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TEXAS A&M UNIVERSITY
Land Air & Space Robotics Laboratory

Uncertainty Quantification of Feature Tracking Algorithms

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I would like to acknowldege the following colleagues for their contributions in various stages:
Xue Iuan Wong, Austin Probe, Abhay Masher, Jeremy Davis and James Doebbler.

Land Air and Space Robotics

ratory

Sensing Systems

iGPS
Localization System

Vicon
Motion Capture System

Phase Space
Motion Capture System

LASR developed during FY2006-2011 via AF, NASA, TAMU & IC Investments

Computational Vision

3D Imaging/Sensing

Ground-Based Robotics

Quad-Rotor Aerial Robotics

Robotics Lab Facility Specs:

- **3** Independent Metrology Systems
- Mm Precision Navigation (100HZ)
- **2000** sq. ft. Flat Floor
- Real Time Information Fusion & Control
- Reconfigurable Wireless Comm.

Recent Innovations to Pipeline:

1. Original pipeline evolved from 5Hz (on Quadcore) to 12 Hz (on single board computer – Atom). Speedups being
2. New navigation algorithms for rate estimation - as a byproduct [ASME JDMC]
    1. Linear algorithm
    2. Iter. EKF for this purpose
3. Algorithms for data driven noise covariance and outlier rejection. [CVPR 2017 paper, Wong Dissertation]

Proximity Operation Emulation Experiments

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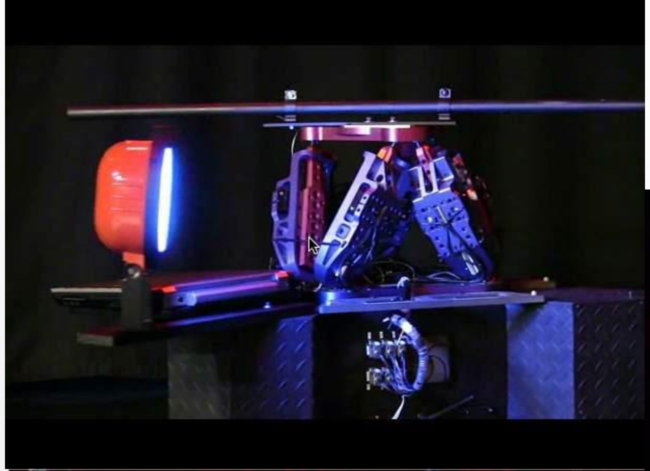
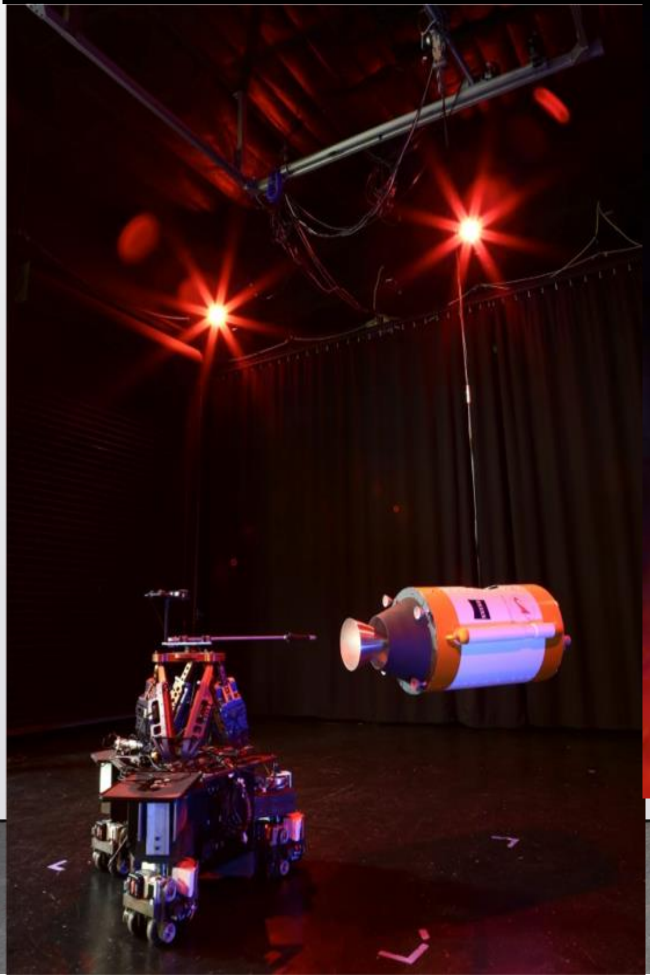
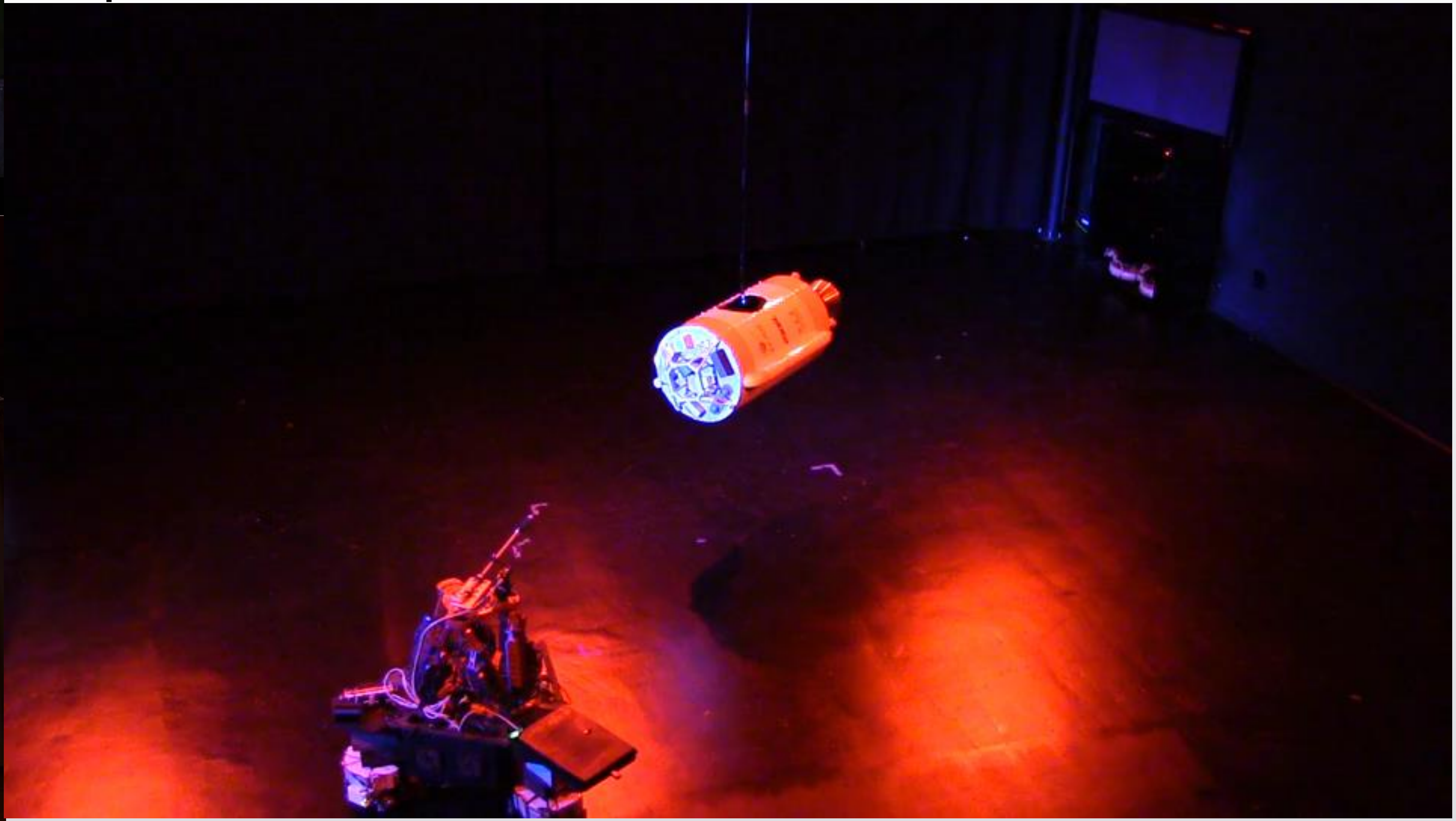






Question: Can I perform rate estimation directly from the images (with a coregistered range sensor)?

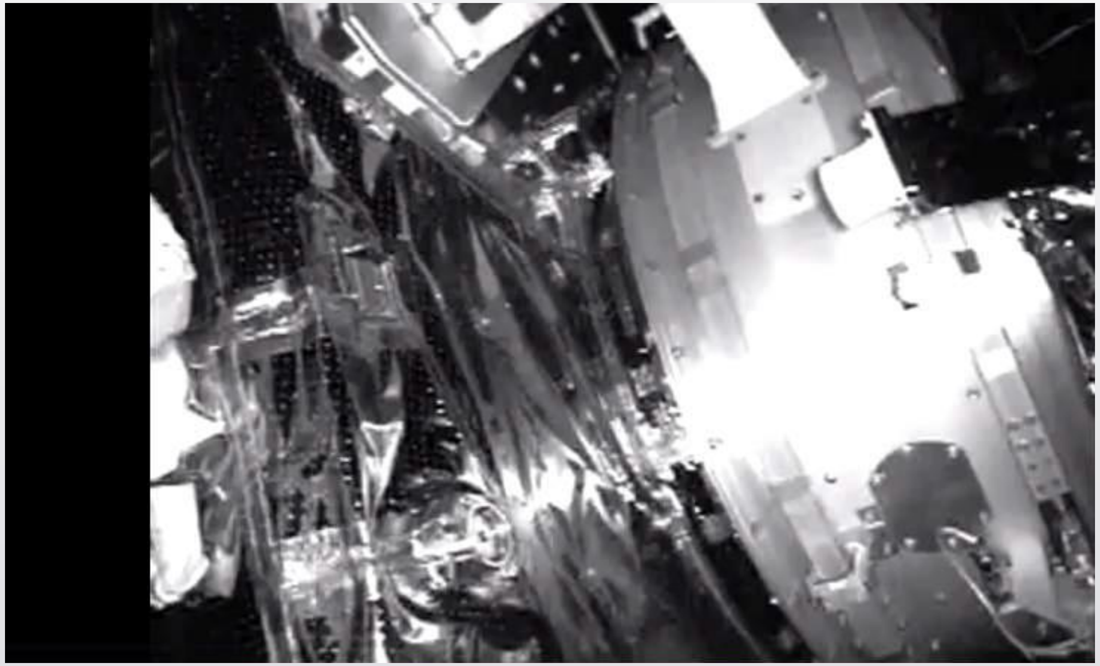
Engineer's Talk Du Jour …

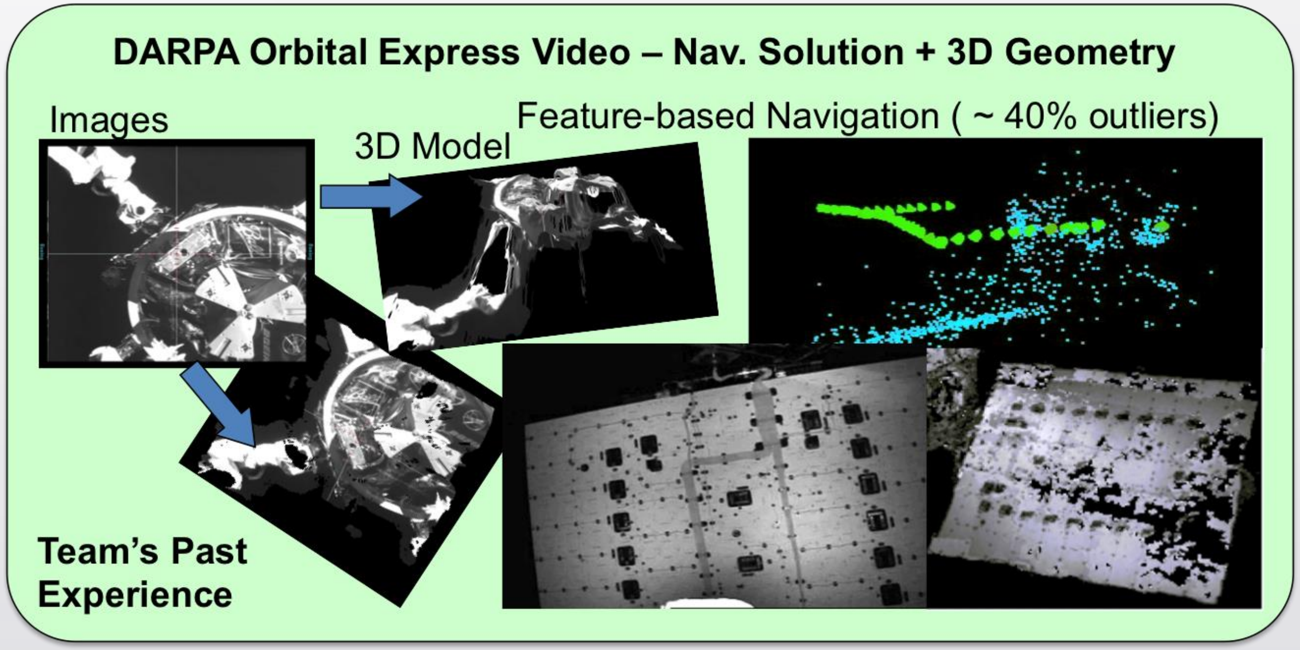
My uncertainty is worse than yours!

- **Image operations**
  - Image formation
  - Feature extraction (complex PAMI algorithms)
  - Feature tracking
- **Unique challenges**
  - Discretization (pixels) – random variables of high dimensions
  - Underlying physical process is continuous…

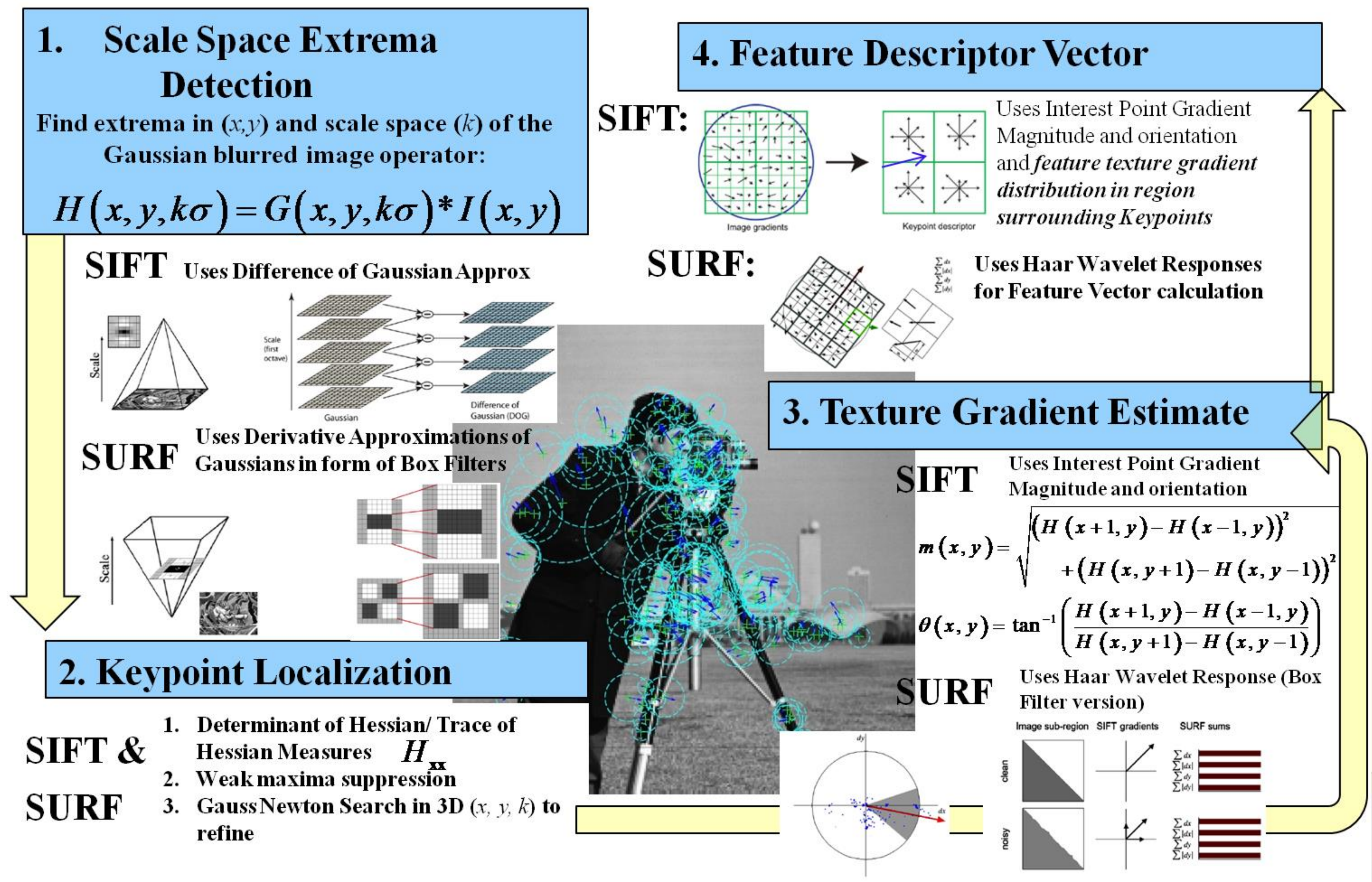
How to derive uncertainty from the Data?
How to rapproache with underlying physics?

Orbital Express Challenge: Close Range and Long Range





Features, features everywhere!



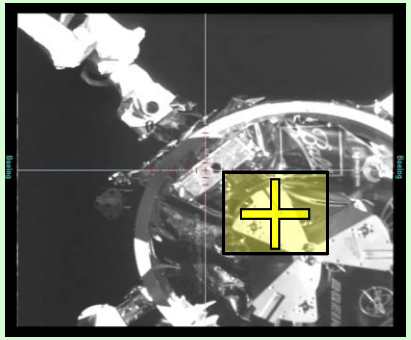
Scale space paradigm merges frequency and spatial domain considerations for extrema detection

Descriptor is a local identification tag to make subsequent identification possible.

It was great to have Profs Pouha and Zucker talk about related decision problems yesterday!

KLT Tracker: Summary

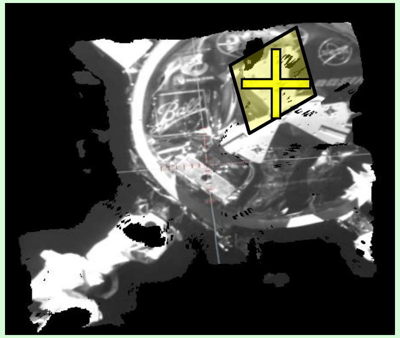
$I_1(x,y)$ @ t



window $W(\mathbf{x}(t))$

Δt

$I_2(x,y)$ @ $t+\Delta t$



window $W(\mathbf{x}(t+\Delta t))$

General deformation models

$$\mathbf{x}(t+\Delta t) = \mathbf{h}(\mathbf{x}(t+\Delta t))$$

Common use:

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \mathbf{d}$$

$$\mathbf{x}(t+\Delta t) = A\mathbf{x}(t) + \mathbf{d}$$

$$I(\mathbf{x}(t+\Delta t), t) = I(\mathbf{x}(t), t) + \nabla I^T \mathbf{d} + I_t$$

Brightness constancy

$$min\ E(\mathbf{d}) = \sum_{W(\mathbf{x})} \left[\nabla I^T \mathbf{d} + I_t\right]^2$$

$$\sum_{W(\mathbf{x})} \nabla I \left[\nabla I^T \mathbf{d} + I_t\right] = 0$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \mathbf{d} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$$A\mathbf{d} = B$$

$$\hat{\mathbf{d}} = (A^T A)^{-1} A^T B$$

Least squares estimate of displacement

Uncertainty Analysis of KLT Tracker

- Exploit local nature and already computed feature track.
  - Any multiple extrema in the window get thrown out already
- Conjecture: prior pdf for the tracked feature is a Gaussian with statistics proportional to the window size.
- Develop local series expansion for the displacement field

$$\delta\mathbf{x} \sim N(\mathbf{d}, \sigma_W)$$

$$\hat{\mathbf{d}} = \mathbf{d} + \left[\frac{\partial \mathbf{d}}{\partial \mathbf{x}}\right]_{\hat{\mathbf{d}}} \delta\mathbf{x} + HOT$$

- Compute error statistics from linear error theory.

$$E(\delta\mathbf{d}\delta\mathbf{d}^T) = \left[\frac{\partial \mathbf{d}}{\partial \mathbf{x}}\right]_{\hat{\mathbf{d}}} P_{\mathbf{x}} \left[\frac{\partial \mathbf{d}}{\partial \mathbf{x}}\right]_{\hat{\mathbf{d}}} + HOT$$

- Nonlinear extensions for full mass function propagations can be carried out (typically only of academic interest).

KLT Sensitivity Calculations

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \mathbf{d} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} \Rightarrow A\mathbf{d} = B$$

$$A\left[\frac{\partial \mathbf{d}}{\partial \mathbf{x}}\right] = \left[\begin{array}{c:c} B_u - A_u\mathbf{d} & B_v - A_v\mathbf{d} \end{array}\right]$$

$$\left[\frac{\partial \mathbf{d}}{\partial \mathbf{x}}\right] = (A^T A)^{-1}\left[\begin{array}{c:c} B_u - A_u\mathbf{d} & B_v - A_v\mathbf{d} \end{array}\right]$$

$$A_u = \sum \begin{bmatrix} 2I_x I_{xx} & I_{xx} I_y + I_x I_{xy} \\ I_{xx} I_y + I_x I_{xy} & 2I_y I_{xy} \end{bmatrix} \qquad B_u = \sum \begin{bmatrix} I_{xx} I_t \\ I_{xy} I_t \end{bmatrix}$$

$$A_v = \sum \begin{bmatrix} 2I_x I_{xy} & I_{xy} I_y + I_x I_{yy} \\ I_{yx} I_y + I_x I_{yy} & 2I_y I_{yy} \end{bmatrix} \qquad B_v = \sum \begin{bmatrix} I_{xy} I_t \\ I_{yy} I_t \end{bmatrix}$$

Uncertainty Analysis of the KLT Tracker

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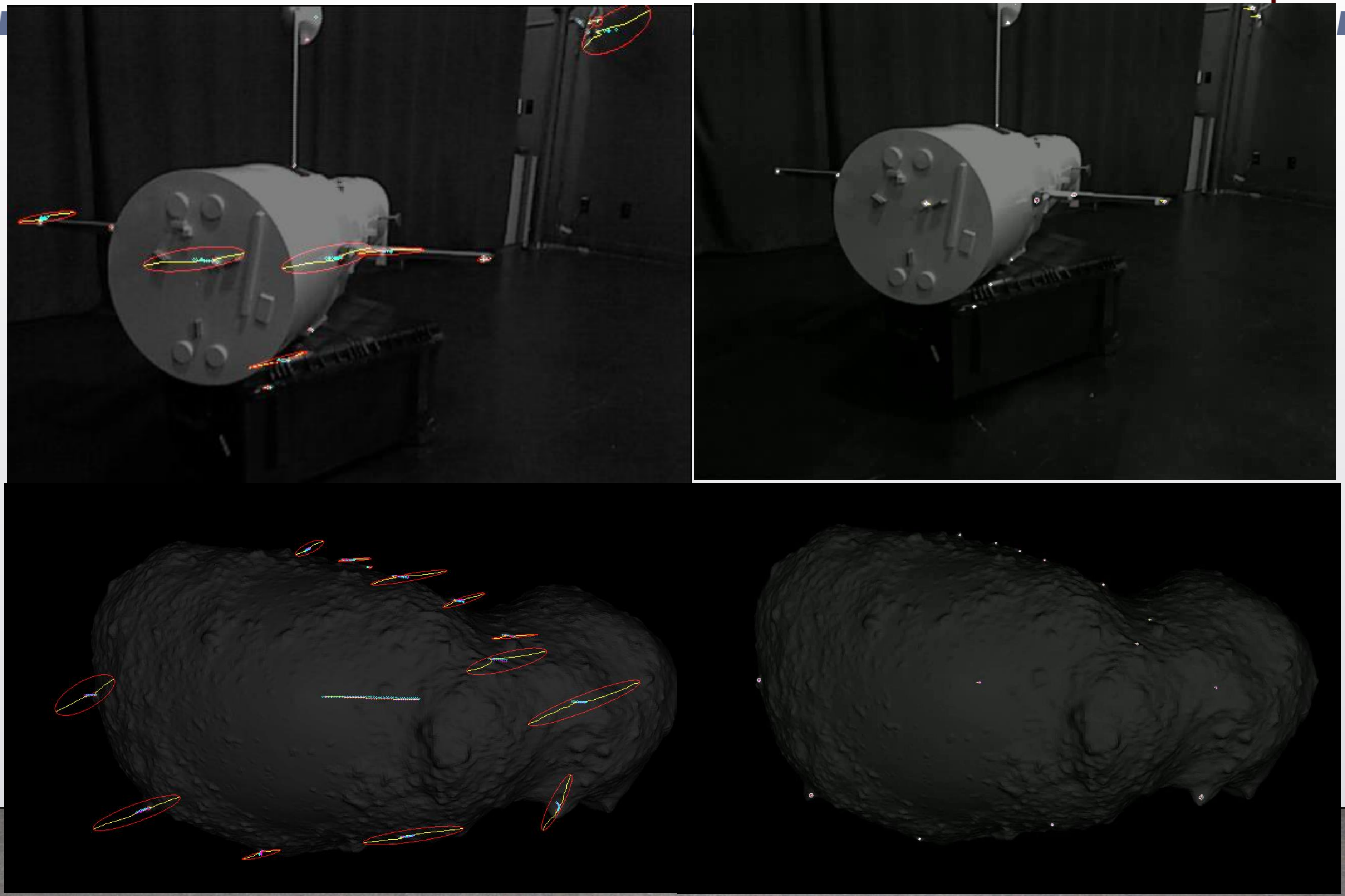

Uncertainty Analysis of KLT Tracker

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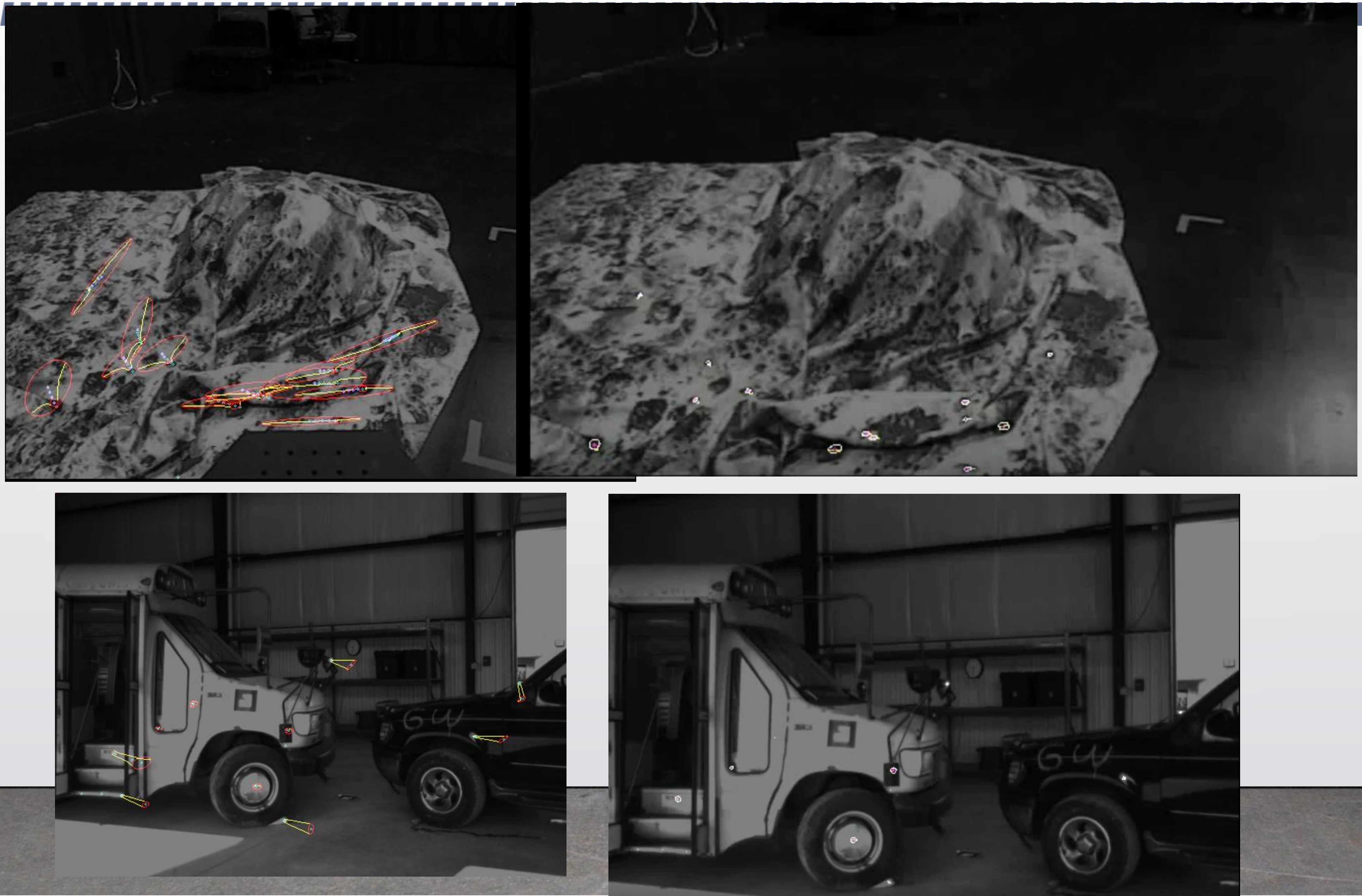
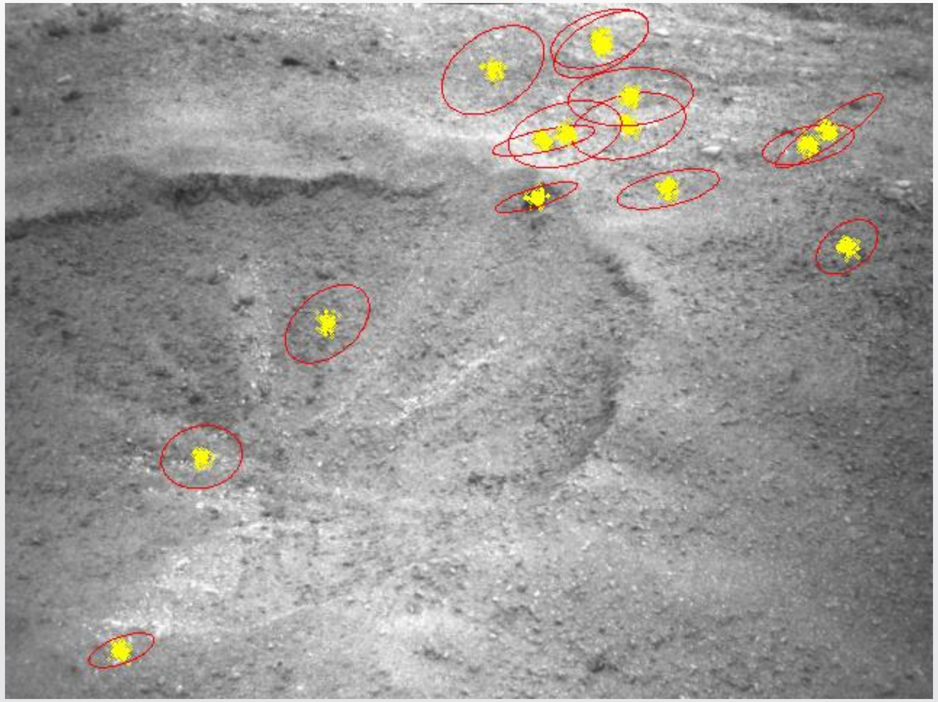
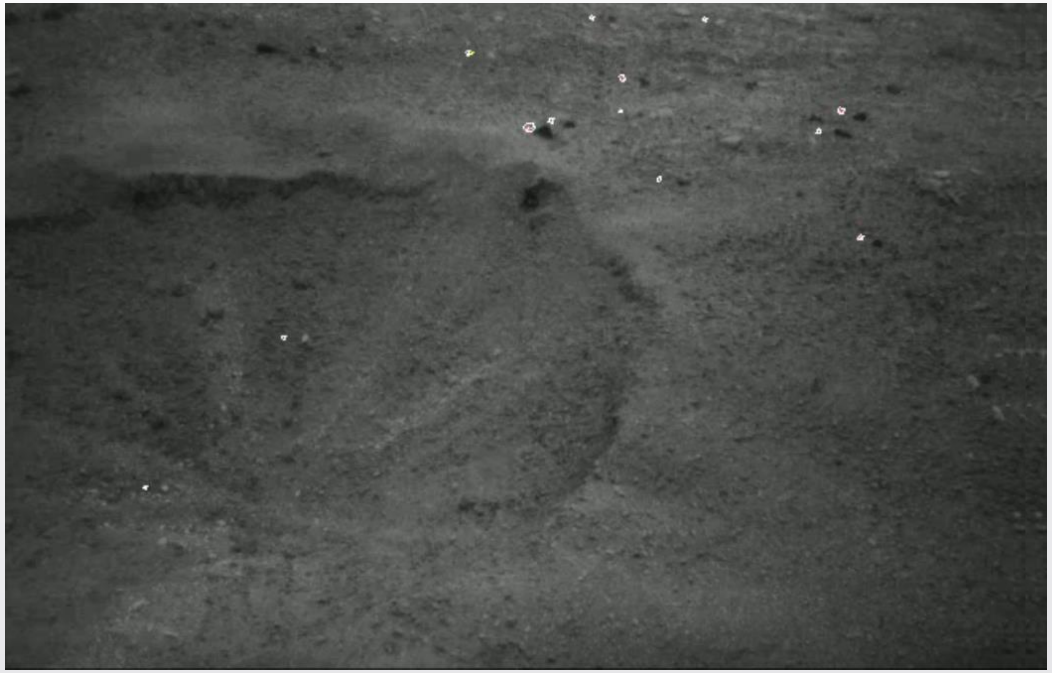


Uncertainty Analysis of KLT

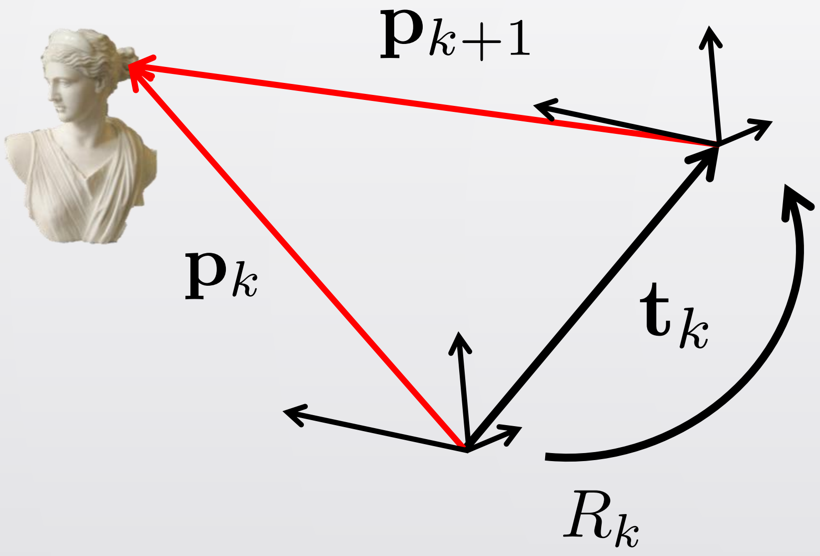




Key Observations

1. When track uncertainty balloons => highly likely we will lose it.
2. Large motion problems => our constant shift model doesn't capture effectively, so UQ is optimistic in that case.

Relative Pose Estimation Problem (3D correspondences)



Euclidean transformation: $\mathbf{p}_{k+1} = R_k \mathbf{p}_k + \mathbf{t}_k$

Cayley transform:

$$R = (I + [\tilde{\mathbf{q}}])^{-1}(I - [\tilde{\mathbf{q}}]) = (I - [\tilde{\mathbf{q}}])(I + [\tilde{\mathbf{q}}])^{-1}$$

Rearranging equations:

$$(I + Q(\mathbf{q}_k))\mathbf{p}_{k+1} = (I - Q(\mathbf{q}_k))\mathbf{p}_k + (I + Q(\mathbf{q}_k))\mathbf{t}_k$$

$$\mathbf{b}_k = [\tilde{\mathbf{a}}_k]\mathbf{q}_k + \mathbf{t}'_k$$

$$\mathbf{b}_k = \mathbf{p}_{k+1} - \mathbf{p}_k$$
$$\mathbf{a}_k = \mathbf{p}_{k+1} + \mathbf{p}_k$$

$$\begin{bmatrix} \mathbf{b}_k^1 \\ \mathbf{b}_k^2 \\ \vdots \\ \mathbf{b}_k^m \end{bmatrix} = \begin{bmatrix} [\tilde{\mathbf{a}}^1] & I_3 \\ [\tilde{\mathbf{a}}^2] & I_3 \\ \vdots & \vdots \\ [\tilde{\mathbf{a}}^m] & I_3 \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ \mathbf{t}'_k \end{bmatrix}$$

Pose Estimates

$$\begin{bmatrix} \mathbf{q}_k \\ \mathbf{t}'_k \end{bmatrix} = B^{\dagger} \begin{bmatrix} \mathbf{b}_k^1 \\ \mathbf{b}_k^2 \\ \vdots \\ \mathbf{b}_k^m \end{bmatrix}$$

Relative pose estimation is a linear algebra problem!

Rate Estimation

Over short interval time periods between measurements, let us consider expansion of relative pose parameters

Translation vector

$$\mathbf{t} = \mathbf{a}_1 + \mathbf{a}_2 \Delta t + \frac{1}{2}\mathbf{a}_3 \Delta t^2$$

Rotation parameterization

$$\mathbf{q} = \mathbf{b}_1 + \mathbf{b}_2 \Delta t + \frac{1}{2}\mathbf{b}_3 \Delta t^2$$

Cross product matrix:

$$\tilde{\mathbf{q}} = Q(\mathbf{b}_1) + Q(\mathbf{b}_2)\Delta t + \frac{1}{2}Q(\mathbf{b}_3)\Delta t^2$$

Rearranging equation:

$$(I + Q(\mathbf{b}_1) + Q(\mathbf{b}_2)\Delta t + \frac{1}{2}Q(\mathbf{b}_3)\Delta t^2)\mathbf{p}_{k+1} = (I - Q(\mathbf{b}_1) - Q(\mathbf{b}_2)\Delta t - \frac{1}{2}Q(\mathbf{b}_3)\Delta t^2)\mathbf{p}_k + (I + Q(\mathbf{q}))(\mathbf{a}_1 + \mathbf{a}_2\Delta t + \frac{1}{2}\mathbf{a}_3\Delta t^2)$$

Redefining: $\mathbf{c}_i = (I + Q(\mathbf{q}))\mathbf{a}_i$

$$(I + Q(\mathbf{b}_1) + Q(\mathbf{b}_2)\Delta t + \frac{1}{2}Q(\mathbf{b}_3)\Delta t^2)\mathbf{p}_{k+1} = (I - Q(\mathbf{b}_1) - Q(\mathbf{b}_2)\Delta t - \frac{1}{2}Q(\mathbf{b}_3)\Delta t^2)\mathbf{p}_k + \mathbf{c}_1 + \mathbf{c}_2\Delta t + \frac{1}{2}\mathbf{c}_3\Delta t^2$$

Rate Estimation

Rearranging further, we get

$$\mathbf{p}_{k+1} - \mathbf{p}_k = -\left(Q(\mathbf{b}_1) + Q(\mathbf{b}_2)\Delta t + \frac{1}{2}Q(\mathbf{b}_3)\Delta t^2\right)(\mathbf{p}_{k+1} + \mathbf{p}_k) + \mathbf{c}_1 + \mathbf{c}_2\Delta t + \frac{1}{2}\mathbf{c}_3\Delta t^2$$

Note the similarity with relative pose estimation problem.

$$\zeta_{k,k+1} = \left[\, Q(\rho_{k,k+1}),\ Q(\rho_{k,k+1})\Delta t,\ \tfrac{1}{2}Q(\rho_{k,k+1})\Delta t^2,\ I_{3\times 3},\ I_{3\times 3}\Delta t,\ \tfrac{1}{2}I_{3\times 3}\Delta t^2 \,\right] \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix}$$

General regression matrix

$$\Xi = H\mathbf{x}$$

$$\Xi = \begin{bmatrix} \zeta_{k-1,k} \\ \zeta_{k-2,k} \\ \zeta_{k-3,k} \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 & \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{bmatrix}^T$$

Least squares solution

$$\mathbf{x} = (H^T W H)^{-1} H^T W \Xi$$

$$H = \begin{bmatrix} Q(\rho_{k-1,k}), & Q(\rho_{k-1,k})\Delta t, & \frac{1}{2}Q(\rho_{k-1,k})\Delta t^2, & I_{3\times 3}, & I_{3\times 3}\Delta t, & \frac{1}{2}I_{3\times 3}\Delta t^2 \\ Q(\rho_{k-2,k}), & Q(\rho_{k-2,k})(2\Delta t), & \frac{1}{2}Q(\rho_{k-2,k})(2\Delta t)^2, & I_{3\times 3}, & I_{3\times 3}(2\Delta t), & \frac{1}{2}I_{3\times 3}(2\Delta t)^2 \\ Q(\rho_{k-3,k}), & Q(\rho_{k-3,k})(3\Delta t), & \frac{1}{2}Q(\rho_{k-3,k})(3\Delta t)^2, & I_{3\times 3}, & I_{3\times 3}(3\Delta t), & \frac{1}{2}I_{3\times 3}(3\Delta t)^2 \end{bmatrix}$$

Experimental Results: Experiment 1

(a) Left frame 1

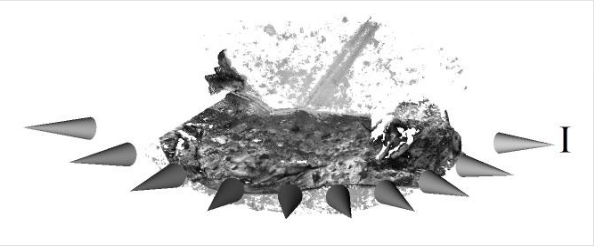
(b) Left frame 100

(c) Left frame 200

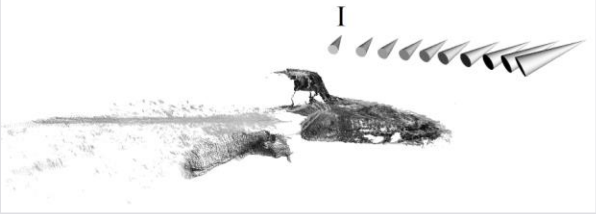
(d) Right frame 1

(e) Right frame 100

(f) Right frame 200

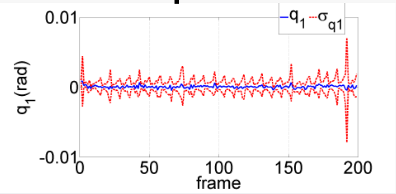


(a)

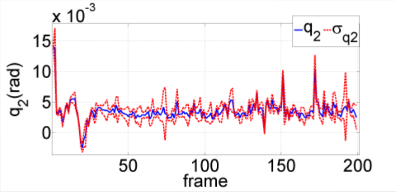


(b)

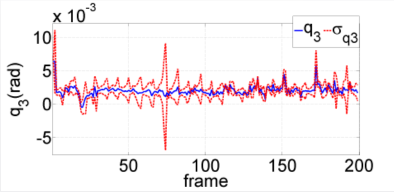
Relative pose estimates



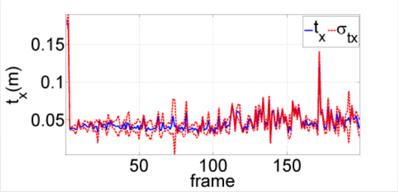
(a) q_1



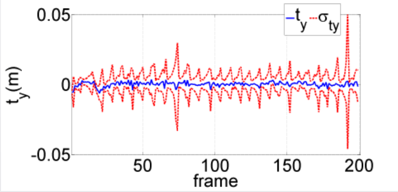
(b) q_2



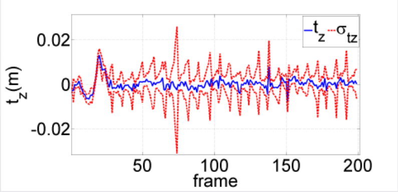
(c) q_3



(d) t_x

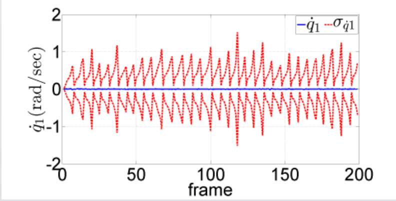


(e) t_y

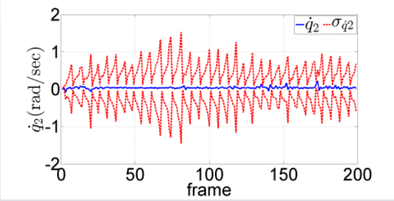


(f) t_z

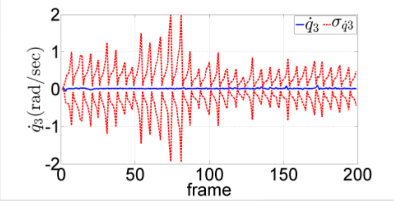
Pose rates:



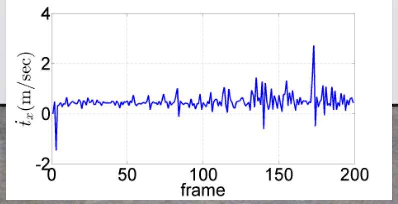
(d) $s_{\dot{q}1}$



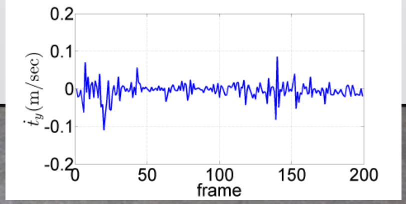
(e) $s_{\dot{q}2}$



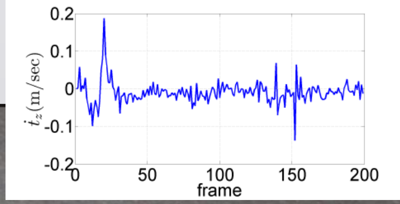
(f) $s_{\dot{q}3}$



(a) $\dot{t}_x$

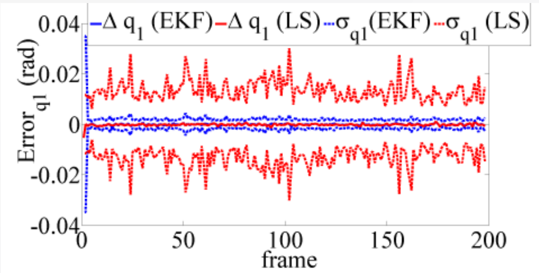


(b) $\dot{t}_y$

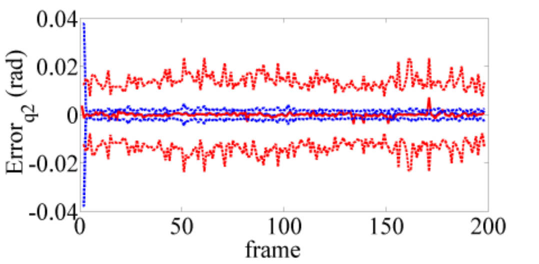


(c) $\dot{t}_z$

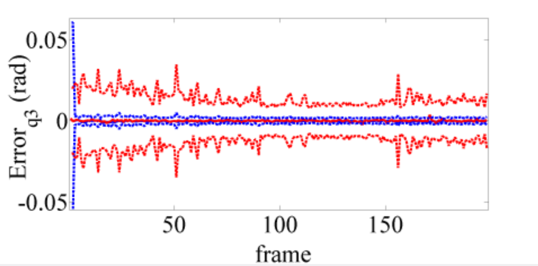
Experimental Results: Comparison with an iEKF



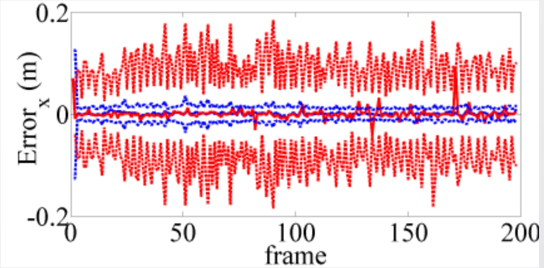
(a) q_1



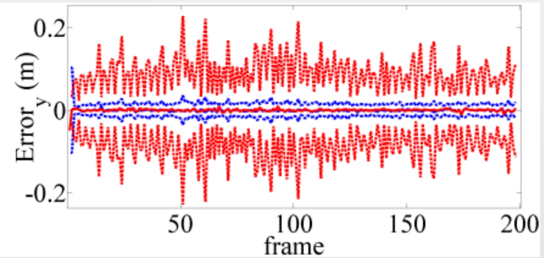
(b) q_2



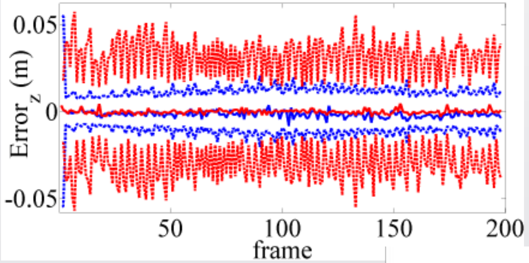
(c) q_3



(d) t_x



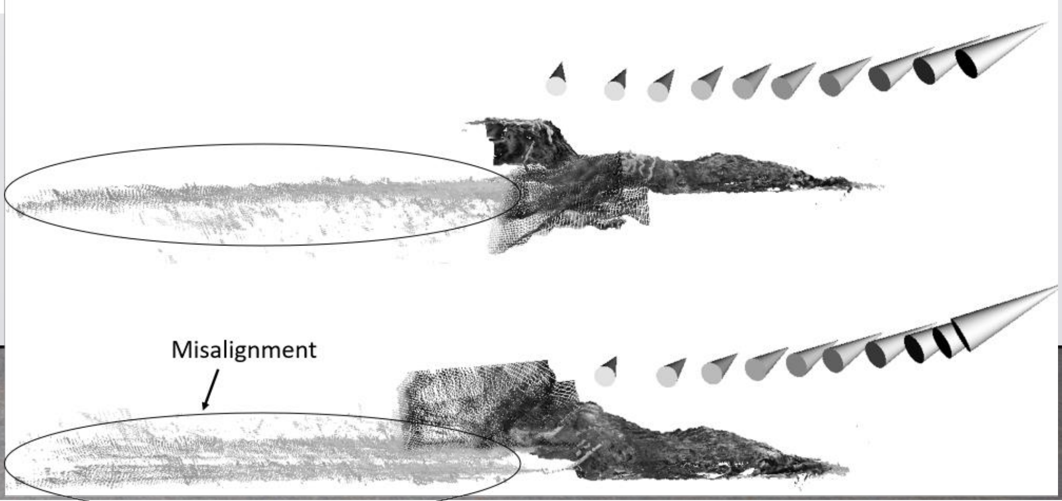
(e) t_y



(f) t_z

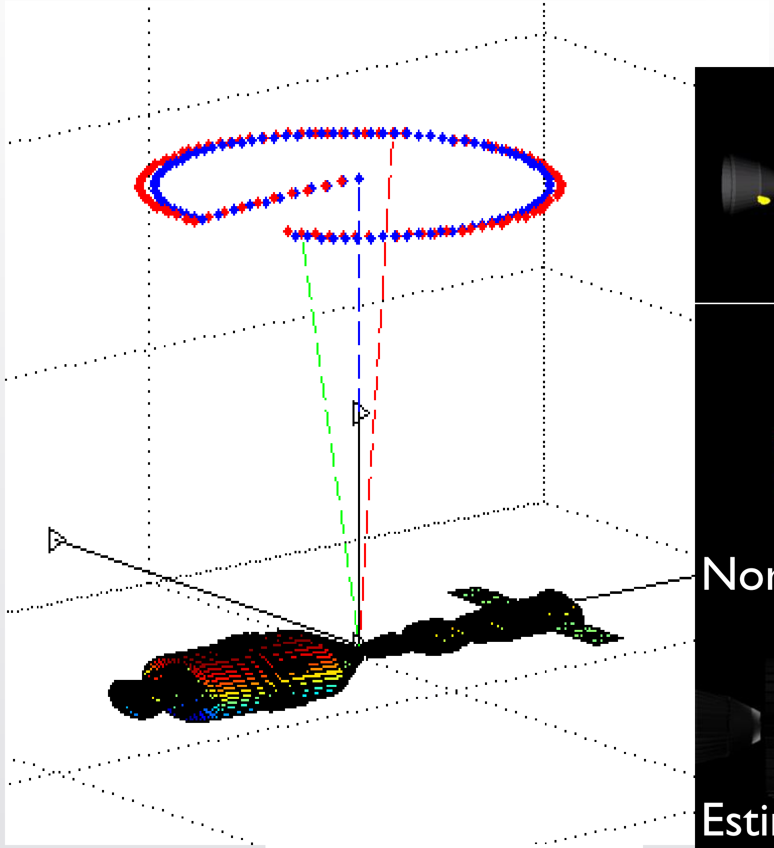
- Iterative EKF implementation based upon solution to linear algebra problem
- Covariance improves in state space
- Alignment improves by dynamical system constraints
- Weights of both problems depend on motion model fidelity

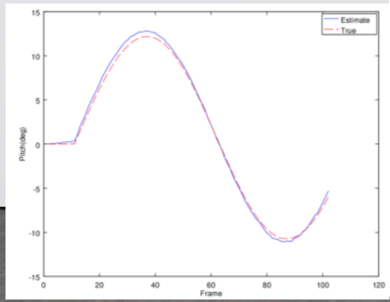
- Degenerates to the relative pose estimation when information is insufficient
- We have implementations that adapt and provide appropriate rate information dependent on information density

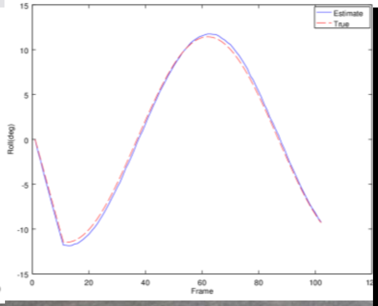


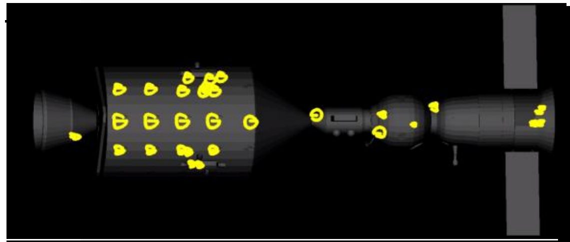
Smoother estimates used as the de-facto ground truth

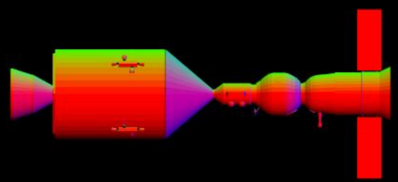
Applications: Photometric Stereo





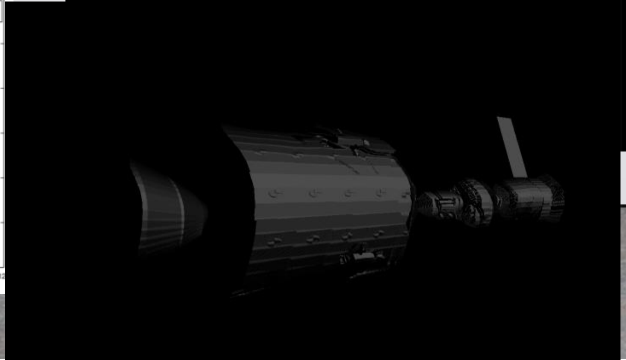




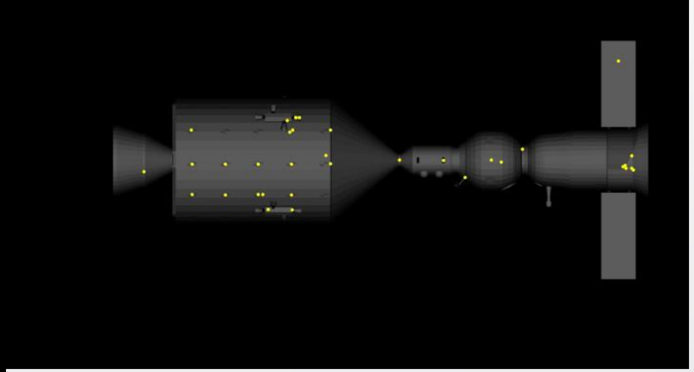


Normal vector estimates

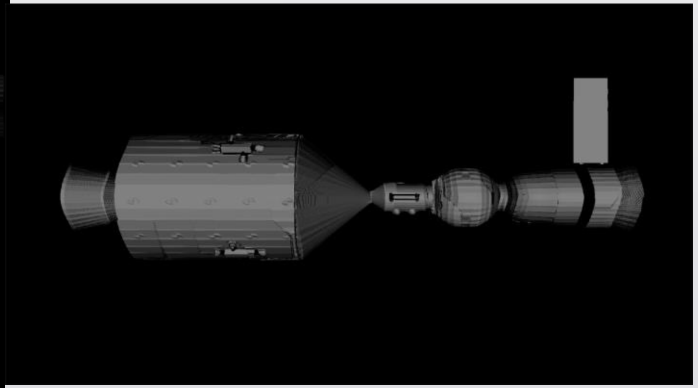
Estimation error



Diffuse Model

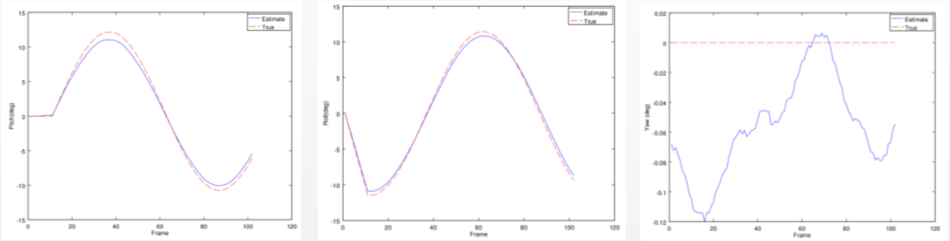


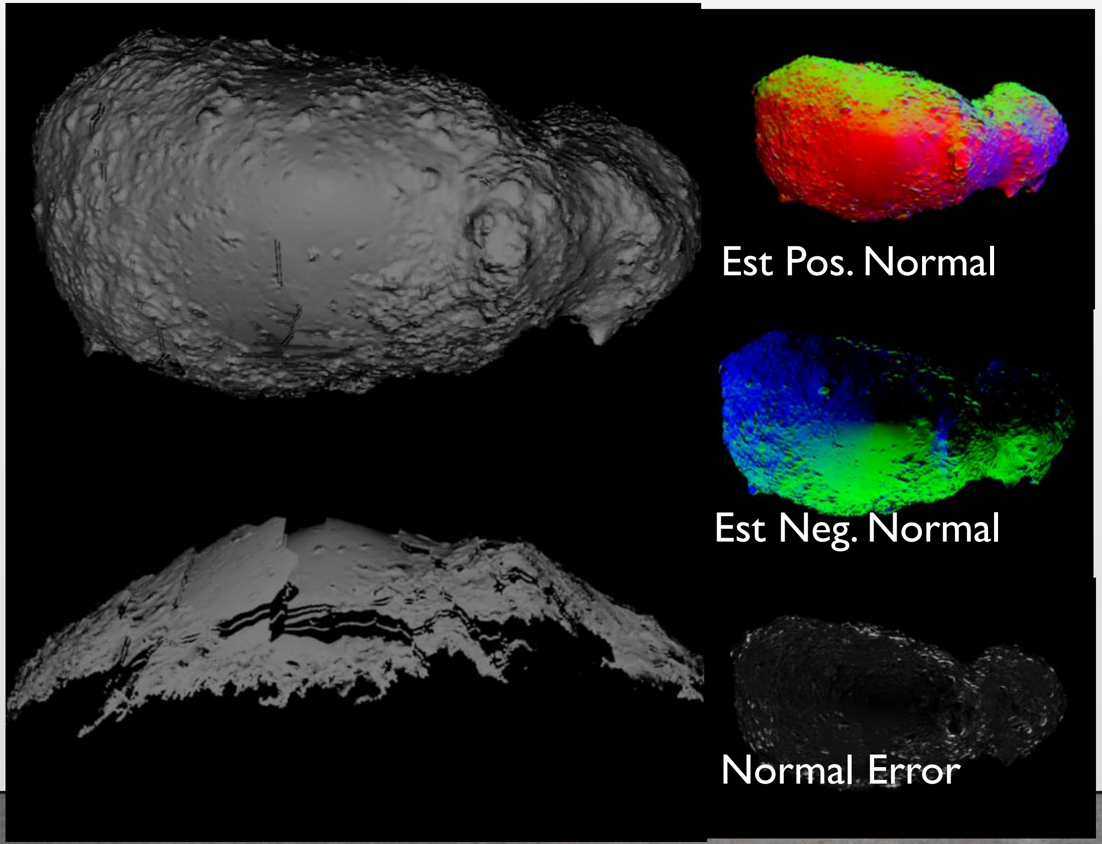
Feature Tracking Process

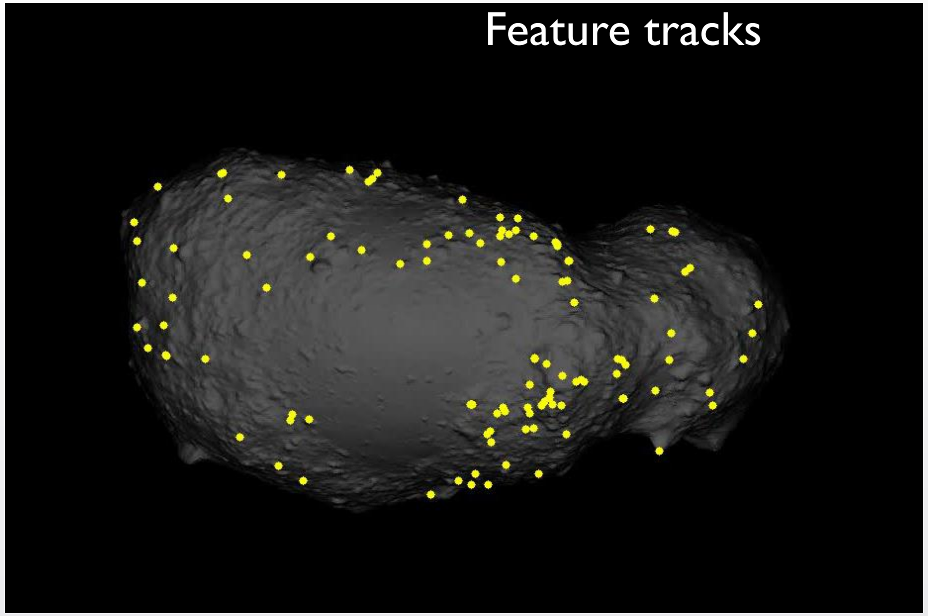


NOTE: not all surfaces have been reconstructed well

Applications: Diffuse Moving Bodies







- Estimation error greater in areas where the relative motion doesn’t add different illumination conditions
- Feature track uncertainties enable UQ of 3D reconstruction.

Photometric Stereo and Mapping

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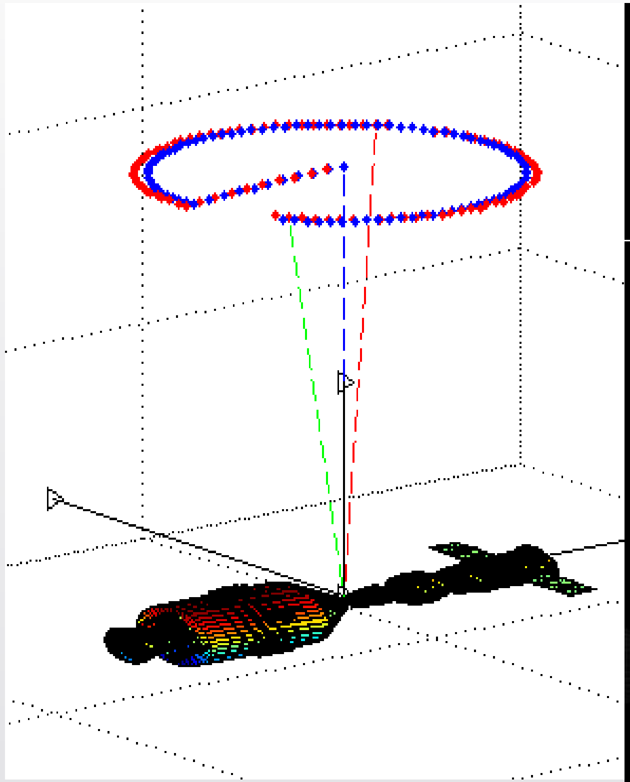
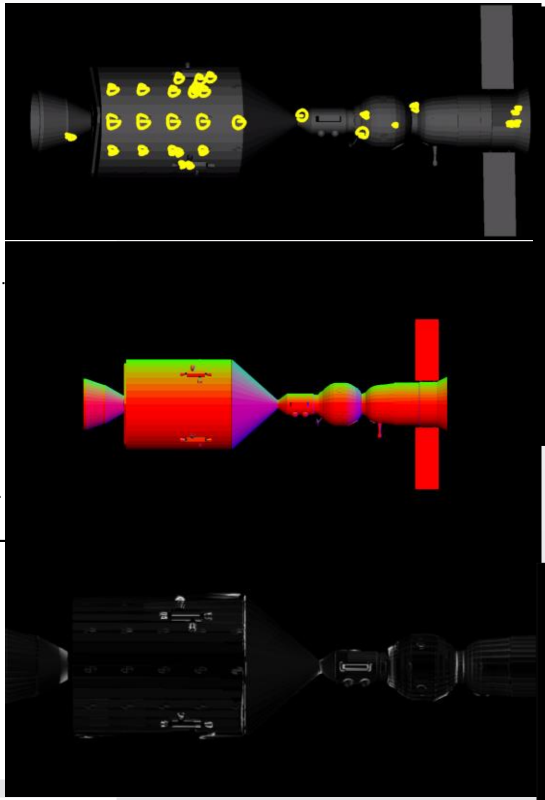
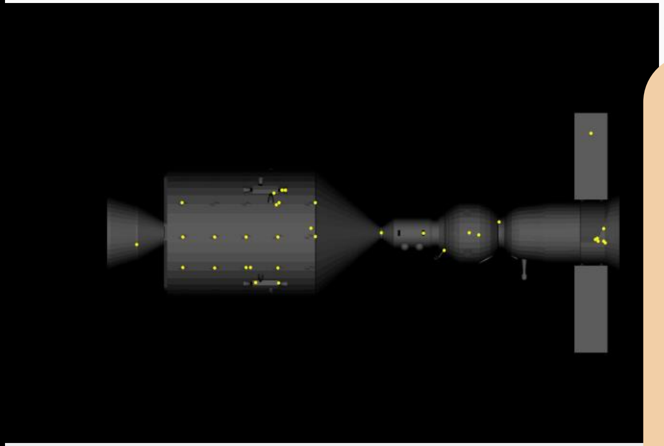
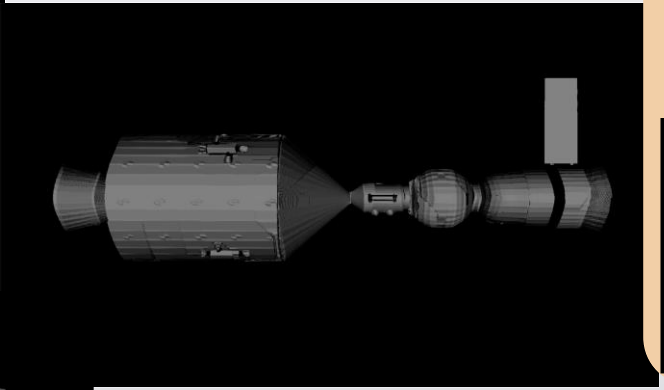


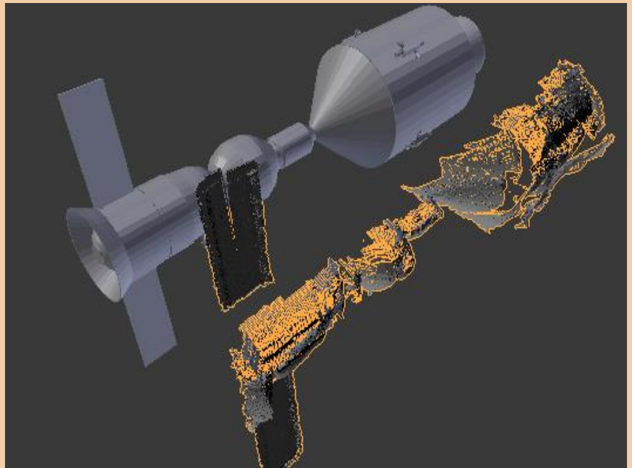




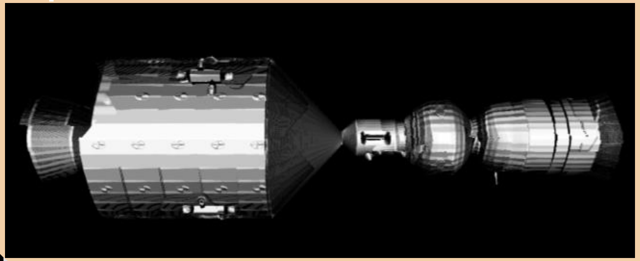
Feature Tracking Process

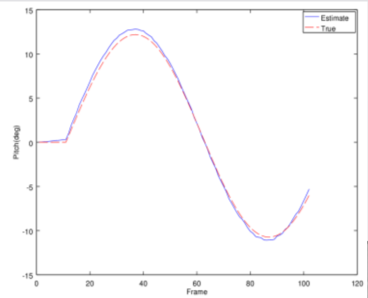


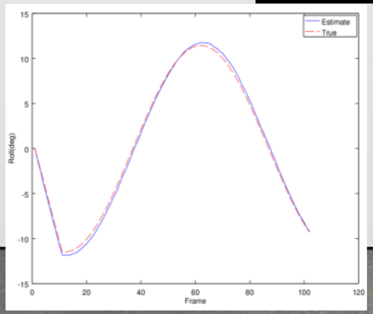
Specular object

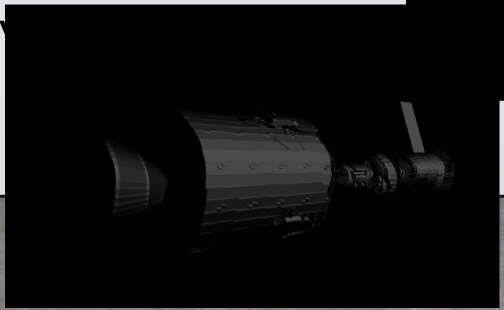


Specular Model/Reconstruction









NOTE: not all surfaces have been reconstructed well

Conclusions

- An approach for uncertainty quantification of KL tracker is developed
- Really useful in various applications – since this approach derives uncertainties from data. Many field robotics applications.
- Integration with guidance, navigation and control.
- Applications involving autonomous aerial refueling, ship landing, planetary exploration, asteroid tracking, debris imaging and satellite servicing. Current LASR work.