A New Control Theory for Dynamic Data Driven Systems

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Joint work with Yuh-Shyang Wang, James Anderson & John C. Doyle
New application areas
New application areas
**Reflex**

- Distributed  
  - Fast (offline computation)
- Rigid  
  - Unstable real dynamics

**Planning**

- Centralized  
  - Slow (online computation)
- Flexible  
  - Stable virtual dynamics
Designing reflex layers for large-scale cyber-physical systems
Power grid: frequency regulation

Topology

Subsystem interaction

\[ x_i \quad x_j \]

\[ u_i \quad u_j \]

\[ \mathbf{R} \quad \mathbf{M} \]

\[ \mathbf{R}_{ij}, d \quad \mathbf{M}_{ij}, d \]

\[ \text{constraint} \]

\[ \text{spectral radius} \]

\[ \text{unstable} \]
Centralized and dense control
Sparse, distributed & localized control w/ delayed communications
new challenges

info exchange constraints

NP-hard problems
new challenges

info exchange constraints

comms channels
new challenges

info exchange constraints

comms channels

infrastructure co-design
new challenges

info exchange constraints

comms channels

infrastructure co-design

large (huge) scale

~125 million homes powered

~100 billion neurons, ~100k miles of axons

billions of nodes

billions of devices
A unified theory for system design

robustness  

efficiency  

resilience  

info exchange  

comms in the loop  

co-design  

scale to  

millions, billions,  

trillions of nodes
DDDAS + SLS
for Automated Control System
Design and Redesign
\[ m \left( \frac{dx}{dt} + q \psi - r \phi \right) = F_x - mg \sin \theta \]

\[ m \left( \frac{dy}{dt} + r \phi - q \psi \right) = F_y + mg \cos \phi \sin \theta \]

\[ m \left( \frac{dz}{dt} - y \theta \right) = F_z + mg \cos \theta \cos \phi \]

\[ M_x - (\alpha - \frac{dy}{dt} - \frac{1}{r} \frac{dz}{dt}) \phi - (\alpha - \frac{1}{r} \frac{dz}{dt}) \psi - (\alpha - \frac{1}{r} \frac{dz}{dt}) \theta \]

\[ M_y - (\frac{dy}{dt} + 2 \frac{r}{d} \psi) - \frac{1}{r} \frac{dz}{dt} \phi - \frac{1}{r} \frac{dz}{dt} \psi - \frac{1}{r} \frac{dz}{dt} \theta \]

\[ M_z - \frac{r}{d} \frac{dr}{dt} + \frac{1}{d} \frac{dz}{dt} + \frac{1}{d} \psi \frac{dz}{dt} + \frac{1}{d} \theta \]

\[ p = \frac{d\psi}{dt} - \frac{d\psi}{dt} \sin \theta \]

\[ q = \frac{d\theta}{dt} \cos \psi + \frac{d\psi}{dt} \sin \psi \cos \theta \]

\[ r = \frac{d\theta}{dt} \sin \psi + \frac{d\psi}{dt} \cos \psi \cos \theta \]
Example 11: Consider the LLQG problem. It can be stated as an optimization problem, i.e.,

\[
\min_{X} \frac{1}{2} \text{tr}(X) - \text{tr}(X^{-1}Q) + \text{tr}(X^{-1}R) + \text{tr}(X^{-1}S)
\]

subject to \( X \geq 0 \) and \( X = F(X) \).

In this section, we will derive the optimal solution to this problem.

Out of the optimal solution, we can evaluate the performance of the system.

In conclusion, the LLQG problem can be solved by finding the \( X \) that minimizes the above objective function subject to the given constraints.
Need a theory for **local, scalable and automated**

Adaptive Control and Sys ID

Fault & Change Detection

Controller (re)design

Actuation/sensing (re)design
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A System Level Approach
A System Level Approach

Novel parameterizations of stabilizing controllers
A System Level Approach

Novel parameterizations of stabilizing controllers

(Approximate) Locality = scalability
controlled output

measurement

disturbance

control input
minimize $\|P_{11} - P_{12}K(I - P_{22}K)^{-1}P_{21}\|$
subject to $K$ stabilizes $P$
$K \in C$ info sharing constraints
\[ u(z) = K(z)y(z) \]

**control input**

**measurement**

\( K \) controller

- act
- sns
- sns
- sns
\[
\begin{bmatrix}
u_1(z) \\
u_2(z)
\end{bmatrix} = \begin{bmatrix}
K_{11}(z) & K_{12}(z) & K_{13}(z) \\
K_{21}(z) & K_{22}(z) & K_{23}(z)
\end{bmatrix} \begin{bmatrix}
y_1(z) \\
y_2(z) \\
y_3(z)
\end{bmatrix}
\]
\[
\begin{bmatrix}
    u_1(z) \\
    u_2(z)
\end{bmatrix}
= 
\begin{bmatrix}
    K_{11}(z) & K_{12}(z) & K_{13}(z) \\
    K_{21}(z) & K_{22}(z) & K_{23}(z)
\end{bmatrix}
\begin{bmatrix}
    y_1(z) \\
    y_2(z) \\
    y_3(z)
\end{bmatrix}
\]
\[
\begin{bmatrix}
    u_1(z) \\
    u_2(z)
\end{bmatrix}
= \left( \frac{1}{z} \begin{bmatrix}
    * & * & 0 \\
    0 & 0 & *
\end{bmatrix} \oplus \frac{1}{z^2} \begin{bmatrix}
    * & * & * \\
    * & * & *
\end{bmatrix} \oplus \cdots \right) \begin{bmatrix}
    y_1(z) \\
    y_2(z) \\
    y_3(z)
\end{bmatrix}
\]
minimize $\|P_{11} - P_{12}K(I - P_{22}K)^{-1}P_{21}\|

subject to $K$ stabilizes $P$

$K \in C$ info sharing constraints
System level approach design entire closed loop response
What we care about

System level approach
design entire closed loop response
What we design

System level approach

design entire closed loop response
What we design

System level approach

design entire closed loop response

Key idea: maintain the notion of state
What we design

System level approach

design entire closed loop response

state $\rightarrow$ structure
\[ x[t + 1] = Ax[t] + B_1 w[t] + B_2 u[t] \]
\[ \bar{z}[t] = C_1 x[t] + D_{11} w[t] + D_{12} u[t] \]
\[ y[t] = C_2 x[t] + D_{21} w[t] + D_{22} u[t] \]
\[ x[t + 1] = A x[t] + B_1 w[t] + B_2 u[t] \]
\[ \bar{z}[t] = C_1 x[t] + D_{11} w[t] + D_{12} u[t] \]
\[ y[t] = C_2 x[t] + D_{21} w[t] + D_{22} u[t] \]

\[
\mathbf{P} = \begin{bmatrix}
A & B_1 & B_2 \\
C_1 & D_{11} & D_{12} \\
C_2 & D_{21} & D_{22}
\end{bmatrix}
\]

\[
\mathbf{P}_{ij} = C_i (z I - A)^{-1} B_j + D_{ij}
\]
parameterize & design entire system response
need to ensure achievable of $\Phi$
need to ensure **that there exists** $K$ such that the above holds
minimize $\bar{z}$

subject to

$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$ stable and achievable

$\Phi \in \mathcal{S}$
\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix}
\] stable and achievable

\[
\begin{bmatrix}
zI - A & -B_2 \\
\Phi_{21} & \Phi_{22}
\end{bmatrix}
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}
\]

\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix}
\begin{bmatrix}
zI - A \\
-C_2
\end{bmatrix} = \begin{bmatrix} I \\
0
\end{bmatrix}
\]

\[\Phi_{11}, \Phi_{12}, \Phi_{21} \in \frac{1}{z}RH_\infty, \Phi_{22} \in RH_\infty\]
\[ \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \text{ stable and achievable} \] in affine space
Theorem
[implementation]

\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix}
\text{ stable and achievable}
\]

\[\implies \exists K \text{ s.t. } \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}\]
sys response structure

\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix} \in \mathcal{S}
\]

controller internal structure

\[\beta = (I - z\Phi_{11})\beta - \Phi_{12}y\]
\[u = z\Phi_{21}\beta + \Phi_{22}y\]
\[
\begin{bmatrix}
  x \\
  u
\end{bmatrix} =
\begin{bmatrix}
  \Phi_{11} & \Phi_{12} \\
  \Phi_{21} & \Phi_{22}
\end{bmatrix}
\begin{bmatrix}
  \delta_x \\
  \delta_y
\end{bmatrix}
\]

controller simulates full system response
minimize $\bar{z}$

subject to 

$$
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix}
$$

stable and achievable

$\Phi \in \mathcal{S}$

system response and controller internal structure
(Convex) System level synthesis

minimize \( g(\Phi_{11}, \Phi_{12}, \Phi_{21}, \Phi_{22}) \) any (convex) functional

subject to \[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix}
\] stable and achievable

\( \Phi \in \mathcal{S} \) SLC: any (convex) set

Convex SLCs: system performance, controller structure & architecture, locality, etc.

Unified framework for custom controller synthesis
State-Feedback System Level Parameterization

response

\[ \mathbf{x} = \Phi_{11} \mathbf{w} \]
\[ \mathbf{u} = \Phi_{21} \mathbf{w} \]

achievability

\[
\begin{bmatrix}
zI - A & -B_2 \\
\Phi_{11} & \Phi_{21}
\end{bmatrix}
= I
\]
Localizability

Sensing / actuation delay: 1
Comm speed: 2 (compared to plant speed)
Localizability
Localizability

State deviation
Localizability

activate

propagate
Localizability
Localizability
Localized!
Localizability

Localized FIR
Idealized centralized control

- Comm Speed: Inf

Space

Time

log|u[t]|
Distributed LQR (Lamperski, Doyle 2013)

Control action

Comm speed: 2 ~Global optimal
LLQR

Control action

Comm speed: 2
~Global optimal

Time

Space

\| \log |u[t]| \|
LLQR

Space

Time

Control action

Comm range: 48 → 3

Log |u[t]|
LLQR

Space - time region

Control action

Comm speed: 2
~Global optimal

Space
Power grid: frequency regulation

Topology

Subsystem interaction

The system is represented with subsystems, each controlled by an actuator and a sensor.

Figure 1: Subsystem interaction diagram.
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Adaptive Control and Sys ID

Fault & Change Detection

Controller (re)design

Actuation/sensing (re)design
LLQR

Space - time region

Control action

Actuation Failure

Comm speed: 2 ~ Global optimal

Time

Log |u[t]|
LLQR

Space

Time

LOCAL Fault Det & Redesign

Control action

Space-time region

Comm speed: 2
~Global optimal

Log|u[t]|
LLQR

Space-time region

Control action

Comm speed: 2
~Global optimal
Novel parameterization:
  • new approach to constrained optimal control
  • new perspective on closed loop sys ID/adaptive?

Locality = scalability:
  • game changer for optimal controller synthesis
  • consequences for DDDAS?

Goal: automated control (re)design for DDDAS


N. Matni and V. Chandrasekaran, Regularization for design, IEEE TAC 2016.