## Dmt Institute

## Developing Mathematical Thinking Institute



Professional
Development
Resources

Assessment

## "The Developing Mathematical

Thinking Institute (DMTI) is dedicated to enhancing students' learning of mathematics by supporting educators in the implementation of researchbased instructional strategies through high-quality professional development, curricular resources and assessments."

For more information contact
Dr. Brendefur at ionathan@dmtinstitute.com

# Engaging Parents in Helping Young Children Develop Mathematical Thinking for STEM Fields 

NATIONAL ESEA CONFERENCE KANSAS CITY, 2019

Jonathan Brendefur, PhD.
DMTI Inc. jonathan@dmtinstitute.com
Boise State University jbrendef@boisestate.edu
Sam Strother, MAE
DMTI Inc. sam@dmtinstitute.com

## Session Overview

$>$ What does it mean to Develop Mathematical Thinking?
$>$ DMT Framework
$>$ Evidence
$>$ Primary Math Assessment
$>6$ predictors of future success
$>$ Parent Portal
$>$ DMTI Professional Development and Resources

# Developing Mathematical Thinking 

## EVIDENCE

## Task to Think About

- Solve the following problem:

Tina saved some money.
Sara saved three times as much money as Tina.
Victor saved $\$ 20$ less than Sara.

How much money did Victor save if they saved \$645 all together?


## Task to Think About

- Solve the following problem:

$$
\begin{aligned}
& T+S+V=\$ 645 \\
& T+(T+T+T)+(T+T+T-20)=\$ 645 \\
& 7 T-20=\$ 645 \\
& 7 T=\$ 665 \\
& T=\$ 95 \quad S=\$ 285
\end{aligned}
$$

## Task to Think About

- Solve the following problem:

$$
\begin{aligned}
& T+S+V=\$ 645 \\
& \frac{1}{3} S+(S)+(S-20)=\$ 645 \\
& 2 \frac{1}{3} S-20=\$ 645 \\
& 2 \frac{1}{3} S=\$ 665 \\
& T=\$ 95 \quad S=\$ 285
\end{aligned}
$$

## DMT Framework



Brendefur, 2008

## DMT Makes a Difference

## Large Scale Evaluation Results

Standardized Achievement Test
Control vs. DMT


## English Language Learners' Proficiency: Grades 3-8



## IES Grant - Randomized Design (2012-2015)

Through an Institute for Educational Science (IES) grant, 8 schools were randomly assigned to receive DMT professional development over a 2 year period compared to a control group.

Using a standardized test, students in grades 2-5 increased their achievement levels with an average of .2 standard deviations above the control group.

$\square$ DMT $\quad$ Control

## DMTI Curricular Resources Make a Difference

Results from a statewide standardized achievement test showed DMTI teachers outperforming their grade level colleagues.

Teacher D participated in DMTI professional development and implemented the curricular resources, while the other teachers did not.

| Legend: Achievement Levels |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Number of Students | Average <br> Scale Score | Percent Proficient | Percentage in Each Achievement Level |
| State | 18882 | $2434 \pm 1$ | 50 | 24 3619 |
| District | 362 | $2430 \pm 3$ | 44 | 223816 <br> 28 |
| School | 124 | $2437 \pm 6$ | 48 | 2131 27 |
| Teacher A | 25 | $2422 \pm 11$ | 44 | 28 28 36 8 |
| Teacher B | 24 | $2441 \pm 12$ | 50 | 13 38 38 17 |
| Teacher C | 24 | $2410 \pm 16$ | 25 | 33 [ $42 \quad 178$ |
| Teacher D | 25 | $2482 \pm 16$ | 76 | 1212\|32 44 |
| Teacher E | 26 | $2431 \pm 14$ | 42 |  |

Dmt Institute

## Developing Mathematical Thinking (DMT) Institute



## What does it mean to Develop Mathematical Thinking?

- DMT is built on a theoretical foundation drawing from three major learning theories:

Cognitive Theories
Social Interactional Theories
Behaviorism

DMT as a structure...
Cognition


# Primary Math Assessment (PMA) 

## Purpose and Overview of the PMA

- 6 Dimensions
- Screener and Diagnostic
- Classroom Reports
- Parent and Community Engagement
- Activities and Resources


## Predictors of Success in Mathematics

What do you think are the greatest math predictors of whether a student will be successful in mathematics later in school and in STEM fields?

```
Interpreting Context Number Facts
```

Number Sequencing Measurement

Relational Thinking Spatial Reasoning

## Predictors of Success in Mathematics

6. Number Sequencing and Identification

7, 8, 9,
32, 31, 30, $\qquad$

## Predictors of Success in Mathematics

5. Number Facts

$$
\begin{aligned}
& 8+5=\square \\
& 13+14=\square
\end{aligned}
$$

## Predictors of Success in Mathematics

4. Interpreting Context

There are 8 black ants and 5 red ants. How many more black ants are there than red ants?

## Predictors of Success in Mathematics

3. Relational Thinking

$$
5+18=\square+4
$$



## Predictors of Success in Mathematics

## 2. Measurement

Which measurement is correct?
Structure

- Unit
- Iterate
- Partition
- Transitivity
- Zero



## Predictors of Success in Mathematics

2. Measurement

If the top line is split into 4 equal size pieces, which of the lengths would match 1 piece?

Structure

- Unit
- Iterate
- Partition
- Transitivity
- Zero



## Predictors of Success in Mathematics

## 1. Spatial Reasoning

Which shape can I make with all the shapes to the right?

## Structure



- Compose
- Decompose
- Rotation
- Congruence


Shape A


Shape B


Shape C

## Predictors of Success in Mathematics

## 1. Spatial Reasoning

Which shapes are exactly the same?
Structure

- Compose
- Decompose
- Rotation
- Congruence



## Predictors of Success in Mathematics

6. Facts
7. Sequencing
8. Interpreting Context
9. Relational Thinking
10. Measurement
11. Spatial Reasoning

## PMA Screener

REPORTS


Class Report provides a color-coded grid to represent the students' quintile placement for each of the six dimensions assessed by the PMA. The quintile ranges are presented by moving the cursor over the individual cell.Teachers have the option to print the report with quintiles shown and can update their student roster by using the Create and Manage tabs.

Sorting Feature is available by selecting the heading for any of the six dimensions. Teachers can immediately have the student roster sorted by quintiles beginning with the lowest quintile. This feature highlights groups of students in greatest need of instructional interventions for the specific dimension.


## Recommended

Suggested Curricular Priorities and Instructional Support

Measurement
Class average: 50\%

First Grade

Me and the Measure of Things
Literature
Joan Sweeney
The energetic narrator of Me Counting Time and Me on the Map is back, this time introducing young readers to the units of measure between a cup and an ounce? What gets measured in bushels and when do you use a scale? Easy-to-understand text and playful corr teach children the differences between wet and dry measurements, weight, size, and length. And all information is conveyed in a uni perspective, using everyday objects and situations. Me and the Measure of Things makes measurement fun-and comprehensible!

Parent Measurement
Activity
Measuring Length
Activity
Match It!
Activity
Find it in the Room
Activity
Covering the Window
Activity
Cover the Shape
Activity

Class Reports
Semester Reports
<< Previous Semester
Winter 2017
1st Grade


Semester Reports show a teacher's classroom results as compared to the school and district. The error bars indicate the norm range for each of the six dimensions. Classrooms are blue, districts are red and the schools are orange.

## Reliabilities and Correlations

PMA - Screener
(20 items)


# Developing Mathematical Thinking 

PARENTS, CAREGIVERS AND COMMUNITY

## Engaging Parents and Community

- Parent/Caregiver Workshops
www.DMTinstitute.com

Topics (2-4 hours)

- K-2 Number and Structure
- Counting
- Fluency for Addition and Subtraction
- 3-5 Number and Structure
- Fluency for Multiplication
- Fractions
- Decimals
- 6-8 Number and Structure
- Ratio and Proportion
- Algebraic Reasoning


## Engaging Parents and Community

- Parent Portal

MyMathSuccess.com

- Primary Math Assessment (PMA) - Screener
- For parents of Kindergarten, first, and/or second grade students
- Provides reports and prioritizes activities
- DMTI Activities
- Specific to each of the six predictive domains
- Includes suggested questions to ask and vocabulary to use
- Focused Work on Each Domain for 2 months
- Notified bi-monthly of 1 DMTI activity to do together
- Suggested Game to purchase and play
- Suggested Book to purchase and read


## Engaging Parents and Community

\author{

- Basic Math Pack
}

- ( 1
- Advanced Math Pack



## Parent Portal

- After the child is screened the results are ready to view and suggestions are automated


Thomas

Thomas has no test results. Get started by taking a screener!
Start Screener

## Screener Results for Kevin



| Summary Table |  |
| :--- | :---: |
| Number Sequencing | $75 \%$ |
| Number Facts | $33.33 \%$ |
| Relational Thinking | $100 \%$ |
| Interpreting Context | $50 \%$ |
| Measurement | $100 \%$ |
| Spatial Reasoning | $66.67 \%$ |



## PMA Parent - K

Number Sequencing and Facts



# Number Sequencing Counting Game 

- You have 8 rocks.

Child: $\quad$ There are 8 rocks. I take 2 rocks and now there are 6 rocks left in the pile.

You:

You:

Child:
You:
Child:

Child: $\quad$ There are 5 rocks. I take 2 rocks and now there are 3 rocks left in the pile.
There are 6 rocks in the pile and I take 1 rock from the pile. There are now 5 rocks in the pile.

There are 3 rocks in the pile. I take 2 rock from the pile. Now there is rock in the pile.
There is 1 rock left in the pile. I take 1 rock and now there are 0 rocks in the pile.
I have 3 rocks total.
I have 5 rocks total.

## Screener Results for Kevin



| Summary Table |  |
| :--- | :---: |
| Number Sequencing | $75 \%$ |
| Number Facts | $33.33 \%$ |
| Relational Thinking | $100 \%$ |
| Interpreting Context | $50 \%$ |
| Measurement | $100 \%$ |
| Spatial Reasoning | $66.67 \%$ |



Number Sequencing

© DMTI (2018) | Resource Materials | www.dmtinstitute.com

## DMTI Curricular

## Resources

focus calendars, unit overviews, modules and ASSESSMENTS

## Central Access Point

- www.dmtinstitute.com
- After logging in, users are given access to all available materials regardless of grade level.
- This supports intervention and extension activities and informs teachers about the progression of content across grades.


Focusing Calendar
Grade K

- Grade 0 Focusing Calendar

Unodules
Grade K

- Unit 1: Counting and Number Sense
- Unit 2: Measurement Comparison
- Unit 3: Part-Whole and Compare Problems
- Unit 4: Geometry
- Unit 5: Operations with Join and Separate Problems

Unit Overviews
Grade K

- Unit 1: Counting, Number and Place Value
- Unit 2: Measurement Comparison
- Unit 3: Number PPW
- Unit 3: Number PPW
- Unit 5: Number. Join \& Seperat
- Unit 6: Measurement Iteration
- Unit 7: Operations and Place Value
- Unit 8: Shape and Space
- 

Assessments
Grade K

- Unit 1: Common Assessment
- Unit 2: Common Assessment - Measuremen
- Unit 3: Common Assessment
- Unit 4 Common Assessmen
- Unit 4. Common Assessmen
- Unit 5: Common Assessmen


## Module Sequence

- Lesson 1: Counting Forward and Back
- Lesson 2: Part-Whole Situations
- Lesson 3: Part-Whole: Practice
- Lesson 4: Part-Whole: Writing Contexts
- Lesson 5: Part-Whole: Iconic Models
- Lesson 6: Solving Compare Situations: Context
- Lesson 7: Solving Compare Situations: Practice
- Lesson 8: Solving Compare Situations: Iconic Models
- Lesson 9: Solving Compare Situations: Symbolic Models
- Lesson 10: Solving Compare Situations: Making Models and Justification
- Lesson 11: Compare Situations: Pocket Survey
- Lesson 12: Part-Whole and Compare Situations: Summary and Varied Practice


## Compare Situations

Tia and Frances are planting a garden. They start by planting carrots and peppers. There are 47 carrot seeds and 81 pepper seeds. How many more pepper seeds did they plant than carrot seeds?

Model this situation.
Are there more carrot or more pepper seeds?
More pepper seeds.
Write a number sentence for this situation.

$$
47+?=81 \quad 81-47=?
$$

Now, answer the question.

## Word Bank

## Compare Situations

Explain how each of Tia's models work using the word bank to the right.

## Unit

Decompose
Compose
Equation


# Compare Situations: Practice Extension 

## Word Bank

Unit
Decompose
Compose
Equation

Using the word bank, explain how you solved each of the four problems.

| Problems | Explanation |
| :--- | :--- |
| 1. Tia planted 26 green peppers and 46 carrots. How <br> many more carrots did she plant than green peppers? |  |
| 2. Frances planted 75 green peppers and 55 carrots. <br> How many more green peppers did he plant than <br> carrots? |  |
| 3. Tia planted 65 green peppers. She planted 35 more <br> carrots than green peppers. How many carrots did she <br> plant? |  |
| 4. Frances planted 70 green peppers. He planted 52 <br> fewer carrots than green peppers. How many carrots <br> did he plant? |  |

## Compare Situations: Practice

Use the compare problem worksheet to solve the following problems. Model each situation first, write an equation and then solve it using one of the methods listed.

| Problems | Number Sets |
| :--- | :--- |
| 1. Tia planted 26 green peppers and 46 carrots. How many more carrots did she <br> plant than green peppers? | $(38,47)(75,125)(107,184)$ |
| 2. Frances planted 75 green peppers and 55 carrots. How many more green <br> peppers did he plant than carrots? | $(84,15)(134,54)(163,89)$ |
| 3. Tia planted 65 green peppers. She planted 35 more carrots than green <br> peppers. How many carrots did she plant? | $(80,25)(72,29)(102,17)$ |
| 4. Frances planted 70 green peppers. He planted 52 fewer carrots than green <br> peppers. How many carrots did he plant? | $(143,52)(185,90)(162,34)$ |

## Compare Situations: Model Matching

Match the following compare models - bar model, equation and story problem.


187


## Lesson 12：Part－Whole Situations <br> －essen

\section*{帾 <br> ，



| Story Problem | Bar Model | Equation | How I solved the problem．．． |  |
| :--- | :--- | :--- | :--- | :--- |
| There were 28 children <br> swimming in the pool．18 of <br> the children were girls．How <br> many were boys？ |  |  |  |  |
|  |  |  |  |  |
| ？？ |  |  |  |  |

dren in the pool． 18 of
girls．How
boys？
Dm Institute

verse 28 children
dren were girls．Ho the pool． 18
ere boys？



D

| Bar Model | Equation |  |
| :--- | :---: | :---: | :---: |
| s． 18 of |  |  |


| Story Problem |
| :--- |
| There were 28 children |
| swimming in the pool． 18 c |
| the children were girls．Ho |
| many were boys？ |
|  |
| Dit institute | $\qquad$



|  |  |  |
| :--- | :--- | :--- |
| $?$ | 14 |  |
|  |  |  |
|  |  |  |



\title{

# <br> <div class="inline-tabular"><table id="tabular" data-type="subtable">
<tbody>
<tr style="border-top: none !important; border-bottom: none !important;"></tr>
<tr style="border-top: none !important; border-bottom: none !important;"></tr>
</tbody>
</table>
<table-markdown style="display: none">|  |
| :--- |
|  |</table-markdown></div> 

五| Story Problem | Bar Model | Equation | How I solved the problem... |
| :--- | :---: | :---: | :---: |
| A farmer has 38 carrots and 53 <br> peppers. How many more <br> peppers does the farmer have <br> than carrots? |  |  |  |
|  |  |  |  |
|  | $\square$ | $36-19=$ |  |
| A farmer has a garden with 85 <br> animals. There are 32 more <br> chickens than pigs. How many <br> chickens and pigs are on the <br> farm? |  | 53 |  |


| LeSSOn Le: conn pare SITUATIONS |
| :--- |
| Story Problem |
| A farmer has 38 carrots and 53 |
| peppers. How many more |
| peppers does the farmer have |
| than carrots? |


#### Abstract



.


$\square$
$\qquad$


|  | Bar Model | Equation |
| :--- | :---: | :---: |
| and 53 |  |  |
| re |  |  |
| r have |  |  |


el
(

| LeSSon 12: Conn are Situations |
| :--- |
| Story Problem |
| A farmer has 38 carrots and 53 |
| peppers. How many more |
| peppers does the farmer have |
| than carrots? |


| LeSSon 12: Conn are Situations |
| :--- |
| Story Problem |
| A farmer has 38 carrots and 53 |
| peppers. How many more |
| peppers does the farmer have |
| than carrots? |


|  |  |  |
| :--- | :--- | :--- |
| 67 |  |  |
| ? |  |  |
|  | $36-19=$ |  | -

Lesson 12: Compare Situations



from
from
Grots and 53
Story Problem Bar Model

## DMT in Schools

- Professional Development Courses and Workshops
-     - Unit Studies
-     - In-Class Support
- Resources
-     - Focusing Calendars
-     - Unit Overviews
-     - Curricular Modules
-     - Common Assessments
-     - Primary Mathematics Assessment: Screener and Diagnostic


## K-2 PMA Data

## Kindergarten - School A



## First Grade - School A



## Second Grade - School A


© DMTI (2018) | Resource Materials | www.dmtinstitute.com

## References

- Number Sense/Sequencing: (Baroody, 1987; Blote, Lieffering, \& Ouwehand, 2006; Butterworth, 2004; Clements \& Sarama, 2007; Desoete, Ceulemans, Roeyers \& Huylebroeck, 2009; Geary et al., 1999; Geary, 2010; Jordan, Glutting \& Ramineni, 2010; LeFevre et al., 2006;)
- Number Facts: (Beishuizen \& Anghileri, 1998; Geary, 2004; Geary, 2004; Passolunghi \& Siegel, 2004)
- Contextual Problems: (Ashkenazi et al., 2013; Hatano, 2003; Carpenter, Franke \& Levi, 2001; Jitendra et al 2013; Montague, 2007; Van Dooren, de Bock, Vleugels \& Verschaffel, 2010; Verschaffel, Greer \& DeCorte, 2007)
- Relational Thinking: (Carpenter, Franke, \& Levi, 2003; Driscoll, 1999; Sarama and Clements, 2008; Sarama \& Clements, 2009; Stephens, 2006)
- Measurement: (Cramer, Post, \& del Mas, 2002; Kamii \& Clark, 1997; Lehrer, 2003; Lehrer, Jaslow \& Curtis, 2003; McClain, Cobb, Gravemeijer \& Estes, 1999; National Research Council, 2001; Watanabe, 2002).
- Spatial Reasoning: (Battista, 1981; (Cheng \& Mix, 2014; Clements \& Sarama, 2007; Geary, Hoard, Bryd-Craven, Nugent \& Numtee, 2007; Holmes, Adams \& Hamilton, 2008; Lee, 2005; McLean \& Hitch, 1999).


## Dmt Institute



กin Curricular
Resources

Assessment

## Brendefur and Strother (2018)

For more information contact Dr. Brendefur at

