Vehicle Modeling and Simulation for determination of Drag Factor in Accident Reconstruction

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ABSTRACT

When trying to determine what happened in a vehicle accident, one of the major questions relate to the conditions of the road in which the accident occurred. The motion of vehicles before and after a collision depends on the coefficient of friction between the tires and the road. When considering the contributions of each tire to the deceleration of the vehicle, the drag factor is a primary factor in what happened. Vehicles are rigid bodies that move in a three dimensional space, but because of gravity that keeps the vehicles in contact with the road (ground), unless they become airborne, are rigid bodies in plane motion. The study of rigid bodies in plane lay out the fundamental principles of "dynamics", a branch of physics. Plane motion means subject to two translations and one rotation. In other words, forward, lateral or rotation "yaw" motion. The decelerating force that slows down a vehicle under braking and skidding conditions is proportional to the coefficient of friction between the road and tires and the weight of the vehicle distributed differently on each tire. When considering the contribution of the tires under each tire condition it is known as "drag factor" The paper presents two approaches used to verify one another in order to determinet the drag factor. Using California Highway Patrol vehicles and a set of accelerometers, the drag factor was found experimentally. Then using computer models for multibody systems and bond graph models, the experimental data was verified or we can look at the other way, the bond graph models were verified. The CAMPG/MATLAB was used for the bond graph models and Working Model 2D was used for the multibody models. The physical geometry from the test vehicle was transferred to the computer programs, using image-processing techniques, which produced an accurate representation of the California Highway Patrol test vehicle in the computer simulation models. The experimental drag factor values were compared to those obtained from the bond graph and multibody models. The simulation results produced some additional useful data because the models can calculate the friction forces at the tires and clearly show the influence of weight shifting and rotation (pitch) during the full braking test.

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I. INTRODUCTION/BACKGROUND

We must establish a convention for the axis attached to a vehicle in motion. Fig 1. Illustrates the roll, pitch and yaw axis used. Whether we look at vehicles from the top, (yaw motion), or whether we look at them from the side, (pitch motion), from the engineering analysis point of view, they are subject to two translations and a rotation. If we use a right hand coordinate system, the roll of the vehicle is around the x-axis, which is the direction of travel. The pitch motion is about the y-axis and the yaw motion is the z-axis pointing down as shown in Fig 1.

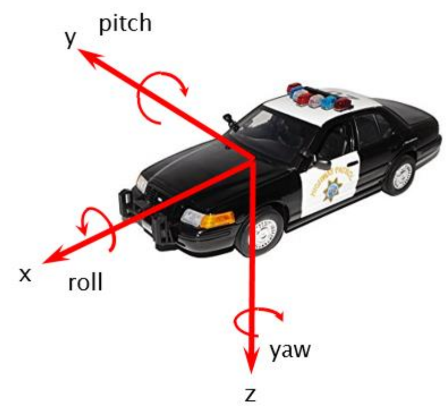

Fig.1 Roll, pitch and yaw axis for a vehicle.

The forces developed by the tires (friction forces), determine the deceleration of the vehicle. Those forces are proportional to the normal force between the vehicle and the ground and the coefficient of friction μ. It is a fundamental principle of physics that the coefficient of friction is a proportional constant between the friction force (decelerating force) and the weight of an object that slides on a surface. This can be expressed also as a ratio of the acceleration of an object to the acceleration of gravity. A detailed explanation of this concept is presented in this paper. If we consider the weight distribution 40% back and 60% in the front, the difference in the weight distribution produces different normal forces at the tires and therefore different friction forces on each tire. This unique set up for vehicles poses a major challenge in accident reconstruction because we must determine what each tire did before and after the accident in order to decelerate the vehicle. In order to achieve that, the coefficient of friction needs to be measured under the road conditions of the accident, to have a realistic value to calculate the deceleration. This data in turn helps us to find the velocities at which the accident occurred by using energy and impulse momentum methods. The determination of this drag factor by experimental means and by computer simulation, using bond graph models and multibody models is the subject developed on this paper. Let us examine the experimental method and then the computer simulation methods in that order.

II. EXPERIMENTAL SKID TEST, FINDING THE DRAG FACTOR

The California Highway Patrol (CHP) test facilities in Sacramento, California were used for the experimental set up. These facilities are used to take measurements, to study vehicle dynamics under various weather conditions and high-speed tests and for training. The set up consists of a CHP Ford Crown Victoria vehicle skid test. Two sets of instruments were installed in a 2004 Crown Victoria Ford Patrol car. The G-Analyst is a device containing accelermeters. It was installed in the middle console, as close as possible to the center of mass of the vehicle as shown in Fig 2.

Fig 2. G-Analyst device mounted on middle console

The vehicle was driven to acquire a velocity of 40 miles/hour. Then the brakes were fully applied. The ABS (Anti-skid braking system) was disconnected. The acceleration data is recorded every 0.1 seconds for a period of three seconds, an interval enough for the vehicle to come to a stop. A test then will produce 30 acceleration points taken at 0.1 sec interval.

Fig 3. Vericom 4000DQA device mounted on the windshield.

The second instrument mounted on the patrol car was a Vericom 4000 DAQ. This device was simultaneously placed with suction cups to the windshield of the vehicle. Once the instrument was calibrated and installed, the vehicle was driven on a flat paved surface until it acquired a velocity of 40 miles/hour. The ABS (Antiskid Braking System) was disconnected and all brakes were applied. The vehicle skidded to a stop with all tires locked. The Vericom 4000DQA measured the deceleration, velocity and stopping distance of the vehicle. It produces a data display as that of Fig 4.

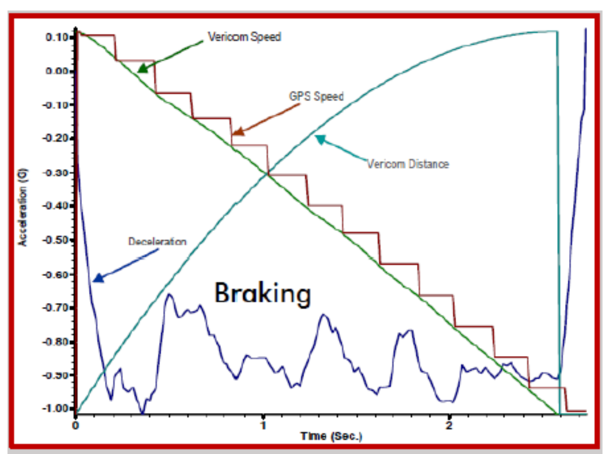

FIG 4. Vericom 4000DQA display measures acceleration, velocity, position. [1] (Vericom Computers 2018)

The blue curve that is labelled "Braking" is the deceleration curve. The y-axis is measuring the deceleration (reason values are negative) and shows values between -0.68 and one with an average deceleration of -0.84 G's. This is a normal value for braking on a concrete road. The green curve that appear diagonal starts with the value of the vehicle velocity and shows a linear declaine until the velocity is zero, when the vehicle stops. The cyan curve labelled "Vercom distance" is measuring the distance the vehicle travelled from the beginning of the full braking action to the stopped position. The brown steps like curve is the measure of the velocity sampling of the GPS at intervals of 0.2 sec. The Vericom 4000DQA has an internal GPS system, which serves to verify the data obtained from the accelerometers. The instrument is capable of displaying the drag factor and the acceleration pulse together with graphs for the velocity and position. All measurements mentioned above were recorded. For the particular test conducted aproximaterly at 11 am in broad day light under dry conditions. The instruments reported the drag factor after applying the brakes to be between 0.7-0.75 G's. The total skidding distance was about 70 ft.

III. THE PHYSICS BEHIND THE DRAG FACTOR TEST.

The forces that act on the vehicle during the skid test are shown in Fig. 5.

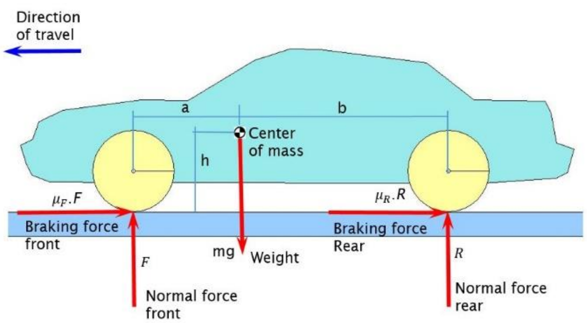

Fig. 5 Forces acting on the vehicle during skid test

Where:
m = mass of the vehicle
g = acceleration of gravity
F = Normal force at one of the front wheels
R = Normal force at one of the back wheels
a = distance from the center of mass to the front wheels
b = distance from the center of mass to the back wheels
h = distance from the road to the center of mass

μ_F = coefficient of friction at the front wheels
μ_R = coefficient of friction at the rear wheels
$\mu_F.F$ = Friction force (braking force) front wheels
$\mu_R.R$ = Friction force (braking force) rear wheels

It is necessary first, to account for the weight distribution, which normally we consider 60% for the front wheels 40% for the back, but this, is specific for each vehicle. Here a general derivation of the equations is presented in symbolic form thus valid for any percentage applicable.

In the vertical direction, the normal forces F and R must equal to the weigh of the vehicle. Their value depends on the location of the center of mass. Relations between them can be derived as well as for their relation to the total weight.

$$2F + 2R = mg \quad (1)$$

Since the vehicle is in equilibrium and on the ground, taken moments about the center of mass.

$$2F.a = 2R.b \quad (2)$$

$$F = \frac{b}{a}.R \quad (3)$$

$$If\ the\ cg\ is\ towards\ teh\ front,$$
$$b\ is\ bigger\ than\ a$$

$$b > a \quad (4)$$

$$\therefore F > R \quad (5)$$

Since F and R are, normal forces and the braking forces are $\mu_F.F$ and $\mu_R.R$ it proves that the braking forces on the front tires will be larger than those of the back tires. Equation (3) is telling us that they will be proportionally larger to b, which determines the location of the center of mass towards the front of the vehicle.

Now let us find out the relation of F and R to the total weight of the vehicle. Substituting equation (3) into (1)

$$2\frac{b}{a}R + 2R = mg \quad (6)$$

$$2R(\frac{b}{a} + 1) = mg \quad (7)$$

$$2R(\frac{b+a}{a}) = mg \quad (8)$$

$$R = \frac{1}{2}(\frac{a}{a+b})mg \quad (9)$$

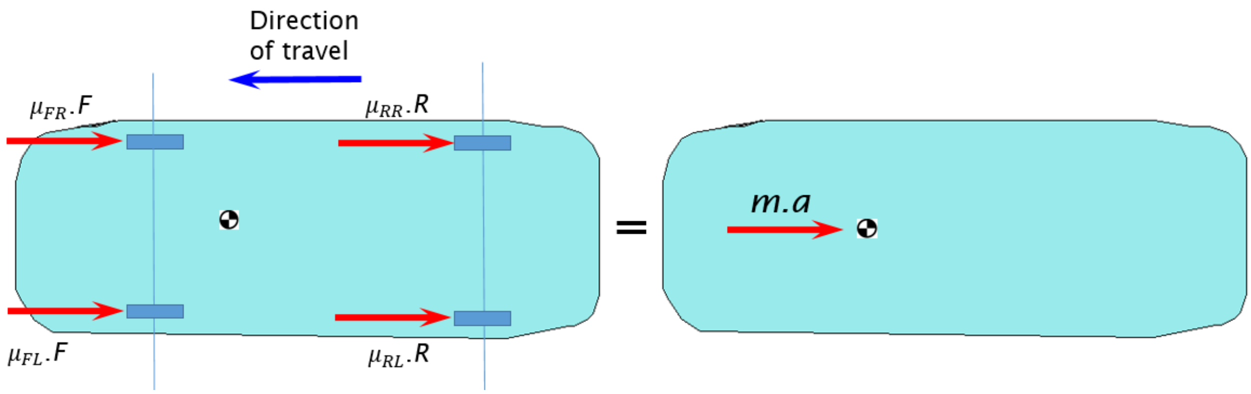

Fig 6. Braking forces and Dynamic forces acting under full braking

and using equation (3) *then*

$$F = \frac{1}{2}\left(\frac{b}{a+b}\right)mg \qquad (10)$$

These are the normal forces at each wheel in the front and each wheel in the back. These are forces used in the calculations of the braking forces $\mu_F.F$ and $\mu_R.R$

The factor $\frac{1}{2}\left(\frac{b}{a+b}\right)$ is indicative of what percentage of the weight each front wheel experiences. In a 60-40, weight distribution $\left(\frac{b}{a+b}\right)$ will be 0.6 or 60% of the weight. The factor then will be 0.3 for each front wheel. For simplicity of the equations to follow, we will call this factor f_{WF} to mean factor due to the weight on each of the front wheels. We assume that both front wheels carry the same percentage of the weight but this analysi will allow each to carry a different amount.

In the same way the factor $\frac{1}{2}\left(\frac{a}{a+b}\right)$ is indicative of what percentage of the weight each rear wheel experiences. In a 60-40, weight distribution $\left(\frac{a}{a+b}\right)$ will be 0.4 or 40% of the weight. The factor then will be 0.2 for each rear wheel. We will call this factor f_{WR} to mean factor due to the weight on each of the rear wheels. We assume that both rear wheels carry the same percentage of the weight but as indicated above this derivation allows for different amounts. So the normal forces or vertical forces that the ground applies on the vehicle can be expressed as:

$$F = f_{WF}.mg \quad \text{and} \qquad (11)$$
$$R = f_{WR}.mg \qquad (12)$$

Now we must apply Newton's second law of motion to the vehicle under deceleration during the skidding test. Fig 6. Shows the applied friction forces (braking forces) $\mu_R.R$ and $\mu_F.F$ on each wheel as external forces applied to the vehicle. The difference from the previous discussion is that now each wheel is allowed its own coefficient of friction to allow the case where each wheel can be just rolling, flat, or locked. Of course for the skidding test they are locked, but lest establish the general case for any situation.

Let us examine in detail exactly what is going on. Fig. 6 is a free body diagram of the four tire forces and the dynamic forces if we apply Newton's second Law of Motion. That principle of physics states, "The sum of the external forces applied on the body is equal to the change of momentum" [2] (Beer, Johnston 1996).

Thus applying Newton's equations, we have sum of the braking forces equals to mass times acceleration. The negative sign indicates they oppose the direction of motion

$$-\mu_{FR}.F - \mu_{FL}F - \mu_{RR}.R - \mu_{RL}.R = -m.a \qquad (13)$$

This equation considers the condition of each time but not the percentage by which the normal forces F and R are affected by the weight distribution. That is expressed in equations (11) and (12). If we implement that, we have:

$$-\mu_{FR}.\, f_{WF}.mg - \mu_{FL}\, f_{WF}.mg - \mu_{RR}.\, f_{WR}.mg - \mu_{RL}.\, f_{WR}.mg = -m.a \qquad (14)$$

Factoring the term mg and suppressing the negative sign because it is on both sides of the equation we have:

$$(\mu_{FR}.\, f_{WF} + \mu_{FL}\, f_{WF} + \mu_{RR}.\, f_{WR} + \mu_{RL}.\, f_{WR}).mg = -m.a \qquad (15)$$

Since the tires produce different forces, the decelerating forces (each tire's friction force) contributes differently to decelerate the vehicle and therefore the coefficient of friction value on each tire is different considering if the tire is rolling or braking under ABS or simply locked or flat after an accident. For this reason, adjustments need to be made for the coefficient of friction value at each tire in order to calculate the contribution of each and then of all tires to the deceleration of the vehicle as shown in equation (15). This unique condition for vehicles lead us to come up with an over all adjusted value for μ which then we call "drag factor" *f*. This adjusted drag factor is the term in parenthesis in equation (15) so the overall drag factor f is:

$$f = (\mu_{FR} .\, f_{WF} + \mu_{FL}\, f_{WF} + \mu_{RR} .\, f_{WR} + \mu_{RL} .\, f_{WR}) \tag{16}$$

Therefore, equation (15) reduces to:

$$f.m.g = ma \tag{17}$$

In addition, the drag factor becomes:

$$f = \frac{m.a}{m.g} \tag{18}$$

Which has significant physical meaning because the drag factor is expressed as a ratio of the dynamic forces decelerating the vehicle to the weight of the vehicle which is in fact the exact definition from the basic concept of the coefficient of dynamic friction which in this case adjusted as shown above is called the drag factor. Canceling the mass in the numerator and denominator in equation (18) yields:

$$f = \frac{a}{g} \tag{19}$$

Which clearly demonstrates what accident reconstruction experts use to express the drag factor as a ratio of the acceleration of the vehicle a to the acceleration of gravity g.

This derivation had another intended purpose. Once we have demonstrated that the drag factor is expressed as a ratio of the acceleration of the vehicle to the acceleration of gravity, we have proven why the G-Analyst or the Vericom 4000DQA can accurately determine the drag factor by measuring the acceleration during the skid test.

For the test, we have considered the fact that the two front tires and the two back tires brake evenly and this is close to reality for the CHP test. However, in case of an accident we must consider what each tire is doing, is it rotating freely, is it flat, is it braking fully or locked? These factors induce adjustments to the coefficient of friction present at each tire. This modifies tremendously the friction forces present at each tire. Therefore, if we consider the adjustment that for each tire and its condition needs to be made, then we are in a position to calculate an overall adjusted coefficient of friction, which in turn considering all four tires and their adjusted coefficients of friction we call it "drag factor". [5] (Fricke 2010) presents its use in accident reconstruction.

One final reflection of this analysis. If all tires brake normally at the same time, even with different weight distribution equations (15) and (16) demonstrate that then the coefficient of friction μ is equal to the drag factor *f*.

IV. MULTIBODY DYNAMICS MODEL

The objective here is to have a computer model that represents exactly the Ford Crown Victoria CHP vehicle and simulate the dynamic conditions or the skid test. The geometry of the actual vehicle was used from an actual photograph that contains the exact geometry, dimensions and scaling instead of data from published model specifications. Using software tools, image processing was used to transfer the photograph into the analysis software Working Model 2-D. This is shown at the top of Fig 7. The exact geometry was transferred to Working Model 2-D using the actual photograph of the CHP test vehicle. In order to achieve this, the image aspect ratio and resolution was maintained. Using the ability of Working Model to produce rigid bodies from multiple points in the actual vehicle, we generated the model while still keeping the original photograph in the background. The bottom picture of Fig 7 shows such model.

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Fig 7 Computer model produced from the actual CHP test vehicle.

At this point, the bodies produced in WM 2-D take physical meaning because the vehicle body and the tires now have mass and inertia properties in translation and rotation. All bodies now acquire the values of the actual CHP vehicle with all dimensions and specifications for weights and moments of inertia. Processing the image in Solidworks or AutoCAD and generating ".DXF" files that Working Model 2-D can process can also achieve the image transfer

Starting at a velocity of 40 miles/hour and hard braking with all wheels locked, the vehicle decelerates to a stop. The computer simulation model shown in Fig 8 was used to simulate the actual braking and skidding test. The display shows the actual vector velocity and acceleration as the tests progresses from the onset of the wheels lock to the stopping of the vehicle. The calculation of all variables displayed occurs every 50 milliseconds and can be displayed in numerical or graphical form as shown in Fig 8.

In order to compare to the experimental test, variables of interest are the position, velocity and acceleration of the vehicle at every instant of time. This in mathematical and simulation terms means to get data at every step of integration. Working Model 2-D allows the simultaneous display of all these variables. The position, velocity and acceleration of the vehicle from the computer simulation are displayed in Fig 9. The results of the experiment were compared to those of the computer simulation.

There is an additional feature of the computer simulation model if we compare it to the experimental method. The computers simulation model displays the braking forces on each tire and the normal reactions between the tires and the groud as a function of time during the entire skidding until it comes to a stop. It demonstrates how the front tires brake more than the back because the weight distribution originally and weight shifting due to the pitch dynamic conditions. In other words, rotation about the y (pitch) axis.

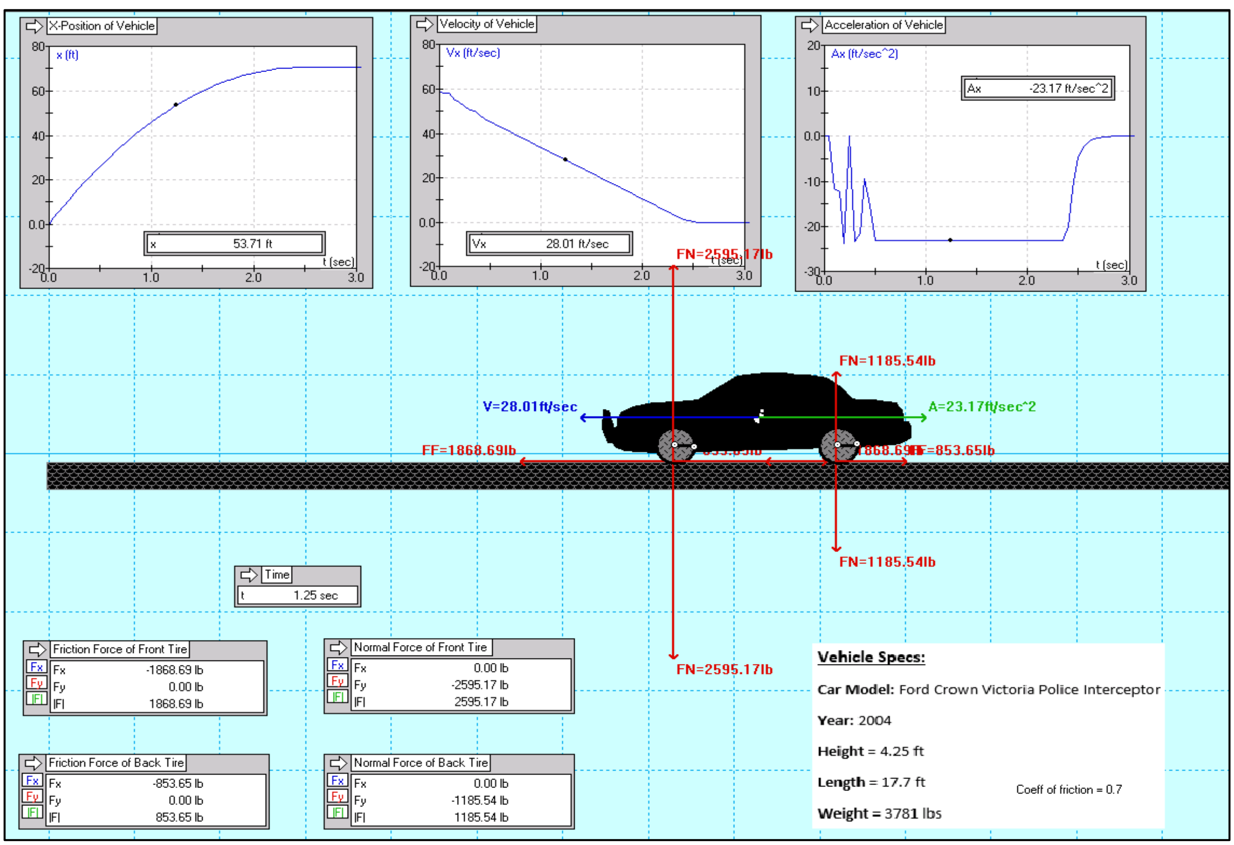


Fig 8. Illustrative display of the computer model skid test simulation. Tire frictional forces and normal forces (red), velocity (blue) and acceleration (green). This display is as it looks at time 1.25 seconds.

The Working Model 2-D skid test simulation, shown I Fig 8, reveals a simulation of a drag factor of 0.72, showing the, tire frictional and normal forces (red), velocity (blue) and acceleration (green) at time 1.25 seconds after the brakes are applied. This computer simulation calculated the forces and velocities of the rigid body (vehicle) in plane motion, distance traveled and drag factor.

Fig 9 shows the position vs time plot from the skid test results where the vehicle starts from a reference position (zero) which is where the car starts applying the brakes and reaches a final position of 70.70 ft. in about 3.1 seconds for a complete stop. Fig. 10 shows the velocity vs time graph demonstrating the car's velocity at the instant the brakes are applied (58.7 ft/s or 40 mph) to a complete stop. Fig. 11 shows the acceleration vs time graph were the car is decelerating at 23.17 ft/s^2 until the car stops. Since we have demonstrated that the critical measure for determining the drag factor is the acceleration, this display becomes the most important for that purpose. Therefore in cases where de do not know the conditions such as in this case of a controlled test, the computer simulation becomes not a verification tool like here but can be used to predict the behavior of the vehicle under other circumstances. Computer simulation can be run to match evidence in a particular case where would be even hard to measure the drag factor experimentally due to change of a scene by the passing of time, modifications of the road or evidence of the accident has been clean up and has disappeared. With actual photographs measurements taken on the day of the accident, a computer simulation can be built to reconstruct the accident and work backwards to match the physical evidence and reveal what actually happened with an accuracy of a valuable tool in accident reconstruction.

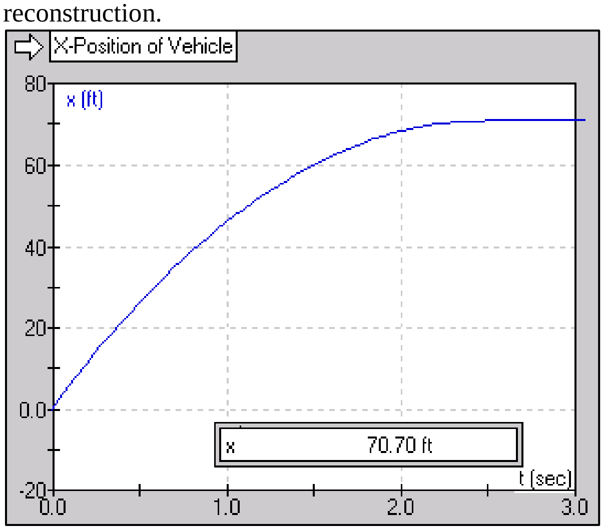


Fig 9. Position vs. time plot showing the vehicle's final position (72.70 ft.) when coming to a fully stop.

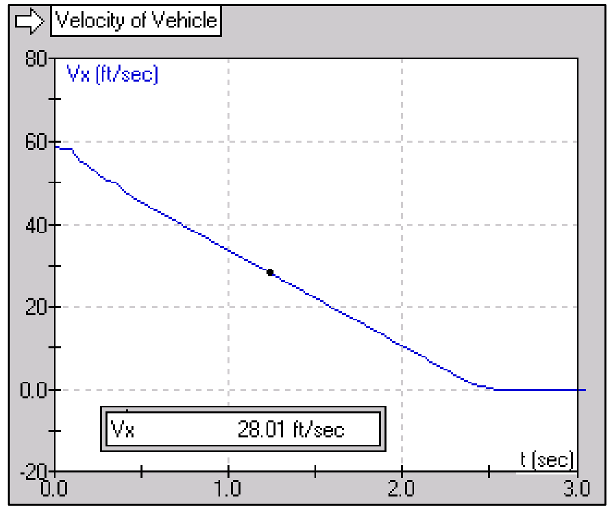


Fig 10. Velocity vs Time plot showing vehicle's initial and final velocities with full braking until comes to a full stop.

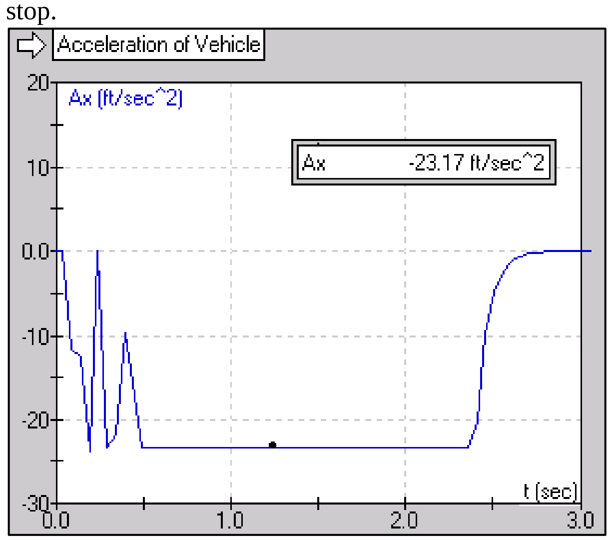
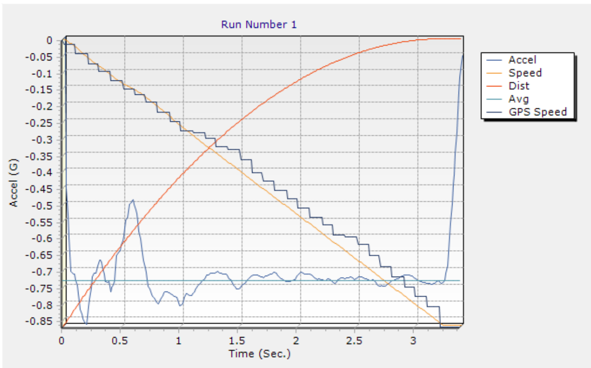


Fig. 11. Acceleration vs time plot showing vehicle's deceleration until it comes to a complete stop.

The simulation of the vehicle starting at 40 miles/hour (58.7 ft/sec) and coming to a stop revealed also the normal and friction forces at each set of tires. The calculated rear tire normal forces were 1185 lb. while the friction forces, were 866 lb. For the front tires, the normal forces were 2596 lb and the friction forces also computed were 1895 lbs. After the vehicle came to a stop, the total distance traveled calculated by the simulation was 72.70 ft with a deceleration of 23.2 ft/s^2 equivalent to 0.72 G's.

The drag factor measured by the G-Analysit and the Verricom 4000DQA was between 0.7-0.75 G's on the experimental tests. The skidding distance about 70 ft. If we compared these values to the computer simulation values of 0.72 G's, and the distance of 70 feet, the obvious conclusion is that there is a very accurate correspondence. If we compare the computer simulation graphs above Fig 9-11 with those of the Vericom 4000DQA, Fig 12, we reach the same conclusion.



Average Table

Vehicle	Run #	Time Secs	Accel G	Speed MPH	Dist ft.	GPS Speed MPH
	1	3.230	-0.727	51.53	122.909	51.81
Averages		3.230	-0.727	51.53	122.909	51.81

Fig 12. Vericom 4000DQA display and data.

Fig. 12 displays the position, velocity, and acceleration and GPS data of the actual vehicle on the experimental skid test conducted on a California Highway Patrol Crown Victoria vehicle. [12] (CHP California Highway Patrol. Enforcement and Planning Division Headquarters MAIT Unit. Sacramento, Ca. (2018)

V. FORWARD DYNAMICS BOND GRAPH MODEL

Section III established two fundamental principles. The skid test is controlled mathematical to differential equations of motion. The second is that if we can find the acceleration experimentally, analytically or throght computer simulation we have resolved the objective of finding the drag factor which is a central aspect of this paper.

We turn now to the bond graph technology, which has proven over 60+ years and hundreds of research papers to be reliable, and allow modelling reality in minute detail whether the system is a mechanical, electric, rotation, hydraulic or thermal system. Here we have a mechcnical system controlled by the equations of motion.

We look at this technology because will allow more detailed models with a mathematical insight and being a new approach to be used in accident reconstruction. For the reader not familiar with bond graphs please refer to reference [3] (Karnopp D, Margolis D, Rosenberg R, 2012) for an extensive treatment from beginner to advanced user. Reference [4] (Granda 2011) explains the modeling process systematically. Reference [6] (Granda JJ, Glockler T, 2016) presents that bond graph model applied to accident reconstruction. This is explained also in [7] (Granda JJ, 2016) and [8] (Granda JJ, 2015).
Let us begin by referring to the physical system shown in Fig 13.

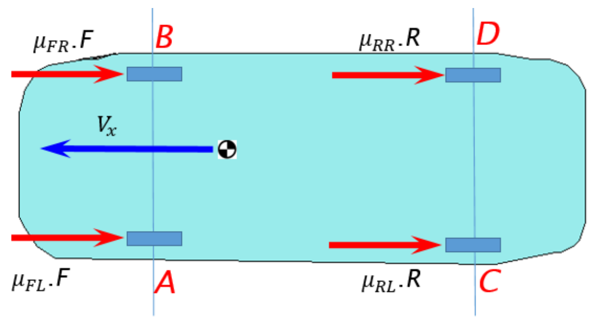


Fig 13 Vehicle under braking, velocities and forces

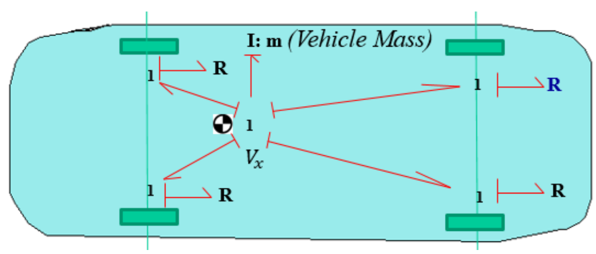


Fig 14 Bond graph of vehicle on skid test

Reference [4] (Granda JJ 2011) explains in detail and systematically the procedure for building a bond graph model. Let us apply that procedure to the problem at hand. We begin building the model by recognizing the distinct velocities and assign them the symbol element 1. Fig 14 shows 1's (symbols) used to represent the velocities. There is 1's at the location of each tire. This means physically that the tires when locked, behave dynamically as if they were part of the vehicle body and the whole system acts as a whole. Realistically however there are stiffness elements on the suspension of the vehicle and small tolerances of motion between the parts because many suspension parts have bushings that will allow for some slack and damping effects. In bond graph modeling terms, the gaps are C element and the damping are R elements.

A. FRICTION FORCES AND DRAG FACTOR

Here the R elements represent the friction elements that dissipate kinetic energy as the vehicle moves with the four wheels locked. Therefore, the R elements represent the friction forces at each tire following the notation used before. However, we propose a simpler notation by labeling the front tires A, B and the back C, and D where the braking forces at each tire are:

$$F_{FL} = F_A = \mu_{FL}.f_{WFL}.mg \quad (21)$$
$$F_{FR} = F_B = \mu_{FR}.\, f_{WFR}.mg \quad (22)$$
$$R_{RL} = R_C = \mu_{RL}.\, f_{WRL}.mg \quad (23)$$
$$R_{RR} = R_D = \mu_{RR}.\, f_{WRR}.mg \quad (24)$$

$F_{FL} = F_A$ =Braking force of front left tire.
$F_{FR} = F_B$ = Braking force of front right tire
$R_{RL} = R_C$ = Braking force rear left tire
$R_{RR} = R_D$ = Braking force rear right tire

$\mu_{FL} = \mu_A$ = Adjusted coefficient of friction front left tire
$\mu_{FR} = \mu_B$ = Adjusted coefficient of friction front right tire
$\mu_{RL} = \mu_C$ = Adjusted coefficient of friction rear left tire
$\mu_{RR} = \mu_D$ = Adjusted coefficient of friction rear right tire

$f_{WFL} = f_A$ = Percentage of weight applied on front left tire
$f_{WFR} = f_B$ = Percentage of weight applied on front right tire
$f_{WRL} = f_C$ = Percentage of weight applied on rear left tire
$f_{WRR} = f_D$ = Percentage of weight applied on rear right tire

These represent the front and back forces at each tire considering their unique condition, whether dragging completely, rolling of being flat, the coefficients μ_{FL}, μ_{FR}, μ_{RL}, μ_{RR}, f_{WF}, and f_{WR} will allow us to do the proper adjustments.

Using the new simplified proposed notation equations (21), (22), (23) and (24) reduce to:

$$F_A = \mu_A.f_A.mg \quad (25)$$
$$F_B = \mu_B.\, f_B.mg \quad (26)$$
$$R_C = \mu_C.\, f_C.mg \quad (27)$$
$$R_D = \mu_D.\, f_D.mg \quad (28)$$

In addition, Equation (16), that defined the drag factor simplifies to:

$$f = (\mu_A.\, f_A + \mu_B\, f_B + \mu_C.\, f_C + \mu_D.\, f_D) \quad (29)$$

B. BOND GRAPH MODEL SIMULATION

In order to perform the simulation with the bond graph model of Fig 14. We used the Computer Aided Modeling Program (CAMPG) to produce the necessary computer code so that it delivers the model to MATLAB which is the simulation language used. Fig. 15 shows the CAMPG model.

The I element on bond 5, we refer to I5, represents the vehicle mass, the R elements on bonds 9, 7, 4 and 3 represent the front tires A, B and the back tires C and D respectively.

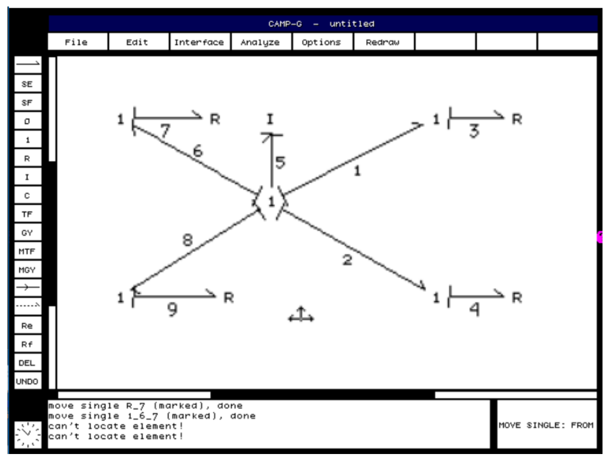


Fig 15. CAMPG Bond graph model of drag factor test.

This is a simple model, which considers the tires and the vehicle as one single rigid body. In reality they are separate because vehicles have suspensions, the have bushings which have flexible components with damping and stiffness properties. Let us consider the simple example first in order to understand the model. Bond graph models are very flexible. We can do simple models at the start and build from there more detailed models with the desired level of complexity. CAMPG performs the derivation of the equations of motion and the production for the computer code that actually runs the simulation. CAMPG processes with ease simple models or complex models automatically. We dedicate our attention to the engineering of the computer model rather that worrying about whether the equations are derived correctly. That is a job for CAMPG.

CAMPG takes the bond graph and produces MATLAB ".m" files. The first one *campgmod.m* contains the physical parameter time controls, output requests. The second, *campgequ.m* contains the differential equations of the system. The simulation uses the specifications for the location of the center of mass and other physical properties of the 2004 Crown Victoria.

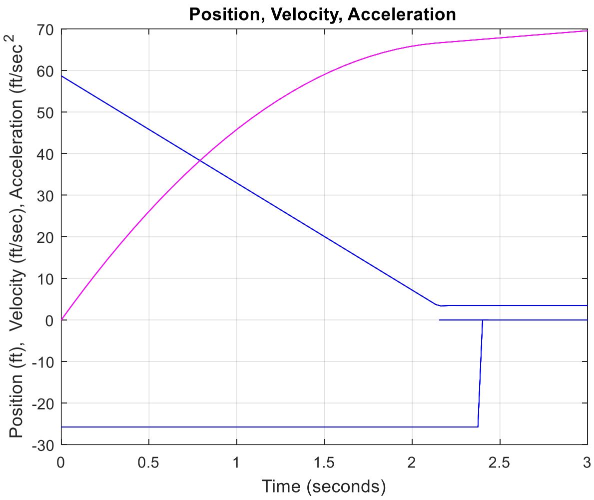


Fig. 16 CAMPG/MATLAB simulation results for position, velocity and acceleration of the vehicle.

We compared the bond graph model results to those of actual experimental test. These are compered also to those of the Multibody Working Model 2-D. The results are very close and validate the simulation. Therefore, it follows that computer simulations can predict the behavior of vehicles under a real accident conditions in which not all data and instrumentation is available.

C. PROGRESSIVE BOND GRAPH MODELS

1. Stiffness Model

The simple model analyzed above illustrated in simple terms the bond graph method. It considered the tires and the body of the vehicle a single rigid body. In real life, the vehicle has suspension systems in the front and in the back.

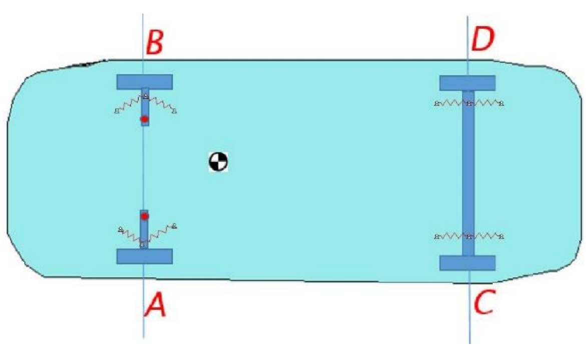


Fig 17 Vehicle with suspension stiffness components

Fig 17 shows a modified model of that shown in Fig 13. This one considers the fact that the body of the vehicle and the wheels of the vehicle are not a single rigid body but they act as a system of four separate tires and the body of the vehicle for five rigid bodies connected via stiffness elements. One factor in real life that we observe in the operation of a vehicle is what "mechanical latency" is. This means the ability of the vehicle to react immediately to a maneuver whether is braking or steering. The vehicle does not react instantaneously as when it is consider a single rigid body. When the brakes are applied, the braking forces acting on each tire need to transmit their effect through four independent suspension components attached to the body of the vehicle by brackets and bushings. While the idea is that the vehicle will react instantaneously, the fact is in reality there is a latency. When developing a more advanced bond graph model we unavoidably need to consider these components of the suspension.

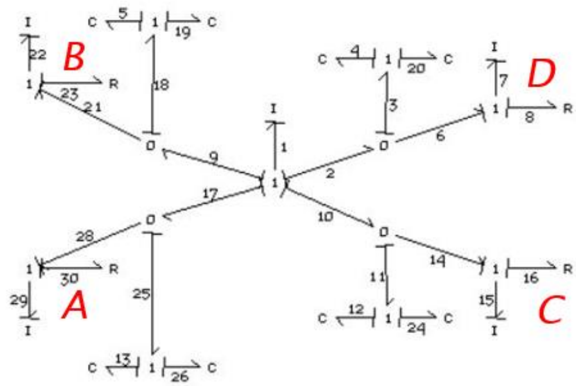


Fig. 18 Bond graph model with suspension stiffens components

The four peripheral I elements represent the mass of the tires rims and suspension assemblies attached to the body of the vehicle with links, springs and bushings. The C elements represent the stiffness elements and represent the slack tolerance on the joints that make up the suspension. These also represent the fact that there are articulations on the suspension parts. The R elements represent the tires "drag forces" (braking forces). At this point we will call them drag forces because the individual conditions of the tires (full braking, flat, rolling, ABS) are factors that have been considered when entering the values describes in equations (21) to (24) or if using the simplified proposed notation here, equations (25) to (28). The model has 8 springs, 4 individual inertia elements that represent the tires and one that represents the vehicle. Each one of this will produce one differential equation so this model is a 13th order system with 13 differential equations.

2. Damping and stiffness model

The model of Fig 19. considered parts with stiffness, but the bushings on the suspension are rubber-metal components that produce damping forces. For this reason we develop progressively a more compete model, which models all those parts that produce damping which act together with the stiffness parts.

The model shown in Fig 19 represents a model that considers the damping and stiffness components of the front and back suspensions.

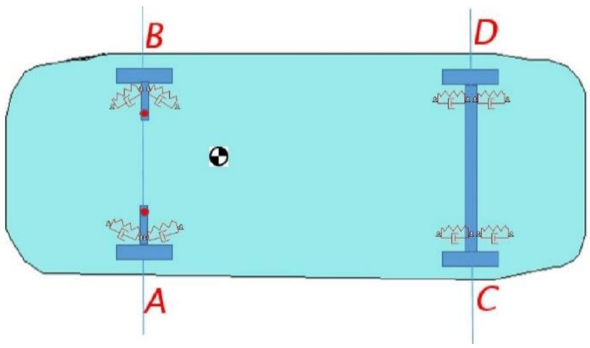


Fig. 19 Model with stiffness and damping components.

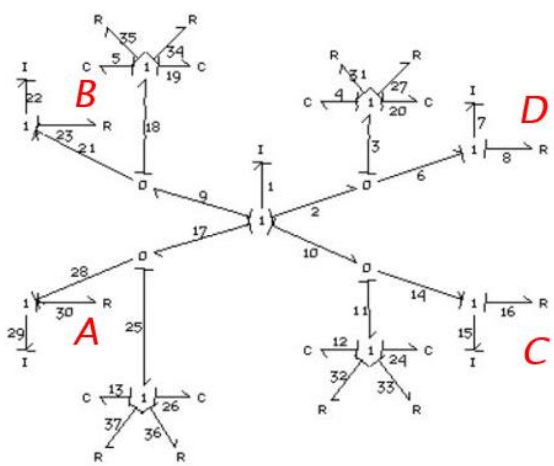


Fig. 20 Bond graph model of skid test vehicle with stiffness and damping components. Sections A, B, (front tires) C, D (back tires) using the simplified notation.

Let us not lose track of our interest on the drag factor determination or verification. These models can be used both ways. I29 on bond 29 is the Front Left tire, in our notation the "A" tire, I22 is the front right tire, the "B" tire, I15 is the rear left tire of "C" tire and I7 is the right rear tire of "D" tire. The difference between this model and that of Fig. 18 is that eight damping components were integrated to the model. However, this did not change the number of differential equations and the system remains at 13 differential equations. The differential equations that are solved on this model come in what is known as State Variable Form. That is first order differential equations in explicit form. Using MATLAB following the method that produce Fig. 16, we obtain the results of the simulation of the system of Fig 20. This time the stiffness and damping elements act and the results produce the oscillations we see on the experimental test due to the pitch and longitudinal effects of the suspension components.

This model illustrates the fact that using bond graph modeling one can represent the system to any degree of detail with the assurance that using a computer-generated model such as the one CAMPG produces automatically for MATLAB the model will be mathematically correct, making the modeling and simulation process very efficient.

VI. CONCLUSIONS

Simulation results were close to the experimental results for the CHP skid test. With a drag factor of 0.72, simulation results showed a braking distance of 70.70 ft. and when compared to the CHP experiment results (drag factor 0.72 and braking distance traveled of 72 ft.) the results are within 2% difference. In addition, comparison of calculated normal and friction forces against simulation results showed only a 1% difference; this again demonstrated a successful verification of vehicle dynamics utilizing computer simulation.

The paper has presented and demonstrated a very good correlation with the experimental data and demonstrates that bond graph computer models not only verify the actual experimatal results, but that they also check with the multibody analysis, giving accident reconstrucion experts a reliable tool to either verify what happened or to predict what would happened under different scenarios.

Models that are more detailed can be developed for the lateral dynamics considering the up and down motion of the suspension. More complex models can be developed to analyst the vehicle not only in skid by also in a combination of skid and yaw motion.

Considering the vehicle as a rigid body in plane motion, we develop the bond graph model. Analysis in "plane motion" is appropriate as long as the vehicle does not leave the ground and becomes airborne. If that were to be the case, the bond graph method also applies. One can consider three-dimensional models with Euler Angles and six degrees of freedom. Vector bond graph models effectively consider the appropriate three-dimensional coordinate transformations.

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