Measurement Selection for Bias Reduction in Structural Damage Parameter Estimation

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Outline

• Introduction the overall project “Progressive Fault Identification and Prognosis in Aircraft Structure Based on Dynamic Data Driven Adaptive Sensing and Simulation”

• Measurement Selection for Bias Reduction in Structural Damage Parameter Estimation
  ✓ Problem formulation
  ✓ Measurement selection algorithms
  ✓ Case studies

• Summary and on-going efforts
Introduction

• Structural health monitoring and management (SHM\(^2\)) is very important for air force applications

• Dynamic responses of an aerospace structure contain rich information

• Limitations of the current dynamic response based SHM\(^2\) systems
  – Sensory system is fixed
  – Applications modeling is either based on mechanistic principles or data driven models using historical data
  – Sensor-structure interaction dynamics are not exploited; Uncertainties not fully addressed
Research objectives

Overarching Goal: To create a new methodology of progressive structural fault identification and prognosis for Air Force applications based on the framework of dynamic data-driven applications systems (DDDAS).

Applications

Applications measurements systems and methods

Applications modeling

Mathematical and statistical algorithms

Aircraft structure

Piezoelectric transducer

Tunable piezoelectric impedance sensor systems

Data collection

Data-driven Sensor tuning

Data driven sensor tuning and structural weakness estimation

Structural weakness growth modeling and prognosis

Structural weakness

Failure threshold

Failure time prognosis

Time
Main Progresses Up to Date

• Measurement systems
  – Development and validation of impedance sensing modality
  – Synthesis of tunable circuitry to amplify measurement signal-to-noise ratio as well as fault features
  – Utilizing impedance measurements to facilitate fault identification in structures
  – Utilizing tunable impedance sensor to enable highly sensitive and robust fault identification (on-going)
Main Progresses Up to Date

• Dynamic data-driven algorithms for structural health monitoring, diagnosis, and prognosis
  – Systematic identifiability study on damage detection using system dynamic responses
  – Integrating first-principle-based inverse sensitivity analysis with measurement data features to enable Bayesian inference-based fault identification (i.e., identification of both fault location and severity) in structures.
  – Identifying structural weakness progression using sensing data and dynamic finite element application models
  – Multiphase Bayesian inference for degradation path tracking and failure prediction.
  – Measurement selection for bias reduction in structural damage parameter estimation (this work)
Background

• Natural frequencies are commonly used for structure damage detection and identification
• Linearization is adopted for problem simplification – induce biases
• The structural dynamic system is an underdetermined system – may vary from true solutions
• Propose new algorithm to address these issues
## Linearization of Damage Isolation

<table>
<thead>
<tr>
<th>Names</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Equation of Structures</td>
<td>( \ddot{\mathbf{x}}(t) + \mathbf{Kx}(t) = \mathbf{F}(t) )</td>
</tr>
<tr>
<td>Eigenvalue Problem for Healthy and Damaged Structures</td>
<td>( (\mathbf{K} - \lambda_i \mathbf{M})\phi_i = 0 )</td>
</tr>
<tr>
<td></td>
<td>( (\mathbf{K}^d - \lambda_i^d \mathbf{M})\phi_i^d = 0 ),</td>
</tr>
<tr>
<td>Change in Stiffness Matrix</td>
<td>( \mathbf{K}^d = \mathbf{K} + \Delta\mathbf{K} )</td>
</tr>
<tr>
<td>Change in Eigenvalues and Eigenvectors</td>
<td>( \lambda_i^d = \lambda_i + \Delta\lambda_i )</td>
</tr>
<tr>
<td></td>
<td>( \phi_i^d = \phi_i + \Delta\phi_i )</td>
</tr>
<tr>
<td>Linear Approximation of the Change in Eigenvalues</td>
<td>( \Delta\lambda_i = \phi_i^T \Delta\mathbf{K}\phi_i )</td>
</tr>
<tr>
<td>Expression in Elemental Stiffness Matrices</td>
<td>( \Delta\mathbf{K} = \sum_{j=1}^{n} \Delta\alpha_j \mathbf{K}_j^{(e)} )</td>
</tr>
<tr>
<td>Final Linear Expression</td>
<td>( \Delta\lambda = \mathbf{S}\Delta\alpha ) and ( \mathbf{S}(i,j) = \phi_i^T \mathbf{K}_j^{(e)} \phi_i )</td>
</tr>
</tbody>
</table>
The Linearization of the Damage Isolation Problem by Natural Frequencies:

\[ \Delta \lambda = S \Delta \alpha \]

\( S \) is the first order sensitivity matrix, \( \Delta \lambda \) is the measured difference of the eigenvalues between healthy and damaged structures and \( \Delta \alpha \) is the unknown damage parameters

Broadly applied to:
- General structure stiffness identification (Law, S. and et al. 1990)
- Railway bridge health diagnosis (Uzgider, E. and et al. 1993)
- Crack detection in beam-type structure (Ki,m J-T and et al. 2003)
- …

Advantages – Easy for solution approach and model simplification

However…..
- The linear system is a underdetermined system with more unknowns than the available measurements
- The true solution is usually sparse
Literature Review

Existing Approach to address the issues in solving $\Delta \lambda = S \Delta \alpha$

1. Include high order terms (J. Tang and et al. 2008)
   • The second order terms included into the linear system
   • Drawbacks – increase the complexity of the model and reduce the efficiency in solution approaching.

2. Regularization by $L_1$ penalty functions (Wang, Y et al. 2013 & Zhou, X. et al. 2015)
   • $L_1$ regularization is applied in the linear equation to constrain the sparsity in the solution
   • Drawbacks – solution may vary from the underlying truth; Sparsity is not sufficiently enforced.
Measurement Selection

**Key observations:** Subset of natural frequencies can have less biased damage estimation compared to employ all available ones.

Mathematically, find a subset $k$ that minimizes the bias

$$d^{(k)} = \|\widehat{\Delta \alpha}^{(k)} - \Delta \alpha^{\text{truth}}\|_2$$

where $\widehat{\Delta \alpha}^{(k)}$ is estimated by

$$\min \|\Delta \lambda^{(k)} - S^{(k)} \Delta \alpha\|_2 + \beta \|\Delta \alpha\|_1$$

**Objectives:**

- Based on the first order approximation with regularization, identify a stable solution method.
- Select proper natural frequencies – reduce biases in the damage estimation.
Solving for $\Delta \alpha$

- $L_0$ regularization should be the ideal penalty function for sparse solution in damage isolation.
- However, $L_0$ penalty is hard to solve and needs high computational cost.
- $L_1$ is good for substitute under conditions in (Candes, E. and et al 2005).

\[
\begin{align*}
\min_{\Delta \alpha} & \left\| \Delta \lambda^{(k)} - S^{(k)} \Delta \alpha \right\|_2 + \beta \left\| \Delta \alpha \right\|_1 \\
\downarrow & \\
\min \left\| \Delta \alpha \right\|_1, \text{ s.t. } & \left\| \Delta \lambda^{(k)} - S^{(k)} \Delta \alpha \right\|_2 \leq \varepsilon \\
\downarrow & \\
& \text{Restricted Isometry Property (RIP)} \\
\min \left\| \Delta \alpha \right\|_1, \text{ s.t. } & \left\| \overline{\Delta \lambda^{(k)}} - \overline{S^{(k)}} \Delta \alpha \right\|_2 \leq \varepsilon \\
\text{where } & \overline{\Delta \lambda^{(k)}} = \Phi \Delta \lambda^{(k)} \text{ and } \overline{S^{(k)}} = \Phi S^{(k)} \\
\Phi & \text{ is a square random matrix}
\end{align*}
\]
Natural Frequencies Selection Algorithm

- **Note:**
  \[ \arg_k \min \| \Delta \lambda^{(k)} - S^{(k)} \Delta \alpha \|_2 \Leftrightarrow \arg_k \min \| \tilde{\alpha}^{(k)} - \Delta \alpha^{\text{truth}} \|_2 \]

- **Replace** \( \Delta \alpha^{\text{truth}} \) **into** the linear equation
  \[ \Delta \lambda = S \Delta \alpha^{\text{truth}} + e \]

  \[ \Delta \lambda^{(k)} = S^{(k)} \Delta \alpha^{\text{truth}} + e^{(k)} \]

- **Apply least square method**
  \[ \Delta \alpha^{\text{truth}} = \left( S^{(k)^T} S^{(k)} \right)^{-1} S^{(k)^T} \left( \Delta \lambda^{(k)} - e^{(k)} \right) \]

  \[ \Delta \tilde{\alpha}^{(k)} = \left( S^{(k)^T} S^{(k)} \right)^{-1} S^{(k)^T} \left( \Delta \lambda^{(k)} \right) \]

- **Approximate** \( d^{(k)} \) **using** \( b^{(k)} \)
  \[ d^{(k)} = \| \tilde{\alpha}^{(k)} - \Delta \alpha^{\text{truth}} \|_2 \]

  \[ \approx \left\| \left( S^{(k)^T} S^{(k)} \right)^{-1} S^{(k)^T} e^{(k)} \right\|_2 = \tilde{b}^{(k)} \]
Procedure to Estimate the Boundary of $\epsilon^{(k)}$

- $\hat{\epsilon}$ could be estimated through the higher order terms in the approximation procedure. General expressions of higher order terms are available in (Wong. C.N and et al. 2004)

- $\Delta \alpha$ is estimated by three loops:
  - The outer loop is used for checking the convergence of damage locations
    - The first inner loop is adopted to stabilize results of regularization
    - The second inner loop is to figure out healthy elements


## Natural Frequencies Selection Algorithm

1. Define $L(\Delta \alpha)$ is the locations of non-zero elements of $\Delta \alpha$ and $L^{\text{new}} = [1, 2, 3, \ldots, n]$
2. Define $C(\Delta \alpha_i = 0)$ is the constrain that $\Delta \alpha_i = 0$ and $C^{\text{new}} = \text{empty}$
3. Do {
   
   3.1 For $l = 1, 2, \ldots, L$
      
      Generate random matrix $\Phi_l$ and compute $\Delta \lambda = \Phi_l \Delta \lambda$ and $\tilde{S} = \Phi_l S$
      
      Solve $\min \|\Delta \alpha\|_1$, s.t. $\|\Delta \lambda - \tilde{S} \Delta \alpha\|_2 \leq \epsilon$ with $C^{\text{new}}$ and record the estimation $\tilde{\alpha}_i, L = [\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n, L]^T$
      
      Define $\tilde{\alpha}_i, L = [\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n, L]$ and $\tilde{\alpha} = [\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n]$
      
      3.2 For $i = 1, 2, \ldots, n$
         
         if $P(\tilde{\alpha}_i \geq -0.05 \geq 0.95)$
            
            $\tilde{\alpha}_i = 0$
            
            $C^{\text{new}} = C^{\text{new}} \cup C(\Delta \alpha_i = 0)$
         
         else
            
            $\tilde{\alpha}_i = \text{mean}(\tilde{\alpha}_i)$
         
         end
      
      end
   
   }While ($L^{\text{old}} \neq L^{\text{new}}$)
4. Calculate the estimated error $\hat{e} = \Delta \lambda - \tilde{S} \tilde{\alpha}$
5. For $k = 1, 2, \ldots, 2^m - 1$
   
   Calculate the estimated bias $\hat{b}^{(k)} = \left\| \left( S^{(k)} \right)^T \left( S^{(k)} \right)^{-1} S^{(k)} \left( e^{(k)} \right)^T \right\|_2$
   
   End
6. Select $k^* = \arg \min \hat{b}^{(k)}$ as the final combination
Case Study

Beam Structures for Case Study

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus</th>
<th>Density</th>
<th>Length</th>
<th>Width</th>
<th>Thickness</th>
<th>$m$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$7.1 \times 10^{10}$ N/m$^2$</td>
<td>2700kg/m$^3$</td>
<td>0.4184m</td>
<td>0.0381m</td>
<td>3.175mm</td>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

Example Damage Case - $\Delta \alpha_8 = -0.3$ and $\Delta \alpha_{17} = -0.1$

Plot of Bias vs Combination

- It can be seen that the least biased estimation is NOT from all the seven natural frequencies.
- The 26$^{th}$ combination is the fifth and sixth natural frequencies.
Numerical Study

The results of randomizing the sensitivity matrix after the first inner loop in the algorithm.

True Damage Locations

Neglectable Damages – Removed in the second inner loop
Numerical Study

The results after the outer loop in the algorithm.
Numerical Study

Plots of \( \hat{b}^{(k)} \) and the \( d^{(k)} = \| \Delta \alpha^{(k)} - \Delta \alpha_{\text{truth}} \|_2 \) for \( k = 8, 9, \ldots, 127 \)

\[ k^* = 26, \text{ the true combination that minimize the bias!} \]
Numerical Study

Comparison of damage parameter estimation using different approaches

- $L_2$ penalty performs the worst with inaccurate estimation of damages distributed along elements.
- $L_1$ penalty performs comparable results at the true damage locations, but has small magnitudes on a few non-damaged elements.
- The proposed iterative random matrix method performs the best.
Extensive Simulation Study

Performance of the Proposed Algorithm in Different Damage Scenarios

<table>
<thead>
<tr>
<th></th>
<th>Single Fault</th>
<th>Two Faults</th>
<th>Three Faults</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k^* = \arg_k \min d^{(k)}$</td>
<td>94.3%</td>
<td>83.5%</td>
<td>80.2%</td>
</tr>
<tr>
<td>$d^{(k^*)} \leq d^{(127)}$</td>
<td>97.1%</td>
<td>94.7%</td>
<td>91.5%</td>
</tr>
<tr>
<td># simulations</td>
<td>160</td>
<td>12160</td>
<td>583680</td>
</tr>
</tbody>
</table>

- $k^* = \arg_k \min d^{(k)}$ indicates the selected combination $k^*$ is the optimal combination that minimizes the bias
- $d^{(k^*)} \leq d^{(127)}$ indicates that the selected combination $k^*$ has smaller bias compared with the case when all natural frequencies are used.
Discussion

- The proposed method is validated for *mild* damages.
- Severe damages highly reduce the accuracy of linear approximation, which hurts the estimation of $\widehat{\Delta \alpha}$.

**Accuracy of Linear Approximation as Function of Severities**

![Graph showing accuracy of linear approximation as a function of severities.](image)
Discussion

Severe Damage Case $\Delta \alpha_3 = -0.9, \Delta \alpha_{10} = -0.9$ and $\Delta \alpha_{18} = -0.2$

Results after the algorithm

Additional Non-damaged Element

Wrong Estimation of Element 18
Summary and Ongoing Work

- **Summary on measurement selection**
  - Improved algorithm with $L_1$ penalty on damage identification
  - Proposed natural frequency selection technique effectively reduces estimation bias
  - Case studies illustrate the effectiveness of the selection method

- **Work planned for near future**
  - Adaptive online tuning of sensor for damage identification
  - Considering change point in the damage progression
  - Experiments and test

- **Long term plan**
  - Developing new sensing systems with both spatial and dynamic tunability
  - Transfer learning enabled structure health management
Publications and Dissemination

Publications:
8. Yuhang Liu , Qi Shuai , Shiyu Zhou, and Jiong Tang, 2017, Prognosis of Structural Damage Growth via Integration of Physical Model Prediction and Bayesian Estimation, IEEE Transactions on Reliability, DOI: 10.1109/TR.2017.2713760

Conferences:
• DDDAS mini track at INFORMS annual conference 2015
• DDDAS session at INFORMNS annual conference 2016

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