Introduction to Thermodynamic Cycles

Part 1

1st Law of Thermodynamics and Gas Power Cycles

by

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1.0 Course Overview

The main focus of this course is to introduce common thermodynamic cycles. The course is divided into three parts. This course is part 1 of the set, and it provides a quick overview of the first law of thermodynamics for both closed and open systems and it covers gas power cycles. The second part of this course covers vapor cycles. Part 3 of this course covers the second law of thermodynamics along with refrigeration cycles.

Each cycle discussed in part 1 of this course is presented in context of an illustrative example problem. Each of these example problems, as well as the exam problems, will utilize the table of Ideal Gas Properties of Air. Thermodynamic tables are available online as well as any thermodynamics textbook. Note that different tables may have slightly different values. For convenience, an abbreviated ideal gas properties table for air is provided in Section 7.0 of this course, which will be used to solve problems for gas power cycles. The abbreviated table provided was modified from property tables in Thermodynamics: An Engineering Approach (Cengel & Boles, 2008). Note that higher accuracy could be obtained by using the more complete thermodynamic tables.

2.0 Basic Concepts of Thermodynamics

Thermodynamics is the study of energy, which is defined as the capacity of a physical system to do work. A system is a quantity of matter to study. The boundary of the system is a real or imaginary surface that separates a system from its surroundings. There are two types of systems: closed systems and open systems. A closed system (also known as a control mass) is a system where mass cannot cross the boundary but energy can cross the boundary. An open system (also called a control volume) is a system that usually involves mass flow across the boundary and energy can also cross the boundary.

2.1 Temperature Conversions

Several temperature conversions are commonly used in thermodynamics problems. For reference, Table 1 lists the temperature conversions.
# Introduction to Thermodynamic Cycles – Part 1

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Table 1  Temperature conversions

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Celsius → Fahrenheit</td>
<td>$T_F = \frac{9}{5}T_C + 32$</td>
</tr>
<tr>
<td>Fahrenheit → Celsius</td>
<td>$T_C = \frac{5}{9}(T_F - 32)$</td>
</tr>
<tr>
<td>Celsius → Kelvin</td>
<td>$T_K = T_C + 273.15$</td>
</tr>
<tr>
<td>Fahrenheit → Rankine</td>
<td>$T_R = T_F + 459.67$</td>
</tr>
</tbody>
</table>

## 2.2 Laws of Thermodynamics

The *zeroth law* of thermodynamics states that if objects A and B are separately in thermal equilibrium with a third object, then A and B are in thermal equilibrium. This law is the basis of temperature measurement devices.

The *first law* of thermodynamics is essentially conservation of energy that includes internal energy. Energy comes in a lot of different forms. Heat flow, for example, is a form of energy transfer occurring when there is a temperature difference between objects. Problems focusing on the first law can be closed systems or open systems. Section 3.0 covers the first law for closed systems and Section 4.0 covers the first law for open systems.

The *second law* of thermodynamics states that energy has quality, and processes will occur in direction of decreasing quality. You can think of this as saying that heat will not spontaneously flow from a cold object to a hot object. Part 2 of this course covers details of the second law of thermodynamics.

## 2.3 Processes and Cycles

Sections throughout this course will discuss different types of processes and different types of cycles, therefore it is important to understand the difference between a process and a cycle. A process, as shown in Figure 1, is a change in the system from one equilibrium state to another. In a cycle, as shown, the system returns to the initial state at the end of the set of processes.
2.4 Properties of Pure Substances

2.4.1 Pure Substances

Before discussing properties of pure substances, we need to define a pure substance as a substance with fixed chemical composition throughout. Note that a pure substance does not need to be a single chemical element. Water, for example, is a pure substance but contains more than one chemical element.

2.4.2 Phases and Phase Changes

It is commonly known that substances can occur in different phases (solids, liquids, and gases). When substances are heated or cooled they can change from one phase to another. A phase change from a solid to a liquid is known as melting, and a phase change from a liquid to a gas is
known as boiling (or vaporization). In the reverse direction, a phase change from a gas to a liquid is known as condensation and from a liquid to a solid is known as freezing.

The ideas of phase change will be illustrated for water in Figure 2. From state $a$ to state $b$ the water is a solid. Once the temperature reaches zero degrees Celsius (the melting point for water), adding more heat will result in a phase change. The phase change from solid to liquid is represented between points $b$ and $c$ on the diagram. The region between points $c$ and $d$ represents the liquid phase. More heat is added until the temperature reaches 100 degrees Celsius (the boiling point for water). As more heat is added the phase change will occur between a liquid and a gas, which is illustrated between points $d$ and $e$. The region between points $e$ and $f$ is the gas phase.

![Diagram showing phase changes for water plotted as temperature vs. heat addition](image)

*Figure 2*  Phase changes for water plotted as temperature vs. heat addition
It is important to note that temperature remains constant during phase changes. From Figure 2 we can see that temperature is a constant value between points \( b \) and \( c \) (phase change from solid to liquid) and between points \( d \) and \( e \) (phase change from liquid to gas).

Consider now the phase diagram shown in Figure 3, which is a plot of temperature (\( T \)) versus specific volume (\( v \)). The process from points 1 to 2 occur in a liquid phase, the phase change is shown between points 2 and 3, and from points 3 to 4 the substance is a gas. Notice again the transformation from one state to another, as shown in Figure 3 from state 2 to state 3, takes place with no change in temperature (horizontal line on the T-v diagram). The diagram will be used to define several important terms associated with phase change processes in pure substances.

**Figure 3** Phase change shown on a \( T-v \) diagram

When a substance occurs in a liquid state it is called a **compressed liquid** (or subcooled liquid). As heat is added to a liquid, it will reach a point where any additional heat added would result in
some of the liquid vaporizing. A liquid at that point where it is about to vaporize is called a **saturated liquid**. State 2 represents the saturated liquid state on Figure 3.

The line between points 2 and 3 on Figure 3 represents the substance in the phase change between a liquid and a vapor. That region is defined as a **saturated liquid-vapor mixture**, because both the liquid and vapor phase exists together in equilibrium.

A vapor that is not about to condense is known as a **superheated vapor**. As heat is removed from the vapor it will reach the point, known as a **saturated vapor**, where the removal of any additional heat will cause some of the vapor to condense. State 3 is the saturated vapor state. At a given pressure, the substance will begin to boil at a temperature known as the saturation temperature. Or, if temperature is held constant, the substance will start to boil at the saturation pressure.

The process line 1-2-3-4 in Figure 3 occurs at a constant pressure. If the pressure is raised or lowered a similar process curve will exist, but the locations of the saturated liquid point and the saturated vapor point would change. The points for different pressures would form a curve defined by the saturated liquid line and the saturated vapor line in Figure 3.

### 2.4.2.1 Property Tables

Properties of a substance are too complex to be represented by equations. Therefore, property tables are developed. An example of a property table would be the abbreviated Ideal Gas Properties of Air table given in Section 7.0. In that table, properties of air are provided for different temperature values ranging from 200 degrees Kelvin up to 2200 degrees Kelvin. Because the table does not provide properties for all values of temperature, interpolation is required to get properties for temperatures not provided in the table. To illustrate the process of interpolation, let’s assume we need to know the value of enthalpy (h) for a temperature of 520 degrees Kelvin. The table given in Section 7.0 gives properties for temperatures of 500 and 550 degrees Kelvin, but properties are not provided for the desired temperature of 520 degrees Kelvin. The table gives values of enthalpy of 503.02 and 554.74 for temperatures of 500 and 550 respectively. Linear interpolation can then be used to determine the enthalpy value for a temperature of 520.

\[
\frac{550 - 500}{554.74 - 503.02} = \frac{520 - 500}{h - 503.02}
\]
The values are summarized in Figure 4. Example problems throughout this course will only give summary tables similar to Figure 4 for any required linear interpolation.

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$h$ (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From table</td>
<td>500</td>
</tr>
<tr>
<td>Interpolated</td>
<td>520</td>
</tr>
<tr>
<td>From table</td>
<td>550</td>
</tr>
</tbody>
</table>

**Figure 4** Interpolation example

### 2.4.2.2 Saturated Liquid and Saturated Vapor States
In property tables, a saturated liquid state is denoted with a subscript $f$ and a saturated vapor state is denoted with a subscript $g$. For example, $v_f$ would indicate the specific volume of the saturated liquid and $h_g$ would indicate the enthalpy of the saturated vapor. Another common subscript in property tables would be $fg$, which indicates the difference between the saturated vapor and the saturated liquid. For example, $v_{fg} = v_g - v_f$. We will see more on this topic in part 2 of this course.
2.4.2.3 Quality
A new property known as quality is important in the saturated mixture region, as shown in Figure 3 between states 2 and 3. The quality is the ratio of the mass of vapor and the total mass of the mixture. This topic will be used for vapor cycles, which will be discussed in detail in part 2 of this course.

2.5 Heat Capacity
Basically, different fluids will ‘heat up’ differently (more specifically they store heat differently). In thermodynamics, you need a way to take the fluid’s properties for storing heat into account. Heat capacity, as shown in Equation 1, is the heat added divided by the change in temperature.

\[
C = \frac{Q}{\Delta T}
\]  

Equation 1

In other words, it is a measure of how the temperature changes for a given amount of heat added. Obviously, the temperature change of the fluid will also depend on the amount of fluid. Therefore, the specific heat (lower case c) is the heat capacity per unit mass as shown in Equation 2.

\[
c = \frac{Q}{m\Delta T}
\]  

Equation 2

The amount of heat required to raise the temperature of a mass is given by rearranging Equation 2 to give Equation 3.

\[
Q = mc\Delta T
\]  

Equation 3
For gases we need to define two separate specific heat values. The temperature change will depend on whether the volume is held constant or if gas is able to expand. We use a subscript $v$ to define specific heat capacity at a constant volume. A subscript $p$ is used to define the specific heat capacity at a constant pressure.

The specific heat ratio $k$ is defined as the ratio of the constant pressure specific heat to the constant volume specific heat. It should be noted that Equation 3 cannot be used during a phase change because temperature remains constant during a phase change (see Figure 2). During phase changes the heat will equal the mass times the change in enthalpy.

### 2.6 Ideal Gas Law

As with many things in engineering, we can make some simplifying assumptions about gases to make them easier to analyze. Ideal gases are assumed to have molecules with no mass and that don’t interact with each other. Many low-density gases, such as air, can be treated as ideal gases.

Pressure in a fixed volume container will be proportional to temperature and mass. If you increase the temperature or add more gas to the container the pressure will increase. Using the gas constant $(R)$ and the specific volume (volume per unit mass) the ideal gas law is

$$Pv = mRT$$  \hspace{1cm} \text{Equation 4}

The terms represent pressure, specific volume, mass, gas constant, and temperature. Note that the pressure is absolute pressure and the temperature is absolute temperature. Different gasses have different gas constants, with some common examples given in Table 2.
### Table 2  Gas constant for various common gases

<table>
<thead>
<tr>
<th>Substance</th>
<th>$R \left( \frac{kJ}{kg \cdot K} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.2870</td>
</tr>
<tr>
<td>Argon</td>
<td>0.2081</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.2968</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.2598</td>
</tr>
<tr>
<td>Steam</td>
<td>0.4615</td>
</tr>
</tbody>
</table>

### 3.0 1st Law for Closed Systems

The first law of Thermodynamics is a statement of conservation of energy. The law considers transfer of heat into a system $(Q)$, work performed by the system $(W)$, and the change in internal energy of the system $(\Delta U)$. If you also include the changes in kinetic energy (KE) and potential energy (PE), the first law equation for a closed system is given in Equation 5

$$Q - W = \Delta U + \Delta KE + \Delta PE$$  \hspace{1cm} \text{Equation 5}

### 3.1 Constant Pressure Process (isobaric)

If the pressure does not change during a process, then the process is known as an isobaric process. You can imagine boiling water in an open container as an example, because the open container is at atmospheric pressure. A cylinder with a moveable piston is another example. The constant weight of the piston provides a constant pressure on the gas within the container.

The P-v diagram (a diagram comparing pressure and volume) in Figure 5 shows a constant pressure processes. Because the pressure remains constant, the process path from state $A$ to state $B$ is a horizontal line.
As stated previously, a common example of a constant pressure process is expansion of a gas in a piston. Increasing the pressure of the gas will cause the piston to move and increase the volume while maintaining a constant pressure. The work done would be equal to the force (pressure times area) multiplied by the distance the piston moves during the process ($W = P\Delta x$). The area times the change in $x$ will give volume. Therefore, the work will be the area under the curve (process path) on a P-v diagram as expressed in Equation 6.

$$W = P\Delta V$$  \hspace{1cm} \text{Equation 6}

A more general form of Equation 6 would be $W = \int PdV$, which is the work done by the gas. However, the simple version shown in Equation 6 exists for a constant pressure process. The work will be positive if we move from state $A$ to state $B$ (adding energy) and will be negative if we move from state $B$ to state $A$ (removing energy).
From the equation of the first law, we can now write Equation 7 for constant pressure processes (assuming the changes in kinetic and potential energy equal zero).

\[ Q = \Delta U + P \Delta V \]  

Equation 7

3.2 Constant Volume Process (isochoric)

If the volume remains constant during the process it is known as an isochoric process. Heating up a gas in a closed container is an example of an isochoric process. Because the container is closed no gas can enter or leave the system resulting in a constant volume process. An isochoric process plotted on a P-v diagram will be a vertical line, as shown in Figure 6.

---

Figure 6  Constant volume process
Adding heat to the system will increase internal energy, but no work will be done because nothing moves (no moving piston for example). Because work is defined as the area under the curve, it is also obvious from the diagram that the work will equal zero.

\[ W = 0 \]  

Equation 8

Because the work is zero, the first law will reduce to the form shown in Equation 9. This again states that adding heat to the system will only increase internal energy.

\[ \Delta U = Q \]  

Equation 9

### 3.3 Constant Entropy Process (isentropic)

A thermodynamic process in which entropy remains constant is known as an isentropic process. However, the isentropic process requires the understanding of a new property known as entropy. Entropy is more closely related to the second law of thermodynamics, and will be covered in more detail in part 2 of this course. A brief description of entropy will be provided here.

Entropy, in a very general description, is a measure of disorder in a system. The second law can be stated more in the sense that the total entropy is always increasing, meaning that disorder in a system will increase. The second law is a statistical law, meaning that on average every system (as the total system) will tend toward the condition of maximum disorder (maximum entropy).

Many engineering systems can be treated as isentropic if they are essentially adiabatic (no heat transfer) and have minimal irreversibilities (such as friction). Below we will examine equations for isentropic processes for ideal gases, because that is what is required for gas power cycles.

#### 3.3.1 Constant Specific Heats

The first case will be using the assumption that specific heat does not change with temperature. The assumption of constant specific heat will often simplify the analysis but will decrease the
accuracy. For an isentropic process assuming constant specific heats we can use the relations expressed in Equation 10.

\[
\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{k-1/k}
\]

Equation 10

Where \( k = \frac{c_p}{c_v} \)

3.3.2 Variable Specific Heats

When accounting for variable specific heats you need to use a table for Ideal Gas Properties of Air. An abbreviated example of this table is given in Section 7.0 and will be used to solve problems for gas power cycles in this course. Two new variables are used for cases when the change in entropy is equal to zero (isentropic). The new variables are relative pressure \( (P_r) \) and relative specific volume \( (v_r) \). For an isentropic process from state 1 to state 2, the ratio of the actual pressures is equal to the ratio of the relative pressures evaluated at states 1 and 2.

\[
\left(\frac{P_2}{P_1}\right)_{v=\text{const}} = \frac{P_{r2}}{P_{r1}}
\]

Equation 11

A similar expression exists for the ratios of specific volumes during an isentropic process.

\[
\left(\frac{v_2}{v_1}\right)_{T=\text{const}} = \frac{v_{r2}}{v_{r1}}
\]

Equation 12

Essentially, accounting for variable specific heats would require equations of specific heat as a function of temperature. Analysis would then require integration of those equations. The Ideal
Gas Properties of Air table includes all the required integrations. However, the tables are based on a certain initial condition. These relative pressure and relative specific volume terms allow you to select your initial conditions and still use the table. Your actual pressure ratios and specific volume ratios will be the same as the relative ones, as shown in Equation 11 and Equation 12, used to develop the table.

3.4 Constant Temperature Process (isothermal)

If the temperature does not change during a process it is an isothermal process. Looking at the ideal gas equation (Equation 4), if temperature is constant (and \( m \) and \( R \) are also constants), then the product of pressure and specific volume must remain constant.

\[
Pv = \text{const.} \quad \text{Equation 13}
\]

Plotting Equation 13 on a P-v diagram will give a process path known as an isotherm (constant temperature). The exact process path will depend on the actual temperature, but a general example is shown in Figure 7.

![Figure 7: Constant temperature process](image)
As the gas expands from state $A$ to state $B$, we must add heat to the system to maintain our constant temperature. If the temperature remains constant, we also know that the internal energy of the system remains constant. Therefore, the change in internal energy equals zero.

\[ \Delta u = 0 \]  
Equation 14

From the first law of thermodynamics, if the internal energy remains constant the heat added will equal the work.

\[ W = Q \]  
Equation 15

3.5 No Heat Transfer (adiabatic)

An adiabatic process is one where no heat transfer occurs. Processes that occur very rapidly can be treated as adiabatic because the heat does not have time to flow in or out of the system. Systems that are fully insulated from the surroundings are also adiabatic.

The solid line process path shown in the P-v diagram in Figure 8 is adiabatic, and the dashed line is an isothermal line for reference. The adiabatic process is defined by Equation 16.

\[ PV^k = \text{const} \]  
Equation 16
The heat transfer is zero in an adiabatic process.

\[ Q = 0 \]  

Equation 17

With the heat transfer being zero, the first law is reduced to the equation shown in Equation 18.

\[ \Delta U = -W \]  

Equation 18

### 4.0 1st Law for Open Systems

Open systems generally have mass flow across the boundary. Because mass flow has energy, the first law equation is different than the one used for closed systems. The conservation of mass principle, which states that the total mass entering a system minus the total mass leaving a system equals the net change in mass within the system, must also be used. The conservation of energy equation then becomes
Equation 19 states that the total energy crossing the boundary as heat and work plus the total energy from mass entering the system minus the total energy from mass leaving the system will equal the net change in energy within the control volume. To make Equation 19 more understandable, we need to develop an expression for the energy of a flowing fluid. I will state without proof that flow work is given by Equation 20.

\[ \dot{w}_{\text{flow}} = P \nu \]  

Equation 20

Therefore, the total energy of a flowing fluid on a unit-mass basis will be \( P \nu + u + ke + pe \). The combination \( P \nu + u \) is known as enthalpy and is designated by \( h \).

The final form of conservation of energy is expressed as the total energy crossing the boundary as heat and work per unit time is equal to the total energy transported out of the control volume with mass per unit time minus the total energy transported into the control volume per unit time. This statement is expressed mathematically in Equation 21.

\[ \dot{Q} - \dot{W} = \sum_{\text{out}} \dot{m} \left( h + \frac{v^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left( h + \frac{v^2}{2} + gz \right) \]  

Equation 21

I will briefly describe each term for Equation 21. The term \( \dot{Q} \) is the rate of heat transfer between the control volume and the surroundings. The term \( \dot{W} \) is the work per unit time or power. As stated previously, \( h \) represents enthalpy. The \( \frac{v^2}{2} \) terms, when multiplied by the mass, will give kinetic energy. Likewise, the \( gz \) term will give potential energy (\( g \) is the gravitational constant and \( z \) is elevation). The term \( \dot{m} \) is a mass flow rate.
4.1 Nozzles and Diffusers

Now that we have the general form of the first law for open systems we can examine some common applications of steady flow devices. The first example will be nozzles and diffusers. A nozzle, as shown in Figure 9 (a), is a device that increases the velocity of a fluid by decreasing the cross-sectional area. A diffuser, as shown in Figure 9 (b), increases the pressure by decreasing the velocity.

Flow through nozzles and diffusers happen fast enough that the rate of heat transfer between the fluid and the surroundings can be neglected \((Q \approx 0)\). The work term is also zero for both nozzles and diffusers. The elevation change is either zero or negligible, so the potential energy terms can be neglected. The first law equation reduces to the form shown in Equation 22.

\[
\left( h + \frac{v^2}{2} \right)_{in} = \left( h + \frac{v^2}{2} \right)_{out}
\]

Equation 22
4.2 Turbines and Compressors

Figure 10 shows schematic representations of a turbine and a compressor. As a fluid passes through a turbine the blades turn and rotate a shaft to produce work. Therefore, the work is positive because it is done by the fluid. A compressor increases the pressure of a fluid. Work is supplied to the compressor from an external source. The work term is negative for compressors.

\[ h_{\text{in}} = h_{\text{out}} + w \]  

Figure 10  
(a) Turbine (b) Compressor

Turbines and compressors are often considered adiabatic \(( Q = 0 \)). Velocity terms can generally be ignored. The elevation change is either zero or negligible, so the potential energy terms can be neglected. The first law reduces to the form shown in Equation 23.
4.3 Throttling Valves

Throttling valves are flow restriction devices used to cause a pressure drop. Throttling valves are commonly used in refrigeration cycles because the pressure drop is accompanied by a temperature drop. For throttling valves there is no work and typically no heat transfer. Ignoring velocity terms and potential energy terms, the first law reduces to the form shown in Equation 24.

\[ h_{in} = h_{out} \]  

Equation 24

5.0 Gas Power Cycles

5.1 Introduction

One of the main categories of thermodynamic applications is in power generation, and the thermodynamic cycles used for power generation are called power cycles. Another distinction in thermodynamic cycles would be based on the phase of the working fluid. In gas cycles, for example, the working fluid will always remain a gas. Gas power cycles, therefore, are cycles for power generation where the working fluid remains a gas. Part 2 of this course will focus on power cycles known as vapor power cycles, which are cycles for power generation where the working fluid changes phase from liquid to vapor in different parts of the cycle. Another main category of thermodynamic applications is in refrigeration, which will be discussed in part 3 of this course.

5.2 Carnot Cycle

The Carnot cycle is an idealized cycle offering the most efficiency operating between two temperature limits. The efficiency of a Carnot cycle is given by Equation 25.
The temperatures used are absolute temperatures (Kelvin scale). The theoretical Carnot engine is the most efficient engine possible. Therefore, the efficiency calculated from Equation 25 is used to define the upper bound of the efficiency possible in any other engine, but it cannot be achieved.

5.3 Air-Standard Assumptions

Actual gas power cycles are complex, so the Otto cycle and diesel cycle make some simplifying assumptions (Air-Standard Assumptions). The air-standard assumptions are listed below:

1. The working fluid is air and is considered an ideal gas (See Section 2.6)
2. All processes are internally reversible
3. The combustion process is replaced with a heat addition process
4. The exhaust process is replaced with a heat rejection process

Before moving on, we should define the idea of reversible processes to better understand assumption 2 listed above. A reversible process is one that can be reversed and have the system and the surroundings return to the initial states. If the original process is completed and followed by the reverse of the original process, a reversible process would then have no net heat or net work. It should be noted that reversible processes do not actually exist, but they are used as idealized cases of processes. All systems will have irreversibilities such as friction.

5.4 Otto Cycle

5.4.1 Introduction to Internal Combustion Engines

Before examining the Otto cycle, it is helpful to discuss some terminology associated with internal combustion engines. Figure 11 shows a cross-section looking at a cylinder in an engine to illustrate the piston and valves.
Figure 12 (a) shows a simplified piston figure with some important terms for the reciprocating piston. The extreme positions of the piston are called top dead center (TDC) and bottom dead center (BDC). The distance between TDC and BDC is the stroke. Figure 12 (b) illustrates a typical four-stroke engine cycle. The four-stroke combustion cycle consists of the compression stroke, combustion stroke, exhaust stroke, and the intake stroke.
5.4.2 Otto Cycle

The Otto cycle is the ideal cycle for spark ignition reciprocating engines. Because the cycle treats the working fluid as an ideal gas, tables for ideal gas properties of air (see Section 7.0) will be used. The cycle is shown in Figure 13 on a P-v diagram along with a brief description of the four internally reversible processes. The combustion process is replaced with a heat addition process.

Figure 12  (a) Piston stroke and terminology (b) Four-stroke combustion cycle (Doane, 2015) [reused with permission]
The following example problem will be used to illustrate the process for solving problems relating to the Otto cycle.

**Example 1**

An ideal Otto cycle has a compression ratio of 8. Air is at 20°C and 95 kPa at the beginning of the compression process. During the heat addition process 750 kJ/kg of heat is transferred to the air. Determine the maximum temperature and pressure during the cycle, the net work output, and the thermal efficiency.

As previously shown in Figure 12, the Otto cycle operates between two volumes: the top dead center (TDC) and the bottom dead center (BDC). TDC is the piston position when the volume in
the cylinder is at a minimum, and BDC is the piston position when volume is at a maximum. The compression ratio is the ratio of maximum and minimum volumes as given in Equation 26.

\[ r = \frac{V_{BDC}}{V_{TDC}} = \frac{v_1}{v_2} \]  
Equation 26

Step 1: Determine properties for state 1

For Example 1, the temperature and pressure of state 1 are given values. Converting the given temperature to Kelvin (see Table 1), the initial properties are

\[ T_1 = 293.15K \quad P_1 = 95kPa \]

Using the initial temperature, we can look up the properties at state 1 using the thermodynamics table for Ideal Gas Properties of Air found in Section 7.0. Using linear interpolation, the values for state 1 are shown below.

<table>
<thead>
<tr>
<th>( T ) (K)</th>
<th>( u ) (kJ/kg)</th>
<th>( v_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>290</td>
<td>206.91</td>
<td>676.1</td>
</tr>
<tr>
<td>315</td>
<td>224.85</td>
<td>549.8</td>
</tr>
</tbody>
</table>

**State 1**

\( 293.15 \quad 209.17 \quad 660.2 \)

Step 2: Determine properties for state 2

The compression ratio can now be used to determine the relative volume at state 2.

\[ r = \frac{v_1}{v_2} \quad \rightarrow \quad (v_r)_2 = \frac{(v_r)_1}{r} \]

\[ (v_r)_2 = \frac{660.2}{8} = 82.52 \]

Again, we use the table for Ideal Gas Properties of Air to get
The ideal gas law can be used to determine the pressure at state 2.

\[
P_2 = P_1 \left( \frac{T_2}{T_1} \right) = 95kPa \left( \frac{659.7K}{293.15K} \right) = 1710.42kPa
\]

Step 3: Determine properties for state 3

The process from state 2 to state 3 is a constant volume process. Using the First Law for the constant volume processes (see Section 3.2) gives

\[
qu = u_3 - u_2 \quad \rightarrow \quad u_3 = q_{in} + u_2
\]

\[
u_3 = 750 \frac{kJ}{kg} + 480.53 \frac{kJ}{kg} = 1230.53 \frac{kJ}{kg}
\]

From the table for Ideal Gas Properties of Air

<table>
<thead>
<tr>
<th>(v_r)</th>
<th>(T)</th>
<th>(u)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(K)</td>
<td>(kJ/kg)</td>
</tr>
<tr>
<td>92.84</td>
<td>630</td>
<td>457.78</td>
</tr>
<tr>
<td><strong>82.52</strong></td>
<td><strong>659.7</strong></td>
<td><strong>480.53</strong></td>
</tr>
<tr>
<td>75.5</td>
<td>680</td>
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</table>

\[
State 2
\]

<table>
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<th>(u)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(K)</td>
<td>(kJ/kg)</td>
</tr>
<tr>
<td>1168.49</td>
<td>1460</td>
<td>7.801</td>
</tr>
<tr>
<td><strong>1230.53</strong></td>
<td><strong>1527.1</strong></td>
<td><strong>6.795</strong></td>
</tr>
<tr>
<td>1260.99</td>
<td>1560</td>
<td>6.301</td>
</tr>
</tbody>
</table>
The pressure at state 3 can be found using the ideal gas law.

\[
P_3 = P_2 \left( \frac{T_3}{T_2} \right) \left( \frac{v_2}{v_3} \right) = 1710.42 \text{kPa} \left( \frac{1527.1 \text{K}}{659.7 \text{K}} \right) (1) = 3958.98 \text{kPa}
\]

The pressure and temperature at state 3 will be the maximum within in the cycle.

\[
T_{\text{max}} = 1527.1 \text{K} \\
P_{\text{max}} = 3.959 \text{MPa}
\]

Step 4: Determine properties for state 4

The volume ratio can be used for the process from state 3 to state 4.

\[
\left( \frac{v_r}{v_3} \right)_{4} = \frac{v_4}{v_3} = r \rightarrow \left( \frac{v_r}{v_3} \right)_{4} = r \left( \frac{v_r}{v_3} \right)_{3}
\]

\[
\left( \frac{v_r}{v_3} \right)_{4} = 8(6.795) = 54.359
\]

From the table for Ideal Gas Properties of Air

<table>
<thead>
<tr>
<th>(v_r)</th>
<th>(T) (K)</th>
<th>(u) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.13</td>
<td>730</td>
<td>536.07</td>
</tr>
<tr>
<td><strong>State 4</strong></td>
<td><strong>54.359</strong></td>
<td><strong>768.7</strong></td>
</tr>
<tr>
<td>48.08</td>
<td>800</td>
<td>592.30</td>
</tr>
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</table>

Step 5: Determine net work and efficiency

The heat rejection is the difference in internal energy.

\[
q_{\text{out}} = u_4 - u_1 = 567.17 - 209.17 = -358 \frac{\text{kJ}}{\text{kg}}
\]

The work output is

\[
w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 750 - 358
\]
The thermal efficiency of the Otto cycle is

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{358}{750}$$

$$\eta_{th} = 0.523$$

### 5.5 Diesel Cycle

The diesel cycle is very similar to the Otto cycle, with the difference being the heat addition process. The Otto cycle has a constant volume heat addition process where the diesel cycle has a constant pressure heat addition process. The diesel cycle is shown on the P-v diagram in Figure 14.

---

**Figure 14**  Diesel cycle
A diesel engine is also known as a compression ignition engine, where the Otto cycle was based on the spark ignition engine. In a diesel engine, the combustion starts by compressing the air to a temperature above the auto ignition temperature of the fuel and injecting fuel into the high temperature air. In the diesel cycle, only air is compressed during the compression process. The combustion process in the cycle is approximated as a constant pressure heat addition process.

As stated in Section 5.3, the diesel cycle is based on the air-standard assumptions. We will illustrate the process for the analysis of diesel cycles with the use of a problem.

---

**Example 2**

An air-standard diesel cycle has a compression ratio of 18 and a cutoff ratio of 2. At the beginning of the compression process air is at 95 kPa and 27°C. Determine the thermal efficiency.

---

The diesel cycle involves another volume ratio. The cutoff ratio, as shown in Equation 27, is defined as the ratio of the volumes after and before the combustion process.

\[ r_c = \frac{v_3}{v_2} \quad \text{Equation 27} \]

**Step 1: Determine properties for state 1**

For **Example 2**, the temperature and pressure of state 1 are given values. Converting the given temperature to Kelvin (see Table 1), the initial properties are

\[ T_1 = 300.15K \quad P_1 = 95kPa \]

Using the initial temperature, we can look up the properties at state 1 using a thermodynamics table for Ideal Gas Properties of Air found in Section 7.0.
Step 2: Determine properties for state 2

Properties for state 2 can be determined using the same procedures as the Otto cycle. The compression ratio is used to determine the relative volume at state 2.

\[ v_{r2} = \frac{v_n}{r} = \frac{624.8}{18} = 34.71 \]

Using the relative volume, the values for the temperature and enthalpy at state 2 can be determined from the Ideal Gas Properties of Air table.

<table>
<thead>
<tr>
<th>( v_r )</th>
<th>( T ) (K)</th>
<th>( h ) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.08</td>
<td>800</td>
<td>821.95</td>
</tr>
<tr>
<td>34.71</td>
<td>897.1</td>
<td>929.69</td>
</tr>
<tr>
<td>34.31</td>
<td>900</td>
<td>932.93</td>
</tr>
</tbody>
</table>

The ideal gas law is used to determine the pressure at state 2.

\[ p_2 = p_1 \frac{T_2}{T_1} = 95kPa \left( \frac{897.1}{300.15} \right)(18) = 5110.79kPa \]

Step 3: Determine properties for state 3

The process from state 2 to state 3 occurs at a constant pressure. The ideal gas law would reduce to
According to Equation 27, the ratio of the volumes is the cutoff ratio. Therefore, the temperature at state 3 will equal the temperature at state 2 times the cutoff ratio.

\[ T_3 = r_c T_2 = 2(897.1) = 1794.16 \text{K} \]

The remaining properties at state 3 can be determined using the Ideal Gas Properties of Air table.

<table>
<thead>
<tr>
<th>( T ) (K)</th>
<th>( h ) (kJ/kg)</th>
<th>( v_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700</td>
<td>1880.1</td>
<td>4.761</td>
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<tr>
<td><strong>1794.16</strong></td>
<td><strong>1996.7</strong></td>
<td><strong>4.11</strong></td>
</tr>
<tr>
<td>1950</td>
<td>2189.7</td>
<td>3.022</td>
</tr>
</tbody>
</table>

Step 4: Determine properties for state 4

The process from state 3 to state 4 is isentropic. Accounting for variation in specific heats, the volume ratio can be determined using Equation 12 from Section 3.3.

\[
\frac{v_4}{v_3} = \frac{v_{r_4}}{v_{r_3}} \rightarrow v_{r_4} = \frac{v_4}{v_3} v_{r_3} \rightarrow v_{r_4} = \left( \frac{v_4}{v_2} \right) \left( \frac{v_2}{v_3} \right) v_{r_3}
\]

\[
v_{r_4} = \frac{r}{r_c} v_{r_3} = \frac{18}{2} (4.11) = 36.95
\]

The temperature and internal energy for state 4 can be determined using the table.

<table>
<thead>
<tr>
<th>( v_r )</th>
<th>( T ) (K)</th>
<th>( u ) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.08</td>
<td>800</td>
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</tr>
<tr>
<td><strong>36.95</strong></td>
<td><strong>880.8</strong></td>
<td><strong>658.78</strong></td>
</tr>
<tr>
<td>34.31</td>
<td>900</td>
<td>674.58</td>
</tr>
</tbody>
</table>

The ideal gas equation can be used to determine the pressure at state 4.
The heat addition occurs at a constant pressure. Referring to Section 3.1, the heat added will equal the change in enthalpy.

\[ q_{in} = h_3 - h_2 = 1996.7 - 929.69 = 1067.02 \text{ kJ/kg} \]

The heat rejection process occurs at a constant volume. Referring to Section 3.2, the heat rejected will equal the change in internal energy.

\[ q_{out} = u_4 - u_1 = 658.78 - 214.19 = 444.58 \text{ kJ/kg} \]

The thermal efficiency of the diesel cycle is

\[ \eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{444.58}{1067.02} \]

\[ \eta_{th} = 0.583 \]

### 6.0 Additional Concepts Related to Otto and Diesel Cycles

#### 6.1 Interpreting Work and Heat from Areas

Section 3.1 first introduced the idea that work can be calculated as the area under a process curve on a P-v diagram. That same concept can be applied to the cycles discussed in this course. Consider the Otto cycle shown in Figure 15 (a). The process from state 1 to state 2 is the isentropic compression phase of the cycle. The area under the process line from state 1 to state 2, as shown in the grey shaded area in Figure 15 (b), represents the work input per unit mass during that compression phase. Similarly, the process from state 3 to state 4 represents the isentropic expansion phase. The area under that process curve, as shown in Figure 15 (c), represents the work output per unit mass. The net work is the difference, which is represented as
the area within the Otto cycle plot shown in Figure 15 (d). The concept is the same for the plot of a diesel cycle.

---

**Figure 15**  (a) Otto cycle on a P-v diagram (b) Input work per unit mass (c) Output work per unit mass (d) Net work per unit mass

The area under the curve on a T-s diagram will represent the net heat. Figure 16 shows a T-s diagram for an Otto cycle. The area under the process path from 2 to 3 would represent the heat added, and the area under the process path from 4 to 1 would represent the heat removed. Therefore, the difference, as shown in the shaded portion, would represent the net heat.
6.2 Increasing Thermal Efficiency

Understanding the areas discussed in the previous section can allow for some optimization, because it can be visually understood how some changes to the system will increase or decrease work values. For example, it can be seen visually that the thermal efficiency of either cycle will increase by increasing the compression ratio. This can be verified mathematically. Equation 28 is given without proof to illustrate that the efficiency of the Otto cycle can be expressed as a function of the compression ratio and the ratio of specific heats. Increasing the compression ratio (while holding the specific heat ratio $k$ constant) would result in an increase in efficiency.

$$\eta = 1 - \frac{1}{\rho^{\frac{k-1}{2}}}$$

Equation 28
A similar equation can be developed for the diesel cycle. It is given as Equation 29, which is again provided without proof. It is again clear that increasing the compression ratio (while keeping $k$ and $r_c$ constant) will increase the efficiency.

\[
\eta = 1 - \frac{1}{r^{k-1}} \left( \frac{r_c^k - 1}{k(r_c - 1)} \right)
\]

Equation 29

### 6.3 Dual Cycle

In the real processes for spark ignition engines, the combustion process is not truly constant volume. Likewise, the process is not truly constant pressure in a diesel engine. Another cycle, known as the dual cycle, is sometimes used to give a better approximation. As shown in Figure 17, a dual cycle consists of five reversible processes: isentropic compression, isochoric heat addition, isobaric heat addition, isentropic expansion, and the isochoric heat removal.

---

![Figure 17](image-url)  
**Dual cycle**
### 7.0 Ideal Gas Properties of Air

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$h$ (kJ/kg)</th>
<th>$P_r$</th>
<th>$u$ (kJ/kg)</th>
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8.0 Works Cited
