Automobile Accident Reconstruction Primer

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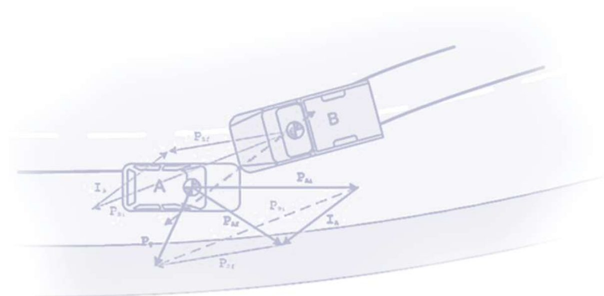


The purpose of this primer is to review some of the fundamental physical and mathematical concepts that are employed when determining physical parameters of automobile accidents.

Automobiles behave like any other deformable objects when they collide with each other or with stationary objects. Energy and momentum are conserved, meaning the total combined energy and momentum of the vehicle(s) is the same just before and just after collision. Keeping track of momentum (a vector quantity) is usually simpler than keeping track of energy (a scalar quantity) since energy is converted into different forms during collision.

Pre-Collision Energy	Post-Collision Energy
Motion (Kinetic) Energy of Vehicles	Motion (Kinetic) Energy of Vehicles + Deformation Energy of Vehicle Body and Frame + Secondary Effects such as heat, vibration, sound, turbulence, environment and roadway damage, etc.

Table 1 - Energy Conservation During Collision

When energy analysis is used in accident reconstruction, vehicle crush displacements are considered for obtaining an approximation of the energy that is converted from kinetic energy to body and frame deformation. The secondary energy conversion (heat, vibration, sound, turbulence, etc.) listed above are usually ignored as a standard practice since the represent a relatively minor portion of the system energy.

Conservation of momentum in the context of a collision between two vehicles states that ***the combined vector sum of each vehicle's momentum remains the same after impact as it was before impact***. This is a very useful concept when determining pre and post impact speeds and directions.

How do we put these physical laws of conservation of energy and conservation of momentum to work in our accident reconstruction practice? Let us proceed by examples. First we need to define some conventions.

Conventions

Vehicles will be referred to as vehicle "A", vehicle "B", etc. Subscripted instants in time will be defined as follows:

0 = instant before driver reaction begins, also used to denote time=0
i = instant just before impact
f = instant just after impact
1 = instant vehicle comes to rest

Table 2 - Subscripts Used

Vector quantities will be shown in bolded capital letters, while scalar quantities will be lower-case and not bolded. Vector and scalar quantities of interest will be defined as follows:

d = Displacement or position (vector)
v = Velocity (vector)
a = Acceleration (vector)
p = Momentum (vector)
F = Force (vector)

Table 3 - Vector Variables Used

s = Speed (scalar)
μ = Coefficient of friction
f = Friction Factor
m = mass
θ, ϕ = Angle
g = Gravitational acceleration constant
x, y, z = spatial coordinates

Table 4 - Scalar Variables Used

Equations of Motion

Before we can dive into analysis of collisions, we must first establish an understanding of objects in motion. An object in motion can be fully described in terms of position, velocity, and acceleration vectors with the following 3D vector equation:

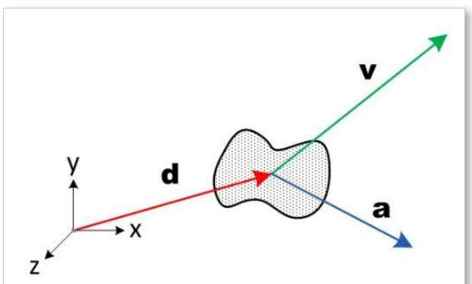


Figure 1 - Motion Vectors for an Object in 3D Space

$$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\,\mathbf{a}\,t^2$$ Eq 1

Where:
t = elapsed time
d = position vector at time = t
$\mathbf{d}_0$ = position vector at time t=0
$\mathbf{v}_0$ = velocity vector at time t=0
a = acceleration vector (assumed constant)

Equation 1 - 3D Vector Equation of Motion for Constant Acceleration

Equation 1 describes the translational position of an object's center of mass in 3 dimensions. Note it does not describe the object's rotational motion. It also should be noted that Equation 1 can only be used when the acceleration vector is not changing (is constant) over the time period being analyzed.

It is usually helpful to split the 3D position vector equation of motion into its x, y, and z component scalar equations:

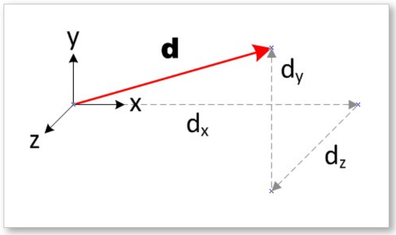


*Figure 2 - x, y, and z components of position vector **d***

$$d_x = d_{0x} + v_{0x}t + \tfrac{1}{2}\, a_x\, t^2$$
$$d_y = d_{0y} + v_{0y}t + \tfrac{1}{2}\, a_y\, t^2$$
$$d_z = d_{0z} + v_{0z}t + \tfrac{1}{2}\, a_z\, t^2$$

Eq 2

Equation 2 - x, y, and z component scalar position equations

These equations of motion can be utilized to determine an object's (vehicle's) position or velocity at specific points in time, forces acting on the object, coefficients of friction, and more, depending on what parameters are known.

A similar (and related) set of equations can be derived for the velocity vector, again assuming constant acceleration:

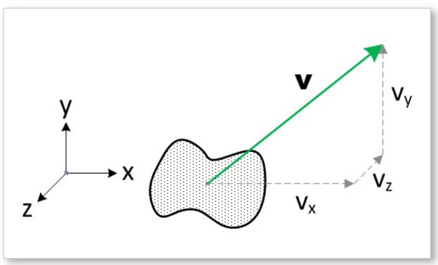


*Figure 3 - x, y, and z components of velocity vector **v***

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}\, t$$
$$v_x = v_{0x} + a_x\, t$$
$$v_y = v_{0y} + a_y\, t$$
$$v_z = v_{0z} + a_z\, t$$

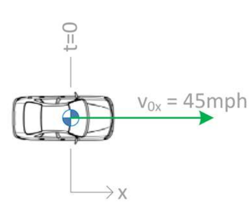
Eq 3

Equation 3 – velocity vector equation and x, y, and z component scalar velocity equations

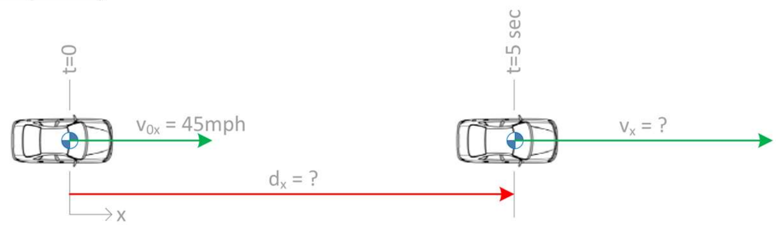
Example 1

An automobile is traveling down a straight road at 45 mph (72.4 kph). At time t=0, the driver steps on the gas and accelerates for 5 seconds at 10 ft/s^2 (3.05 m/s^2). Determine the following: a) How far did the vehicle travel in those 5 seconds? b) How fast was the vehicle travelling at t=5s? c) How much time elapsed before the vehicle's speed reached 65 mph?

Solution:
We choose a coordinate system that aligns the x-axis with the direction of travel.



Solution (cont.)



a) We can then use the x-component of Eq 2:

$$d_x = d_{0x} + v_{0x}\,t + ½\,a_x\,t^2$$

We also assume t=0 when the driver steps on the gas, and we assume the origin of our coordinate system is located at the position of the vehicle when t=0, therefore $d_{0x}=0$.

$$d_x = d_{0x} + v_{0x}\,t + ½\,a_x\,t^2$$

0

$$= v_{0x}\,t + ½\,a_x\,t^2$$

$$= (45\text{ mi/hr})(5280\text{ ft/mi})(1\text{ hr}/3600\text{ s})(5\text{ s}) + (0.5)(10\text{ ft/s}^2)(5\text{ s})^2$$

$$= \boxed{455\text{ ft}}\quad (139\text{ m})$$

b) Using the x-component of Eq 3:

$$v_x = v_{0x} + a_x\,t$$

$$= (45\text{ mi/hr})(5280\text{ ft/mi})(1\text{ hr}/3600\text{ s}) + (10\text{ ft/s}^2)(5\text{ s})$$

$$= 116\text{ ft/s} = \boxed{79.1\text{ mph}}\quad (127\text{ kph})$$

c) Rearranging the x-component of Eq 3 and solving for t at v_x=65mph:

$$t = (v_x - v_{0x}) / a_x$$

$$= \frac{(65\text{ mi/hr} – 45\text{ mi/hr})(5280\text{ ft/mi})(1\text{ hr}/3600\text{ s})}{(10\text{ ft/s}^2)}$$

$$= \boxed{2.93\text{ s}}$$

Momentum and Forces

Isaac Newton's Laws of Motion lie at the foundation of the analytical side of vehicular accident reconstruction. The First Law of Motion states that an object's momentum will remain unchanged unless it is acted upon by an external force.

An object's momentum is a vector quantity that is determined by multiplying its velocity vector by its mass. An object's momentum vector is always in the same direction as its velocity vector.

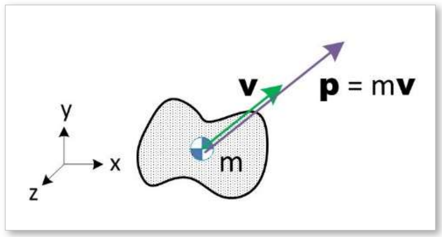
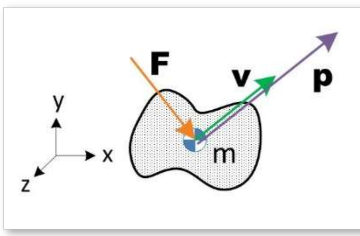


Figure 4 – Newton's 1st Law - Momentum Vector

Newton's Second Law of Motion states that a force applied to an object will result in an equivalent rate of change in momentum:



$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} \qquad \text{Eq 4}$$

Figure 5 – Newton's 2nd Law - Applied Force and Derivation of $\mathbf{F}=m\mathbf{a}$

Since momentum is the product of velocity and mass, and in most cases we can assume mass is constant, this gives rise to the familiar and ubiquitous $\mathbf{F}=m\mathbf{a}$.

Newton's Third Law of Motion states that two bodies in contact will exert equal opposing forces on one another. The resulting forces are not necessarily normal (perpendicular) to the contacting surfaces.

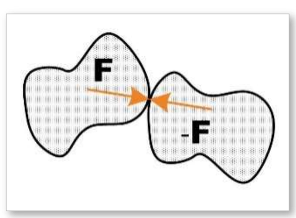


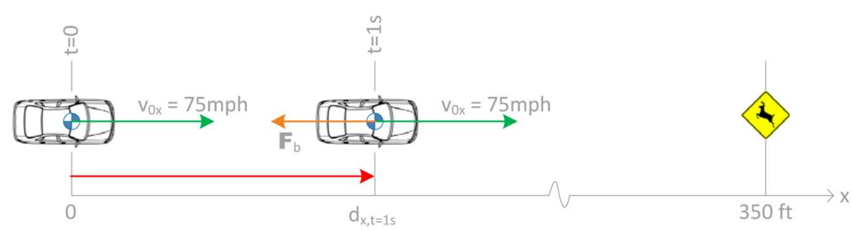
Figure 6 - Newton's 3rd Law - objects in contact

Example 2

A 2500 lb car is traveling at 75 mph on a straight road and a deer steps into the road 350 feet in front of the car. After a driver reaction time of 1 second the brakes are applied and a constant braking force of 2000 lb is applied. Does the car stop in time to avoid hitting the deer?

Solution:

This is another 1-dimensional problem, so we choose a coordinate system that aligns the x-axis with the direction of travel.



We first need to know how far the car travels in the initial 1 second reaction time of the driver. Since our coordinate system sets our initial position at x=0, and since there is no acceleration during this reaction period, the initial position and acceleration terms are zero in Eq 2:

$$d_{x,t=1s} = \underset{0}{\cancel{d_{0x}}} + v_{0x}\, t + ½ \underset{0}{\cancel{a_x}}\, t^2 = v_{0x}\, t = (75\text{mph})\ (1.47\ \text{fps/mph})\ (1.0\ \text{s}) = 110\ \text{ft}$$

The next phase is with the brakes applied, and a 2000 lb braking force applied to the car in the negative x direction. The acceleration produced by this braking force can be determined by Eq 4. We can drop the vector notation since this is a 1D problem:

$$F_b = m\, a_x \ \longrightarrow\ a_x = F_b/m = [\ -(2000\ \text{lbf}) / (2500\ \text{lbm})\ ]\ (32.2\ \text{lbm ft/lbf s}^2)$$
$$a_x = -25.8\ \text{ft/s}^2$$

Now we can determine the time required to stop the car by Eq 3, setting v_x=0:

$$v_x = v_{0x} + a_x\, t \ \longrightarrow\ 0 = v_{0x} + a_x\, t \ \longrightarrow\ t = -v_{0x} / a_x$$
$$t = -\ [\ (75\text{mph})\ (1.47\ \text{fps/mph})\ ] / (-\ 25.8\ \text{ft/s}^2)$$
$$t = 4.27\ \text{s}$$

And finally, we use Eq2 again to determine the total distance travelled, including reaction time and braking:

$$d_{x,t=4.27s} = d_{0x} + v_{0x}\, t + ½\, a_x\, t^2$$
$$= (110\ \text{ft}) + (75\text{mph})\ (1.47\ \text{fps/mph})\ (4.27\ \text{s})$$
$$+ (0.5)\ (-25.8\ \text{ft/s}^2)\ (4.27\ \text{s})^2$$

$d_{x,t=4.27s}$ = 346 ft (<350 ft so Bambi lives)

There are two fundamental forces acting on a vehicle at any given time during normal operation: gravity and contact forces on the tires from the road. We ignore wind drag under most circumstances.

Gravity (weight) is a body force that acts in a distributed manner according the mass distribution of the vehicle. To simplify analysis, we treat the gravitational force as a single force vector applied at the vehicle's center of mass. The gravitational force always acts in a vertical direction (relative to earth) regardless of the orientation of the vehicle or the angle of the driving terrain.

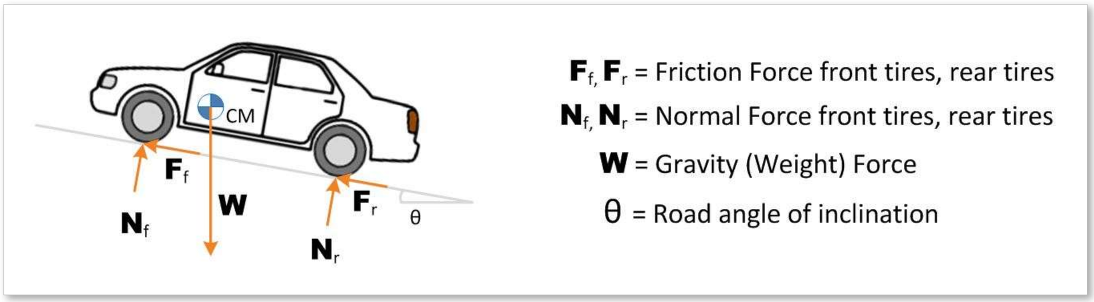


Figure 7 - Forces acting on a vehicle during normal operation

The contact forces on the tires by the road surface are usually split into two components: Normal and Frictional forces. Normal forces always act perpendicular to the contact surface of the road, and Frictional forces always act parallel to the contact surface of the road.

Figure 7 shows how, in general, these forces are different for each tire. Front and rear tire normal and frictional forces are shown, but in general each of the four tires would have a unique combination of normal and frictional forces acting on them. These many force vectors depend upon many factors including path curvature, weight distribution of vehicle, front vs. rear wheel drive, accelerating vs. braking, consistency of braking effectiveness between the 4 wheels, suspension dynamics, road surface uniformity, and the list goes on.

It is easy to see how analyzing these forces individually for each wheel would quickly become overwhelming and this purist approach would be prohibitive and beyond the scope of most accident reconstruction analyses. *Fortunately there are shortcuts we can take to simplify our models and reduce the number of variables to a manageable level, while remaining within an acceptable envelope of accuracy.*

<u>Tire Friction</u>

The frictional force on a tire acts in the plane of contact (road surface). This force can also be broken into components: **roll** friction and **lateral** friction forces. The roll friction force vector acts in the direction of roll of the tire, in the plane of contact. The lateral friction force vector acts perpendicular to the roll friction vector, in the plane of contact The lateral friction force is what enables the car to follow a curve without sliding off of the road. The roll friction force is what causes the car to accelerate when the throttle is applied, or to slow down when the brakes are applied.

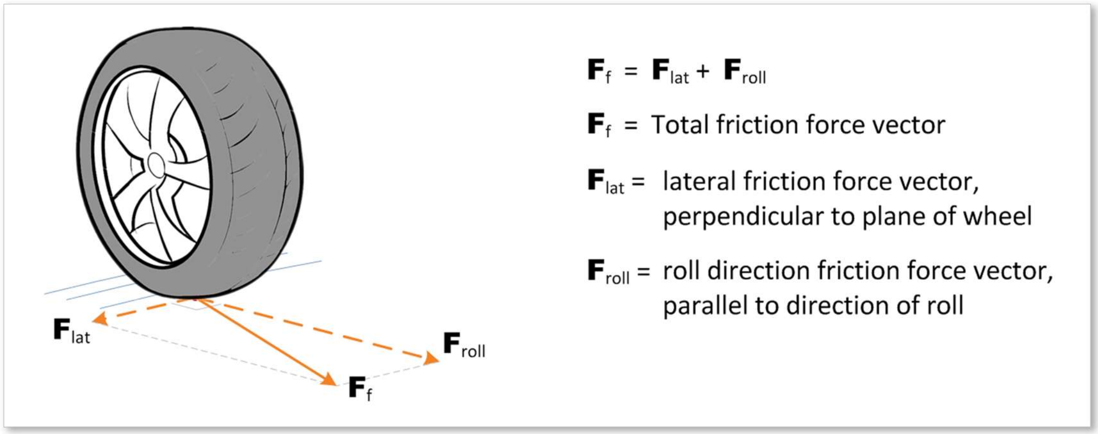


Figure 8 - Friction forces on a tire

Friction forces are continually changing during normal operation of a vehicle depending on driver behavior and geometry of the road. The magnitude of frictional force that two contacting surfaces are capable of exerting on each other is determined by the material of the surfaces, the Normal force (perpendicular to plane of contact) that the surfaces are exerting on each other, and whether the surfaces are in static or sliding contact with one another. This maximum friction force magnitude is governed by Equation 5 below.

Eq 5

$$F_f = \mu N$$

Where:
μ = coefficient of friction
(property of mating materials)
N = Normal contact force
(perpendicular to plane of contact)

The numerical value for the coefficient of friction depends upon the mating materials, but also on whether the surfaces are in static or sliding contact. If the surfaces (tire and road) are in static contact, as is the case under normal operation of the vehicle, Equation 5 tells you the maximum friction force that the tire can carry without breaking into sliding contact (skid).

If the surfaces are in sliding contact (tire is skidding) Equation 5 tells you the resulting frictional force on the tire. Note in a skidding scenario this frictional force does not depend upon the vehicle's speed. Note also that Equation 5 only tells us the actual frictional force if we are skidding. If we are not skidding it only tells us what the maximum can be before skidding begins.

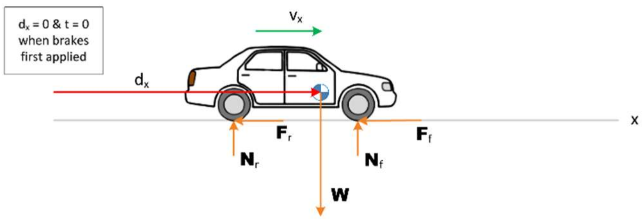
Generally speaking, the coefficients of friction (μ) for static and sliding contact are not the same. The static coefficient is usually higher than the sliding coefficient. Fortunately, in accident reconstruction we mostly use Equation 5 in situations where the vehicle is in a skid, in which cases we only require the sliding coefficient of friction.

Example 3

Two cars are traveling at 65 mph on a straight, level road in the same lane. The front car starts braking and the driver of the trailing car is texting and doesn't notice that the car in front is slowing down. When the driver finally looks up he brakes hard, locking all four wheels in a skid. The trailing car weighs 3000 lb and the pavement is dry – assume 0.75 sliding coefficient of friction. Determine the following:

a) What is the average friction force on each tire? b) How fast is the second car traveling after skidding for 50 feet?

Solution



a) The road is level, so the average Normal force on each tire is:

$N_{ave} = W/4 = (3000\ lb)/4 = 750\ lb$

We can then use Eq 5 to determine the average sliding friction force per tire:

$F_{f,ave} = \mu N_{ave} = (0.75)(750\ lb)$

$F_{f,ave} = 563\ lb$

b) We aligned our coordinates such that the x-axis is in the direction of travel. First step is to determine the acceleration caused by the sliding friction forces on all 4 tires. We use Eq 4 and solve for acceleration:

$a_x = F_x/m = [\ (4)(-563\ lb_f) / (3000\ lb_m)\ ]\ (32.2\ lb_m\text{-}ft/lb_f\text{-}s^2) = -24.2\ ft/s^2$

Next we use Eq 2 to determine the time it takes to skid 50 feet:

$d_x = d_{0x} + v_{0x}t + ½\ a_x\ t^2$

$50\ ft = 0 + (65mph)\ (1.47\ fps/mph)\ t + (0.5)\ (-24.2\ ft/s^2)\ t^2$

Rearranging, we have a quadratic formula to solve for t:

$12.1\ t^2 – 95.3\ t + 50 = 0$

Keeping the smaller root (the larger root corresponds to the vehicle eventually reversing direction which is nonsensical for this problem):

$t = 0.565\ s$

Using this elapsed time in Eq 3:

$v_x = v_{0x} + a_x\ t = (65mph)\ (1.47\ fps/mph) + (-24.2\ ft/s^2)\ (0.565\ s)$

$= 81.9\ ft/s$

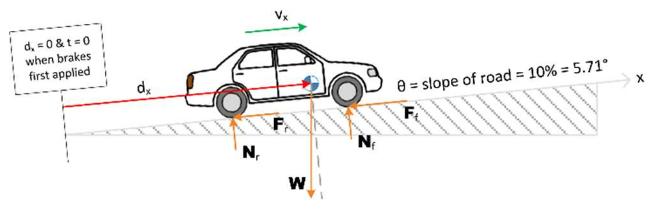
$v_x = 55.8\ mph$

It is interesting to note that the vehicle only slows by 9.2 mph (14%) after 50 feet of skidding!

Example 4

The same problem as Example 3 except the car is traveling up a steady 10% grade.

Solution



a) We realign our x-axis to be coincident with the road. The weight force vector always acts vertically, regardless of the slope of the road. The normal forces on the tires act perpendicular to the road surface. Their combined magnitudes equal the component of the weight force that is also perpendicular to the road. Thus:

$$N_{ave} = W\cos(\theta)/4 = (3000\ lb)(\ \cos(5.71°)\ )\ /\ 4 = 746.3\ lb$$

We can then use Eq 5 to determine the average sliding friction force per tire:

$$F_{f,ave} = \mu\ N_{ave} = (0.75)(746.3\ lb)$$

$$\boxed{F_{f,ave} = 559.7\ lb}$$

b) We aligned our coordinates such that the x-axis is in the direction of travel. Now that we are on a slope the braking force is no longer the only force acting in the x-direction – we also have the x-component of the weight force:

$$W_x = -\ W \sin(\theta) = -\ (3000\ lb)\ \sin(5.71°) = -\ 298.5\ lb$$ (negative since it acts in -x direction)

Next we determine the acceleration caused by the summed sliding friction forces and x-component of the weight force. We use Eq 4 and solve for acceleration:

$$a_x = F_x/m \qquad F_x = (4)\ (-559.7\ lb_f) - 298.5\ lb_f = -\ 2537\ lb_f$$

$$a_x = [\ (-\ 2537\ lb_f)\ /\ (3000\ lb_m)\ ]\ (32.2\ lb_m\text{-}ft/lb_f\text{-}s^2)$$

$$a_x = -\ 27.2\ ft/s^2$$

And repeating the same steps as in Example 3:

$$d_x = d_{0x} + v_{0x}t + \tfrac{1}{2}\ a_x\ t^2$$

$$50\ ft = 0 + (65mph)\ (1.47\ fps/mph)\ t + (0.5)\ (-27.2\ ft/s^2)\ t^2$$

$$13.6\ t^2 - 95.3\ t + 50 = 0$$

$$t = 0.571\ s \quad \text{(discarding larger root)}$$

$$v_x = v_{0x} + a_x\ t = (65mph)\ (1.47\ fps/mph) + (-27.2\ ft/s^2)\ (0.571\ s)$$

$$= 79.8\ ft/s$$

$$\boxed{v_x = 54.4\ mph}$$

The 10% grade only affects the velocity after skidding by about 2.5%

Energy

The concept of energy plays a central role in vehicular accident reconstruction. It is energy which has the capacity to do harm to property and person. More precisely, it is the uncontrolled conversion of energy from one form to another which releases the destructive potential of object (automobile) in motion.

Energy is a scalar quantity (it has magnitude only, with no associated direction), unlike force, impulse, position, velocity, acceleration, and momentum which are vector quantities which have both magnitude and direction associated with them. Common forms of energy in an operating vehicle include:

Eq 6

Gravitational Potential Energy – the amount of energy represented by an object's elevation relative to some reference elevation, represented by:

$$PE = mgh$$

where:

m = mass of object
g = gravitational acceleration constant
h = height (elevation) of object

Chemical Potential Energy – in the case of an automobile, this is the amount of energy contained in the fuel in the tank. This can be a considerable amount of energy. For example, there is enough energy in a full 15 gallon tank to lift the car 2000 miles straight up, if converted with 100% efficiency. That's almost 1% of the distance to the moon! Fortunately this fuel storage energy rarely comes into play in accident reconstruction analysis.

Kinetic Energy – the energy associated with mass in motion. There are two basic ways we categorize kinetic energy:

Eq 7

Translational KE, motion energy associated with mass moving at velocity v:

$$KE_T = \frac{1}{2}mv^2$$

where:

m = mass of object
v = velocity (speed)

Eq 8

Rotational KE – motion energy associated with mass rotating at angular velocity ω:

$$KE_R = \frac{1}{2}I\omega^2$$

where:

I = mass moment of inertia of object, or "rotational mass"
ω = angular velocity

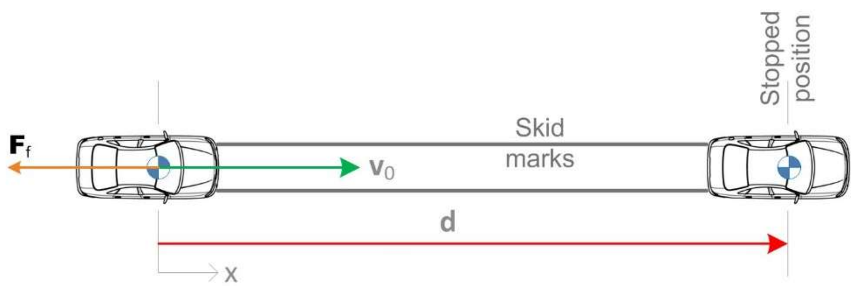
Deformation Energy – energy absorbed by materials that are forced to change shape. Strictly speaking this is not a form of energy independent of all others. When deformation occurs in as a result of an applied force, the energy put into the deformed object is converted mostly to heat, vibration, and noise. Some of it may also be converted to gravitational potential energy if the center of mass changes.

During a collision, kinetic energy of the vehicle(s) is converted rapidly into other forms. You will frequently hear in the accident reconstruction community that energy is *not* conserved in a collision, but momentum *is* conserved. This is misleading – both energy and momentum are conserved in a collision, it's just that energy is converted to different forms and it not possible to trace these energy conversions forensically. We will discuss conservation of momentum in a later section.

When investigating an accident, we are often presented with the challenge of determining how fast the vehicles were travelling prior to the initial reaction of the drivers. The evidence used in this determination often include skid marks. What do skid marks tell us about the speed of a car prior to braking?

Example 5

Take the simplest example of a car traveling on a straight road, driver applies the brakes locking all four wheels, and skids to a stop. Determine the initial speed of the car based on the skid mark evidence by a) energy method and b) equations of motion



Solution

a) The initial translational kinetic energy of the car is $KE_0 = \frac{1}{2}mv_0^2$

The car comes to a stop when the work done by the opposing friction force is equal to the initial kinetic energy. The frictional work is $W_f = F_f\, d = \mu\, m\, g\, d$

Since the frictional work energy is equal to the initial kinetic energy, we equate them and solve for v_0:

Eq 9

$$\frac{1}{2}mv_0^2 = \mu\, m\, g\, d$$

$$v_0 = \sqrt{2\, \mu\, g\, d}$$

μ = coefficient of sliding friction between tires and road

g = acceleration of gravity constant

d = length of skid

Solution (cont.)

b) Using the equations of motion, we first determine the acceleration due to the frictional force using F=ma: $a_x = -F_f/m$ (negative since F_f acts in –x direction)

Then using the x-component of Eq2, we substitute this expression for the acceleration term:

$$d_x = d_{0x} + v_{0x}t + \frac{1}{2}a_x t^2$$

$$d_x = v_{0x}t - \frac{1}{2}\frac{F_f}{m}t^2$$

Then using the x-component of Eq3, solving for t and doing a lot of algebraic substitution and rearranging we eventually come up with the same expression:

$$v_0 = \sqrt{2\,\mu\,g\,d}$$

The point being that the energy method is much simpler to implement.

Rotational Energy

Sometimes a vehicle will have rotational motion (yaw) that must be analyzed. This rotational motion can, in general, be analyzed separately from its translational motion. Yaw motion can result from oblique impacts with other objects, non-uniform braking, steering over-correction, etc.

In this section we will consider the kinetic energy associated with yaw motion and attempt to illustrate how the concept of rotational energy can be used in our analyses. We will also attempt to justify a simplification that is often used by accident reconstructionists.

For an object in rotational motion, as mentioned above in Eq 8 the kinetic energy can be calculated as Eq 8

$$KE_R = \frac{1}{2}I\omega^2$$

In this equation the term "*I*" represents the object's mass moment of inertia, a property of a given vehicle which accounts for the way in which the mass of the object is distributed. The ω term represents the speed of rotation.

Properties like weight, center of mass location, and mass moments of inertia are available for individual models of automobiles. Contents of the vehicle, including passengers and driver, must also be accounted for and these properties adjusted accordingly in order to maintain accuracy in the calculations.

In general after a collision, if a car has rotational motion there will likely be translational motion as well, which means there will be both translational and rotational kinetic energy involved.