Lesson 15 & Introduction

Numerical Expressions with Exponents

Use What You Know

In previous lessons, you learned about powers of 10. Take a look at this problem.

A sports store orders a box of table tennis balls. The dimensions of the box are 10 inches by 10 inches by 10 inches. Inside the box, there are 6 layers of table tennis balls. Each layer is made up of 6 rows and 6 columns of table tennis balls.

How many table tennis balls are in the box?

Use the math you know to solve the proble:

	·
a.	Write a multiplication expression to represent the volume of the box in cubic inches.
b.	Write the expression you wrote for the volume of the box using exponents.
c.	Write a multiplication expression to represent the number of table tennis balls in one
	layer of the box.
d.	Write a multiplication expression to represent the number of table tennis balls in all
	6 layers of the box
e.	Write an expression using exponents to represent the total number of table tennis
	balls in the box
f.	Explain how you could find the number of table tennis balls in the box.

> Find Out More

You already know that multiplication is a shorter way to write repeated addition. You use exponents to write repeated multiplication in a shorter way.

	Repeated Addition	Repeated Multiplication	
Problem	5 + 5 + 5 + 5	5 • 5 • 5	
Shorter way to write it	5 • 4	5 ⁴	
How to read it	5 times 4	5 raised to the fourth power	

Numbers raised to the second or third power are often read in specific ways.

5² is read "five squared."

5³ is read "five cubed."

Expressions written with exponents are called **exponential expressions**.

The number being multiplied by itself is called the **base**.

The **exponent** shows how many times to use the base as a factor.

5 is the base.

3 is the exponent.

The base of an exponential expression can be any kind of number.

$$7^2 = 7 \cdot 7$$

$$\left(\frac{1}{3}\right)^5 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$$
 $(0.2)^3 = 0.2 \cdot 0.2 \cdot 0.2$

$$(0.2)^3 = 0.2 \cdot 0.2 \cdot 0.2$$

When you multiply measurements with units, the units are also multiplied. In the example on the previous page, to find the volume of the box, you multiply 10 inches • 10 inches • 10 inches.

10 inches • 10 inches • 10 inches =
$$10^3$$
 inches³
= $10 \cdot 10 \cdot 10$ inches³
= $1,000$ in.³

This is why area is measured in square units and why volume is measured in cubic units.

Reflect

1 Is 2^7 equal to $2 \cdot 7$? Explain.

Learn About Writing and Evaluating Expressions with Exponents

Read the problem below. Then explore different ways to understand how to write and evaluate expressions with exponents.

Julie's brother says that instead of paying her the \$40 he owes her, he will give her \$2 today and double the amount she has each day for 6 days. Should Julie accept her brother's offer? Why or why not?

Model It Use multiplication to represent the problem.

Find the amount of money Julie has each day and then **double** it to find the amount she has the next day.

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
2	2 • 2 = 4	4 • 2 = 8	8 • 2 = 16	16 • 2 = 32	32 • 2 = 64

Model It Represent the problem with repeated multiplication.

Each day, Julie gets two times the amount of money she got the previous day.

Day	Amount of Money Julie Has from Her Brother
1	2
2	2 • 2
3	2 • 2 • 2
4	2•2•2 •2
5	2 • 2 • 2 • 2
6	2 • 2 • 2 • 2 • 2

Look at the first <i>Model It</i> on the previous page. Why do you double 8 to find the
amount of money Julie will have on Day 4?
Look at the second <i>Model It</i> . How can you tell that the amount from the previous day is being doubled?
In the second <i>Model It</i> , which expression represents the amount of money Julie will have on Day 6?
How can you write an expression for the amount of money Julie has on Day 6 using exponents?
How much money will Julie have on Day 6? Should she take her brother's offer? Explain
Matt thinks that if Julie starts with \$2, she will have \$12 after 6 days. Explain what
Matt is doing wrong.
y It Use what you just learned to solve these problems. Show your work on a parate sheet of paper.
Maria is buying new carpet for her bedroom. Her bedroom is in the shape of a squar and the length of each side is 12 feet. Write and simplify an exponential expression t
find how much carpet she needs.

Learn About Evaluating Expressions with Exponents

Read the problem below. Then explore different ways to understand evaluating expressions with exponents.

Michael, Morgan, and Keegan are in the same class. The teacher asks the students to write and simplify the expression "6 plus 4 cubed." Here are the expressions each person wrote.

Michael	Morgan	Keegan
$(6 + 4)^3$	$6^3 + 4^3$	$6 + 4^3$

Which students will get the correct answer? Which, if any, of the expressions are equivalent? Explain.

Model It Use the order of operations to simplify Michael's expression.

First add 6 and 4.

$$6 + 4 = 10$$

Then raise 10 to the third power. $10^3 = 10 \cdot 10 \cdot 10$

$$10^3 = 10 \cdot 10 \cdot 10$$

Model It Use the order of operations to simplify Morgan's expression.

$$6^3 = 6 \cdot 6 \cdot 6$$

$$4^3 = 4 \cdot 4 \cdot 4$$

Then add
$$6^3$$
 and 4^3 .

Model It Use the order of operations to simplify Keegan's expression.

First find 4³.

$$4^3=4\cdot 4\cdot 4$$

Then add 6.

Connect It Now you will solve the problem from the previous page using the order of operations with exponents.

- 10 Look at Michael's expression. Why do you add 6 + 4 first?
- 11 Look at Morgan's expression. Why do you simplify each exponential expression first?
- 12 The teacher says the expression equals 70. Which student wrote the expression correctly?
- How might the teacher have said the expression if she wanted students to write Michael's expression?
- 14 Corwin tries to evaluate $4^2 \cdot 5 + 6$, as shown below.

$$4^2 \cdot 5 + 6$$

$$= 4^2 \cdot 11$$

$$= 176$$

Explain why his answer is incorrect. __

Try It Use what you just learned to solve these problems. Show your work on a separate sheet of paper.

- Evaluate: $\frac{6^2}{3}$
- 16 Evaluate: 3 + 5² 4 _____

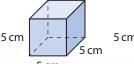


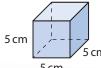
Practice Evaluating Expressions with Exponents

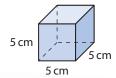
Study the example below. Then solve problems 17–19.

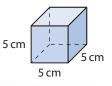
Example

Write and evaluate an expression using exponents to find the total volume of the 4 cubes shown below.









The volume of 4 cubes is 4 times the volume of 1 cube.

Look at how you could show your work using labeled equations.

Volume of 1 cube: $= \ell wh$

 $= 5 \text{ cm} \cdot 5 \text{ cm} \cdot 5 \text{ cm}$

 $= 5^3 \text{ cm}^3$

Volume of 4 cubes: $V = 4 \cdot 5 \cdot 5 \cdot 5 \text{ cm}^3$

 $V = 4 \cdot 125 \text{ cm}^3$

 $V = 500 \text{ cm}^3$

Solution $V = 4 \cdot 5^3$; 500 cm³



Pair/Share

Is $(4 \cdot 5)^3$ equal to $4 \cdot 5^3$? Why or why not?

17 In an old story, a man puts 2 grains of rice on the first space of a chess board. He puts 4 grains on the second space, 8 grains on the third space, and so on. Write and evaluate an expression with exponents to find how many grains of rice the man puts on the tenth space.

Show your work.



What is the relationship between the number of the space and the power that shows the number of grains on that space?



Pair/Share

What do you think is meant by the phrase "exponential growth"?

Solution

18 Moira does one-half of a homework assignment on Monday. On Tuesday, Wednesday, and Thursday, she does one-half of the homework she has left over from the day before. Write and evaluate an expression using exponents to find the fraction of the original assignment Moira will have to do on Friday in order to completely finish her homework.



Try using a picture to model the assignment.

Show your work.



Pair/Share

If Moira kept on doing one-half of the remaining assignment instead of finishing it on Friday, would she ever finish her homework?

Solution _

- 19 Evaluate: $\frac{(6+2)^3-12}{5}$

 - **B** $90\frac{2}{5}$
 - **C** 100
 - **D** $509\frac{3}{5}$

Jerry chose **A** as the correct answer. How did he get that answer?



How does the order of operations apply to evaluating this expression?



Pair/Share

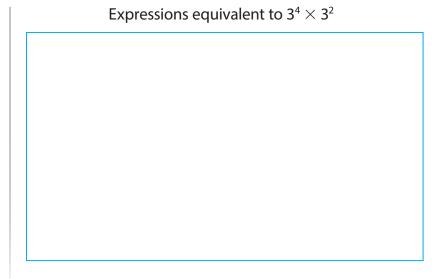
Why does everyone need to follow the order of operations when evaluating expressions?

Solve the problems.

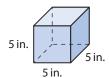
- 1 Which **best** describes the first step in evaluating the expression $4 \cdot \frac{53+1}{(8-5)^3}$?
 - A simplify 5³
 - B multiply 4 53
 - **C** subtract 8-5
 - **D** add 53 + 1
- 2 In a science-fiction story, a spaceship travels 3 times faster each minute than it traveled during the minute before. If the ship travels at 3 km/hr during its first minute of flight, which expression shows the ship's speed in kilometers per hour during the 15th minute?
 - A 3 15
 - **B** 15^3
 - $C 3^{15}$
 - **D** 3 3
- 3 From the choices on the left, write inside the box each expression equivalent to $3^4 \times 3^2$.

 $3^2 \times 3^4$ 12×6 $3^3 \times 3^3$ 38

 81×9



4 Look at the cube below.



Which statement correctly calculates the volume of the cube? Choose True or False for each statement.

- **a.** Simplify the expression 5 + 5 + 5. True **False**
- **b.** Simplify the expression $5 \times 5 \times 5$. True **False**
- **c.** Simplify the expression 3×5 . **False** True
- **d.** Simplify the expression 5×25 . **False** True
- **e.** Simplify the expression 5^3 . **False** True
- 5 Write each of the numbers 1, 4, 9, 16, and 25 as a base raised to the second power. Explain why these numbers sometimes are called "perfect squares."

6 Trey knows that 3+4=4+3 and $3 \cdot 4=4 \cdot 3$. He says that $3^4=4^3$. Is Trey correct? Explain your answer.

Self Check Go back and see what you can check off on the Self Check on page 143.