Acceleration

Unit: Kinematics (Motion)

Knowledge/Understanding Goals:

• what acceleration means
• what positive vs. negative acceleration means

Skills:

• calculate position, velocity and acceleration for problems that involve movement in one direction

Notes:

acceleration: a change in velocity over a period of time.

If an object is “speeding up,” we say it has positive acceleration.
If an object is “slowing down,” we say it has negative acceleration.

Variables Used to Describe Acceleration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Quantity</th>
<th>Common Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>acceleration</td>
<td>$\frac{m}{s^2}$, $\frac{cm}{s^2}$</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
<td>$\frac{m}{s^2}$, $\frac{cm}{s^2}$</td>
</tr>
</tbody>
</table>

By convention, physicists use the variable $g$ to mean acceleration due to gravity, and $a$ to mean acceleration caused by something other than gravity.
Remember that the way the units are arranged must always be the way the formula is arranged. Notice that the units for acceleration are a distance divided by a time squared.

Recall that velocity was distance divided by time, so this means acceleration is velocity (distance/time) divided by time.

This means the formula for acceleration must be the change in velocity divided by the change in time:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{\Delta v}{t}$$

and the change in velocity is acceleration times the time:

$$\Delta v = \bar{a}t$$

If the object started from rest ($v_0 = 0$), then,

$$v_f = \Delta v = \bar{a}t$$

Note that when an object's velocity is changing, $v$ is not the same as $\bar{v}$. (This is a common mistake that first-year physics students make.)

If the object was already moving at an initial velocity $v_o$, then the acceleration would add to the initial velocity. The final velocity ($v_f$) would therefore be:

$$v_f = v_o + \bar{a}t$$

[In a physics class that does not use calculus, we will limit ourselves to problems in which acceleration is constant. This means that from this point forward, we will use $a$ instead of $\bar{a}$ for acceleration.]

Because $a = \frac{\Delta v}{\Delta t}$, acceleration is the slope of a graph of velocity vs. time:
In the graph below, between 0 s and 4 s the object is accelerating at a rate of (positive) 2.5m/s².

Between 4 s and 6 s the object is moving at a constant velocity of 10 m/s, so the acceleration is zero.

![Velocity vs. Time Graph]

To show the relationship between \( v \) and \( \bar{v} \) we can combine the formula for average velocity with the formula for acceleration in order to get a formula for the position of an object that is accelerating.

\[
d = \bar{v}t
\]

\[
v_f = at
\]

However, the problem is that \( v_f \) in the formula \( v_f = at \) is the velocity at the end, which is not the same as the average velocity \( \bar{v} \). If the object starts at rest (not moving) and accelerates at a constant rate, the average velocity means the average of zero and the final velocity. This equals exactly half of the final velocity. This means in the case of an object starting from rest:

\[
\bar{v} = \frac{1}{2}v_f
\]

Combining all of these, we get:

\[
d = \bar{v}t = \frac{1}{2}vt = \frac{1}{2}(at) t = \frac{1}{2}at^2
\]

Note that the area under a velocity/time graph equals the total distance traveled:
If an object was moving before it started to accelerate, it had an initial velocity, or a velocity at time $t = 0$. We will represent this initial velocity as either $v_i$ or $v_o$. ($v_o$ is usually pronounced either "v zero" or "v naught"). When this happens, our formula becomes:

$$d = v_o t + \frac{1}{2} at^2$$

**distance the object would travel at its initial velocity**

**additional distance the object will travel because it is accelerating**

Finally, if the object started out at a position (location) other than zero, the object's displacement is $x - x_o$. This gives us the generic equation for the position of an object that is accelerating:

$$d = x - x_o = v_o t + \frac{1}{2} at^2$$
This equation can be combined with the equation for velocity to give the following equation, which relates initial and final velocity and distance:

\[ v_f^2 - v_o^2 = 2ad \]

**Free fall:** when an falling object is accelerating because of gravity.

On earth, the average acceleration due to gravity is approximately 9.807 m/s\(^2\) (which we will usually round to 9.8 m/s\(^2\), or sometimes just 10 m/s\(^2\)). Any time gravity is involved (assuming the problem takes place on Earth), you can assume that \(a = g = 9.8 \text{ m/s}^2\).

### Problem Solving Strategy

Many motion problems in physics involve gravity. This means the acceleration is 9.8 m/s\(^2\) downwards. These problems usually fall into one of two categories:

1. If you have an object in free fall, the problem will probably give you either the distance it fell (\(d\)), or the time it fell (\(t\)). Use the formula

\[ d = \frac{1}{2} gt^2 \]

   to calculate whichever one you don’t know.

2. If an object is thrown upward, it will decelerate at a rate of -9.8 m/s\(^2\) (assuming “up” is the positive direction) until it stops moving (velocity = zero). Then it will fall. This means you need to split the problem into two parts:

   a. When the object is moving upward, the initial velocity, \(v_0\), is given and \(v_f\) (at the top) is zero. From these, you can figure out the distance it traveled, which gives you the maximum height.

   b. Once you know the maximum height, you know the distance to the ground and you can use

\[ d = \frac{1}{2} gt^2 \]

   (this time with an acceleration of +9.8 m/s\(^2\)) to find the time.
Sample Problems:

Q: If a cat jumps off a 1.8 m tall refrigerator, how long does it take to hit the ground?

A: The cat is starting from rest, and gravity is accelerating the cat at a rate of 9.8 m/s². The formula is \( d = \frac{1}{2} gt^2 \), and we need to solve for \( t \). Rearranging, we get:

\[
\begin{align*}
t^2 &= \frac{d}{\frac{1}{2}g} = \frac{2d}{g} \\
2(1.8) &= \frac{3.6}{9.8} = 0.367 = t^2 \\
t &= \sqrt{0.367} = 0.61 s
\end{align*}
\]
Q: An apple falls from a tree branch at a height of 5 m and lands on Isaac Newton’s head. (Assume Isaac Newton was 1.8 m tall.)

How fast was the apple traveling at the time of impact?

A: There are two ways to solve this problem.

One is to use \( v = at \). However, we don’t know \( v \) or \( t \). Therefore, we need to:

1. Find \( t \).
2. Use \( t \) to find \( v \).

We also know that \( d = \frac{1}{2}gt^2 \) and we know that the total distance the apple fell was 5 m − 1.8 m = 3.2 m. Rearranging, we get:

\[
 t^2 = \frac{d}{\frac{1}{2}g} = \frac{2d}{g}
\]

\[
 \frac{2(3.2)}{9.8} = \frac{6.4}{9.8} = 0.653 = t^2
\]

\[
 t = \sqrt{0.653} = 0.808 \text{ s}
\]

\[
 v = at = 9.8(0.808) = 7.9 \text{ m/s}
\]

The other approach is to use the formula \( v_f^2 - v_0^2 = 2ad \). The distance the apple fell is \( d = 5 \text{ m} - 1.8 \text{ m} = 3.2 \text{ m} \), and the initial velocity is \( v_0 = 0 \). This gives:

\[
 v_f^2 = v_0^2 = 2ad
\]

\[
 v_f^2 = 2ad
\]

\[
 v_f^2 = 2gd = (2)(9.8)(3.2) = 62.7
\]

\[
 v_f = 7.9 \text{ m/s}
\]