Newton's Law of Gravitation Alternate Treatment

**Newton's Law of Gravitation** (Gravity) states that ‘Every particle’ attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the distance between them given by \( r \) and sometimes \( d \). This is a vector force with direction and magnitude and is given by:

\[
F_{\text{gravity}} = \frac{G \ m_1 \ m_2}{r^2}
\]

where \( F_{\text{gravity}} \) represents the force due to gravity between two objects

\( G \) called ‘Big G’ is the Universal Gravitational Constant and is equal to \( 6.6726 \times 10^{-11} \text{N-m}^2/\text{kg}^2 \)

\( m_1 \) is the mass of object 1

\( m_2 \) is the mass of object 2

\( r \) represents the distance between the two masses \( m_1 \) and \( m_2 \) centers

**Newton's Law of Gravity Formula**

**Gravitational Force:** This is the force of attraction between two bodies separated by a distance \( r \) with mass 1 and mass 2.

\[
F = \frac{Gm_1m_2}{r^2}
\]

**Mass of Object 1:** This is the formula from Newton’s Law solved for the mass \( m_1 \) of object 1

\[
m_1 = \frac{Fr^2}{Gm_2}
\]

**Mass of Object 2:** This is the formula from Newton’s Law solved for the mass \( m_2 \) of object 2

\[
m_2 = \frac{Fr^2}{Gm_1}
\]

**Distance Between the Objects:** This is the formula from Newton’s Law solved for the radius or distance of separation between object 1 and object 2

\[
r = \sqrt{\frac{Gm_1m_2}{F}}
\]
Newton's Law of Gravity Examples:

Example 1:
Determine the force of gravitational attraction between the earth $5.98 \times 10^{24}$ kg and a 70 kg boy who is standing at sea level, a distance of $6.38 \times 10^6$ m from earth's center. $m_1 = 5.98 \times 10^{24}$ kg, $m_2 = 70$ kg, $r = 6.38 \times 10^6$ m, $G = 6.6726 \times 10^{-11}$ N-m$^2$/kg$^2$

Solution:
Step 1: Substitute the values in the below Gravitational Force formula:

$F = \frac{G m_1 m_2}{r^2}$

$= \frac{6.6726 \times 10^{-11} \times 5.98 \times 10^{24} \times 70}{(6.38 \times 10^6)^2}$

$= \frac{6.6726 \times 5.98 \times 70 \times 10^{24-11}}{40.744 \times 10^{12}}$

$= \frac{279.15 \times 10^3}{40.744 \times 10^{12}} = 685.54$ N

Example 2:
Find the mass of one object if the magnitude of the gravitational force acting on each particle is $2 \times 10^{-8}$, the one mass is 25 kg and the objects are 1.2 meters apart $F = 2 \times 10^{-8}$, $m_2 = 25$ kg, $r = 1.2$ m, $G = 6.6726 \times 10^{-11}$ N-m$^2$/kg$^2$.

Solution:
Step 1: Substitute the values in the below Mass formula:

$m_1 = \frac{F r^2}{G m_2}$

$= \frac{2 \times 10^{-8} \times (1.2)^2}{6.6726 \times 10^{-11} \times 25}$

$= \frac{2 \times (1.2)^2}{6.6726 \times 10^{-3} \times 25}$

$= \frac{2 \times 1.44}{0.166815} = \frac{2.88}{0.166815} = 17.27$ kg

Example 3:
Determine the force of gravitational attraction between the earth ($m = 5.98 \times 10^{24}$ kg) and a 70-kg physics student if the student is standing at sea level, a distance of $6.38 \times 10^6$ m from earth's center.
Solution: The solution of the problem involves substituting known values of \( G \) (6.673 \( \times \) \( 10^{-11} \) N m\(^2\)/kg\(^2\)), \( m_1 \) (5.98 \( \times \) \( 10^{24} \) kg), \( m_2 \) (70 kg) and \( d \) (6.38 \( \times \) \( 10^{6} \) m) into the universal gravitation equation and solving for \( F_{grav} \). The solution is as follows:

\[
F_{grav} = \frac{(6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(70 \text{ kg})}{(6.38 \times 10^{6} \text{ m})^2}
\]

\[
F_{grav} = 686 \text{ N}
\]

Example 4:
Determine the force of gravitational attraction between the earth (\( m = 5.98 \times 10^{24} \text{ kg} \)) and a 70-kg physics student if the student is in an airplane at 40000 feet above earth's surface. This would place the student a distance of 6.39 \( \times \) \( 10^{6} \) m from earth's center.

Solution: The solution of the problem involves substituting known values of \( G \) (6.673 \( \times \) \( 10^{-11} \) N m\(^2\)/kg\(^2\)), \( m_1 \) (5.98 \( \times \) \( 10^{24} \) kg), \( m_2 \) (70 kg) and \( d \) (6.39 \( \times \) \( 10^{6} \) m) into the universal gravitation equation and solving for \( F_{grav} \). The solution is as follows:

\[
F_{grav} = \frac{(6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(70 \text{ kg})}{(6.39 \times 10^{6} \text{ m})^2}
\]

\[
F_{grav} = 684 \text{ N}
\]

Example 5: What is the force exerted by Big Ben on the Empire State building? Assume that Big Ben has a mass of \( 10^8 \) kilograms and the Empire State building10^9 kilograms. The distance between them is about 5000 kilometers and Big Ben is due east of the Empire State building.

Solution: The direction of the force clearly attracts the Empire State towards Big Ben. So the direction is a vector pointing due east from New York. The magnitude is given by Newton's Law:

\[
F = GM_{ben}M_{empire} / r^2 = 6.67 \times 10^{-11} \times 10^8 \times 10^8 / (5000000)^2 = 2.67 \times 10^{-7} \text{ N}
\]

Clearly, the gravitational force is negligibly small, even for quite large objects.
Example 6: What is the gravitational force that the sun exerts on the earth? The earth on the sun? In what direction do these act? \((M_e = 5.98 \times 10^{24} \text{ and } M_s = 1.99 \times 10^{30} \text{ and the earth-sun distance is } 150 \times 10^9 \text{ meters})\).

Solution: First, consider the directions. The force acts along the direction such that it attracts each body radially along a line towards their common center of mass. For most practical purposes, this means a line connecting the center of the sun to the center of the earth. The magnitude of both forces is the same, as we would expect from Newton's Third Law, and they act in opposite directions, both attracting each other mutually. The magnitude is given by:

\[
F = \frac{GM_e M_B}{r^2} = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 1.99 \times 10^{30}}{(150 \times 10^9)^2} = 3.53 \times 10^{22} \text{ N}
\]

Example 7 Advanced: If Mercury, Venus and the sun are aligned in a right triangle, as shown, then calculate the vector sum of the forces on Venus due to both Mercury and the Sun. What is the direction and magnitude of the resulting force? (Sun-Venus distance \(r_v = 108 \times 10^9 \text{ meters}, \) Sun-Mercury distance \(r_m = 57.6 \times 10^9 \text{ meters}, \) mass of Sun \(M_s = 1.99 \times 10^{30} \text{ kilograms}, \) mass of Mercury \(M_m = 3.3 \times 10^{23} \text{ kilograms}, \) mass of Venus \(M_v = 4.87 \times 10^{24} \text{ kilograms})\).

Solution: The magnitude of the force on Venus due to the sun is given by:

\[
F = \frac{GM_B M_v}{(r_v)^2} = 5.54 \times 10^{22} \text{ N}
\]

The distance between Mercury and Venus is given by:

\[
r_{mv} = [(r_v)^2 + (r_m)^2]^{1/2} = 1.08 \times 10^{11} \text{ meters}.
\]

The magnitude of the force from Mercury, then, is:

\[
F_m = \frac{GM_m M_v}{(r_{mv})^2} = 9.19 \times 10^{15} \text{ N}
\]

The directions of these forces are along the lines connecting the planets. If the size of the forces was comparable, we would have to resolve each vector force into components perpendicular and parallel to some direction, and then sum these components in order to
find the final direction of the force. In this case however, the force due to the sun is more than a million times greater than the force due to Mercury, and so the net force is very well approximated by the magnitude and direction of the force due to the sun.

Example 8 Advanced: It is possible to simulate "weightless" conditions by flying a plane in an arc such that the centripetal acceleration exactly cancels the acceleration due to gravity, see Vomit Comet. Such a plane was used by NASA when training astronauts. What would be the required speed at the top of an arc of radius 1000 metres?

Solution: We require an acceleration that exactly cancels that due to gravity -- that is, exactly 9.8 m/sec$^2$. Centripetal acceleration is given by $a_c = v^2 / r$. We have been given

\[ r = 1000 \text{ meters}, \, \text{so} \, \, v = (ra_c)^{1/2} = (9.8 \times 1000)^{1/2} \sim 99 \text{ m/s}. \]