The advantages that composite piles have over traditional material piles include their higher strength to weight ratio, corrosive resistance, durability and their immunity to decay and deterioration in marine environments.

**Disadvantages:** The disadvantages of composite piles relate to the cost of production, installation and long-term structural performance. Since these piles are relatively new to the civil engineering industry, manufacturers have yet to find cheap production methods, making these piles generally more expensive than traditional piles.
c) Steel H Piles

Steel H-piles may be obtained in a wide variety of sizes and lengths and may be easily handled, spliced, and cut off as required. H-piles displace little soil and are relatively easy to drive.

**Advantages:** The advantages of Steel H-piles are there high axial working capacity, 400+ kips. They can penetrate obstacles better than most piles, with less damage to the pile from the obstacle or from driving.

**Disadvantages:** The disadvantages of steel H-piles are the high material costs for steel and possible long delivery time. H-piles may also be subject to excessive corrosion in certain environments. Pile shoes are required when driving in dense sand strata, gravel strata, cobble-boulder zones, and when driving piles to refusal on a hard layer of bedrock.
IV. Pile Cap Design

Pile Spacing and Layout Pattern

For the load on each individual pile to be transferred effectively to soil the piles should be spaced at a minimum of 3 times the pile diameter. If the spacing is less than 3 times of diameter, pile group settlement and bearing capacity should be checked.

Symmetry is also an important consideration when placing piles with respect to the location of the load and the shape of the pile cap so as to transfer the load as evenly as possible onto each pile. Examples of pile layout patterns are shown below:
(A) Pile Load Calculation

Figure 5 below depicts the loading which the piling and the pile cap may be required to resist. The pile load can be calculated as

\[ P_i = \frac{P_z}{n} + \frac{M_x d_x}{I_x} + \frac{M_y d_y}{I_y} \]

where \( P_i \) is the axial load for an individual pile
\( P_z \) is column load
\( M_x \) and \( M_y \) are moment from column and/or from eccentricity between center of column and center of pile group
\( n \) is total number of piles
\( d_x \) and \( d_y \) are x and y distance from center of pile group
\( I_x \) and \( I_y \) are moment of inertia of pile group in x and y directions

\( I_x \) is the sum of the \( d_{yi}^2 \) (\( I_x = \sum d_{xi}^2 \))
\( I_y \) is the sum of the \( d_{yi}^2 \) (\( I_y = \sum d_{yi}^2 \))

The procedure for the design of a pile cap can be most easily be presented by using an example.

**Design Data:**
- Column dead load: \( P_D = 175 \) kip
- Column live load: \( P_L = 50 \) kip
- Column dead load moment: \( M_{Dy} = 20 \) ft-kip
- Column Live load moment: \( M_{Ly} = 25 \) ft-kip
- Column size: 12”x12” concrete column
- Pile: 8 in diameter pile
- Ultimate pile compression capacity: \( P_{cu} = 120 \) kip;
- Using Safety Factor of 2.0 yields allowable pile compression capacity: \( P_{ca} = 60 \) kip
- Compressive strength of concrete: \( f_{c'} = 4,000 \) psi
- Tensile strength of reinforcing steel: \( f_y = 60,000 \) psi
Solution:
1. Estimate number of pile and select pile layout pattern

Total allowable pile vertical load: \( P_z = P_D + P_L = 225 \) kip

Estimate number of pile: \( n = \frac{P_z}{P_{ca}} = \frac{225}{60} = 3.75 \)

The pile cap weight plus the effect of the moments \( M_{Dy} + M_{Ly} \) will increase the load on piles, therefore, a six-pile layout pattern \( (n = 6) \) will be attempted for the design.

Minimum spacing of pile: \( s = 10 \text{ in} \times 3 = 30 \text{ in} \); use 36 in

A six-pile layout pattern should be adequate for the design. Figure 6 below shows the pile spacing and the pile cap dimensions, note that the depth of the pile cap is an assumed depth and the edge distance is normally governed by punching shear capacity of corner piles. The thickness of the Pile cap is normally determined by the required shear strength. It is common practice for the designer to increase the pile caps thickness to provide adequate shear capacity rather than include rebar for shear reinforcement. For smaller pile caps, the thickness is normally governed by deep beam shear. For large pile cap, the thickness is governed by direct shear. When necessary, shear reinforcement may be used to reduce the thickness of the pile cap. ACI318-14, Section 13.4.2.1 states that “Overall depth of pile cap shall be selected such that the effective depth of bottom reinforcement is at least 12 in.”

Note: All Tables included in this course are from ACI 318-14, for reference to notes in Tables see Building Code Requirements for Structural Concrete (ACI 318-14), hereafter referred to as “The Code”. This course does not address Earthquake and Seismic design considerations.

A pile cap thickness of 2'-0 will be used as a starting point for this design example. The top of piles are at 6" above bottom of pile cap and the reinforcement is at 2" above top of piles, hence, the effective depth is \( h = 24 \text{ in.} - 6 \text{ in.} - 2 \text{ in.} = 16 \text{ in.} \).
Fig. 6
Check unfactored pile capacity:

d_{y1} = -3 \text{ ft}
d_{y2} = -3 \text{ ft}
d_{y3} = 0 \text{ ft}
d_{y4} = 0 \text{ ft}
d_{y5} = 3 \text{ ft}
d_{y6} = 3 \text{ ft}

I_y = d_{y1}^2 + d_{y2}^2 + d_{y3}^2 + d_{y4}^2 + d_{y5}^2 + d_{y6}^2 = 36 \text{ ft}^2.

**Column allowable load moment:**

M_X = 0 \text{ ft-kip}; \quad M_Y = M_{Dy} + M_{Ly} = 45 \text{ ft-kip}

**Maximum unfactored pile compression load:**

The equation for determining the load on any pile is shown below

\[ p_i = \frac{P_z}{n} + \frac{M_X d_x}{I_x} + \frac{M_Y d_y}{I_y} \]

\[ p_1 = \frac{225}{6} + \frac{45x-3}{36} = 33.75 \text{ Kips}; \quad p_2 = \frac{225}{6} + \frac{45x-3}{36} = 33.75 \text{ Kips}; \quad p_3 = \frac{225}{6} + \frac{45x0}{36} = 37.50 \text{ Kips} \]

\[ p_4 = \frac{225}{6} + \frac{45x0}{36} = 37.50 \text{ Kips}; \quad p_5 = \frac{225}{6} + \frac{45x3}{36} =41.25 \text{ Kips}; \quad p_6 = \frac{225}{6} + \frac{45x3}{36} = 41.25 \text{ Kips} \]

The allowable pile compression capacity \( P_{ca} = 60 \text{ kip} \) which is > the maximum unfactored pile comp. load of 41.25 kips

To determine the shear capacity the factored column loads are required.
Table 5.3.1 shown below shows the Load factors and combinations.

### Table 5.3.1 Load combinations

<table>
<thead>
<tr>
<th>Load combinations</th>
<th>Equation</th>
<th>Primary load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U = 1.4D$</td>
<td>(5.3.1 a)</td>
<td>$D$</td>
</tr>
<tr>
<td>$U = 1.2D + 1.6L + 0.5(L_r, or S or R)$</td>
<td>(5.3.1 b)</td>
<td>$L$</td>
</tr>
<tr>
<td>$U = 1.2D + 1.6(L_r, or S or R) + (1.0L or 0.5W)$</td>
<td>(5.3.1 c)</td>
<td>$L_r, or S or R$</td>
</tr>
<tr>
<td>$U = 1.2D + 1.0W + 1.0L + 0.5(L_r, or S or R)$</td>
<td>(5.3.1 d)</td>
<td>$W$</td>
</tr>
<tr>
<td>$U = 1.2D + 1.0E + 1.0L + 0.2S$</td>
<td>(5.3.1 e)</td>
<td>$E$</td>
</tr>
<tr>
<td>$U = 0.9D + 1.0W$</td>
<td>(5.3.1 f)</td>
<td>$W$</td>
</tr>
<tr>
<td>$U = 0.9D + 1.0E$</td>
<td>(5.3.1 g)</td>
<td>$E$</td>
</tr>
</tbody>
</table>

Equation 5.3.1 b [$U = 1.2D + 1.6L + 0.5(L_r, or S or R)$] will be used in this design example

Where,

- $D =$ unfactored dead load
- $E =$ effect of horizontal and vertical earthquake-induced forces
- $L =$ live load
- $L_r =$ roof live load
- $R =$ rain load
- $S =$ snow load
- $W =$ wind load

Where,

- Column dead load: $P_D = 175$ kip
- Column live load: $P_L = 50$ kip
- Column dead load moment: $M_{Dy} = 20$ ft-kip
- Column Live load moment: $M_{Ly} = 25$ ft-kip

Factored column load $P_u = 1.2(175) + 1.6(50) = \textbf{290 kips}$

and

Factored column moment $M_u = 1.2(20) + 1.6(25) = \textbf{64 ft-kips}$
Maximum factored pile compression load (ultimate pile load):

\[ p_1 = \frac{290}{6} + \frac{64x-3}{36} = 43.0 \text{ Kips}; \quad p_2 = \frac{290}{6} + \frac{64x-3}{36} = 43.0 \text{ Kips}; \quad p_3 = \frac{290}{6} + \frac{64x0}{36} = 48.33 \text{ Kips} \]

\[ p_4 = \frac{290}{6} + \frac{64x0}{36} = 48.33 \text{ Kips}; \quad p_5 = \frac{290}{6} + \frac{64x3}{36} = 53.67 \text{ Kips}; \quad p_6 = \frac{290}{6} + \frac{64x3}{36} = 53.67 \text{ Kips} \]

The ultimate pile compression capacity \( P_{cu} = 120 \text{ kip} \) which is \( > \) the maximum factored pile comp. load of 53.67 kips.

(B) Flexural Reinforcement

To determine the area of reinforcement steel, five quantities are required in addition to the value of the moment in either the short or long direction:

From Table 21.2.2 \( \phi = 0.90, f'c = 4,000 \text{ psi}, f_y = 60,000 \text{ psi} \),
b = 72 inches for the long direction reinforcement steel and
d = 16 inches the solution of the quadratic equation shown below will provide the solution for \( \rho \),

where \( \rho \) represents the ratio of the \( \frac{\text{area of steel}}{\text{area of concrete}} \)

the value of \( M_u \) in the long direction = 2.5 ft x 117.3 kips

\[ = 293.3 \text{ ft-kips} \] (See Figure 8)
\[ Ru = \frac{M_u}{b d^2} = \frac{293.3 \times 12,000}{0.9 \times 72 \times 16^2} = 212.2 \text{ psi} \]

\[ \rho = \left( \frac{0.85 f'c}{f_y} \right) \left( 1 - \sqrt{1 - \frac{2 Ru}{0.85 f'c}} \right) \]

\[ \rho = \left( \frac{0.85 \times 4,000}{60,000} \right) \left( 1 - \sqrt{1 - \frac{2 \times 212.2}{0.85 \times 4,000}} \right) = 0.00365 \]

\[ A_s = \rho \times b \times d = 0.00365 \times 72 \times 16 = 4.20 \text{ in}^2 \]

The minimum allowable value for \( \rho \) is \( \rho_{\text{min}} \) which is equal to the greater of

(a) \[ \frac{3 \sqrt{f'/c}}{f_y} = \frac{3 \sqrt{4,000}}{60,000} = 0.00316 \]
(b) \[ \frac{200}{f_y} = \frac{200}{60,000} = 0.0033 \]

\[ A_{s,\text{min}} = \rho_{\text{min}} \times b \times d = 0.0033 \times 72 \text{ in.} \times 16 \text{ in.} = 3.80 \text{ in}^2 \]

\( A_{s,\text{min}} \) is less than the calculated \( A_s \), therefore, the required area of reinforcement steel in the long direction is 4.20 in\(^2\).
Checking Temperature and Shrinkage reinforcement area for gross area of pile cap from ACI 318-14 Table 24.4.3.2 shown below results in $A_{s,\text{min}} = 0.0018 \times 72 \text{ in.} \times 24 \text{ in.} = 3.11 \text{ in}^2$

Table 24.4.3.2-Minimum ratios of deformed shrinkage and temperature reinforcement area to gross area

<table>
<thead>
<tr>
<th>Reinforcement Type</th>
<th>$f_y$, psi</th>
<th>Minimum reinforcement ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformed bars</td>
<td>&lt; 60,000</td>
<td>0.0020</td>
</tr>
<tr>
<td>Deformed bars or welded wire reinforcement</td>
<td>$\geq 60,000$</td>
<td>Greater of: $\frac{0.0018 \times 60,000}{f_y}$</td>
</tr>
<tr>
<td></td>
<td>$\geq 60,000$</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Use the $A_s$ value of 4.20 in$^2$ of reinforcing steel in the long direction of the pile cap. To determine the area of reinforcement steel in the short direction:

\[ \phi = 0.90, f'_c = 4,000 \text{ psi}, f_y = 60,000 \text{ psi} , b = 108 \text{ inches for the short direction reinforcement steel and } d = 16 \text{ inches} \]

The value of $M_u$ in the short direction = 1.0 ft x (43.0 + 48.3 + 53.7) kips = 145.0 ft-kips (See Figure 9)

\[ R_u = \frac{M_u}{\phi b d^2} = \frac{145 \times 12,000}{0.9 \times 108 \times 16^2} = 69.9 \text{ psi} \]
\[ \rho = \left( \frac{0.85 f'c}{f_y} \right) \left( 1 - \sqrt{1 - \frac{2 Ru}{0.85 f'c}} \right) \]

\[ \rho = \left( \frac{0.85 \times 4,000}{60,000} \right) \left( 1 - \sqrt{1 - \frac{2 \times 69.9}{0.85 \times 4,000}} \right) = 0.00118 \]

\[ A_s = \rho \times b \times d = 0.00118 \times 108 \times 16 = 2.04 \text{ in}^2 \]

\[ A_{s,\text{min}} = \rho_{\text{min}} \times b \times d = 0.0033 \times 108 \times 16 \text{ in.} = 5.70 \text{ in}^2 \]

\( A_{s,\text{min}} \) is greater than the calculated \( A_s \), therefore, the required area of reinforcement steel in the short direction is 5.70 in\(^2\).

Checking Temperature and Shrinkage reinforcement area for gross area of footing from ACI 318-14 Table 24.4.3.2 results in \( A_{s,\text{min}} = 0.0018 \times 108 \times 24 \text{ in.} = 4.67 \text{ in}^2 \)

Use the \( A_s \) value of 5.70 in\(^2\) of reinforcing steel in the short direction of the pile cap.

The flexural reinforcement steel requirement for the long direction is 4.20 in\(^2\) and for the short direction is 5.70 in\(^2\).

Figure 10 below shows that for the long direction flexural reinforcement steel the development length, \( l_d = 45 \text{ in.} \) and for the short direction flexural reinforcement steel the development length, \( l_d = 27 \text{ in.} \). Typically, the minimum spacing for reinforcement steel or rebar in a pile cap is 6 in. on center (OC), although, this is not a requirement of the code. If the spacing of the rebar is excessive there is a higher propensity for cracking to occur. When practical, it is best to use a larger number of smaller diameter rebar than visa-versa.
Using ACI 318-14 Tables 25.4.2.2 and 25.4.2.4 for development length, $l_d$ for $\lambda = 1.0$ for Normal weight concrete, $\Psi_e = 1.0$ for Uncoated or zinc-coated (galvanized) reinforcement, $\Psi_f = 1.0$ for “Other”, $\sqrt{f'c} = \sqrt{f_{y}/4,000} = 63.25$ and $f_{y} = 60,000$ yields the following table for development length of #3 through #11 rebar.

<table>
<thead>
<tr>
<th>Rebar Size</th>
<th>$l_d$ (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>14</td>
</tr>
<tr>
<td>#4</td>
<td>19</td>
</tr>
<tr>
<td>#5</td>
<td>24</td>
</tr>
<tr>
<td>#6</td>
<td>43</td>
</tr>
<tr>
<td>#7</td>
<td>62</td>
</tr>
<tr>
<td>#8</td>
<td>71</td>
</tr>
<tr>
<td>#9</td>
<td>80</td>
</tr>
<tr>
<td>#10</td>
<td>89</td>
</tr>
<tr>
<td>#11</td>
<td>98</td>
</tr>
</tbody>
</table>

From the table to the right for the rebar in the short direction which is limited to a development length of 27 in., the maximum size rebar which provides the full tension capacity is a #5 rebar. The #5 rebar has an area of 0.31 in$^2$, the required area for the flexural reinforcement steel in the short direction is 5.70 in$^2$. Therefore, 5.70 in$^2$/0.31 in$^2 = 18.4$ or 19 #5 rebars spaced @ 5½” OC.

For the rebar in the long direction which is limited to a development length of 45 in., the maximum size rebar which provides the full tension capacity is a #6 rebar. The #6 rebar has an area of 0.44 in$^2$, the required area for the flexural reinforcement steel in the long direction is 4.20 in$^2$. Therefore, 4.20 in$^2$/0.44 in$^2 = 9.5$ or 10 #6 rebars spaced @ 6½” OC.

Note: The use of 90° or 180° hooks could also be used to provide the required development length.
Figure 11 shows the rebar layout for the pile cap.
(C) The equations for Punching Shear for Column or Piles

If the punching shear on a column or pedestal results from axial forces without lateral forces and/or moment(s), the calculation is based on the length of the perimeter at a distance of d/2 from the column as indicated by ①.

The equations in Table 22.6.5.2 on page 24 provide the value for punching shear at the critical section, \( b_0 \) at the column location and is the perimeter depicted by ① in Figure 12.

The equations in Table 22.6.5.2 on page 24 provide the value for punching shear at the critical section, \( b_0 \) at the pile location and is the perimeter depicted by ② in Figure 12.

The value \( \phi \) \( v_c \) is the shear capacity for the column or piling which the concrete can resist in units of Lbs or Kips and the value for \( \phi \) from Table 21.2.1 for Strength Reduction Factors is 0.75.

To determine the punching shear for an interior column or pedestal which is resisting lateral forces and/or moment(s), Section 8.4.2.3.2 States that "The fraction of factored slab moment resisted by the column, \( \gamma_f M_{sc} \), shall be assumed to be transferred by flexure, where \( \gamma_f \) shall be calculated by:"

\[
\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} \quad (8.4.2.3.2)
\]

where, \( b_1 = c_1 + d \) and \( b_2 = c_2 + d \) in the figure R8.4.4.2.3 below

Section 8.4.4.2.2 States that "The fraction of \( M_{sc} \) transferred by eccentricity of shear, \( \gamma_v M_{sc} \), shall be applied at the centroid of the critical section in accordance with 8.4.4.1, where:"

\[
\gamma_v = 1 - \gamma_f \quad (8.4.4.2.2)
\]
For a square interior column or pedestal equation (8.4.2.3.2) results provide that 60% of the moment is resisted by flexure and equation (8.4.4.2.2) results provide that 40% is resisted by shear. Combining the two above equations gives

$$\gamma_v' = 1 - \gamma_f' = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = 0.40$$

As shown in Figure R8.4.4.2.3, the shear stress resulting from moment transfer by the eccentricity of the shear varies linearly about the axis C-C which is the centroid of the critical section. The maximum shear stress due to the factored shear force and moment is:

$$\nu_u = \frac{V_u}{A_c} + \frac{\gamma_V M_u (b_{1/2})}{J_c}$$  (Note: $P_u = V_u = 290$ kips)

$A_c$ = the area of the critical surface which is shown in the isometric graphic on the right side of R8.4.4.2.3 above and $A_c = b_0d = 2(b_1+b_2)d$ where, $b_0 = 2(b_1+b_2)$

$J_c$ = is a property analogous to the polar moment of inertia for the area $A_c$

$$J_c = \frac{db_1^3}{6} + \frac{d^3b_1}{6} + \frac{db_2b_1^2}{2}$$

† Figure R8.4.4.2.3 used with permission from ACI
The maximum shear stress $v_u$ must not exceed the critical permissible shear stress $\phi V_n$ given by

$$v_u \leq \frac{\phi V_c}{b_o d}$$

in this example

d=16 in, $b_1 = b_2 = 12$ in + 16 in = 28 in, then $b_o = 2(b_1 + b_2) = 2(28+28) = 112$ in, Therefore, $A_c = 112 \times 16 = 1,792$ in$^2$

$$J_c = \frac{16 \times 28^3}{6} + \frac{16^3 \times 28}{6} + \frac{16 \times 28 \times 28^2}{2} = 58,539 + 19,115 + 175,616 = 253,270$ in$^4$

Therefore, $v_u = \frac{V_u}{A_c} + \frac{\gamma v M_u (b_1/2)}{J_c} = \frac{290}{1,792} + \frac{0.4 \times 64 \times 12 \times (28/2)}{253,270} = 0.1618 + 0.0170 = 0.179$ ksi or 179 psi
Table 22.6.5.2- Calculation of $V_c$ for two-way shear

<table>
<thead>
<tr>
<th>$V_c$</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4\lambda\sqrt{f'_c}$</td>
<td>(a)</td>
</tr>
<tr>
<td>$\left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f'_c}$</td>
<td>(b)</td>
</tr>
<tr>
<td>$\left(2 + \frac{\alpha_s d}{b_o}\right)\lambda\sqrt{f'_c}$</td>
<td>(c)</td>
</tr>
</tbody>
</table>

Note: $\beta$ is the ratio of long side to short side of the column, concentrated load, or reaction area and $\alpha_s$ is given in 22.6.5.3.

From Table 22.6.5.2 above for a square interior column $\lambda=1$, $\alpha_s=40$, $b_o=112$ in and $\lambda=1.0$

The coefficient of equation (a) from Table 22.6.5.2 = $4\lambda = 4$

The coefficient of equation (b) from Table 22.6.5.2 = $\left(2 + \frac{4}{\beta}\right)\lambda = \left(2 + \frac{4}{1}\right)x1 = 6$

The coefficient of equation (c) from Table 22.6.5.2 = $\left(2 + \frac{\alpha_s d}{b_o}\right)\lambda = \left(2 + \frac{40 \times 16}{112}\right)x1 = 7.7$

The coefficient of equation (a) controls as it is the least value, hence

$$\phi_V n = \frac{\phi V_c}{b_o d} = (0.75) 4\sqrt{f'_c} = (0.75) 4\sqrt{4,000} = 189 \text{ psi}$$

$$\nu_n = 179 \text{ psi} < \phi_V V_c = 189 \text{ psi}, \text{ OK}$$

The pile cap has the shear capacity to resist a factored axial load of 290 kips and a moment of 64 ft-kips on the column or pedestal.
Checking the shear capacity of the pile cap for the piling for equations (a) from Table 22.6.5.2 above yields the piling punching shear value for $\phi \nu_c$ using normal weight concrete for equation (a), however, if there is no reinforcement in the top of the pile cap, then for the consideration of pile punching the slab is considered unreinforced. The value for $\phi$ from Table 21.2.1 for Strength Reduction Factors for “plain concrete elements” is 0.60.

$$\phi \nu_c = (0.60) 4\sqrt{f'c}$$

The punching shear value for $\phi\nu_c$ using normal weight concrete and square or round piles, plain concrete and for $\beta = 1$ for equation (b) from Table 22.6.5.2 above is:

$$\phi \nu_c = (0.60) \left(2 + \frac{4}{\beta}\right) \sqrt{f'c} b o d ; \text{ the coefficient } \left(2 + \frac{4}{\beta}\right) = 6$$

The punching shear value for $\phi \nu_c$ using normal weight concrete and edge piles for equation (c) from Table 22.6.5.2 above is:

$$\phi \nu_c = (0.60) \left(2 + \frac{30d}{b o}\right) \sqrt{f'c} b o d , \text{ From Figure 13 below, the } b o \text{ for the edge pile shown about piling “p3” = 74 in.}$$

$$\nu_c = (0.60) \left(2 + \frac{30 \times 16}{74}\right) \sqrt{4,000} (74 \times 16) = 635,491 \text{ Lbs or 635 kips}$$

$$\nu_u = 48.33 \text{ kips } < \phi\nu_c = 635 \text{ Kips, OK}$$
The punching shear value for $\phi \, v_c$ using normal weight concrete and corner piles for equation (c) from Table 22.6.5.2 above is:

$$\phi \, v_c = (0.60) \left( 2 + \frac{20d}{b_o} \right) \sqrt{f'_c \, b_o \, d},$$

From Figure 13 below, the $b_o$ for the corner pile shown about piling “p5” = 55 in.

$$\phi \, v_c = (0.60) \left( 2 + \frac{20 \times 16}{55} \right) \sqrt{4,000} \, (55 \times 16) = 348,104 \text{ Lbs or 348 kips}$$

$v_u = 53.67 \text{ kips} < \phi \, v_c = 348 \text{ Kips, OK}$
(D) Direct Shear
The direct shear at the critical section adjacent to the column is located at the distance $d$ from the column face and is depicted by the vertical dashed line in Figure 14. The direct shear at the critical section adjacent to the corner pile is located at the distance $d$ from the corner pile and is depicted by the diagonal dashed line in Figure 14.

Section 22.5.5.1 in the Code states that: “For nonprestressed members without axial force, $V_c$ shall be calculated by:"

$$V_c = 2\sqrt{f'c} \cdot b_w \cdot d$$

"unless a more detailed calculation is made in accordance with Table 22.5.5.1", Table 22.5.5.1 is shown below.

**Table 22.5.5.1- Detailed Method for Calculating $V_c$**

<table>
<thead>
<tr>
<th>$V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least of (a), (b), and (c)</td>
</tr>
<tr>
<td>$\left(1.9\lambda\sqrt{f'c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w \cdot d$</td>
</tr>
<tr>
<td>$\left(1.9\lambda\sqrt{f'c} + 2500\rho_w\right) b_w \cdot d$</td>
</tr>
<tr>
<td>$3.5\sqrt{f'c} \cdot b_w \cdot d$</td>
</tr>
</tbody>
</table>

[1] $M_u$ occurs simultaneously with $V_u$ at the section considered.

For the direct shear at the critical section adjacent to the column the value of $b_w = 72$ in. and $d = 16$in.
\[ \phi V_c = \phi 2 \sqrt{f'c} b_w d = (0.75) 2 \sqrt{4,000} \ (72 \times 16) = 109,288 \text{ Lbs or } 109.3 \text{ kips}; \]

\( V_u = \) the sum of the factored pile loads for pile \( p_5 + p_6 \)

(partial weight of pile cap is not considered in this example)

\[ V_u = 2(53.67) = 107.3 \text{ kips} \]

\[ V_u = 107.3 \text{ Kips} < \phi V_c = 109.3 \text{ Kips}, \text{ OK} \]

For the direct shear at the critical section adjacent to the corner pile the value of \( b_w \approx 91 \text{ in.} \) and \( d = 16\text{in.} \)

By inspection it is obvious that the direct shear for the corner pile is adequate because the \( b_w \) value (91 in.) is greater than the \( b_w \) value (72 in.) for the column and there is only one pile load (\( p_5 \)) for the critical section to resist.

The Detailed Method for Calculating \( V_c \) can be used to evaluate the direct shear, however, as stated in the Commentary above, the value for \( \rho_w \) (the reinforcement ratio) and \( (V_u d/M_U) \) at the section considered must be known.

Typically, the equation 22.5.5.1, \( V_c = 2\lambda \sqrt{f'c} b_w d \) is used.
(E) Deep Beam Shear

Deep beam shear is evaluated at the face of column when $\omega < d$ and $\frac{V_u d}{M_u} \geq 1$, both of the two criteria must be meet for deep beam shear evaluation.

Figure 15 below shows that deep beam shear should be investigated for the longitudinal direction based on the first criteria of $\omega < d$ where

$\omega = 12$ in. and $d = 16$ in.

The evaluation of the second criteria $\frac{V_u d}{M_u} \geq 1$ for

$V_u = (p_1 + p_2 + p_3) = (43.00 + 48.33 + 53.67) = 145$ kips

and for

$M_u = \omega (p_1 + p_2 + p_3) = 12(43.00 + 48.33 + 53.67) = 1,740$ in-kips

$\frac{V_u d}{M_u} = \frac{145 \times 16}{1,740} = 1.333 \geq 1$

Therefore, both criteria for deep beam shear evaluation have been met.

The critical location where shear is checked is taken at the face of the support when $d > \omega$ and the shear strength is calculated as follows:

$\phi V_c = \phi \left( \frac{d}{\omega} \right) \left\{ 3.5 - 2.5 \left( \frac{M_u}{V_u d} \right) \left[ (1.9 \lambda \sqrt{f'c} + 2500 \rho_w V_u d M_u) \right] \right\} \leq \phi 10 \sqrt{f'c}$

The above equation is from CRSI Design Handbook 2008 Eqn 13-2 on P.13-26
where;

\[ 1.0 \geq \frac{M_u}{V_u d} > 0, \text{ hence, } 1.0 \geq \frac{1,740}{145 \times 16} = 0.75 > 0 \quad \text{OK} \]

and \[ \frac{V_u d}{M_u} = 1.333 \geq 1 \quad \text{OK} \]

The value for \( \rho_w \) (the reinforcement ratio) would have to be calculated to use the equation above, however, as stated in the Commentary on page 27 “For most designs, it is convenient to assume that the second term in expression (a) and (b) of Table 22.5.5.1 equals \( 0.1 \lambda \sqrt{f'c} \), which yields the following equation for normal weight concrete:

\[
\phi v_c = \phi \left( \frac{d}{\omega} \right) \left[ 3.5 - 2.5 \left( \frac{M_u}{V_u d} \right) \right] \left[ \left( 1.9 \sqrt{f'c} + 0.1 \sqrt{f'c} \frac{V_u d}{M_u} \right) \right] b_w d \leq \phi 10 \sqrt{f'c} \ b_w d
\]

\[
\phi v_c = 0.75 \left( \frac{16}{12} \right) \left[ 3.5 - 2.5 \left( \frac{1,740}{145 \times 16} \right) \right] \left[ 1.9 \sqrt{4,000} + 0.1 \sqrt{4,000} \left( \frac{145 \times 16}{1,740} \right) \right] 72 \times 16 = 1,027 \text{ kips}
\]

\[
\phi v_c = 0.75 \times 10 \sqrt{4,000} \times 72 \times 16 = 546,442 \text{ Lbs or 546 kips;}
\]

\[
v_u = 145 \text{ Kips} < \phi v_c = 546 \text{ Kips}, \quad \text{OK}
\]
(F) Bearing Capacity of Column or Pedestal on Pile Cap

The calculation of bearing strength of a column or pedestal on a pile cap requires that the factored loads \( B_u \) are less than \( \phi B_n \) for each applicable load combination.

\[ \phi B_n \geq B_u \quad (22.8.3.1) \]

From Table 21.2.1 \( \phi = 0.65 \)

**Table 22.8.3.2- Nominal bearing strength**

<table>
<thead>
<tr>
<th>Geometry of bearing area</th>
<th>( B_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supporting surface is wider on all sides than the loaded area</td>
<td>Least of (a) and (b) [ \sqrt{A_2/A_1} \ (0.85 f'c A_g) ] (a)</td>
</tr>
<tr>
<td></td>
<td>[ 2(0.85 f'c A_1) ] (b)</td>
</tr>
<tr>
<td>Other cases</td>
<td>[ 0.85 f'c A_1 ] (c)</td>
</tr>
</tbody>
</table>

Where \( A_1 \) is the loaded area and \( A_2 \) is the area of the lower base;

**Note:** \( A_g = A_1 \) for a solid column or pedestal

\( A_1 \) in this example = 12 in x 12 in = 144 in\(^2\)

\( A_2 \) in this example = (9 x 12) in x (6 x 12) in = 7,776 in\(^2\)

and \( \sqrt{A_2/A_1} = \sqrt{7,776/144} = 7.35 \leq 2.0 \); use 2.0 as shown in equation (b)

\[ \phi B_n = 0.65 \times 2.0 \times (0.85 \times 4,000 \times 144) = 636,480 \text{ Lbs or 636 Kips} \]

\[ \phi B_n = 636 \text{ Kips} \geq B_u = 290 \text{ Kips}, \text{ OK} \]
(G) Dowels Required Between Cast-in-place Column or Pedestal and Foundation

Section 16.3.4.1 states “For connections between a cast-in-place column or pedestal and foundation, $A_s$ crossing the interface shall be at least $0.005A_g$, where $A_g$ is the gross area of the supported member.”

**Note:** Section 16.3.4.1 is the provision for the minimum $A_s$, see Section 16.3.5 *Details for connections between cast-in-place members and foundation* for additional information.

Even though the bearing strength in this example is adequate at the interface between the column or pedestal and the pile cap a minimum steel area of $A_s$ is required.

$$A_s = 0.005 \times 12 \text{ in} \times 12 \text{ in} = 0.72 \text{ in}^2$$

Using 4 – 4 dowels, $4 \times 0.20 \text{ in}^2 = 0.80 \text{ in}^2$

The development length of the dowels must be checked in both the column or pedestal and the pile cap.

From Section 25.4.9 *Development of deformed bars and deformed wires compression*

Section 25.4.9.1 states “Development length $\ell_{dc}$ for deformed bars and deformed wires in compression shall be the greater of (a) and (b)

-(a) Length calculated in accordance with 25.4.9.2
(b) 8 in.”
and

Section 25.4.9.2 states that “$\ell_{dc}$ shall be the greater of (a) and (b), using the modification factors of 25.4.9.3:”

(a) $\left( \frac{f_y \Psi_r}{50 \lambda \sqrt{f'_c}} \right) \cdot d_b$  (b) $0.0003 f_y \Psi_r \cdot d_b$
Table 25.4.9.3- Modification factors for deformed bars and wires in compression

<table>
<thead>
<tr>
<th>Modification factor</th>
<th>Condition</th>
<th>Value of factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lightweight $\lambda$</td>
<td>Lightweight concrete</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Lightweight concrete, if $f_{ct}$ is specified</td>
<td>In accordance with 19.2.4.3</td>
</tr>
<tr>
<td></td>
<td>Normalweight concrete</td>
<td>1.0</td>
</tr>
<tr>
<td>Confining Reinforcement $\Psi_r$</td>
<td>Reinforcement enclosed within (1), (2), (3), or (4):</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(1) a spiral</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) a circular continuously wound tie with $d_b \geq 1/4$ in. and pitch 4 in.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3) No. 4 bar or D20 wire ties in accordance with 25.7.2 spaced $\leq 4$ in. on center</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4) hoops in accordance with 25.7.4 spaced $\leq 4$ in. on center</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The development length, $\ell_{dc}$ in compression for a #4 rebar per equation (a)

$$\ell_{dc} = \left( \frac{f_y \Psi_r}{50 \lambda \sqrt{f'_{c}}} \right) d_b = \left( \frac{60,000 \times 0.75}{50 \sqrt{4,000}} \right) \times 0.5$$

$\ell_{dc} = 7.1$ in

The development length, $\ell_{dc}$ in compression for a #4 rebar per equation (b)

$$\ell_{dc} = 0.0003 f_y \Psi_r d_b$$

$$\ell_{dc} = 0.0003 \times 60,000 \times 0.75 \times 0.5 = 6.75$$ in

Therefore, per Section 25.4.9.1

$\ell_{dc} = 8$ in

The #4 rebars must protrude a minimum of 8 in into the column or pedestal and the pile cap.

Figure 16 below shows the dowels in the pedestal and pile cap, notice that the dowels are in an “L” configuration for construction purposes. The hoops for this example are #3 rebar and are in accordance with (4) under “Condition” in Table 25.4.9.3 above, see Figure 17 below.
Fig. 16

Fig. 17
V. Pile Cap Support Conditions

A pile supported foundation which has its bottom surface against soil (see Figure 18 below) may actually receive a resistance to a vertically downward load, however, the resistance to vertically downward load from the soil below the pile cap is always considered to be negligible. The piling are the only elements which resist any loading applied to the pile cap.

For pile supported foundations which does not have its bottom surface against adequate soil (see Figure 19 below), Section 13.4.3.1 states that “Portions of deep foundation members in air, water, or soils not capable of providing adequate restraint throughout the member length to prevent lateral buckling shall be designed as columns in accordance with the applicable provisions of Chapter 10.

Fig. 18

Fig. 19
VI. Critical Section & Pile Location

Section 13.4.2.5 states that “Calculation of factored shear on any section through a pile cap shall be in accordance with (a) through (c):”

(a) Entire reaction from any pile with its center located \( \frac{d_{\text{pile}}}{2} \) or more outside the section shall be considered as producing shear on that section.

Figure 20 depicts the condition (a).

(b) Reaction from any pile with its center located \( \frac{d_{\text{pile}}}{2} \) or more inside the section shall be considered as producing no shear on that section.

Figure 21 depicts the condition (b).
(c) For intermediate positions of pile center, the portion of the pile reaction to be considered as producing shear on the section shall be based on a linear interpolation between full value at $d_{\text{pile}}/2$ outside the section and zero value at $d_{\text{pile}}/2$ inside the section.

Figure 22 depicts the condition (c).

VII. Uplift on Piles

Figure 23 depicts an uplift connector on a timber piling. Uplift on piles typically requires that shear be checked for pile loads for both uplift and compression as the uplift is usually a transient load. Flexural reinforcement will also be necessary as top steel in the pile cap, in other words, the pile cap will be doubly reinforced.
VIII. Horizontal Forces on Pile Caps

Figure 24 depicts a pile cap resisting a horizontal force “H”. Battered piles can be used to resist some or all of the horizontal forces imposed on a pile cap. When battered piles are used the vertical component $V_1$ of the allowable pile load is equal to $P_1 \cos \theta$ and the horizontal component $H_1$ of the allowable pile load is equal to $P_1 \sin \theta$.

The efficiency of the pile to resist vertical loads is proportional to $\cos \theta$, and the values of $n$ (total number of piles) and $I_x$ & $I_y$ (moment of inertia of pile group in x and y directions) must be adjusted to reflect the loss of efficiency due to the batter.

Fig. 24