Gravitational potential energy

ergopedia Essential Physics

Objectives

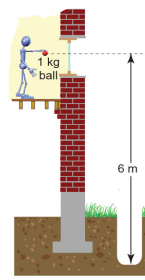
- Investigate examples of gravitational potential energy.
- Calculate the potential energy, mass, or height of an object using the gravitational potential energy equation.
- Choose the reference frame and coordinate system best suited to a particular problem.

Assessment

1. What does each of the symbols mean in this equation: $E_P = mgh$?
2. Translate the equation $E_P = mgh$ into a sentence with the same meaning.
3. How much E_P does a 1 kg mass gain when raised by a height of 10 m?
4. How high would a 2.0 kg mass have to be raised to have a gravitational potential energy of 1,000 J?
5. Mountain climbers at the Everest base camp (5,634 m above sea level) want to know the energy needed reach the mountain's summit (altitude 8,848 m). What should they choose as zero height for their energy estimate: sea level, base camp, or the summit?

Assessment

6. Which location is most convenient to choose as the zero height reference frame if the robot tosses the ball into the hole?



Physics terms

- potential energy
- gravitational potential energy
- mechanical energy

Equations

$$E_p = mgh \quad \text{or} \quad PE = mgh$$

The change in gravitational potential energy of an object is its mass multiplied by "g" and by the change in height.

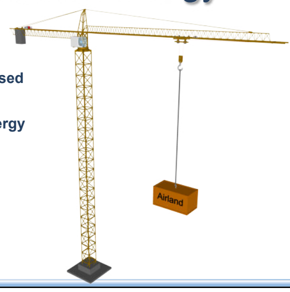
At Earth's surface, g = 9.8 N/kg, or 9.8 kg m/s^2

Gravitational potential energy

This heavy container has been raised up above ground level.

Due to its height, it has stored energy—gravitational potential energy.

How do we know that the energy is there?

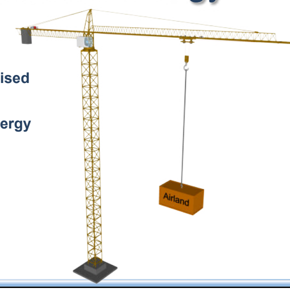


Gravitational potential energy

This heavy container has been raised up above ground level.

Due to its height, it has stored energy—gravitational potential energy.

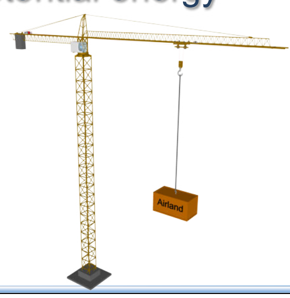
If the container is released, the stored energy turns into kinetic energy.



Gravitational potential energy

If the *mass* of the container increases, its potential energy will also increase.

If the *height* of the container increases, its potential energy will also increase.



Gravitational potential energy

$$E_p = mgh$$

$$PE = mgh$$

The gravitational potential energy of an object is . . .

Gravitational potential energy

$$E_p = mgh$$

$$PE = mgh$$

The gravitational potential energy of an object is the mass m in kilograms . . .

Gravitational potential energy

$$E_p = mgh$$

$$PE = mgh$$

The gravitational potential energy of an object is the mass m in kilograms multiplied by the local acceleration due to gravity g (which is 9.8 m/s^2 near Earth's surface) . . .

Gravitational potential energy

$$E_p = mgh$$

$$PE = mgh$$

The gravitational potential energy of an object is the mass m in kilograms multiplied by the local acceleration due to gravity g (which is 9.8 m/s^2 near Earth's surface), multiplied by the height h in meters.

Gravitational potential energy

How can you give an object gravitational potential energy?

Gravitational potential energy

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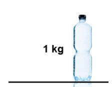
Gravitational potential energy comes from *work done against gravity* …

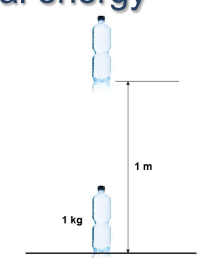


Gravitational potential energy

How can you give an object gravitational potential energy?

Gravitational potential energy comes from *work done against gravity* …

… such as the work you do when you lift this bottle of water.



Gravitational potential energy

Where does the formula for gravitational potential energy come from?

$$E_p = mgh$$

Gravitational potential energy

Where does the formula for gravitational potential energy come from?

$$E_p = mgh$$

The gravitational potential energy stored in an object equals the work done to lift it.

Deriving the formula

Work is force times distance.

$$W = Fd$$

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Work is force times distance.

$$W = Fd$$

$$F = mg$$

To lift an object, you must exert an upward force equal to the object's weight.

Deriving the formula

Work is force times distance.

$$W = Fd$$

$$F = mg \qquad d = h$$

The distance you lift it is the height *h*.

Deriving the formula

Work is force times distance.

$$W = Fd = mgh = E_p$$

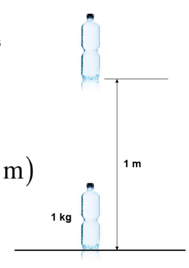
$$F = mg \qquad d = h$$

force = weight　　distance = height

An example

A 1.0 kg mass lifted 1.0 meter gains 9.8 joules of gravitational potential energy.

$$E_p = mgh$$
$$= (1.0\ \text{kg})(9.8\ \text{N/kg})(1.0\ \text{m})$$
$$= 9.8\ \text{J}$$



Exploring the ideas

Click this interactive calculator on page 259

Equation for potential energy

How much potential energy does the brick have? If the brick has more mass, the force of gravity gives it more weight and more potential energy. If it has more height, it can fall farther and also has more potential energy. By these arguments we deduce that potential energy depends, at least, on weight and height. Checking the units supports that conjecture—the product of weight times height has units of force times distance, the same as work. The equation for **gravitational potential energy** is exactly what we surmised. The potential energy of any object of mass, *m*, raised by a height, *h*, is given by equation (9.3). We will see in Chapter 10 that this energy is *exactly equal to the minimum work it takes to raise the brick from the floor to the tabletop*.

Calculator

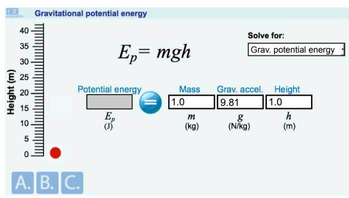
(9.3) $E_p = mgh$

E_p = potential energy (J)
m = mass (kg)
g = strength of gravity, typ. 9.8 N/kg
h = change in height (m)

Gravitational potential energy

Engaging with the concepts

What is the potential energy of a 1.0 kg ball when it is 1.0 meter above the floor?

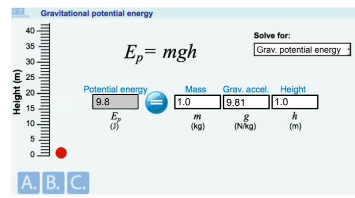


Engaging with the concepts

What is the potential energy of a 1.0 kg ball when it is 1.0 meter above the floor?

E_p = 9.8 J

What is the energy of the same ball when it is 10 m above the floor?



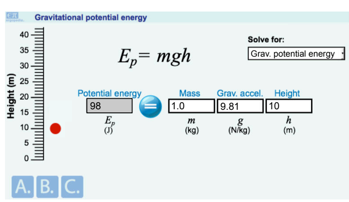
Engaging with the concepts

What is the potential energy of a 1.0 kg ball when it is 1.0 meter above the floor?

E_p = 9.8 J

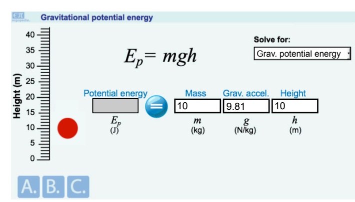
What is the energy of the same ball when it is 10 m above the floor?

E_p = 98 J



Engaging with the concepts

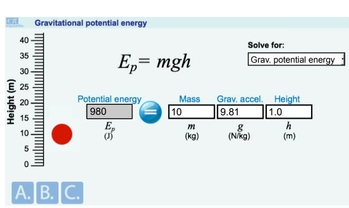
How does the potential energy of a 10 kg ball raised 10 m off the floor, compare to the 1 kg ball?



Engaging with the concepts

How does the potential energy of a 10 kg ball raised 10 m off the floor, compare to the 1 kg ball?

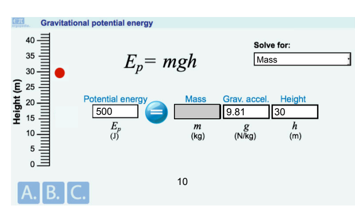
It is 10 times greater, or 980 J.



Engaging with the concepts

Suppose a battery contains 500 J of energy.

What is the heaviest object the battery can raise to a height of 30 meters?



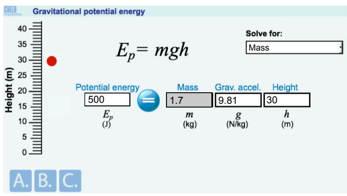
Engaging with the concepts

Suppose a battery contains 500 J of energy.

What is the heaviest object the battery can raise to a height of 30 meters?

1.7 kg

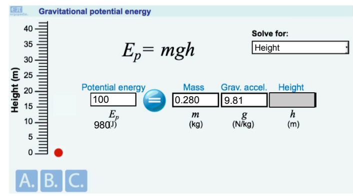
$$m = \frac{E_p}{gh} = \frac{500 \text{ J}}{(9.8 \text{ m/s}^2)(30 \text{ m})} = 1.7 \text{ kg}$$



Engaging with the concepts

The energy you use (or work you do) to climb a single stair is roughly equal to 100 joules.

How high up is a 280 gram owlet that has 100 J of potential energy.



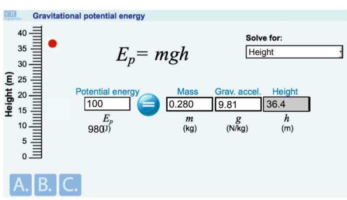
Engaging with the concepts

The energy you use (or work you do) to climb a single stair is roughly equal to 100 joules.

How high up is a 280 gram owlet that has 100 J of potential energy.

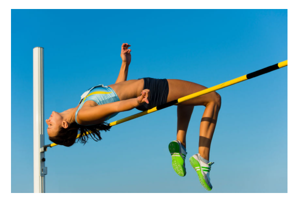
36.4 meters

$$h = \frac{E_p}{mg} = \frac{100 \text{ J}}{(0.280 \text{ kg})(9.8 \text{ m/s}^2)} = 36.4 \text{ m}$$



Athletics and energy

How much energy does it take to raise a 70 kg (154 lb) person one meter off the ground?

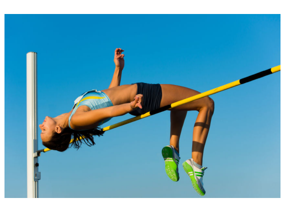


Athletics and energy

How much energy does it take to raise a 70 kg (154 lb) person one meter off the ground?

$$\begin{aligned} E_p &= mgh \\ &= (70 \text{ kg})(9.8 \text{ N/kg})(1.0 \text{ m}) \\ &= 686 \text{ J} \end{aligned}$$

This is a good reference point. It takes 500 to 1,000 joules for a very athletic jump.



Typical potential energies

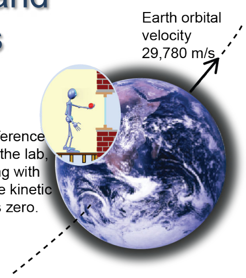
	mass (kg)	height (m)	g (N/kg)	E_P (J)
baseball	0.15	1	9.8	1.47
baseball	0.15	10	9.8	14.7
arrow	0.01	100	9.8	9.8
seagull	1.75	50	9.8	857
diver	50	10	9.8	4,900
helicopter	6,000	1,000	9.8	58,800,000

Reference frames and coordinate systems

When calculating kinetic energy, you need to chose a reference frame.

- **Typically, we choose the Earth as our reference frame.**
- **We treat the Earth *as if* it is at rest.**

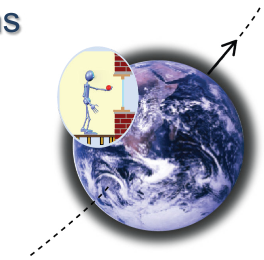
In the reference frame of the lab, co-moving with Earth, the kinetic energy is zero.



Reference frames and coordinate systems

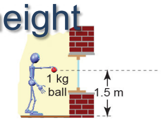
When calculating gravitational potential energy, we need to choose where to put the origin of our coordinate system.

In other words, where is height equal to zero?



Determining height

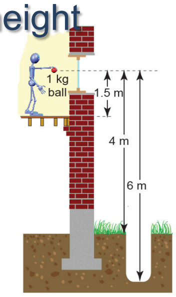
$$E_p = mgh$$



Where is zero height?

Determining height

$$E_p = mgh$$



Where is zero height?

the floor?

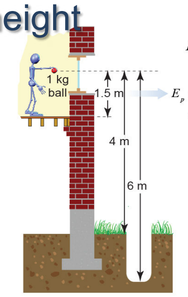
the ground outside?

the bottom of the hole?

Determining height

$$E_p = mgh$$

If *h* = 1.5 meters, then the potential energy of the ball is 14.7 joules.



$E_p = mgh$

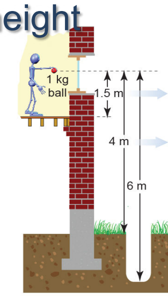
$E_p = (1\ \text{kg})(9.8\ \text{N/kg})(1.5\ \text{m})$

$= 14.7\ \text{J}$

Determining height

$$E_p = mgh$$

If *h* = 4 meters, then the potential energy is 39.2 J.



$E_p = mgh$

$E_p = (1\ \text{kg})(9.8\ \text{N/kg})(1.5\ \text{m})$

$= 14.7\ \text{J}$

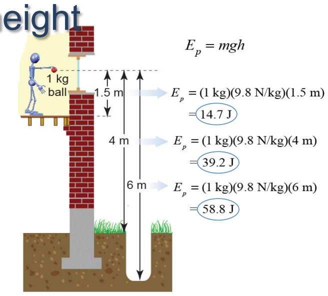
$E_p = (1\ \text{kg})(9.8\ \text{N/kg})(4\ \text{m})$

$= 39.2\ \text{J}$

Determining height

$$E_p = mgh$$

If h = 6 meters, then the potential energy is 58.8 J.

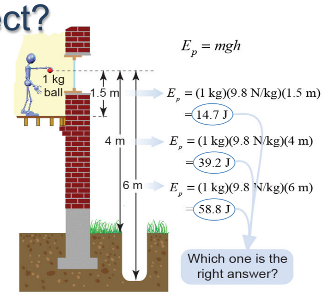


Which is correct?

$$E_p = mgh$$

14.7 J? 39.2 J? 58.8 J?

Which answer is correct?

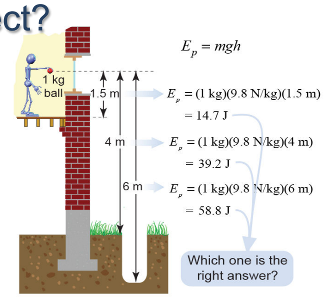


Which is correct?

$$E_p = mgh$$

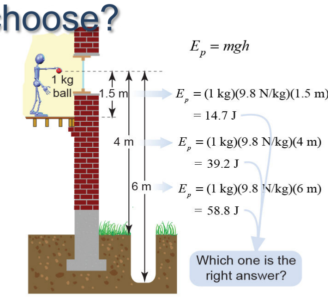
14.7 J? 39.2 J? 58.8 J?

All are correct!



How do you choose?

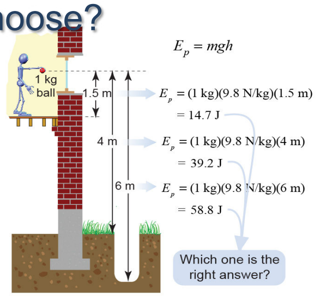
The height you use depends on the problem you are trying to solve …



How do you choose?

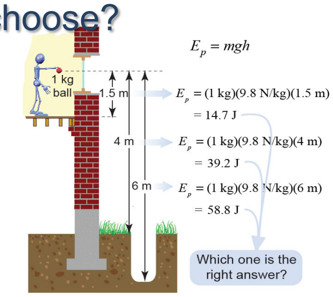
The height you use depends on the problem you are trying to solve …

… because only the *change* in height actually matters when solving potential energy problems.



How do you choose?

So how do you know where h = 0?

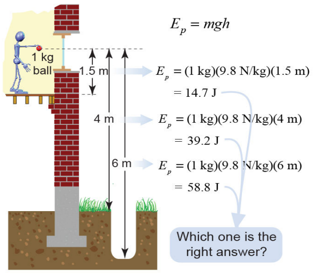


You decide

So how do you know where h = 0?

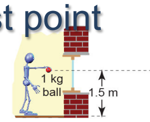
YOU get to set h = 0 wherever it makes the problem easiest to solve.

Usually, that place is the lowest point the object reaches.



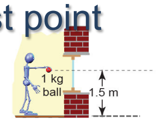
Pick the lowest point

If the ball falls only as far as the floor, then the floor is the most convenient choice for zero height (that is, for h = 0).



Pick the lowest point

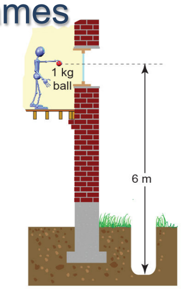
In this case, the potential energy at the position shown here (at the level of the dashed line) is



$$E_p = mgh$$
$$= (1\text{ kg})(9.8\text{ N/kg})(1.5\text{ m})$$
$$= 14.7\text{ J}$$ ***relative to the floor.***

Reference frames

If the ball falls to the bottom of the hole, then the bottom of the hole is the best choice for zero height (that is, for h = 0).

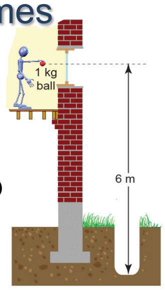


Reference frames

In this case, the potential energy at the position shown here (at the level of the dashed line) is

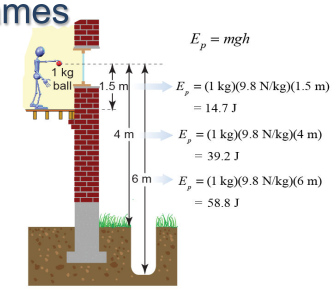
$$E_p = mgh$$
$$= (1\text{ kg})(9.8\text{ N/kg})(6\text{ m})$$
$$= 58.8\text{ J}$$

relative to the bottom of the hole.



Reference frames

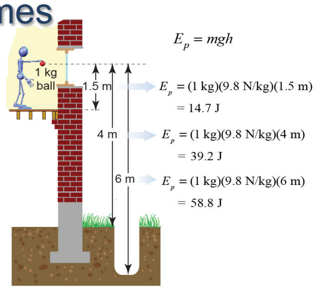
Gravitational potential energy is *always* defined relative to your choice of location for zero height.



Reference frames

Gravitational potential energy is *always* defined relative to your choice of location for zero height.

And unlike kinetic energy, it can even be negative!

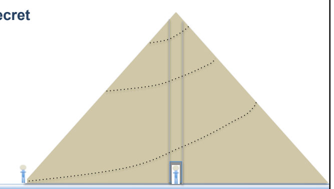


Does the path matter?

A set of identical twins wants to get to the top of a mountain.

- One twin hikes up a winding trail.
- The second twin takes the secret elevator straight to the top.

Which twin has the greatest potential energy at the top?

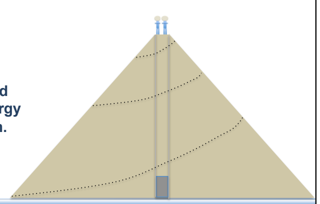


Path independence

The twins have the SAME potential energy at the top.

$$E_p = mgh$$

It doesn't matter HOW they gained height. Changes in potential energy are independent of the path taken.



Assessment

1. What does each of the symbols mean in this equation: $E_p = mgh$?

Assessment

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   m = mass in kg
   g = the strength of gravity in N/kg
   h = the change in height in meters
2. Translate the equation $E_p = mgh$ into a sentence with the same meaning.

Assessment

1. What does each of the symbols mean in this equation: $E_p = mgh$?
   m = mass in kg
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2. Translate the equation $E_p = mgh$ into a sentence with the same meaning.
   The change in gravitational potential energy of an object is its mass multiplied by "g" and multiplied by the change in height.
3. How much E_p does a 1 kg mass gain when raised by a height of 10 meters?

Assessment

1. What does each of the symbols mean in this equation: $E_P = mgh$?
   m = mass in kg
   g = the strength of gravity in N/kg
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2. Translate the equation $E_P = mgh$ into a sentence with the same meaning.
   The change in gravitational potential energy of an object is its mass multiplied by "g" and multiplied by the change in height.
3. How much E_P does a 1 kg mass gain when raised by a height of 10 meters?
   $E_P = mgh$ = 98 joules

Assessment

4. How high would a 2 kg mass have to be raised to have a gravitational potential energy of 1,000 J?

Assessment

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   $h = E_P/mg$ = 51 m
5. Mountain climbers at the Everest base camp (5,634 m above sea level) want to know the energy needed reach the mountain's summit (altitude 8,848 m). What should they choose as zero height for their energy estimate: sea level, base camp, or the summit?

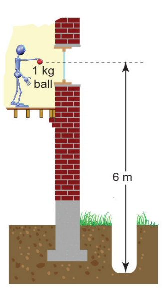
Assessment

4. How high would a 2 kg mass have to be raised to have a gravitational potential energy of 1,000 J?
   $h = E_P/mg$ = 51 m
5. Mountain climbers at the Everest base camp (5,634 m above sea level) want to know the energy needed reach the mountain's summit (altitude 8,848 m). What should they choose as zero height for their energy estimate: sea level, base camp, or the summit?

   The climbers are located at the base camp, so their change in gravitational potential will be relative to the base camp. They should therefore set the base camp's altitude as zero height.

Assessment

6. Which location is most convenient to choose as the zero height reference frame if the robot tosses the ball into the hole?



Assessment

6. Which location is most convenient to choose as the zero height reference frame if the robot tosses the ball into the hole?

Setting h = 0 at the *lowest* place that the object reaches means the potential energy will always be positive. This makes the problem easier to solve.