Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Student Outcomes

- Students use visual models, such as fraction bars, number lines, and area models, to show the quotient of whole numbers and fractions and to show the connection between them and the multiplication of fractions.
- Students divide a fraction by a whole number.

Classwork

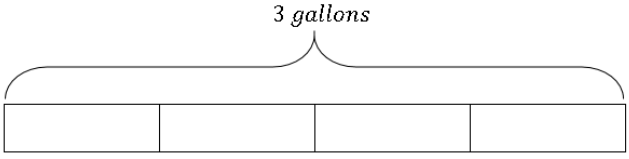
Opening Exercise (5 minutes)

At the beginning of class, have students work in pairs to complete the following problems. Partner A completes the problems in column A, and Partner B completes the problems in column B.

Opening Exercise

A	B
Write a division sentence to solve each problem.	**Write a multiplication sentence to solve each problem.**
1. 8 gallons of batter are poured equally into 4 bowls. How many gallons of batter are in each bowl?	**1. One fourth of an 8-gallon pail is poured out. How many gallons are poured out?**
2. 1 gallon of batter is poured equally into 4 bowls. How many gallons of batter are in each bowl?	**2. One fourth of a 1-gallon pail is poured out. How many gallons are poured out?**
Write a division sentence *and* draw a model to solve.	**Write a multiplication sentence *and* draw a model to solve.**
3. 3 gallons of batter are poured equally into 4 bowls. How many gallons of batter are in each bowl?	**3. One fourth of a 3-gallon pail is poured out. How many gallons are poured out?**

After three minutes, have students share their division and multiplication sentences for each problem. Post the completed division and multiplication sentences alongside one another to allow for easy comparison. Have students discuss what they notice. Emphasize the fact that dividing by 4 and multiplying by $\frac{1}{4}$ are equivalent. Use student models for Problem 3 to demonstrate this fact.



Scaffolding:

Each class should have a set of fraction tiles. Students who are struggling may benefit from using the fraction tiles to see the division until they are better at drawing the models.

Also, look for students who show work using unit language. For example in Problem 2 of column A, a student might recognize that in order to divide the gallon of batter equally among 4 bowls, it is helpful to think of 1 gallon as being equal to 4 fourths gallon (4 quarts). Likewise, in Problem 3 of column A, 3 gallons of batter is equal to 12 fourths gallons of batter.

- 2. $1 \div 4$
  $4 \textit{ fourths} \div 4 = 1 \textit{ fourth} = \frac{1}{4}$
- 3. $3 \div 4$
  $12 \textit{ fourths} \div 4 = 3 \textit{ fourths} = \frac{3}{4}$

To conclude the Opening Exercise, have students describe how their models would look if Problem 3 started with less than 1 gallon of batter being poured into 2 bowls.

Example 1 (7 minutes)

This lesson focuses on fractions divided by whole numbers. Students learned how to divide unit fractions by whole numbers in Grade 5. Teachers can become familiar with what was taught on this topic by reviewing the materials used in the Grade 5 Module 4 lessons and assessments.

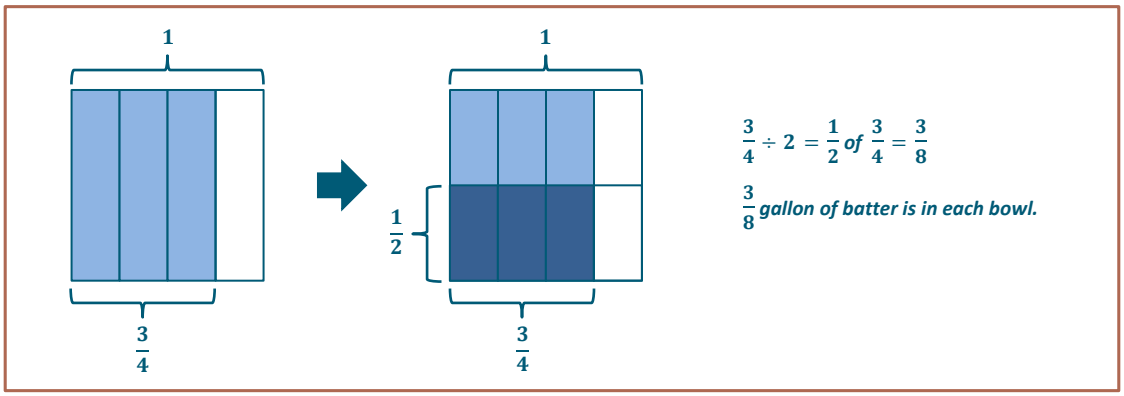
Example 1

$\frac{3}{4}$ gallon of batter is poured equally into 2 bowls. How many gallons of batter are in each bowl?

- Since the whole is being put into 2 equal parts or bowls, this is the partitive model of division. What division expression can we write to match this story?
  - $\frac{3}{4} \div 2$
- We can also think of this problem as asking, "3 fourths is 2 groups of what?" Let's take a look at how to solve this using an area model. What is the whole? How much batter is being shared?
  - *Three-fourths gallon*
- The story tells us that the batter is poured equally into 2 bowls. How can we show this in the model?
  - *We can draw another line, horizontally, to partition the model into 2 equal units.*
- Now our model shows 3 fourths being partitioned into 2 equal parts. Could we also say that our model shows $\frac{1}{2}$ of 3 fourths?
  - *Yes, it's just like in the Opening Exercise where dividing by 2 and multiplying by $\frac{1}{2}$ are the same.*
- Let's label our model to show that. What is half of 3 fourths?
  - *Half of 3 fourths is 3 eighths.*
- Yes, and what is 3 fourths divided by 2?
  - *It's also 3 eighths!*

- We said that we could also think of this problem as, "3 fourths is 2 groups of what?" Is it true that 3 fourths is 2 groups of 3 eighths? Use a repeated addition or multiplication sentence to support your response.
  - *Yes, it's true. 3 eighths plus 3 eighths is 6 eighths, which is equal to 3 fourths. → It's true.*

    $2 \times \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$.



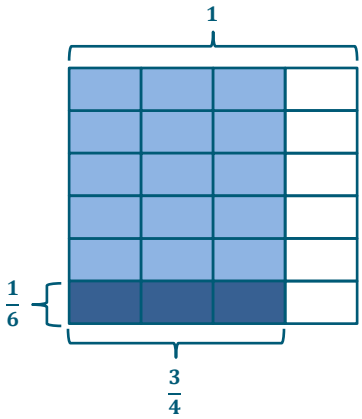
Example 2 (4 minutes)

Example 2

$\frac{3}{4}$ pan of lasagna is shared equally by 6 friends. What fraction of the pan will each friend get?

- Again, this is partitive division problem since we're told that there are 6 *parts*, or that the lasagna is being shared equally among 6 friends. Write a division expression to represent this story problem.
  - $\frac{3}{4} \div 6$
- Remember when solving this problem, we can also think, "3 fourths is 6 groups of what?" Let's draw another area model to solve. Think, what is our whole? Draw a model to show how much lasagna is being shared.
- Now, partition your model again to show how it can be equally shared by 6 friends.
- Remind me; dividing by 6 is the same as multiplying by what?
  - *Multiplying by* $\frac{1}{6}$
- Look at our model. Explain to your neighbor how it shows division by 6 and also shows multiplication by $\frac{1}{6}$.
- Now write a multiplication expression that is equal to 3 fourths divided by 6.
  - $\frac{1}{6}$ *of* $\frac{3}{4}$ *or* $\frac{1}{6} \times \frac{3}{4}$

$\frac{3}{4} \div 6 = \frac{1}{6}$ *of* $\frac{3}{4} = \frac{3}{24} = \frac{1}{8}$



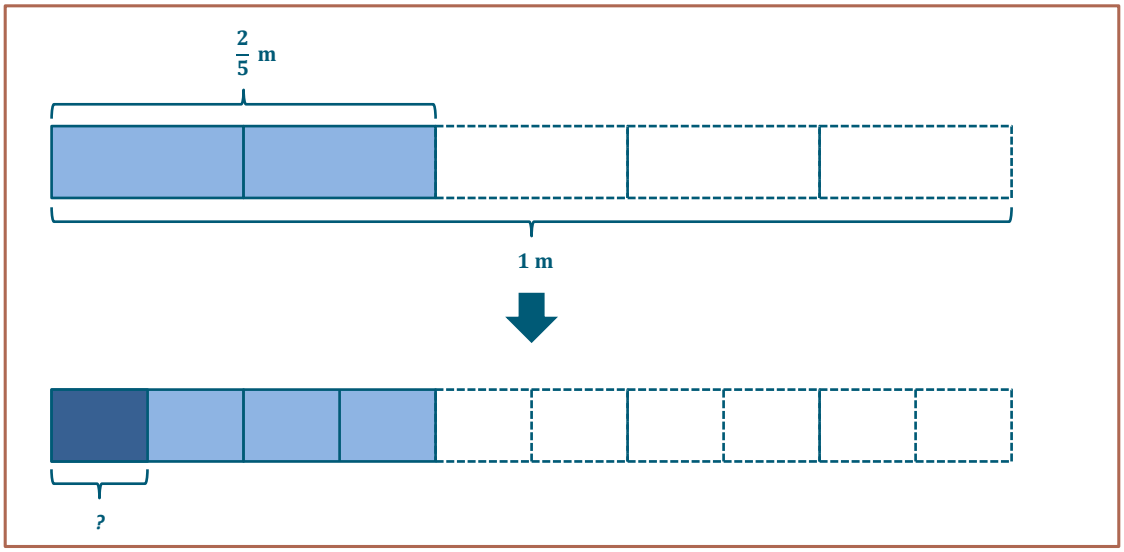
- According to our model, what fraction of the pan will each friend get?
  - 3 *twenty-fourths of the pan*
- Express 3 twenty-fourths in its simplest form.
  - 1 *eighth*

Example 3 (5 minutes)

Example 3

A rope of length $\frac{2}{5}$ m is cut into 4 equal cords. What is the length of each cord?

- Again, this is a partitive division problem since we're told that there are 4 *parts* or that the rope is cut equally into 4 cords. Write a division expression to represent this story problem.
  - $\frac{2}{5} \div 4$
- For this example, let's draw a tape diagram and a number line.
- The length of rope is cut into 4 equal cords. How can we show that in our models?
  - *Partition the 2 shaded units into 4 equal parts.*



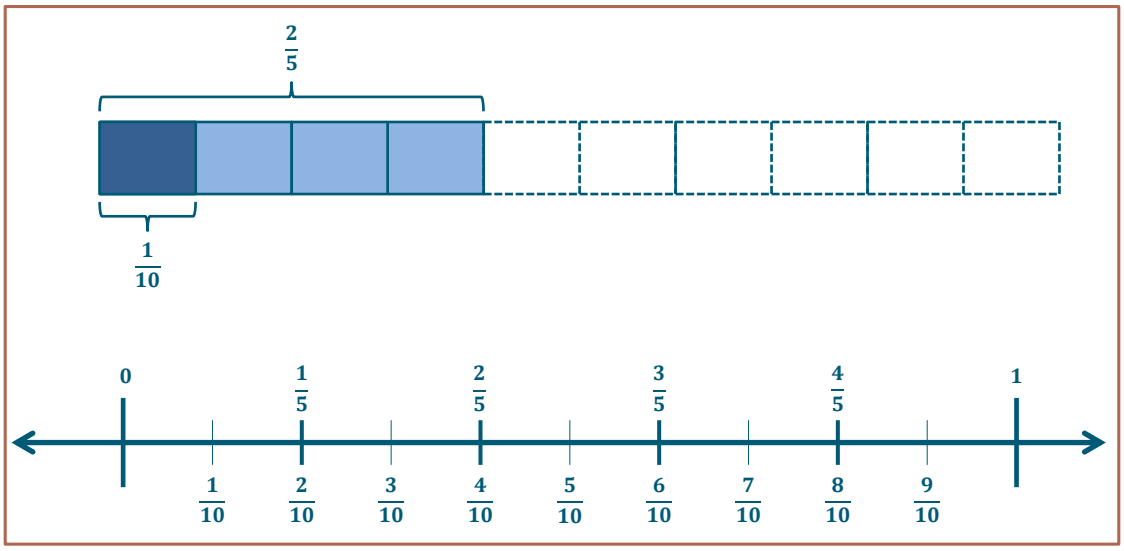
- We originally drew fifths and shaded 2 of them to represent the length of the rope, but now what fractional unit does our model show?
  - *Tenths*
- Answer the question. What is the length of each cord?
  - *Each cord is* $\frac{1}{10}$ *meter long.*

$\frac{2}{5} \div 4 = \frac{1}{4}$ *of* $\frac{2}{5} = \frac{2}{20} = \frac{1}{10}$

Each cord is $\frac{1}{10}$ m.

$\frac{2}{5} \div 4 =$ *4 tenths* $\div 4 =$ *1 tenth* $= \frac{1}{10}$

- Take a look at the division sentence I've written using unit language. How do our models support this thought? How does the use of unit language support your understanding of this division problem?
- We can also construct a number line to support our solution. When the number line is drawn beneath the tape diagram, we can see the similarities between the two models.



Encourage students to study the models and discuss the similarities between dividing by 4 and multiplying by $\frac{1}{4}$. Students should articulate that both operations yield the same result.

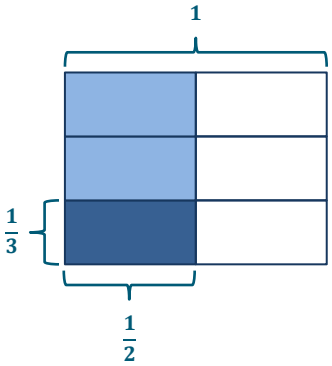
Exercises 1–6 (14 minutes)

Students work in pairs to solve the following questions.

Exercises 1–6

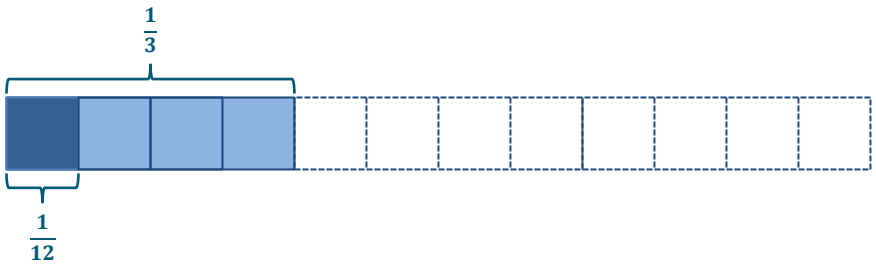
Fill in the blanks to complete the equation. Then, find the quotient and draw a model to support your solution.

1. $\frac{1}{2} \div 3 = \frac{\square}{3} \times \frac{1}{2}$



$\frac{1}{2} \div 3 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

2. $\frac{1}{3} \div 4 = \frac{1}{4} \times \frac{1}{\square}$



$\frac{1}{3} \div 4 = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

$\frac{1}{3} \div 4 = 4$ *twelfths* $\div 4 = 1$ *twelfth* $= \frac{1}{12}$

Find the value of each of the following.

3. $\frac{1}{4} \div 5$

$\frac{1}{4} \div 5 = \frac{1}{5}$ *of* $\frac{1}{4} = \frac{1}{20}$

4. $\frac{3}{4} \div 5$

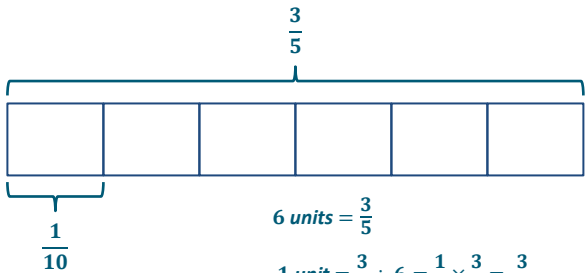
$\frac{3}{4} \div 5 = \frac{1}{5}$ *of* $\frac{3}{4} = \frac{3}{20}$

5. $\frac{1}{5} \div 4$

$\frac{1}{5} \div 4 = \frac{1}{4}$ *of* $\frac{1}{5} = \frac{1}{20}$

Solve. Draw a model to support your solution.

6. $\frac{3}{5}$ **pt. of juice is poured equally into 6 glasses. How much juice is in each glass?**



6 units $= \frac{3}{5}$

1 unit $= \frac{3}{5} \div 6 = \frac{1}{6} \times \frac{3}{5} = \frac{3}{30}$

1 unit $= \frac{1}{10}$

Each glass has $\frac{1}{10}$ *pint of juice.*

Closing (5 minutes)

- Discuss the relationship between dividing by 4 and multiplying by $\frac{1}{4}$.
  - *Students should articulate that division by 4 and multiplication by $\frac{1}{4}$ are the same thing.*

Exit Ticket (5 minutes)

Name _____ Date _____

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Exit Ticket

Write an equivalent multiplication expression. Then, find the quotient in its simplest form. Use a model to support your response.

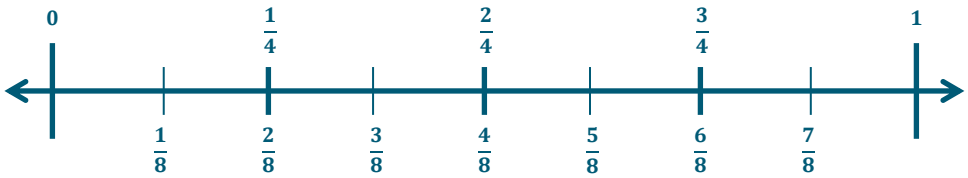
1. $\frac{1}{4} \div 2$

2. $\frac{2}{3} \div 6$

Exit Ticket Sample Solutions

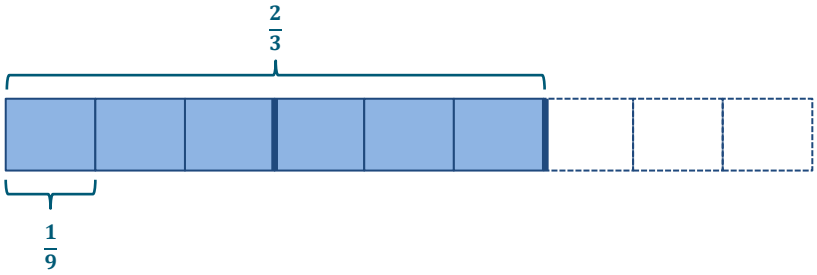
Write an equivalent multiplication expression. Then, find the quotient in its simplest form. Use a model to support your response.

1. $\frac{1}{4} \div 2$



$\frac{1}{4} \div 2 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

2. $\frac{2}{3} \div 6$



$\frac{2}{3} \div 6 = \frac{1}{6} \times \frac{2}{3} = \frac{2}{18} = \frac{1}{9}$

Problem Set Solutions

Find the value of each of the following in its simplest form.

1.

a. $\frac{1}{3} \div 4 = \frac{1}{12}$ b. $\frac{2}{5} \div 4 = \frac{1}{10}$ c. $\frac{4}{7} \div 4 = \frac{1}{7}$

2.

a. $\frac{2}{5} \div 3 = \frac{2}{15}$ b. $\frac{5}{6} \div 5 = \frac{1}{6}$ c. $\frac{5}{8} \div 10 = \frac{1}{16}$

3.

a. $\frac{6}{7} \div 3 = \frac{2}{7}$ b. $\frac{10}{8} \div 5 = \frac{1}{4}$ c. $\frac{20}{6} \div 2 = \frac{5}{3}$

4. **4 loads of stone weigh $\frac{2}{3}$ ton. Find the weight of 1 load of stone.**

$\frac{2}{3} \div 4 = \frac{1}{6}$

Each load of stone weighs $\frac{1}{6}$ ton.

5. **What is the width of a rectangle with an area of $\frac{5}{8}$ in^2 and a length of 10 inches?**

$\frac{5}{8} \div 10 = \frac{1}{16}$

The width of the rectangle is $\frac{1}{16}$ in.

6. **Lenox ironed $\frac{1}{4}$ of the shirts over the weekend. She plans to split the remainder of the work equally over the next 5 evenings.**

a. **What fraction of the shirts will Lenox iron each day after school?**

$\frac{3}{4} \div 5 = \frac{3}{20}$

Lenox will iron $\frac{3}{20}$ of the shirts each day after school.

b. **If Lenox has 40 shirts, how many shirts will she need to iron on Thursday and Friday?**

$\frac{3}{20}(40) = 6$

Lenox will need to iron 6 shirts on Thursday and 6 shirts on Friday.

Therefore, Lenox will need to iron 12 shirts on Thursday and Friday.

7. Bo paid bills with $\frac{1}{2}$ of his paycheck and put $\frac{1}{5}$ of the remainder in savings. The rest of his paycheck he divided equally among the college accounts of his 3 children.

   a. What fraction of his paycheck went into each child's account?

   $\frac{1}{2}\left(\frac{1}{5}\right) = \frac{1}{10}$. *This means* $\frac{1}{10}$ *of Bo's paycheck goes into savings.* $\frac{1}{2} + \frac{1}{10} = \frac{5}{10} + \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$.

   This means that $\frac{3}{5}$ *of Bo's paycheck goes to bills and savings, which leaves* $\frac{2}{5}$ *of his paycheck for college accounts.*

   $\frac{2}{5} \div 3 = \frac{2}{15}$

   Therefore, Bo put $\frac{2}{15}$ *of his paycheck in each child's account.*

   b. If Bo deposited $\$400$ in each child's account, how much money was in Bo's original paycheck?

   $\$400$ is $\frac{2}{15}$ group of what size?

   2 *units* $= \$400$

   1 *unit* $= \$400 \div 2 = \$200$

   15 *units* $= 15 \times \$200 = \$3,000$ *Bo was originally paid* $\$3,000$.