# Applications for Active Search in Material Fatigue Lifetime Estimation

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# Table of Contents

1. Overview and Background
   - Intermittent Loading and Fatigue
   - Linear Methods
2. Analytical Calculation for Fatigue Lifetime
   - Serebrinsky-Ortiz Model
   - Determininstic Loads
3. Active Search for Fatigue Lifetime
   - Problem Setup
   - Results
   - Conclusion

Oil Risers exhibit intermittent responses to forcing due to Vortex Induced Vibrations.

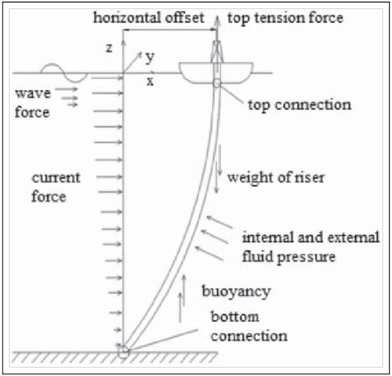


Figure: Schematic sketch of a Top Tensioned Riser (taken from Wang et al (2017) [1]).

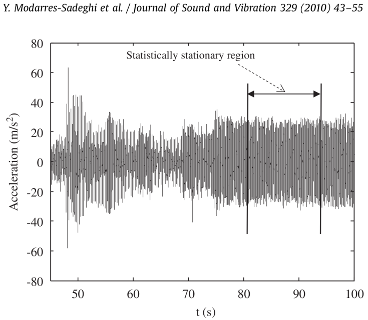


Figure: Sample riser response data, exhibiting intermittent behavior. Taken from Modarres-Sadeghi et al (2010) [2].

Two goals:

1. We want a **model** that considers time ordering effects for load signals that vary intermittently.
2. We want an experimental protocol that **minimizes** the number of expensive fatigue experiments

Simplest material loading is *harmonic* forcing.

The *SN* curve is a material property that compares the difference between $S_{\text{min}}$ and $S_{\text{max}}$ with the number of cycles until material failure.

Correction terms have been developed for variations, such as loads with finite mean stress.

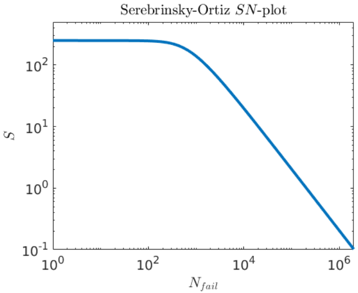


Figure: Sample SN-plot for the Serebrinsky-Ortiz constitutive model.

Simple extension to non-harmonic loading: The Palgrem-Miner rule:

$$C = \sum_i \frac{n_i \times S_i}{N_i \times S_i} \tag{1}$$

- Each cycle of magnitude $S_i$ adds $N_i^{-1}$ “accumulated fatigue damage.”
- The load signal includes $n_i$ cycles of magnitude $S_i$
- When $C \geq 1$, enough fatigue damage has accumulated to lead to material failure.

The rainflow counting algorithm is useful to construct a histogram of the stress increments/decrements.

Serebrinsky-Ortiz (SO) Model [3, 4]: constitutive relationship the describes the change in the material stiffness $K$ as a function of total history of the applied load.

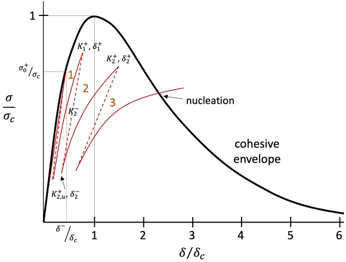


Figure: Cartoon evolution of the material stiffness. Taken from Serebrinksy and Ortiz (2005), [3].

- $\sigma$ – applied stress
- $\delta$ – material strain
- $K^+$ and $K^-$ – material loading and unloading stiffness
- $\delta_a$ – fatigue endurance length (material property)

$$\dot{\sigma} = \begin{cases} K^- \dot{\delta} & \dot{\delta} < 0 \\ K^+ \dot{\delta} & \dot{\delta} > 0 \end{cases} \tag{2}$$

$$\dot{K}^+ = \begin{cases} (K^+ + K^-) \frac{\dot{\delta}}{\delta_a} & \dot{\delta} < 0 \\ -K^+ \frac{\dot{\delta}}{\delta_a} & \dot{\delta} > 0 \end{cases} \tag{3}$$

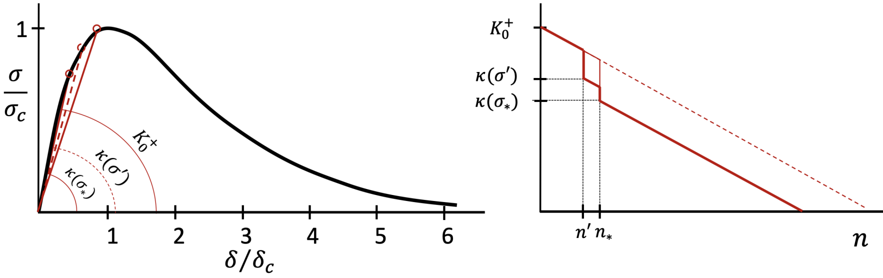


Figure: Sample time evolution of $K^+$ for the Serebrinsky-Ortiz model. Not the correspondance between intersection with the coherent envelope (left) and jumps in the value of $K^+$ (right).

How can we integrate the SO response to load signals quickly?

Two steps:

1. First, explicitly integrate equations 2 and 3, which replaces continuous ODE with a discrete update equation that depends only on the sequence of local maxima and minima.
2. Second, approximate the change in $K_n^+$ for load peaks we are confident do not intersect the coherent envelope.

$$K_{n,u}^{+} = K_n^{-} - \exp\left(\frac{\Delta\sigma_n^{-}}{\delta_a K_n^{-}}\right)(K_n^{-} - K_{n-1}^{+}), \tag{4}$$

$$K_n^{+} = K_{n,u}^{+} - \frac{\Delta\sigma_n^{+}}{\delta_a}, \tag{5}$$

$$\Delta K_n^{+} = \left(1 - \exp\left(\frac{\Delta\sigma_n^{-}}{\delta_a K_n^{-}}\right)\right)(K_n^{-} - K_{n-1}^{+}) - \frac{\Delta\sigma_n^{+}}{\delta_a}. \tag{6}$$

When we linearize the exponential in equation 6, and restrict to the case where $\frac{\Delta\sigma_n^{-}}{\delta_a K_n^{-}} \ll 1$ and $\left(\frac{K_{n-1}^{+}}{K_n^{-}} - 1\right) \ll 1$ (i.e., the case where the load peak is not too large), we get

$$\mathbb{E}[\Delta K^{+}] \approx \frac{\mathbb{E}[\Delta\sigma^{+}]}{\delta_a} \tag{7}$$

Graphical method:

$$\Delta K^+ = \frac{K_m - K_n}{m - n}$$

Gaussian method:

$$f_{\sigma^+}(a) = \frac{a}{s^2} \exp(\frac{-a^2}{2s^2}), \qquad a \geq 0$$

Rice Formula:

$$v_\sigma^+(a) = \int_0^\infty \dot{\sigma} f_{\sigma\dot{\sigma}}(a, \dot{\sigma}) d\dot{\sigma}$$

$$f_{\sigma^+}(a) = \frac{-1}{v_\sigma^+(0)} \frac{dv_\sigma^+(a)}{da}, \qquad a \geq 0$$

Sørensen full increment: [5]

$$f_E(e) = \int_0^\infty f_T(\tau) \times \int_\infty^\infty f_{P,V}(a, a-e, \tau) da d\tau.$$

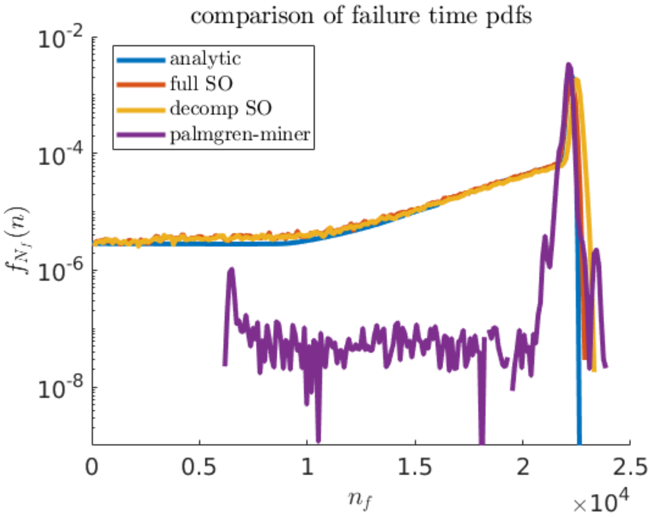


Figure: Comparison of predicted failure time pdf using Miner's rule and SO integration.

Miner's rule and the SO model :

- agree about the median failure time
- disagree about the frequency of *early failure*

In particular, the two methods predict different failure times when:

- there is an *unusually large load* increment
- near the *beginning or end* of the material's fatigue lifetime

# Active Search Experimental Model

- We *don't* know material properties (coherent envelope, etc)
- We *do* know statistical properties of the load
- We want to know the (approximate) pdf of the failure times, particularly frequency of extremely early failure times

Carefully choose input signals (experimental parameters) to maximize learning!

1-spike model for intermittent signals:

$$\sigma^1(t) = \sum_i^p a_i \cos(\omega_i t + \phi_i) + \alpha G(t - \beta T_{max}) \tag{8}$$

- Low-amplitude narrow-banded background.
- Exactly 1 large spike, with amplitude-location pair $(\alpha, \beta)$.

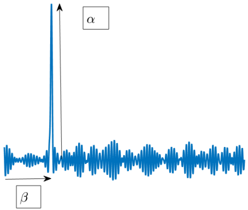


Figure: Sample signal from the 1-spike model.

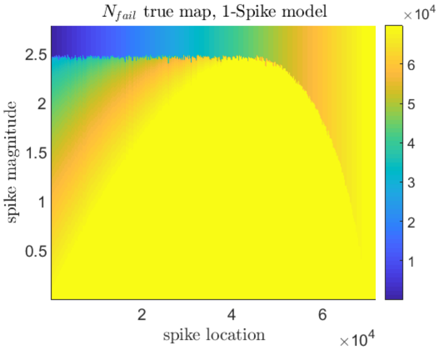


Figure: True map of the 1-spike model failure times.

The true map reproduces three important features:

- For large amplitude $\alpha$, immediate failure
- For moderate early spikes, smooth dependence
- For moderate late spikes, threshold dependence

Load distribution is given:

- (spike amplitude) $\alpha \sim$ Rayleigh
- (spike location) $\beta \sim$ Uniform

Response map allows us to transform distribution of load signals into distribution of failure times.

Need to perform experiments to recover the response map!

Typical active search methods attempt to maximize Mutual Information or minimize variance [6], which prioritize the mode of the distribution.

Q-Criterion (T. Sapsis (2020) [7]):

$$Q[\sigma_N^2] = \int \frac{p_x(x)}{p_{N_0}(N_0(x))} \sigma_N^2(x; h) dx.$$

Weighting factor $p_x(x)/p_{N_0}(N_0(x))$ prioritizes regions in experimental design space that lead to rare outcomes!

We borrowed the active search Python code from A. Blanchard, T. Sapsis (2020) [8].

1. Update a surrogate Gaussian Mixture Model (GMM) to fit results of previous experimental designs
2. Estimate the Q-criterion across the surrogate
3. Choose a proposal design that extremizes the Q-criterion
4. Perform the (simulated) experiment

Essentially, surrogate GMM model will attempt to fit previously displayed true map.

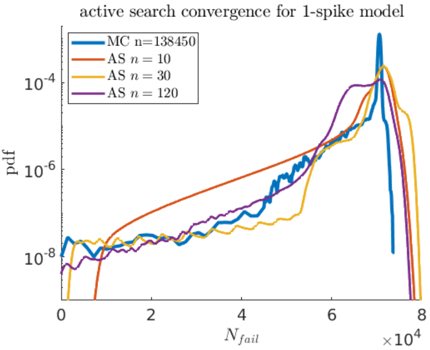


Figure: Comparison of recovered failure time PDF for Monte Carlo and active search. 1-spike model.

# Comparison of Methods

- Monte Carlo requires an obscene number of samples to recover long tail
- Active search requires a very small number of samples to recover long tail

Choice of error metrics:

- global error ($l_2$) of the recovered pdf ignores the long tail
- global error ($l_2$) of the recovered mapping includes the long tail, but is extremely sensitive to choice of parametrization
- tail mass error is narrowly tailored to target the accuracy of the long tail

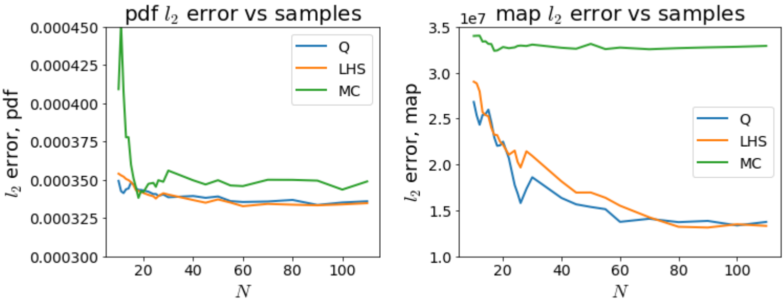


Figure: $l_2$ error of a) the recovered pdf and b) the recovered mapping. Note that $l_2$ error of the pdf is a very poor error metric, because it ignores the long tail.

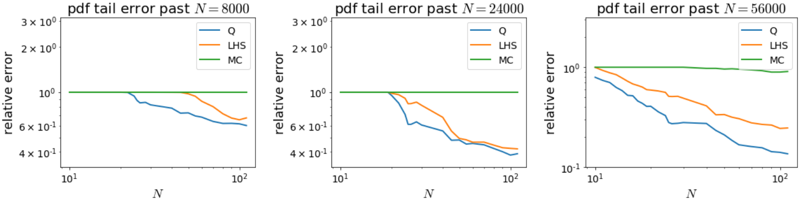


Figure: Error in the recovered pdf tail mass, for three different thresholds a) $N = 8000$, b) $N = 24000$, c) $N = 56000$. Note that while both Latin hypercube sampling (LHS) and $Q$-criterion active search outperform simple random samples, active search generally outperforms LHS in the region of interest ($20 < N < 100$ samples).

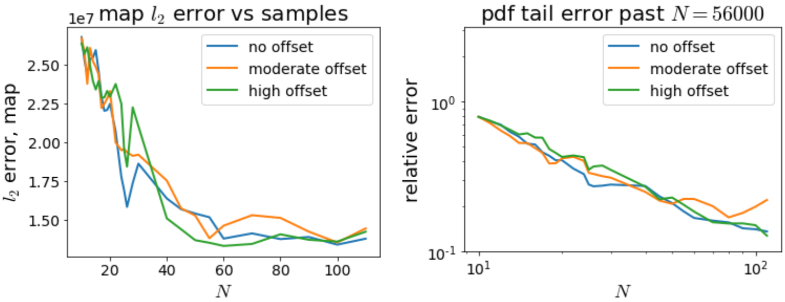


Figure: Comparison of the recovered 1) map and b) tail error for different choices of the GMM fitting offset. Dependence on the zero value for GMM fitting was nearly nonexistent.

Active search recovers pdf tail shapes

- significantly faster than Monte Carlo sampling,
- also faster than deterministic designs.

Active search also extends naturally to more parameters:

- Probe 2-spike correlation effects.
- Adjust magnitude of background loading

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