Inexact relative-error proximal point algorithms

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Based on joint works with
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The problem

- Monotone inclusion:
  
  \[
  \text{find } z \in \mathcal{H} \text{ such that } 0 \in T(z)
  \]

  where \( T : \mathcal{H} \rightrightarrows \mathcal{H} \) is a maximal monotone operator.

- \( T : \mathcal{H} \rightrightarrows \mathcal{H} \) is a **monotone operator** if

  \[
  \langle z - z', v - v' \rangle \geq 0 \quad \forall v \in T(z), v' \in T(z').
  \]

- \( T : \mathcal{H} \rightrightarrows \mathcal{H} \) is a **maximal monotone operator** if \( T \) is monotone and there exists no monotone operator \( S : \mathcal{H} \rightrightarrows \mathcal{H} \) such that \( G(S) \) properly contains \( G(T) \).
The proximal point method

- Monotone inclusion:

  \[ \text{find } z \in \mathcal{H} \text{ such that } 0 \in T(z) \]

  where \( T : \mathcal{H} \rightrightarrows \mathcal{H} \) is maximal monotone.

- Resolvent computation:

  \[ z_+ = (\lambda T + I)^{-1} z \iff 0 \in \lambda T(z_+) + z_+ - z. \]

- Rockafellar (1976):

  \[ \| z_k - (\lambda_k T + I)^{-1} z_{k-1} \| \leq e_k, \quad \sum_{k=1}^{\infty} e_k < \infty. \]
The hybrid proximal extragradient (HPE) method

- Decoupling:

\[ z_+ = (\lambda T + I)^{-1} z \iff \exists v \in T(z_+), \quad \lambda v + z_+ - z = 0. \]

- Note that, in this case,

\[ z_+ = z - \lambda v. \]

- HPE iteration (Solodov-Svaiter): \( \sigma \in [0, 1), \)

\[ v_k \in T(\tilde{z}_k), \quad \|\lambda_k v_k + \tilde{z}_k - z_{k-1}\| \leq \sigma \|\tilde{z}_k - z_{k-1}\|, \]

\[ z_k = z_{k-1} - \lambda_k v_k. \]

- If \( \sigma = 0, \) then \( z_k = \tilde{z}_k = (\lambda T + I)^{-1} z_{k-1}. \)
The hybrid proximal extragradient (HPE) method

- More general version with enlargements (Solodov-Svaiter):
  
  \[ v_k \in T^\varepsilon_k(\tilde{z}_k), \quad \| \lambda_k v_k + \tilde{z}_k - z_{k-1} \|^2 + 2\lambda_k\varepsilon_k \leq \sigma^2 \| \tilde{z}_k - z_{k-1} \|^2, \]
  
  \[ \tilde{z}_k = z_{k-1} - \lambda_k v_k. \]

- For \( \varepsilon \geq 0 \),
  
  \[ T^\varepsilon(z) := \{ v \in \mathcal{H} \mid \langle z - z', v - v' \rangle \geq -\varepsilon \quad \forall v' \in T(z') \}. \]

- \[ T^\varepsilon(z) \supset T(z). \]
The hybrid proximal extragradient (HPE) method

- Monteiro, Ortiz and Svaiter (2014): Numerical experiments on large scale conic semidefinite programming problems ($\sigma = 0.99$).

- Eckstein and Yao (2018): Douglas-Rachford and ADMM relative-error HPE-type algorithms. Numerical experiments on LASSO and Logistic Regression ($\sigma = 0.90; \sigma = 0.99$).

- Some special instances of the HPE method/framework: Forward-backward, Tseng’s modified forward-backward, Korpolevich, ADMM.
The hybrid proximal extragradient (HPE) method

- Monteiro and Svaiter (2010): Iteration-complexity; global $\mathcal{O}(1/\sqrt{k})$ pointwise and $\mathcal{O}(1/k)$ ergodic convergence rates.

- Is it possible to obtain a global $\mathcal{O}(1/k)$ pointwise rate?

- A., Monteiro and Svaiter (2016): Regularized HPE method with $\mathcal{O}(\rho^{-1} \log(\rho^{-1}))$ pointwise iteration-complexity.
Regularized HPE-type methods for solving monotone inclusions with improved pointwise iteration-complexity bounds.
Thank you for your attention!

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