Projectile Motion

Unit: Kinematics, Motion in 2 Dimensions  
Skills: solve problems involving motion in two dimensions

Motion in 2D Projectiles

Projectile: an object that is propelled (thrown, shot, etc.) horizontally and also falls due to gravity.

Assuming we can neglect friction and air resistance (which is usually the case in first-year physics problems), we make two important assumptions:

• All projectiles have a constant horizontal velocity (in the x direction).
• All projectiles have a constant downward acceleration of $g = 9.8 \text{ m/s}^2$ (in the $-y$ direction), due to gravity.

The consequences of these assumptions are:

• The time that the object takes to fall is determined by its movement only in the y-direction.
• The horizontal distance that the object travels is determined by the time (calculated above) and its velocity in the x-direction.

Therefore, the general strategy for solving problems are:

1. Solve the y-direction problem first to get the time.
2. Use the time from the y-direction to solve the x-direction problem.

The x- and y-Components of the Velocity Vector

If the object is thrown/launched at an angle, you will need to use trigonometry to separate the velocity vector into its horizontal ($x$) and vertical ($y$) components:

Thus:

• horizontal velocity $= v_x = v \cos \theta$
• initial vertical velocity $= v_{0,y} = v \sin \theta$

Note that the vertical component of the velocity, $v_y$, is constantly changing because of acceleration due to gravity.
An example of Projectile Motion would be represented in the plot below which is a plot of the height of the object’s motion vs its horizontal motion, an upside down parabola:
Example problems:

**Problem 1:** A ball is thrown horizontally at a velocity of \( 5 \text{ m/s} \) from a height of 1.5 m. How far does the ball travel (horizontally)?

**Solution:** Because the ball is thrown horizontally, it has no initial velocity in the y-direction.
In the y-direction, the ball simply falls a distance of 1.5 m. The time it takes to do this is given by:

\[
s = \frac{1}{2} a t^2
\]

\[
t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1.5}{10}} = 0.55 \text{ s}
\]

Now that we know the time, we apply it to the x-direction. The ball travels at a velocity of 5 m/s for 0.55 s
\[
s = v_{\text{average}} t = (5)(0.55) = 2.75 \text{ m}
\]

The motion of the ball looks like this:

![Graph showing the motion of the ball](image)

**Note:** Initial \( y_0 = 1.5 \text{ m} \) and \( v_{\text{ox}} = \text{horizontal initial velocity which is in the x-direction and } v_{\text{oy}} = \text{vertical y direction} = 0 \)

**Problem 2:** A ball is thrown upward at an angle of 30° from a height of 1 m with a velocity of 18 m/s. How far does the ball travel?

**Solution:** The initial vertical component of the velocity is:

\[
v \sin \theta = 18 \sin 30^\circ = 18(0.5) = 9 \text{ m/s}.
\]
We need to split the time into two parts: the time the ball is traveling upwards, and the time it is traveling downwards.

When the ball reaches its maximum height, its upward velocity is zero. This means:

\[ v_i + at = 0 \]
\[ 9 + (-10)t = 0 \]
\[ 10t = 9 \]
\[ t = 9/10 = 0.9 \text{ s} \]

At this point, the ball has reached a height of:

\[ s = s_i + v_i T + \frac{1}{2} a t^2 \]
\[ s = 1 + (9)(0.9) + (1/2)(10)(0.9)^2 = 12.2 \text{ m} \]

Now the ball falls from its maximum height of 12.25 m to the ground. The time this takes is:

\[ s = \frac{1}{2}at^2 \]
\[ t = \sqrt{2s/a} = 1.56 \text{ s.} \]

Thus the total elapsed time is 0.9 + 1.6 = 2.5 s.

The horizontal distance is the horizontal velocity times the time. The horizontal component of the velocity is:

\[ v \cos \theta = 18 \cos 30^\circ = 18(0.866) = 15.6 \text{ m/s} \]
\[ s = v \cos \theta \cdot t = v_{\text{average}} t = (15.6)(2.5) = 39 \text{ m} \]

Note: these problems are using \( a = -g \) where \( g = -9.81 \text{ m/s}^2 \) rounded up to 10 \text{ m/s}^2. Remember that \( g \) is the acceleration due to gravity.