Dynamic Data-Driven Adaptive Observations
in Data Assimilation for Multiscale Systems

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Research Objectives

(I) Develop theory for lower-dimensional nonlinear filtering equation in important cases of multi-scale dynamics and correlated signal-sensor noise.

(II) Develop efficient and robust algorithms to solve lower-dimensional recursive nonlinear filtering equations driven by the observations.

(III) Develop an integrated framework that combines the ability to dynamically steer the measurement process, extracting useful information, with nonlinear filtering for inference and prediction of large scale complex systems.
Objectives and Motivation

Previous Efforts; Information Flow, Adaptive Observations


- Namachchivaya, N. Sri; Random dynamical systems: addressing uncertainty, nonlinearity and predictability; Meccanica, (51, 2975-2995); 2016 https://link.springer.com/article/10.1007%2Fs11012-016-0570-4

- Beeson, R., et al., Dynamic Data-Driven Adaptive Observations in a Vortex Flowfield; 9th European Nonlinear Dynamics Conference; Budapest Hungary; June 2017

- Yeong, H.C., et al. Particle Filters with Nudging in Multiscale Chaotic Systems: with Application to the Lorenz-96 Atmospheric Model; Budapest Hungary; June 2017
Two time-scale Problem, Filter Definition

Let \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{Q})\) be a filtered probability space supporting the following SDEs:

\[
\begin{align*}
    dX^e_t &= b(X^e_t, Z^e_t)dt + \sigma(X^e_t, Z^e_t)dW_t & X^e_0 &= x \in \mathbb{R}^m \\
    dZ^e_t &= \frac{1}{\epsilon}f(X^e_t, Z^e_t)dt + \frac{1}{\sqrt{\epsilon}}g(X^e_t, Z^e_t)dV_t & Z^e_0 &= z \in \mathbb{R}^n \\
    dY^e_t &= h(X^e_t, Z^e_t)dt + dU_t & Y^e_0 &= 0 \in \mathbb{R}^d
\end{align*}
\]

Normalized Conditional Measure

The filter is defined as,

\[
\pi^e_t (\varphi(X^e_t, Z^e_t)) \equiv \mathbb{E}_\mathbb{Q} [\varphi(X^e_t, Z^e_t) | y^e_t],
\]

where \(\varphi\) is an integrable function and

\[
y^e_t \equiv \sigma(\{Y^e_s - Y^e_0 | s \in [0, t]\}) \lor \mathcal{N}.
\]
CORRELATED SENSOR NOISE

Typical Atmosphere-Ocean Sensors

1. Drifters
2. Dropsondes
3. Weather Balloons
4. Remote Sensing (Satellite)

Image References

2. https://www.jamstec.go.jp/e/about/press_release/20160129/
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Multiscale Correlated Noise Problem, Filter Definition

Let \((\Omega, \mathcal{F}, \{\mathcal{F}\}_{t \geq 0}, \mathbb{Q})\) be a filtered probability space supporting the following SDEs:

\[
\begin{align*}
    dX_t^e &= \left[ b(X_t^e, Z_t^e) + \frac{1}{e} b_1(X_t^e, Z_t^e) \right] dt + \sigma(X_t^e, Z_t^e) dW_t \\
    X_0^e &= x \in \mathbb{R}^m \\
    dZ_t^e &= \frac{1}{e^2} f(X_t^e, Z_t^e) dt + \frac{1}{e} g(X_t^e, Z_t^e) dV_t \\
    Z_0^e &= z \in \mathbb{R}^n \\
    dY_t^e &= h(X_t^e, Z_t^e) dt + \alpha dW_t + \beta dV_t + \gamma dU_t \\
    Y_0^e &= 0 \in \mathbb{R}^d
\end{align*}
\]

Normalized Conditional Measure

The filter is defined as,

\[
\pi_t^e (\varphi(X_t^e, Z_t^e)) \equiv \mathbb{E}_{\mathbb{Q}} [\varphi(X_t^e, Z_t^e) \mid y_t^e],
\]

where \(\varphi\) is an integrable function and

\[
y_t^e \equiv \sigma(\{Y_s^e - Y_0^e \mid s \in [0, t]\}) \vee \mathcal{N}.
\]
**Problem Statement and Theoretical Results**

**Weak Convergence of the Slow Process**

**Theorem (Pardoux and Veretennikov Theorem 4 [5])**

*Under appropriate assumptions for $b, b_I, \sigma, f, g$, then*

$$X^e_s \Rightarrow X^0_s \text{ as } \epsilon \to 0,$$

$X^0_s$ a Markov process with generator

$$\hat{\mathcal{G}} \equiv \sum_{i=1}^{m} \tilde{b}_i(x) \frac{\partial}{\partial x^i} + \frac{1}{2} \sum_{i,j} (\tilde{\sigma}\tilde{\sigma}^*)_{ij}(x) \frac{\partial^2}{\partial x^i \partial x^j},$$

*where*

$$\tilde{b}(x) \equiv \bar{b}(x) + \sum_{i=1}^{m} \int b_{I,i}(x, z) \frac{\partial}{\partial x^i} g_F^{-1}(-b_I)(x, z)p_\infty(dz; x),$$

$$\tilde{\sigma}\tilde{\sigma}^*(x) \equiv \bar{\sigma}\bar{\sigma}^*(x) + \int b_I(x, z) g_F^{-1}(-b_I)(x, z)^* + g_F^{-1}(-b_I)(x, z)b_I(x, z)^*p_\infty(dz; x)$$

$$\bar{h}(x) \equiv \int h(x, z)p_\infty(dz; x), \quad \bar{b}(x) \equiv \int b(x, z)p_\infty(dz; x), \quad \bar{\sigma}\bar{\sigma}^*(x) \equiv \int \sigma\sigma^*(x, z)p_\infty(dz; x).$$
Therefore if \( \varphi = \varphi(X_t^\epsilon) \), then it would be advantageous to know if

\[
\pi_t^{\epsilon,x} \rightarrow \pi_t^0,
\]

in some appropriate sense, where

**Marginal Normalized Conditional Measure**

\[
\pi_t^{\epsilon,x}(\varphi) \equiv \int \varphi(x)\pi_t^\epsilon(dx, dz)
\]

**Homogenized Normalized Conditional Measure**

\[
\pi_t^0(\varphi(X_t^0)) \equiv \mathbb{E}_{\mathcal{Q}} \left[ \varphi(X_t^0) \mid y_t^\epsilon \right]
\]

Since \( \dim(\text{supp}(\pi_t^0)) < \dim(\text{supp}(\pi_t^{\epsilon,x})) \) and direct numerical integration of \((X_t^\epsilon, Z_t^\epsilon)\) is tasking when \( \epsilon \ll 1 \).
After Girsanov, Kallianpur-Striebel and with $h$ bounded, defining the unnormalized conditional measures,

$$
\pi_t^\varepsilon(\varphi) = \rho_t^\varepsilon(\varphi)/\rho_t^\varepsilon(1), \quad \pi_t^{\varepsilon,x}(\varphi) = \rho_t^{\varepsilon,x}(\varphi)/\rho_t^{\varepsilon,x}(1), \quad \pi_t^0(\varphi) = \rho_t^0(\varphi)/\rho_t^0(1).
$$

### Unnormalized Conditional Measure SDE

$$
d\rho_t^\varepsilon(\varphi) = \rho_t^\varepsilon(\mathcal{G}^\varepsilon \varphi)dt + \rho_t^\varepsilon(h^* \varphi + \partial_x \varphi \sigma \alpha^* + \frac{1}{\varepsilon} \partial_z \varphi \beta^*) dY_t^\varepsilon.
$$

### Reduced Order Unnormalized Conditional Measure SDE

$$
d\rho_t^0(\varphi) = \rho_t^0(\mathcal{G} \varphi)dt + \rho_t^0(h^* \varphi + \partial_x \varphi \bar{\sigma} \alpha^*) dY_t^\varepsilon.
$$
**Multiscale Slow-Correlated Noise Problem, Result**

\[
\begin{align*}
    dX_t^e &= b(X_t^e, Z_t^e)dt + \sigma(X_t^e, Z_t^e)dW_t & X_0^e &= x \in \mathbb{R}^m \\
    dZ_t^e &= \frac{1}{e^t}f(X_t^e, Z_t^e)dt + \frac{1}{\sqrt{e^t}}g(X_t^e, Z_t^e)dV_t & Z_0^e &= z \in \mathbb{R}^n \\
    dY_t^e &= h(X_t^e, Z_t^e)dt + \alpha dW_t + \gamma dU_t & Y_0^e &= 0 \in \mathbb{R}^d
\end{align*}
\]

**Theorem**

*Under appropriate assumptions, for every \( p \geq 1, T \geq 0, \) there exists \( C > 0, \) such that for every \( \varphi \in C_b^4 \)

\[
(E_Q \left[ |\pi_T^{\epsilon,x}(\varphi) - \pi_T^0(\varphi)|^p \right])^{1/p} \leq \sqrt{\epsilon}C\|\varphi\|_{4,\infty}.
\]

*In particular, there exists a metric \( d \) on the space of probability measures, such that \( d \) generates the topology of weak convergence, and such that for every \( T \geq 0 \) there exists \( C > 0, \) such that

\[
E_Q \left[ d(\pi_T^{\epsilon,x}, \pi_T^0) \right] \leq \sqrt{\epsilon}C.
\]
Convergence Proof Sketch

Convergence Proof Diagram

\[ \pi_t \in \mathcal{P} \rightarrow \rho_t \in \mathcal{M} \rightarrow v_t \in \mathcal{F}(\mathbb{R}^{m+n}, \mathbb{R}) \rightarrow (X_t, Z_t, \theta_t, \eta_t) \in (\mathbb{R}^m, \mathbb{R}^n, \mathbb{R}, \mathbb{R}^{w+v}) \]

Convergence Relations

\[
\mathbb{E} \left[ \left| \pi_T^{e,x}(\varphi) - \pi_T^0(\varphi) \right|^p \right] \leq \mathbb{E} \left[ \left| \rho_T^{e,x}(\varphi) - \rho_T^0(\varphi) \right|^p \right] = \mathbb{E} \left[ \left| \int v_0^e(x, z) - v_0^0(x) Q_0 \, dx, dz \right|^p \right] \\
\leq \mathbb{E} \left[ \int \left| v_0^e(x, z) - v_0^0(x) \right|^p Q_0 \, dx, dz \right] = \int \mathbb{E} \left[ \left| v_0^e(x, z) - v_0^0(x) \right|^p \right] Q_0 \, dx, dz \\
\leq \int \mathbb{E} \left[ \left| \psi_t(x, z) \right|^p \right] Q_0 \, dx, dz + \int \mathbb{E} \left[ \left| R_t(x, z) \right|^p \right] Q_0 \, dx, dz
\]

Formal Expansion

\[ v_t^e = v_t^0 + \psi_t + R_t \]
**Convergence Proof Sketch**

**Convergence Proof Diagram**

\[ \pi_t \in \mathcal{P} \rightarrow \rho_t \in \mathcal{M} \rightarrow v_t \in \mathcal{F}(\mathbb{R}^{m+n}, \mathbb{R}) \rightarrow (X_t, Z_t, \theta_t, \eta_t) \in (\mathbb{R}^m, \mathbb{R}^n, \mathbb{R}, \mathbb{R}^{w+v}) \]

**Convergence Relations**

\[
\mathbb{E} \left[ \left| \pi_T^{\epsilon,x}(\varphi) - \pi_T^0(\varphi) \right|^p \right] \leq \mathbb{E} \left[ \left| \rho_T^{\epsilon,x}(\varphi) - \rho_T^0(\varphi) \right|^p \right] = \mathbb{E} \left[ \left\| v_0^\epsilon(x, z) - v_0^0(x) \mathbb{Q}_0(dx, dz) \right\|^p \right] \\
\leq \mathbb{E} \left[ \int \left| v_0^\epsilon(x, z) - v_0^0(x) \right|^p \mathbb{Q}_0(dx, dz) \right] = \int \mathbb{E} \left[ \left| v_0^\epsilon(x, z) - v_0^0(x) \right|^p \right] \mathbb{Q}_0(dx, dz) \\
\leq \int \mathbb{E} \left[ |\psi_t(x, z)|^p \right] \mathbb{Q}_0(dx, dz) + \int \mathbb{E} \left[ |R_t(x, z)|^p \right] \mathbb{Q}_0(dx, dz)
\]

**Formal Expansion**

\[ v_t^\epsilon = v_t^0 + \psi_t + R_t \]
**Convergence Proof Sketch**

**Convergence Proof Diagram**

\[
\pi_t \in \mathcal{P} \rightarrow \rho_t \in \mathcal{M} \rightarrow \nu_t \in \mathcal{F}(\mathbb{R}^{m+n}, \mathbb{R}) \rightarrow (X_t, Z_t, \theta_t, \eta_t) \in (\mathbb{R}^m, \mathbb{R}^n, \mathbb{R}, \mathbb{R}^{w+v})
\]

**Convergence Relations**

\[
\mathbb{E} \left[ |\pi_t^{e,x}(\varphi) - \pi_0^{e,x}(\varphi)|^p \right] \leq \mathbb{E} \left[ |\rho_t^{e,x}(\varphi) - \rho_0^{e,x}(\varphi)|^p \right] = \mathbb{E} \left[ \left| \int v_0^e(x, z) - v_0^0(x) Q_0(dx, dz) \right|^p \right]
\]

\[
\leq \mathbb{E} \left[ \int |v_0^e(x, z) - v_0^0(x)|^p Q_0(dx, dz) \right] = \int \mathbb{E} \left[ |v_0^e(x, z) - v_0^0(x)|^p \right] Q_0(dx, dz)
\]

\[
\leq \int \mathbb{E} \left[ |\psi_t(x, z)|^p \right] Q_0(dx, dz) + \int \mathbb{E} \left[ |R_t(x, z)|^p \right] Q_0(dx, dz)
\]

**Formal Expansion**

\[
\nu_t^e = \nu_t^0 + \psi_t + R_t
\]
**Multiscale Noise Problem and Expansion**

\[
\begin{align*}
\dot{X}_t^e &= \left[ b(X_t^e, Z_t^e) + \frac{1}{\epsilon} b_1(X_t^e, Z_t^e) \right] dt + \sigma(X_t^e, Z_t^e) dW_t \\
\dot{Z}_t^e &= \frac{1}{\epsilon^2} f(X_t^e, Z_t^e) dt + \frac{1}{\epsilon} g(X_t^e, Z_t^e) dV_t \\
\dot{Y}_t^e &= h(X_t^e, Z_t^e) dt + \gamma dU_t
\end{align*}
\]

\[
X_0^e = x \in \mathbb{R}^m, \quad Z_0^e = z \in \mathbb{R}^n, \quad Y_0^e = 0 \in \mathbb{R}^d
\]

\[
u^e = u_0 + u_1 + u_2 + R.
\]

\[
\begin{align*}
\partial_t u_0 &= \left( \bar{G}_S + \bar{G} \right) u_0 + \bar{h} u_0 \partial_t B \\
\partial_t u_1 &= \frac{1}{\epsilon^2} G_F u_1 + \frac{1}{\epsilon} G_I u_0 \\
\partial_t u_2 &= \frac{1}{\epsilon^2} G_F u_2 + (G_S - \bar{G}_S) u_0 + \left( \frac{1}{\epsilon} G_I u_1 - \bar{G} u_0 \right) \\
\partial_t R &= G^e R + \left( \frac{1}{\epsilon} G_I + G_S \right) u_2 + G_S u_1 + \left( h - \bar{h} \right) u_0 \partial_t B + h (u_1 + u_2 + R) \partial_t B.
\end{align*}
\]
**Multiscale Correlated Noise Problem, Filter Definition**

\[
\begin{align*}
  dX^e_t &= \left[ b(X^e_t, Z^e_t) + \frac{1}{\epsilon} b_1(X^e_t, Z^e_t) \right] dt + \sigma(X^e_t, Z^e_t) dW_t \\
  X^e_0 &= x \in \mathbb{R}^m \\
  dZ^e_t &= \frac{1}{\epsilon^2} f(X^e_t, Z^e_t) dt + \frac{1}{\epsilon} g(X^e_t, Z^e_t) dV_t \\
  Z^e_0 &= z \in \mathbb{R}^n \\
  dY^e_t &= h(X^e_t, Z^e_t) dt + \alpha dW_t + \gamma dU_t \\
  Y^e_0 &= 0 \in \mathbb{R}^d \\

  u^e &= u_0 + u_1 + u_2 + R + Q. \\

  \partial_t u_0 &= \left( \overline{g}_S + \overline{g} \right) u_0 + \bar{h} u_0 \partial_t B + \partial_x u_0 \bar{\sigma} \alpha \partial_t B \\
  \partial_t R &= \mathcal{G}^e R + \left( \frac{1}{\epsilon} g_L + g_S \right) u_2 + g_S u_1 + \left( h - \bar{h} \right) u_0 \partial_t B + h (u_1 + u_2 + R) \partial_t B \\
  &\quad + \partial_x u_0 (\sigma - \bar{\sigma}) \alpha \partial_t B + \partial_x (u_1 + u_2) \sigma \alpha \partial_t B \\
  \partial_t Q &= \mathcal{G}^e Q + ((\partial_x R + \partial_x Q) \sigma \alpha + hQ) \partial_t B.
\end{align*}
\]
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6 Accomplishments and Reporting
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Particle Filters [8], [9], [10]

1. \( \{A^j \in \mathbb{R}^{m \times n}\}_{j \in A} \), an ensemble of particles.
2. \( \{w^j\} \), normalized weights: \( \sum_{j \in A} w^j = 1 \).

Approximation of posterior distribution at time \( t_k \)

\[
p(\bar{x}_k|y_k) = \sum_{j \in A} w^j_k \delta_k^i(\bar{x}_k),
\]
1a) At time $t_k$, set $w_k^j = 1/N, \forall j \in \mathcal{A}$ and

$$p(\xi_k|y_k) = \sum_{j \in \mathcal{A}} w_k^j \delta_k^j (\xi_k).$$

1b) Calculate $\bar{b}, \bar{\sigma}, \bar{h}$.

2) Generate the $t_{k+1}$ prior,

$$p(\xi_{k+1}) = \sum_{j \in \mathcal{A}} w_k^j \delta_{k+1}^j (\xi_{k+1}).$$

3) Collect observation $y_{k+1}$.

4) Update the weights,

$$w_{k+1}^j \propto w_k^j p(y_{k+1}|\delta_{k+1}^j).$$

5a) If $N_{\text{eff},k} < \delta_{\text{eff}}$, apply (universal) resampling and re-normalize.

5b) Otherwise, re-normalize with $l_2$ norm.
Homogenized Hybrid Particle Filter

1a) At time $t_k$, set $w_k^j = 1/N, \forall j \in \mathcal{A}$ and

$$p(\xi_k | y_k) = \sum_{j \in \mathcal{A}} w_k^j \delta_k^j(\xi_k).$$

1b) Calculate $\bar{b}, \bar{\sigma}, \bar{h}$.

2) Generate the $t_{k+1}$ prior,

$$p(\xi_{k+1}) = \sum_{j \in \mathcal{A}} w_k^j \delta_{k+1}^j(\xi_{k+1}).$$

3) Collect observation $y_{k+1}$.

4) Update the weights using $\bar{h}$,

$$w_{k+1}^j \propto w_k^j p(y_{k+1} | \delta_{k+1}^j).$$

5a) If $N_{\text{eff},k} < \delta_{\text{eff}}$, apply (universal) resampling and re-normalize.

5b) Otherwise, re-normalize with $l_2$ norm.
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Stochastic FitzHugh-Nagumo [12], [13]

A two-timescale, two-dimensional signal

\[
dX_t^\varepsilon = \left[-(Z_t^\varepsilon)^3 + \sin(\pi t) + \cos(\sqrt{2}\pi t)\right] dt + \sigma dW_t
\]  
(4.1)

\[
dZ_t^\varepsilon = -\frac{1}{\varepsilon}(Z_t^\varepsilon - X_t^\varepsilon) dt + \frac{1}{\sqrt{\varepsilon}} dV_t.
\]  
(4.2)

And discrete-time observation process,

\[
Y_{t_k}^\varepsilon = X_{t_k}^\varepsilon + \gamma U_{t_k} + \alpha \int_{t_k-1}^{t_k} dW_s,
\]  
(4.3)

\[U_{t_k} \sim \mathcal{N}(0, \Delta t), \quad \Delta t \text{ the observation step-size and correlation is model using an extension of Saha and Gustafsson [11].}
\]

For fixed $X_t^\varepsilon = x$, $Z_t^\varepsilon$ becomes an Ornstein-Uhlenbeck process; transition density is Gaussian and approaches $\mu_\infty(z; x)$ at an exponential rate. The effective dynamics for $X_t^\varepsilon$ is,

\[
dX_t^0 = -(X_t^0)^3 - \frac{3}{2}X_t^0 + \sin(\pi t) + \cos(\sqrt{2}\pi t) + \sigma dW_t.
\]  
(4.4)
Likelihood for Correlated Sparse Observations

\[
p(y_k|x_k, x_{k-1}, \ldots, x_{j-1}) \propto \mathcal{N}\left(h(x_k) + \tilde{S}^T\tilde{Q}^{-1}(X_{j-1:k} - f(X_{j-1:k})), R_k - \tilde{S}^T\tilde{Q}^{-1}\tilde{S}\right).
\]
FitzHugh-Nagumo Simulation Results

Filter | PF | HHPF | PF | HHPF
--- | --- | --- | --- | ---
α | 0 | 0 | $\sqrt{0.1}$ | $\sqrt{0.1}$
RMSE | 0.779 | 0.825 | 0.766 | 0.808
Run-Time | 55.68 s | 10.44 s | 56.83 s | 10.42 s
**Multiscale Stochastic Lorenz ’96 Model** [14], [15], [16], [17]

1. Mid-Latitude Atmospheric Dynamics
2. Linear Dissipation
3. External Forcing $F$
4. Quadratic Advection-Like Terms (Conserve Total Energy)
5. Chaotic for a wide range of $F, h_x, h_z$

\[
\begin{align*}
\frac{dX_t^k}{dt} &= (X_t^{k-1} (X_t^{k+1} - X_t^{k-2}) - X_t^k + F + \frac{h_x}{J} \sum_{j=1}^{J} Z_t^{k,j})dt + \sigma_x dW_t^k, \quad k = 1, \ldots, K \\
\frac{dZ_t^{k,j}}{dt} &= \frac{1}{\varepsilon} \left( Z_t^{k,j+1} (Z_t^{k,j-1} - Z_t^{k,j+2}) - Z_t^{k,j} + h_z X_t^k \right) dt + \frac{1}{\sqrt{\varepsilon}} \sigma_z dV_t^j, \quad j = 1, \ldots, J \\
\dot{Y}_t^e &= X_t^e + \sqrt{1 - \alpha^2} U_t^e + \alpha \int_{t_{k-1}}^{t_k} \sigma_x dW_s, \\
U_t^e &\sim \mathcal{N}(0_{K \times 1}, \Delta t \sigma_x \sigma_x^*).
\end{align*}
\]
Lorenz ’96 Simulation Results

In (gray), the $X_t^1$ marginal density when $\epsilon = 1E-2$ and $\epsilon = 1E-3$.

Transition densities of $Z_t^\epsilon$:

$\mu_{15, \Delta_m} (Z_0^1; X_0 = x, Z_0^\epsilon)$, for randomly generated $Z_0^\epsilon$; the first component is $Z_0^1$. 
**Lorenz ’96 Simulation Results**

![Graphs showing time series data with error](image)

<table>
<thead>
<tr>
<th>Filter</th>
<th>PF</th>
<th>HHPF</th>
<th>HHPF&lt;sub&gt;c&lt;/sub&gt;</th>
<th>enKF</th>
<th>henKF</th>
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<td><strong>RMSE</strong></td>
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<td>15.1619</td>
<td>5.6847</td>
<td>2.7767</td>
<td>2.8725</td>
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<td><strong>Run-Time</strong></td>
<td>338 s</td>
<td>35 s</td>
<td>59 s</td>
<td>412 s</td>
<td>30 s</td>
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<tr>
<td><strong>RMSE</strong></td>
<td>14.2202</td>
<td>15.3279</td>
<td>5.7458</td>
<td>2.9318</td>
<td>2.8172</td>
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<tr>
<td><strong>Run-Time</strong></td>
<td>315 s</td>
<td>34 s</td>
<td>60 s</td>
<td>400 s</td>
<td>28 s</td>
</tr>
</tbody>
</table>
Presented:

(i) Quantitative rate of convergence for slow signal-sensor correlation case.
(ii) Results toward a quantitative rate of convergence for intermediate case with and without slow signal-sensor correlation.
(iv) Numerical investigations with reduced order (homogenized hybrid) particle based filters with correlated sensor noise.

Future Efforts:

(I) Estimates for intermediate and slow signal-sensor noise correlation.
(II) Development of techniques to address the fast signal-sensor noise correlation problem as well as the full multi-scale correlated noise case.
(III) Develop efficient and robust methods to produce lower-dimensional recursive nonlinear filtering equations driven by the observations; particle filters for the integration of observations with the simulations of large-scale complex systems.
(IV) Develop an integrated framework that combines the ability to dynamically steer the measurement process, extracting useful information, with nonlinear filtering for inference and prediction of large scale complex systems.
Accomplishments:

- Full funding of Hoong Chieh Yeong (2017 Ph.D.), partial funding of Nishanth Lingala (2018 Ph.D.), continuing students: Ryne Beeson (Ph.D.) and Kyle Cochran (M.S.).

Book Chapters:


Journal Articles:

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