Discovering Laws from Observations: A Data-driven Approach

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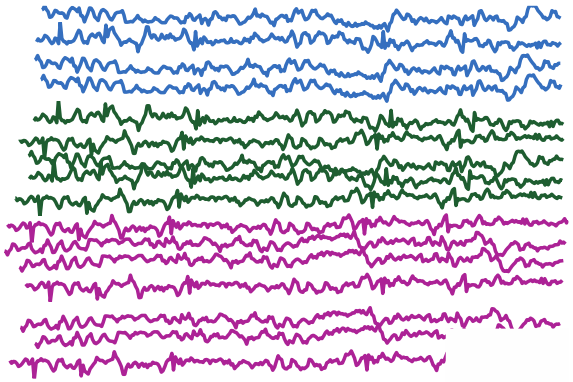
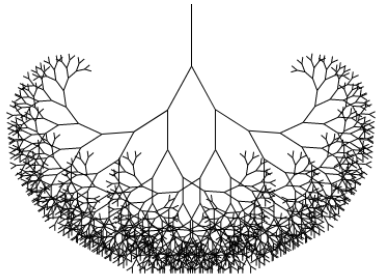
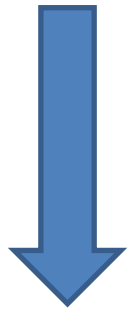
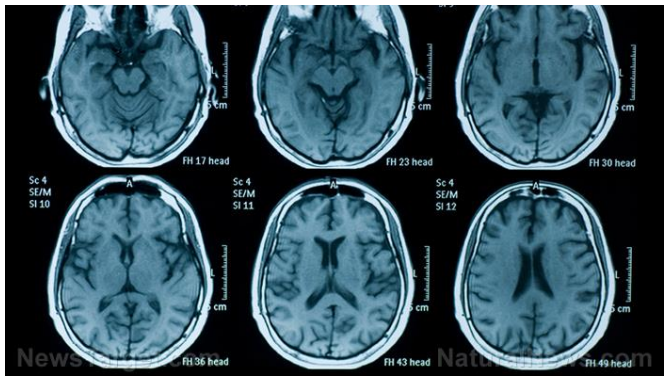
Introduction





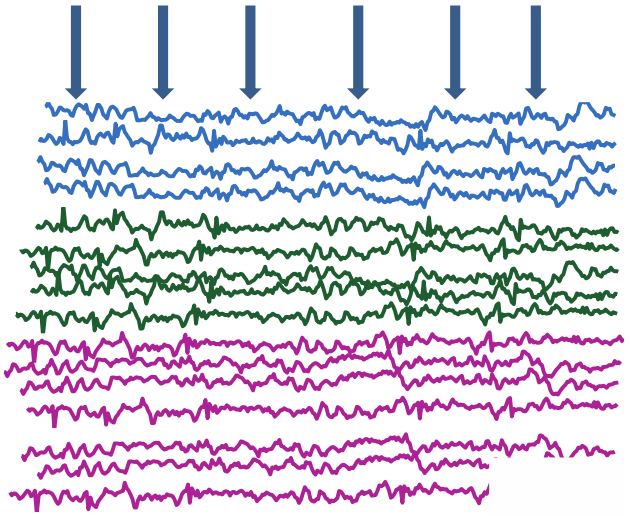


- Analyze the time-series data to predict the system.
- Fractal characteristic in the time-series data
- Early disease prediction

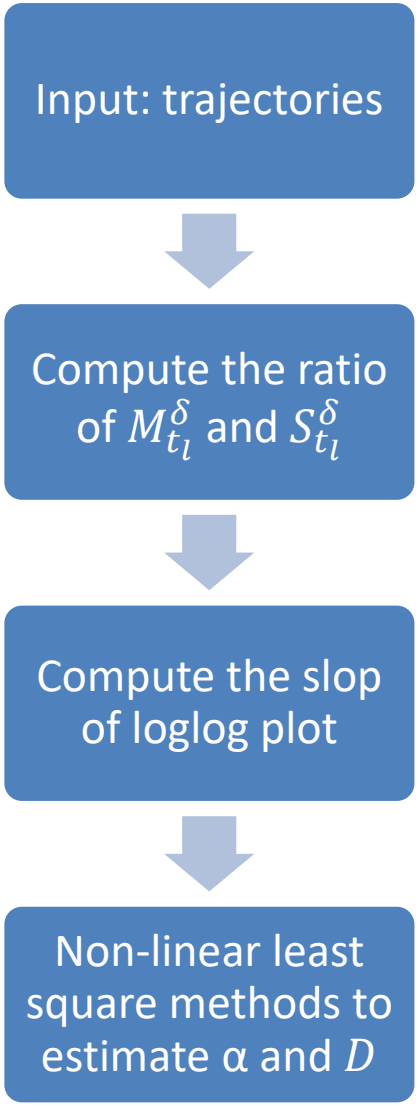


Methods

- **Space-time fractional diffusion equation**
  - $${}_tD_*^{\beta} = D \times {}_xD_{\theta}^{\alpha} u(x,t) + D'u(x,t) \quad (1)$$
- **Problem formulation:**
  - Given N trajectories, predict the equation (1)
- **Solution:**
  - Fractional order Moment based approach

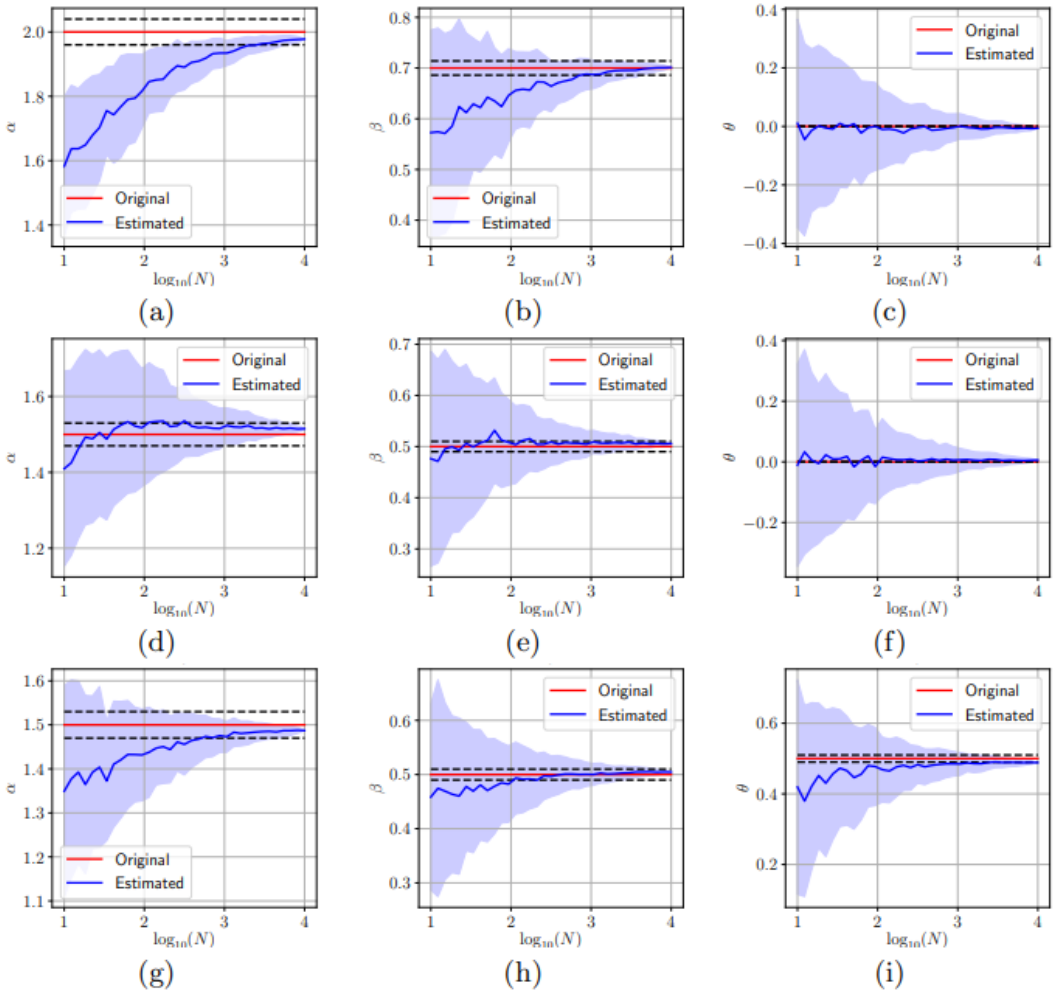


Algorithm


Input: trajectories
Compute the ratio of $M_{t_l}^{\delta}$ and $S_{t_l}^{\delta}$
Compute the slop of loglog plot
Non-linear least square methods to estimate α and D

Experiments

Experimental Results (Simulations)



- Fig.1. Estimating parameters of the space-time fractional diffusion with advection via our approach while varying the number of trajectories. The dotted line indicate 2% error tube around the original parameter value in the red: (1a, 1b, 1c) ($\alpha = 2, \beta = 0.7, \theta = 0$), (1d, 1e, 1f) ($\alpha = 1.5, \beta = 0.5, \theta = 0$), (1g, 1h, 1i) ($\alpha = 1.5, \beta = 0.5, \theta = 0.5$).

Experiments

Experimental Results (Real-World Data)

- This dataset is a voltage intervals generated from Drosophila brain neuron.
- 80% data are combined as the test set. 20% data are combined as the validation set
- Use the algorithm to retrieve the parameters (α, β, θ, and D) on both sets for 20 times. The following table expresses estimation results of the first 9 epochs and the mean values denote the results for all 20 epochs.

	Epoch									Mean
	1	2	3	4	5	6	7	8	9	
Test set (α)	0.511	0.511	0.511	0.510	0.511	0.511	0.511	0.511	0.512	0.511
Valid set (α)	0.505	0.507	0.506	0.508	0.505	0.507	0.507	0.506	0.505	0.507
Test set (β)	0.011	0.011	0.011	0.011	0.011	0.010	0.012	0.011	0.010	0.011
Valid set (β)	0.009	0.010	0.010	0.011	0.010	0.010	0.010	0.097	0.011	0.010
Test set (θ)	0.488	0.493	0.489	0.489	0.489	0.490	0.489	0.488	0.488	0.489
Valid set (θ)	0.495	0.493	0.494	0.492	0.495	0.493	0.493	0.494	0.495	0.494
Test set (D)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Valid set (D)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Thank You!