Use of Operationally Flexible Robust Optimization in Dynamic Data Driven Application Systems

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• Goal
  – Observation platform
  – Integrated array of sensors
  – High efficiency
• Target
  – 40 hours of flight
  – 2000-nautical mile range
• Features
  – Atypical designs
  – Thin and flexible wings
  – Joined-wing configuration

SensorCraft Tradeoff

• Extension of performance envelope
• Stability concerns
  – Flutter
  – Aerodynamic buckling
• Interference with surveillance and reconnaissance
Mission Optimization

- Using aeroelastic model, optimize for:
- Objectives – Maximize performance criteria
  - Flight time, range, payload, etc.
- Constraints – Safe operation to avoid
  - Material yield, flutter, buckling
- Uncertainty – Stochastic environment

Example

Variables
- Hardpoints at five locations along fore wing ($x_d$)
- Six pairs of symmetric flaps ($x_{op}$)
- Uncertain headwind ($p$)

Objectives
- Maximize payload ($f_1$)
- Maintain target lift ($f_2$)

Constraints
- Maintain acceptable stress levels ($g$)
Conceptual Example

\[ x_d: \text{design variable} \]
\[ x_{op}: \text{operational variable} \]
\[ p: \text{uncertain parameter} \]
\[ f: \text{objective function} \]
\[ g: \text{constraint function} \]

\[ x_{op} = \text{lateral position} \]
\[ x_d = \text{vertical position} \]

\[ \min_{x_d} f(x_d) = x_d \]

\[ g(x_d, x_{op}, p) \leq 0 \]
\[ p = p_{\text{min}} \]

\[ g(x_d, x_{op}, p) \leq 0 \]
\[ p = p_{\text{max}} \]
Optimization With Uncertainty

• **Deterministic Approach**: Optimize design and operational variables for fixed (nominal) realization of uncertain parameters
Deterministic

$x_d$

$p_{\text{max}}$

$p_{\text{min}}$

$p_0$

$p_{\text{max}}$
Deterministic
Deterministic

\[ x_d \]

\[ p_{\text{min}} \quad p_0 \quad p_{\text{max}} \]
Optimization With Uncertainty

• **Deterministic Approach**: Optimize design and operational variables for fixed (nominal) realization of uncertain parameters

• **Robust Approach**: Optimize design and operational variables for all realizations of uncertain parameters
Robust - Discretized

\[ x_d \]

\[ p_{\text{min}} \quad p_0 \quad p_{\text{max}} \]
Robust - Discretized
Optimization With Uncertainty

• Deterministic Approach: Optimize design and operational variables for fixed (nominal) realization of uncertain parameters

• Robust Approach: Optimize design and operational variables for all realizations of uncertain parameters

• Robust with Operational Flexibility (ROOF) Approach: Optimize design for all realizations of uncertain parameters while using operational variables to mitigate effect of uncertainty
Operational Flexibility - Discretized

Operational Flexibility - Discretized

$x_d$

$p_{min}$ $p_0$ $p_{max}$

Operational Flexibility - Discretized

Operational Flexibility - Discretized

Operational Flexibility - Discretized

Robust - Sequential

$x_d$

$\rho_{min}$ $\rho_0$ $\rho_{max}$

Robust - Sequential

$\rho_{\min} \quad \rho_0 \quad \rho_{\max}$

$x_d$
Robust - Sequential

Robust - Sequential

$x_d$

$p_{min}$  $p_0$  $p_{max}$

Operational Flexibility - Sequential

\[ x_d \]

\[ p_{\text{min}} \quad p_0 \quad p_{\text{max}} \]
Operational Flexibility - Sequential

\[ x_d \]

\[ p_{\min} \quad p_0 \quad p_{\max} \]
Operational Flexibility - Sequential

\[ p_{\text{min}} \quad p_0 \quad p_{\text{max}} \]

\[ x_d \]
Operational Flexibility - Sequential

\[ x_d \]

\[ p_{\min} \quad p_0 \quad p_{\max} \]
Basic Aircraft Example

- **Variables**
  - Hardpoints at five locations along wing \( \left( x_d \right) \)
  - Two pairs of symmetric flaps \( \left( x_{op} \right) \)
  - Uncertain headwind \( (p) \)

- **Objectives**
  - Maximize payload \( (f_1) \)
  - Maintain target lift \( (f_2) \)

- **Constraints**
  - Maintain acceptable stress levels \( (g) \)
Representative Multi-Objective Results

Pareto Optimality Comparison

- Payload (kg)

- Lift Deviation (N)

-0.3

0 20 40 60 80 100 120

-0.2

-0.25

-0.15

Deterministic Frontier
Flexible Frontier
Robust Frontier Sequential
Flexible Discretized
Robust Discretized
Implementation with DDDAS

- **ROOF assumptions**
  - Interval uncertainty
  - Accurate online knowledge of current state
- **High fidelity aeroelastic models not computationally feasible for online implementation**
- **Have combination of sensor measurements and low fidelity simulations**
Implementation Framework

**Offline**
- High fidelity aeroelastic simulations
- Predict
  - worst conditions
  - optimal maneuvers
- Determine optimally flexible design

**Online**
- Design fixed
- Low fidelity simulations
- Sensor data
- Moving window co-Kriging surrogate model
- Determine optimal maneuvers
Concluding Remarks

• Sequential robust optimization with operational flexibility (ROOF) produces less conservative results than standard robust optimization.

• In the DDDAS framework one can take advantage of high and low fidelity simulations online.

• Future work
  – Online training of surrogate models (coKriging)
  – Application to UVLM-FEM aeroelastic simulation
  – Use in online decision support
Co-Kriging

• Expensive – high fidelity data
  • High fidelity simulations
• Cheap – low fidelity data
  • Sensor readings, low fidelity simulations

THE END
Back Up Slides
Aeroelastic Modeling

- Coupled structural modeling (Finite Element) and aerodynamic modeling (Unsteady Vortex Lattice Method)
- Predict aeroelastic effects
  - Aerodynamic forces
  - Internal stress
  - Excitation frequencies
- High fidelity-offline simulations
Optimization Formulations

### Deterministic Optimization

Minimize

$$\min_{x_d, x_{op}} f_1(x_d, x_{op}, p), f_2, \ldots, f_n$$

Subject to

$$g_j(x_d, x_{op}, p) \leq 0$$

$$\max_{j \in J} g_j(x_d, x_{op}, p) \leq 0$$

Solve for single expected outcome

- $p$: (uncertain) parameters
- $x_d$: design variables
- $f$: objective function
- $g$: constraint function

### Robust Optimization

Minimize

$$\min_{x_d, x_{op}} E[f_1(x_d, x_{op}, p_i)], E[f_2], \ldots, E[f_n]$$

Subject to

$$g_j(x_d, x_{op}, p_i) \leq 0, \quad \forall i = 1, 2, \ldots, m$$

$$\max_{p \in T} \max_{j \in J} g_j(x_d, x_{op}, p) \leq 0$$

Solve for all possible outcomes

- $J$: set of constraint indices
- $T$: set of all discretized $p$
- $x_{op}$: operational variables
Robust Optimization

\[
\begin{align*}
\text{minimize} & \quad E[f_1(x_d, x_{op}, p_i)], E[f_2], \ldots, E[f_n] \\
\text{subject to} & \quad g_j(x_d, x_{op}, p_i) \leq 0, \\
& \quad \forall i = 1, 2, \ldots, m \\
& \quad \max_{p \in T} \max_{j \in J} g_j(x_d, x_{op}, p) \leq 0
\end{align*}
\]

Solve for all possible outcomes

Robust Optimization with Operational Flexibility

\[
\begin{align*}
\text{minimize} & \quad E[f_1(x_d, x_{opi}, p_i)], E[f_2], \ldots, E[f_n] \\
\text{subject to} & \quad g_j(x_d, x_{opi}, \theta_i) \leq 0, \\
& \quad \forall i = 1, 2, \ldots, m \\
& \quad \max_{p \in T} \min_{x_{op}} \max_{j \in J} g_j(x_d, x_{op}, p) \leq 0
\end{align*}
\]

Add operational variables and reduce constraints

\[p: \text{(uncertain) parameters}\]
\[x_d: \text{design variables}\]
\[f: \text{objective function}\]
\[g: \text{constraint function}\]

\[J: \text{set of constraint indices}\]
\[T: \text{set of all discretized p}\]
\[x_{op}: \text{operational variables}\]

Sequential ROOF Formulation

Initially, $p_0 \in S_g$

First step:

\[
\begin{align*}
\mathbf{x} &= \mathbf{x}_d, \mathbf{x}_{op1}, \mathbf{x}_{op2}, \ldots, \mathbf{x}_{opn} \quad n = |S_g| \\
\min_{\mathbf{x}} f(\mathbf{x}, p_0) \\
g_i(\mathbf{x}_d, \mathbf{x}_{opi}, p_i) &\leq 0, \forall p_i \in S_g \\
x_d &\in [x^l_d, x^u_d], x_{opi} &\in [x^l_{op}, x^u_{op}] 
\end{align*}
\]

Second step:

\[
\max_p \left[ \max_k g(x_d, x_{opi}, p) \right], \forall p \in [p^l, p^u] 
\]

$k = \text{constraint index}$
Flexibility Visualization

- Increased dimensionality
- Lateral movement
- Can actively satisfy constraints
DDDAS Scheme

Sensor Data → Aeroelastic Model → Model Refinement

Model Refinement → Sensor Data → Forecasted States

Forecasted States → Model Verification → Optimize for Next States

Optimize for Next States → Maneuver/Action