

Solving Inequalities



Use What You Know

You've learned how to replace values for variables into equations to see if they are true. Take a look at this problem.

A grocery store is giving a reusable bag to every person who donates more than \$5 to charity. Let x equal the amount that a person donates. Use words and symbols to solve all of the following problems about this situation.

Use the math you already know to solve these problems.

a. Ella donates \$5.50. Will she get a bag? Explain how you know.

b. Daniel donates \$5. Will he get a bag? Explain how you know.

c. Courtney donates \$1.25. Will she get a bag? Explain how you know.

d. Name 2 other amounts people could contribute and get a bag.

e. To get a bag, are the amounts greater than or less than \$5?

f. Use the symbols $>$ or $<$ to show x is greater than \$5.

g. Explain how you know when any person should receive a free bag.

Find Out More

A sentence such as $x > 5$ is called an **inequality**. On the previous page you identified values for x that make $x > 5$ true, like 5.50 or 10, but there are too many possible values for x to be counted. Unlike the equations we've seen that have one solution, an inequality may have infinitely many solutions.

Here are some symbols and words to describe an inequality:

$>$	$\geq$	$<$	$\leq$
• is more than • is greater than • is above	• is greater than or equal to • is at least • is no less than • minimum	• is less than • is below	• is less than or equal to • is at most • is no more than • is no greater than • maximum
x is greater than 5 $x > 5$	x is at least 5 $x \geq 5$	x is below 5 $x < 5$	x is at most 5 $x \leq 5$

Reflect

1 Explain the difference between an equation, like $x = 5$, and an inequality, like $x \geq 5$.

Learn About **Writing and Solving an Inequality**

Read the problem below. Then explore how to write and solve an inequality.

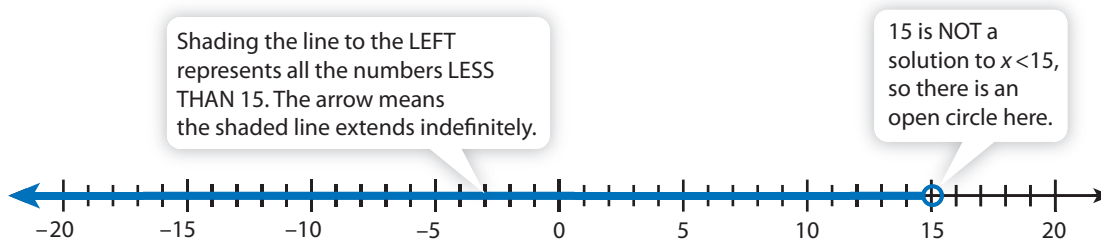
When the temperature drops below 15°C in a building, the furnace turns on. At what temperatures will the heater turn on? Write an inequality to represent this situation, and graph the solution on a number line.

Model It You can use words and symbols to represent the problem.

Let x equal the temperature in a building. When x is **less than** 15, the heater turns on.

$$x < 15$$

Graph It You can graph the inequality on a number line to show all solutions.



A closed or shaded circle on the above graph would show the solutions for $x \leq 15$.

Check It You can substitute values for x to check your solution.

The graph shows that -5°C is a possible solution. You can replace x with -5 to check.

$$x < 15$$

$$-5 < 15 \quad \text{TRUE}$$

The graph shows that 16°C is not a solution. You can replace x with 16 to check.

$$x < 15$$

$$16 < 15 \quad \text{FALSE}$$

Connect It Now you will solve the problem using the model and graph.

2 What words in the problem help you know which inequality symbol to use?

3 Look at the graph. Why is there an open circle at 15? What does it mean?

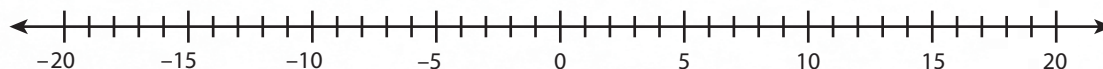
4 Explain the meaning of the shaded line and arrow to the left of 15.

5 Would the heater turn on if the temperature was 2.5°C ? Explain how you know.

6 Suppose the heater turns on when the temperature is at 15°C or below.

Write an inequality for this new situation. _____

Graph the solution on the number line below.



7 Explain when to use an open or closed circle when graphing an inequality.

Try It Use what you just learned about graphing inequalities to solve this problem.

8 This graph shows the solution of what inequality? _____



Learn About **More Inequalities**

Read the problem below. Then explore how to write and solve an inequality.

Cooper spent at least \$25 at a music concert. What are some possible amounts of money Cooper could have spent? Write an inequality to represent the amount of money Cooper spent, and graph the solution on a number line.

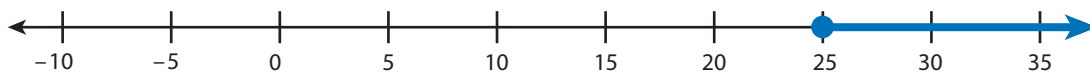
► **Model It** You can use words and symbols to represent this problem.

The amount Cooper spent, x , is **greater than or equal to** \$25.

$$x \geq 25$$

Another way to think about this problem is that \$25 is **less than or equal to** the amount Cooper spent, so $25 \leq x$.

► **Model It** You can graph the inequality on a number line to solve the problem.



► **Check It** You can substitute values for x to check your solution.

The graph shows that \$25.50 is a possible solution. You can replace x with 25.5 to check.

$$x \geq 25$$

$$25.5 \geq 25 \quad \text{TRUE}$$

The graph shows that \$30 is a possible solution. You can replace x with 30 to check.

$$25 \leq x$$

$$25 \leq 30 \quad \text{TRUE}$$

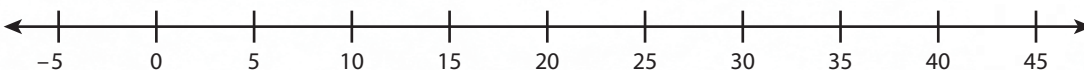
Connect It Now you will solve the problem using the model and graph.

- 9 Explain how $x \geq 25$ and $25 \leq x$ both represent the amount Cooper spent at the concert.

- 10 Look at the graph of $x \geq 25$. What does the closed circle at 25 mean?

- 11 What is a possible amount of money Cooper could have spent? Check your solution.

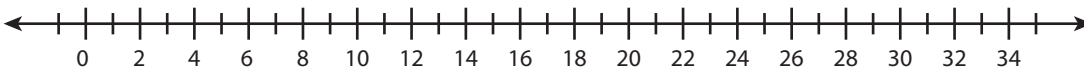
- 12 How would the inequality be different if the problem said that Cooper spent more than \$25? Write and graph the inequality on the number line below.



- 13 Explain how a shaded line means that fractions, decimals, and numbers not labeled on a number line can be part of the solution to an inequality.

Try It Use what you just learned about writing and graphing inequalities to solve this problem.

- 14 Some sources say that Earth's atmosphere does not exist above $30\frac{1}{2}$ km. Write an inequality to represent where the atmosphere exists. _____
Now graph the solution on the number line below.



Practice **Solving Inequalities**

Study the example below. Then solve problems 15–17.

ExampleWhich of the following values is not a solution of $x - 4 < 15$?0, 19, 18.9, $15\frac{1}{4}$

Look at how you can show your work using a model.

$0 - 4 < 15$

$-4 < 15$ TRUE

$18.9 - 4 < 15$

$14.9 < 15$ TRUE

$19 - 4 < 15$

$15 < 15$ FALSE

$15\frac{1}{4} - 4 < 15$

$11\frac{1}{4} < 15$ TRUE

Solution 19 is not a solution of $x - 4 < 15$.The student replaced the variable, x , with a value to see if the inequality was true.**Pair/Share**

How could you justify your answer by graphing the solution on a number line?

15 Which of the following values is a solution of $12.6 \leq 3x$?

4, 4.2, 3, 10

Show your work.

When you replace the x with each value, does that value make the inequality true or false?**Pair/Share**

How is this problem similar to and different from the student model?

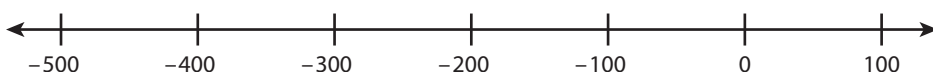
Solution _____

- 16 Algae cannot survive at depths greater than 300 meters below sea level. The inequality that represents depths at which algae cannot survive is $x < -300$. Graph the solution on a number line.



Will your graph have an open or a closed circle?

Solution



Pair/Share

How could you use substitution to check your graph?

- 17 Which inequality represents the situation: Hailey has at most \$500 in her bank account?

- A $x > 500$
- B $x \geq 500$
- C $x < 500$
- D $x \leq 500$



Does “at most 500” include 500 as a possible solution?

Tessa chose **C** as the correct answer. How did she get that answer?



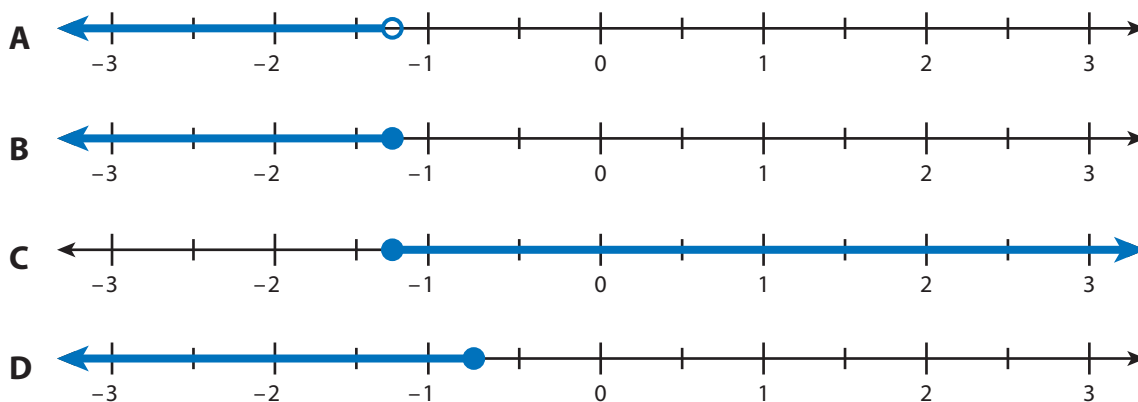
Pair/Share

What are some words that help you know when to use $<$ and when to use $\leq$?

Practice Solving Inequalities

Solve the problems.

1 Which is a correct graph of $x \leq -1\frac{1}{4}$?



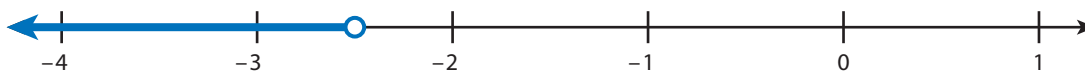
2 Mark cannot read traffic signs that are more than 50 meters away. Which inequality represents distances at which Mark cannot read signs?

- A** $x > 50$
- B** $x \geq 50$
- C** $x < 50$
- D** $x \leq 50$

3 Look at the inequality $5x + 1 \geq 35$. Choose *True* or *False* for each statement.

- | | | |
|---|-------------------------------|--------------------------------|
| a. 100 is not a solution because it is much greater than 35. | ☐ True | ☐ False |
| b. 7.04 cannot be a solution because only whole numbers can be solutions. | ☐ True | ☐ False |
| c. $6\frac{4}{5}$ is not a solution because the expression on the left side cannot equal 35. | ☐ True | ☐ False |
| d. There are an infinite number of solutions. | ☐ True | ☐ False |
| e. 34 is not a solution because it is less than 35. | ☐ True | ☐ False |

4 Which inequality does this graph represent?



Insert the appropriate inequality symbol to complete the statement.

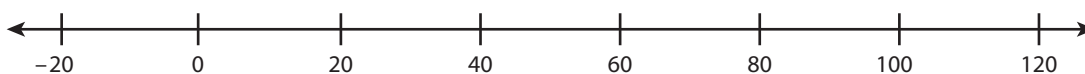
< > ≤ ≥

x -2.5

5 Eric has practiced more than 40 hours with his band.

Write an inequality to express this situation. _____

On the graph below, graph Eric's situation.



6 Judah's family wants to ride the bumper boats at a water park.

- There are 4 children (c) and 2 adults (a) in Judah's family.
- The boats are safe if $40c + 120a \leq 500$.

Can Judah's family ride the bumper boats safely? Explain.

Show your work.

Answer _____

Self Check Go back and see what you can check off on the Self Check on page 143.