Force Applied at an Angle

Unit: Forces

Skills:

• calculate forces applied at different angles using trigonometry

Notes:

Many physics problems involve a force applied at an angle, but the object can only move horizontally.

In this situation, only the horizontal (parallel) component of the applied force actually causes the object to move. If the total force is \( F \), then the horizontal component (in the \( x \)-direction) is given by:

\[
F_x = F \cos \theta
\]

This the only component of the force involved in the motion. Only forces and components of forces in the direction of motion are involved in the calculations.

Example: Force at angle (see picture)

Resolving force components:
Force \( F = 18 \text{ N} \) at angle 27° where
\[
F = (F_x, F_y) = F_x i + F_y j
\]
\[
F_x = \cos 27°(18 \text{ N}) = 17.8 \text{ N}
\]
\[
F_y = \sin 27°(18 \text{ N})
\]

Example: Worker pushing a crate

Suppose the worker in the picture pushes on a crate on the floor of a warehouse with a force \( F \) of 200 N, at an angle \( \theta \) of 21°. The force in the direction of
motion (horizontally) would be:

\[ F \cos \theta = 200 \cos (21^\circ) = (200)(0.9336) = 186.72 \text{ N} \]

In other words, the worker has to apply 200 N of force at an angle of 21° to get a force of 186.72 N horizontally.

**Example:** Puppy being pulled by a leash shown in Picture

![Diagram of puppy being pulled by a leash](image)

\[ \sin 40^\circ = \frac{F_{\text{vert}}}{60 \text{ N}} \quad \cos 40^\circ = \frac{F_{\text{horiz}}}{60 \text{ N}} \]

\[ F_{\text{vert}} = 60 \text{ N} \times \sin 40^\circ \quad F_{\text{horiz}} = 60 \text{ N} \times \cos 40^\circ \]

\[ F_{\text{vert}} = 38.6 \text{ N} \quad F_{\text{horiz}} = 45.9 \text{ N} \]

**Ramp (inclined plane) Problems:**

Notice that in the picture (diagram) that the direction of the normal force does not always directly oppose gravity. For example, if a block is resting on a (frictionless) ramp, the weight of the block is \( F_g \), in the direction of gravity. However, the normal force is perpendicular to the ramp, not to gravity. Remember that the Normal force is a reaction due to contact and is always perpendicular to that point.

If we were to add the vectors representing the two forces, we would see that the resultant — the net force — acts down the ramp as shown in the diagram.
From geometry, we can determine that the angle of the ramp \( \theta \) is the same as the angle between gravity and the normal force.

Then, from trigonometry, we can calculate that the component of gravity parallel to the ramp (which equals the net force down the ramp) is:

\[
F_{\text{net}} = F_g \sin \theta
\]

The component of gravity perpendicular to the ramp is \( F_g \cos \theta \), which means the normal force is:

\[
F_N = -F_g \cos \theta
\]

**Example Problem:**

A block with a mass of 2.5 kg sits on a frictionless ramp with an angle of inclination of 35°. How fast does the block accelerate down the ramp?

**Answer:** The weight of the block is

\[
F_g = ma = (2.5)(9.8) = 24.5 \text{ N}
\]

However, the component of the force of gravity in the direction that the block slides down the ramp is

\[
F_g \sin \theta = 24.5 \sin 35^\circ = (24.5)(0.574) = 14.1 \text{ N}
\]

Now that we know the net force (in the direction of motion), we can apply Newton’s Second Law:

\[
F = ma \quad \text{Solving for the acceleration } a \text{ yields:}
\]

\[
a = \frac{F}{m}
\]

\[
a = \frac{14.1}{2.5} = 5.64 \text{ m/sec}^2
\]