Warm Up

1. **Vocabulary** The simplest form of \( \frac{3}{6} \) is \( \frac{1}{2} \). When you ________ a fraction, you are writing it in simplest form.

Simplify.

\[
\begin{align*}
2. & \quad 2.3 - 3.6 ÷ 4 - 1.7 \\
3. & \quad \frac{-0.4 + 1.3 \cdot 4}{0.5 - 5.1 ÷ 3}
\end{align*}
\]

Solve.

\[
\begin{align*}
4. & \quad 8x = 112 \\
5. & \quad 2.5y = 62.5
\end{align*}
\]

New Concepts

A **ratio** is a comparison of two quantities using division.

Examples: 2 boys to 3 girls  \( 2 \) to \( 3 \), \( \frac{2}{3} \)

A **rate** is a ratio that compares quantities measured in different units.

Examples: 5 feet per 30 seconds \( 15 \) apples for \$6.00 \( 25,000 \) hits per month

A **unit rate** is a rate whose denominator is 1. A unit price is the cost per unit.

Examples: 55 miles per hour \$2.50 per box

**Example 1** Finding Unit Rates

Which is the better buy: 5 cans of tuna for \$4.95 or 6 cans for \$5.75?

**SOLUTION**

\[
\begin{align*}
\text{total cost} & \quad \frac{\text{number of cans}}{5} = \frac{\$4.95}{5} = \$0.99 \text{ per can} \\
\text{total cost} & \quad \frac{\text{number of cans}}{6} = \frac{\$5.75}{6} = 0.96 \text{ per can}
\end{align*}
\]

\(\$0.96 < \$0.99\)

Compare the unit prices.

6 cans for \$5.75 is the better buy.

**Example 2** Converting Rates

a. A bus driver drives at 30 miles per hour. What is the rate of the bus in miles per minute?

**SOLUTION**

\[
\begin{align*}
\frac{30 \text{ miles}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} & \quad \text{Multiply by a conversion factor.} \\
\frac{\frac{1}{2} \text{ miles}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} & \quad \text{Cancel like units of measure.}
\end{align*}
\]

If the driver drives 1 mi/2 min, then he drives \( \frac{1}{2} \) mi/min.

**Hint**

When working with rates, usually you will see the words **per**, **for**, and **each** indicated by a forward slash (/).

1 mile per 2 minutes
1 mi/2 min

**Online Connection**

www.SaxonMathResources.com
b. An engineer opens a valve that drains 60 gallons of water per minute from a tank. How many quarts were drained per second?

**SOLUTION**

\[
\begin{align*}
\text{60 gallons} & \quad \frac{? \text{ quarts}}{1 \text{ minute}} \\
\text{1 minute} & \quad \frac{4 \text{ quarts}}{1 \text{ gallon}} = \frac{240 \text{ quarts}}{1 \text{ minute}} \\
\text{Use a conversion factor to change gallons to quarts. Then simplify.}
\end{align*}
\]

\[
\begin{align*}
\frac{240 \text{ quarts}}{1 \text{ minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} & = \frac{240 \text{ quarts}}{60 \text{ seconds}} \\
\text{Use a conversion factor to change minutes to seconds. Then simplify.}
\end{align*}
\]

The tank drains at a rate of 4 quarts per second.

A **proportion** is an equation that shows two ratios are equal. The equation \(\frac{3}{5} = \frac{9}{15}\) is a proportion.

<table>
<thead>
<tr>
<th>Cross Products Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>If (\frac{a}{b} = \frac{c}{d}) and (b \neq 0) and (d \neq 0), then (ad = bc).</td>
</tr>
<tr>
<td>In (\frac{a}{b} = \frac{c}{d}), (ad) and (bc) are the <strong>cross products</strong>.</td>
</tr>
</tbody>
</table>

**Example 3**  
Solving Proportions Using Cross Products

Solve each proportion.

\[\text{a. } \frac{x}{15} = \frac{2}{3}\]

**SOLUTION**

\[
\begin{align*}
\frac{x}{15} & = \frac{2}{3} \\
3 \cdot x & = 15 \cdot 2 \\
3x & = 30 \\
x & = 10
\end{align*}
\]

\[\text{b. } \frac{x - 1}{12} = \frac{1}{6}\]

**SOLUTION**

\[
\begin{align*}
\frac{x - 1}{12} & = \frac{1}{6} \\
6(x - 1) & = 12(1) \\
6x - 6 & = 12 \\
6x & = 18 \\
x & = 3
\end{align*}
\]
Proportions are used to represent many real-world situations that require finding a missing value. Using the cross products is an efficient method for solving the proportions.

**Example 4 Solving Multi-Step Proportions**

**a.** The ratio of boys to girls in a math class is 3:2. The class has 25 students in all. How many boys and how many girls are in the class?

**SOLUTION**

The ratio of boys to girls is 3 to 2. There are 3 boys in each group of 5 students.

\[
\frac{\text{number of boys}}{\text{total in group}} = \frac{3}{5}
\]

Write a ratio.

Write and solve a proportion. Let \( b \) represent the number of boys in the class.

\[
\frac{3}{5} = \frac{b}{25}
\]

There are \( b \) boys to 25 students.

\[
3 \cdot 25 = 5 \cdot b
\]

Write the cross products.

\[
5b = 75
\]

Simplify.

\[
b = 15
\]

Solve.

There are 15 boys in the class. So, there are 25 − 15 or 10 girls in the class.

**b.** On the map, Albany to Jamestown measures 12.6 centimeters, Jamestown to Springfield measures 9 centimeters, and Springfield to Albany measures 4.75 centimeters. What is the actual distance from Albany to Jamestown to Springfield and back to Albany?

![Map diagram]

**SOLUTION**

\[
12.6 + 9 + 4.75 = 26.35 \text{ cm}
\]

Find the total distance on the map.

\[
\frac{26.35 \text{ cm}}{x \text{ km}} = \frac{1 \text{ cm}}{25 \text{ km}}
\]

Set up a proportion using the map scale.

\[
26.35 \cdot 25 = 1 \cdot x
\]

Write the cross products.

\[
658.75 = x
\]

Solve.

The actual distance is 658.75 kilometers.

Proportions are frequently used to solve problems involving variations of the distance formula \( d = rt \).

\[
\text{rate} = \frac{\text{distance}}{\text{time}} \quad \text{or} \quad \text{time} = \frac{\text{distance}}{\text{rate}}
\]
Example 5  Application: Trucking

Mr. Jackson drove a truck 300 miles in 6 hours. If he drives at a constant speed, how long will it take him to drive 450 miles?

SOLUTION

Let \( x \) represent the number of hours it will take to drive 450 miles.

\[
\frac{\text{rate}}{\text{distance}} = \frac{\text{distance}}{\text{time}} \quad \text{Set up a proportion.}
\]

\[
\frac{300 \text{ miles}}{6 \text{ hours}} = \frac{450 \text{ miles}}{x \text{ hours}} \quad \text{Write cross products.}
\]

\[
300x = 2700 \quad \text{Solve.}
\]

\[
x = 9
\]

Mr. Jackson will drive 450 miles in 9 hours.

Lesson Practice

a. Which is the better buy: 8 boxes for $4.96 or 5 boxes for $3.25?

b. A chemist raised the temperature of a liquid 45°F in 1 minute. What is this amount in degrees Fahrenheit per second?

c. Jamie typed 20 pages of a document in 2 hours. How many pages did she type in 1 minute?

Solve each proportion.

d. \( \frac{c}{7} = \frac{3}{21} \)

e. \( \frac{5}{n + 2} = \frac{10}{16} \)

f. The ratio of blue chips to red chips in a bag is 5:7. The bag has 60 chips in all. How many blue chips and how many red chips are in the bag?

g. A map shows a 5.5-inch distance between Orange City and Newtown, and a 3.75-inch distance from Newtown to Westville. The scale on the map is 1 inch:100 miles. What is the actual distance, if you drive from Orange City via Newtown to Westville?

h. If Jeff walks 4 miles in 48 minutes, how far can he walk in 72 minutes?

Practice

Simplify.

1. \( 7 - 4 - 5 + 12 - 2 - | -2 | \)

2. \( -6 \cdot 3 + | -3(-4 + 2^3) | \)
Solve.

3. \(-0.05n + 1.8 = 1.74\)

4. \(-y - 8 + 6y = -9 + 5y + 2\)

**Multiple Choice** What is the value of \(x\) when \(2x - 4.5 = \frac{1}{2}(x + 3)\)?

A 9  
B 2.4  
C 2  
D 4

6. Solve for \(y\): \(4 + 2x + 2y - 3 = 5\).

7. Simplify \(4k(2c - a + 3m)\).

Evaluate.

8. \(3x^2 + 2y\) when \(x = -2\) and \(y = 5\)

9. \(2(a^2 - b)^2 + 3a^3b\) when \(a = -3\) and \(b = 2\)

*10. If 10 boxes of cereal sell for $42.50, what is the unit price?*

11. Geometry In the diagram, \(\triangle ABC\) and \(\triangle XYZ\) are similar triangles. What is the value of \(n\)?

12. Predict An estimate of the number of tagged foxes to the total number of foxes in a forest is 3:13. A forest warden recorded 21 tagged foxes. About how many foxes are in the forest?

13. Multi-Step A skydiver falls at a rate given by \(s = 1.05\sqrt{w}\), where \(s\) is the falling speed in feet per second and \(w\) is the weight of the skydiver with gear in pounds. What is the approximate falling speed of a 170-pound man with 40 pounds of gear? (Round to the nearest whole number.)

14. Copy and complete the table for \(y = x^2 + 2\). Then use the table to graph the equation.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-3</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>11</td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*15. Shopping* Glenn buys 4 computers for $2800. How much will 6 computers cost?
16. **Probability** A spinner is divided into 5 sections labeled $A$ through $E$. The bar graph shows the results of 50 spins. What is the experimental probability that the next spin will land on $A$ or $D$?

![Spin Results Bar Graph]

17. **Mileage** The table shows how far a car travels for each gallon of gasoline it uses.

<table>
<thead>
<tr>
<th>Number of Gallons, $x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Traveled, $f(x)$</td>
<td>33</td>
<td>66</td>
<td>99</td>
<td>132</td>
</tr>
</tbody>
</table>

a. Use the table to make a graph.
b. Write a rule for the function.
c. How far will the car travel using 10 gallons of gasoline?

18. **Multi-Step** Students are paid $d$ dollars per hour for gardening and $g$ dollars per hour for babysitting and housework. Sally babysat for 6 hours and mowed lawns for 3 hours. Her brother weeded gardens for 5 hours and mopped floors for 1 hour.

a. Write an expression to represent the amount each student earned.
b. Write expressions for the total amount they earned together.
c. If they are paid $5 an hour for gardening and $4 an hour for babysitting and housework, how much did they earn together?

19. **Carpentry** A carpenter has propped a board up against a wall. The wall, board, and ground form a right triangle. What will be the measures of the three angles?

![Carpentry Diagram]

20. Give the domain and range of the relation.

$$\{(12, 2); (11, 10); (18, 0); (19, 1); (13, 4)\}$$

21. Use a graphing calculator to make a table of values for $f(x) = x^2 - 1$. Graph the function and determine the domain and range.
22. Write Why is there no conclusive value for $x$ in the equation \( \frac{3}{2}x + 5 = 2x - \frac{1}{2}x + 5 \)? Explain.

23. Multi-Step Amy works in a kitchen appliance store. She earns $65 daily and a commission worth $10 less than one-fifth of the value of each appliance she sells. Let $m$ equal the value of an appliance.
   a. Write an expression for Amy’s daily salary if she sells one appliance every day.
   b. Write an expression for Amy’s daily salary if she sells $n$ appliances every day.
   c. If you know the value of $m$ and $n$, which part of the expression would you solve first?

24. Multiple-Choice Which expression is equivalent to $4(x^2 - 4) + 3z^3(4z^7)$?
   A. $4x^2 - 16 + 3z^{10}$
   B. $4x^2 - 16 + 12z^{10}$
   C. $4x^2 - 16 + 12z^{21}$
   D. $4x - 16 + 12z^{10}$

25. Error Analysis Two students solved $\frac{3z}{2} - \frac{4q}{3} = 6$ for $z$. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3z}{2} - \frac{4q}{3} = 6$</td>
<td>$\frac{3z}{2} - \frac{4q}{3} = 6$</td>
</tr>
<tr>
<td>$3(3z) - 2(4q) = 6$</td>
<td>$3(3z) - 2(4q) = 6(6)$</td>
</tr>
<tr>
<td>$9z - 8q = 6$</td>
<td>$9z - 8q = 36$</td>
</tr>
<tr>
<td>$9z = 6 + 8q$</td>
<td>$9z = 36 + 8q$</td>
</tr>
<tr>
<td>$z = \frac{6 + 8q}{9}$</td>
<td>$z = 4 + \frac{8}{9}q$</td>
</tr>
</tbody>
</table>

26. Finance Compound interest is calculated using the formula $A = P\left(1 + \frac{r}{100}\right)^t$, where $P = \text{principal (amount originally deposited)}$, $r = \text{the interest rate}$, and $t = \text{time in years}$. If $1500$ is deposited into a account and compounded annually at $5.5\%$, how much money will be in the account after $10$ years?

27. Verify Suppose that $\frac{3}{4} = \frac{x}{100}$. Show that $x$ equals $75$.

28. Verify Is $m = \frac{3}{2}$ a solution for $\frac{1}{3}m + \frac{5}{6} = \frac{11}{18}$? Explain. If false, provide a correct solution and check.


30. The Great Pyramid The base of the great Pyramid of Giza is almost a perfect square. The perimeter of the base measures about $916$ meters. What is the length of each side of the pyramid’s base?
Simplifying and Evaluating Expressions with Integer and Zero Exponents

Warm Up

1. **Vocabulary** The _________ of a power is the number used as a factor.

   Simplify.

2. \(3^4\)
3. \(x^5 \cdot x^6\)
4. \(26 + (-18)\)
5. \(-34 - 19\)

New Concepts

Algebraic expressions may contain exponents that are positive, negative, or zero. The relationship between the different exponents can be understood by looking at successive powers of a positive integer greater than 1. Look at the powers of 2.

<table>
<thead>
<tr>
<th>Power of 2</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^1)</td>
<td>2</td>
</tr>
<tr>
<td>(2^2)</td>
<td>4</td>
</tr>
<tr>
<td>(2^3)</td>
<td>8</td>
</tr>
<tr>
<td>(2^4)</td>
<td>16</td>
</tr>
</tbody>
</table>

In the left column, each entry is found by decreasing the exponent in the previous entry by one. In the right column, each entry is found by halving the previous entry, or by dividing it by 2. Use this pattern to find the next three powers.

<table>
<thead>
<tr>
<th>Power of 2</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^5)</td>
<td>(\frac{1}{2}) or (\frac{1}{2^5})</td>
</tr>
<tr>
<td>(2^6)</td>
<td>(\frac{1}{4}) or (\frac{1}{2^6})</td>
</tr>
</tbody>
</table>

The pattern illustrates the properties for negative and zero exponents.

<table>
<thead>
<tr>
<th>Negative and Zero Exponent Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Negative Exponent Property</strong></td>
</tr>
<tr>
<td>For every nonzero number (x), (x^{-n} = \frac{1}{x^n}) and (x^n = \frac{1}{x^{-n}}).</td>
</tr>
<tr>
<td><strong>Zero Exponent Property</strong></td>
</tr>
<tr>
<td>For every nonzero number (x), (x^0 = 1).</td>
</tr>
</tbody>
</table>

An algebraic expression is not considered simplified if it contains negative or zero exponents. The Product Property of Exponents applies to negative and zero exponents.
Example 1  Simplifying Expressions with Negative Exponents

Simplify each expression. All variables represent nonzero real numbers.

a. \( x^{-3} \)

SOLUTION

\[ x^{-3} = \frac{1}{x^3} \]
Write with only positive exponents.

b. \( \frac{y^{-4}}{x^2} \)

SOLUTION

\[ \frac{y^{-4}}{x^2} = \frac{1}{x^2 y^4} \]
Write with only positive exponents.

c. \( \frac{1}{w^{-4}} \)

SOLUTION

\[ \frac{1}{w^{-4}} = w^4 \]
Write with only positive exponents.

Example 2  Evaluating Expressions with Negative and Zero Exponents

Evaluate each expression for \( a = -2 \) and \( b = -3 \).

a. \( a^2 b^0 \)

SOLUTION

\[ a^2 b^0 = a^2 \cdot 1 \]
Simplify using the Zero Exponent Property.

\[ = a^2 \]
Multiplicative Identity

\[ = (-2)^2 \]
Substitute \(-2\) for \( a \).

\[ = 4 \]
Simplify.

b. \( 3b^{-3} \cdot b \)

SOLUTION

\[ 3b^{-3} \cdot b = 3b^{-2} \]
Product Property of Exponents

\[ = \frac{3}{b^2} \]
Simplify using the Negative Exponent Property.

\[ = \frac{3}{(-3)^2} \]
Substitute \(-3\) for \( b \). Then simplify.

\[ = \frac{3}{9} = \frac{1}{3} \]
Simplify.
The Quotient Property of Exponents is used when dividing algebraic expressions. This property states that to divide two algebraic expressions with the same base, subtract their exponents.

### Quotient Property of Exponents

If $m$ and $n$ are real numbers and $x \neq 0$, then

$$\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$$

**Example 3** Using the Quotient Property of Exponents

Simplify each expression. All variables represent nonzero real numbers.

**a.** $\frac{x^7}{x^3}$

**SOLUTION**

$$\frac{x^7}{x^3} = x^{7-3} = x^4$$

**b.** $\frac{x^3}{x^{-7}}$

**SOLUTION**

$$\frac{x^3}{x^{-7}} = x^{3-(-7)} = x^{10}$$

**c.** $\frac{x^{-5}y^6z}{z^{-3}y^2x}$

**SOLUTION**

$$\frac{x^{-5}y^6z}{z^{-3}y^2x} = \frac{1}{x^{-5-1}y^{6-2}}z^{1-(-3)} = \frac{x^6y^4z^4}{x^6}$$

Write with only positive exponents.
Example 4  Application: The Intensity of Sound

The intensity of sound can be measured in watts per square meter. The table below lists intensity levels for some common sounds.

<table>
<thead>
<tr>
<th>Intensity of Sound</th>
<th>Watts/Square Meter</th>
<th>Common Sound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10^1 to 10^7</td>
<td>Rocket Liftoff</td>
</tr>
<tr>
<td></td>
<td>10^7 to 10^2</td>
<td>Jet Liftoff</td>
</tr>
<tr>
<td></td>
<td>10^-2 to 10^0</td>
<td>Loud Music</td>
</tr>
<tr>
<td></td>
<td>10^-6 to 10^-4</td>
<td>Vacuum Cleaner</td>
</tr>
<tr>
<td></td>
<td>10^-9 to 10^-6</td>
<td>Regular Speech</td>
</tr>
<tr>
<td></td>
<td>10^-10 to 10^-9</td>
<td>Soft Whisper</td>
</tr>
</tbody>
</table>

How many times more intense is the sound of a rocket liftoff at 10^3 watts per square meter than that of regular speech at 10^-7 watts per square meter?

Express the answer in exponential and standard form.

SOLUTION

\[
\frac{10^3}{10^{-7}} = \frac{10^{3-(-7)}} = 10^{10}
\]

The sound of a rocket liftoff is 10^{10} or 10,000,000,000 times more intense than that of regular speech.

Lesson Practice

Simplify each expression. All variables represent nonzero real numbers.

a. \(x^{-5}\)  

b. \(\frac{p^3}{q^4}\)  

c. \(\frac{1}{d^8}\)

evaluate each expression for \(a = 4\), \(b = 6\), and \(c = 3\).

d. \(a^b c^2\)  

e. \(4d^{-2}\)

Simplify each expression. All variables represent nonzero real numbers.

f. \(\frac{x^{10}}{x^4}\)  

g. \(\frac{x^9}{x^2}\)  

h. \(\frac{xy^{-3}z^5}{y^3x^2z}\)

i. Refer to the table in Example 4. How many times more intense is the sound of a jet liftoff at 10^1 watts per square meter than that of a vacuum cleaner at 10^{-5} watts per square meter? Express the answer in exponential and standard form.
Simplify.

1. \( y^6/y^5 \)
2. \( m^3p^2q^{10}/m^3p^4q^{10} \)

Solve.

3. \( 9x - 2 = 2x + 12 \)

4. \( 3y - y + 2y - 5 = 7 - 2y + 5 \)

5. \( 2y + 3 = 3(y + 7) \)

6. \( 5(r - 1) = 2(r - 4) - 6 \)

7. **Geometry** Express the ratio of the area of the circle to the area of the square.

8. The sum of twice a number and 17 is 55. Find the number.

9. **Error Analysis** Two students solved the proportion \( \frac{3}{8} = \frac{x}{4} \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{8} = \frac{x}{4} )</td>
<td>( \frac{3}{8} = \frac{x}{4} )</td>
</tr>
<tr>
<td>( 3 \cdot x = 8 \cdot 4 )</td>
<td>( 8 \cdot x = 3 \cdot 4 )</td>
</tr>
<tr>
<td>( x = 10 \frac{2}{3} )</td>
<td>( x = 1 \frac{1}{2} )</td>
</tr>
</tbody>
</table>

10. **Health** The circle graph shows the prevalence of all listed types of allergies among people who suffer from allergies. What about the graph may lead someone to an inaccurate conclusion?

11. **Write** Why is it best to combine like terms in an equation, such as \( 3n + 9 - 2n = 6 - 2n + 12 \), before attempting to isolate the variable?

12. **Verify** Is \( n = 9 \) a solution for \( -28 = -4n + 8 \)? Explain. If false, provide a correct solution and check.
13. The table lists the ordered pairs from a relation. Determine whether the relation represents a function. Explain why or why not.

<table>
<thead>
<tr>
<th>Domain (x)</th>
<th>Range (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

14. If there are 60 dozen pencils in 12 cartons, how many are in 1 carton?

15. Multi-Step How many seconds are in 1 day?

16. **Roller Coasters** The table shows the number of roller coasters in several countries. Suppose one student displays the data in a bar graph, and another student makes a circle graph of the data. Compare the information that each type of display shows.

<table>
<thead>
<tr>
<th>Country</th>
<th>Japan</th>
<th>United Kingdom</th>
<th>Germany</th>
<th>France</th>
<th>China</th>
<th>South Korea</th>
<th>Canada</th>
<th>United States</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>240</td>
<td>160</td>
<td>108</td>
<td>65</td>
<td>60</td>
<td>54</td>
<td>51</td>
<td>624</td>
</tr>
</tbody>
</table>

17. If there are 720 pencils in 6 cartons, how many dozen pencils are in 10 cartons?

18. Multi-Step How many centimeters are in 1 kilometer?

*19. (Geography) On a map, Brownsville and Evanstown are 2.5 inches apart. The scale on the map is 1 inch:25 miles. How far apart are the two towns?

20. Copy and complete the table for \( y = |x| + 10 \). Then use the table to graph the equation. Is the graph of the equation a function?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

*21. Multiple Choice Which expression is simplified?

A. \( \frac{6xy^2}{z^0} \)  
B. \( \frac{6x^3y^{-2}}{z} \)  
C. \( \frac{6x^3y^2}{z} \)  
D. \( \frac{6x^3y^2z}{z} \)

*22. (Chemistry) An electron has a mass of \( 10^{-28} \) grams and a proton has a mass of \( 10^{-24} \) grams. How many times greater is the mass of a proton than the mass of an electron?
23. **Multi-Step** A border is being built along two sides of a triangular garden. The third side is next to the house.

![Image of a triangular garden with a border along two sides and the third side next to the house]

a. The second side of the garden is 4 feet longer than first side. Write an expression for the length of the second side.

b. If the total amount of border is 28 feet, how long are the sides of the garden that are not next to the house?

24. **Multi-Step** The temperature of a liquid is 72°F. The first step of a set of instructions requires that a scientist cools the liquid by 15°F. The second step requires that she warms it until it reaches 85°F. By how many degrees will she warm the liquid in the second step?

25. **Analyze** Megan and Molly have an age gap of 6 years. Megan is older. If Molly is 8 years old, then how old is Megan?

26. **Fuel Costs** It cost Rayna $73.25 to fill her truck with gas, not including tax. The gasoline tax is $0.32 per gallon. If the price for gasoline including tax is $3.25 per gallon, how many gallons of gas did she buy?

   a. Write an equation to represent the problem.

   b. How many gallons of gas did she buy?

27. Expand the expression \((5p - 2c)4xy\) by using the Distributive Property.

28. **Solve each proportion.**

   \[ \frac{7}{x} = \frac{1}{0.5} \]

   \[ \frac{1}{x} = \frac{-3}{x + 2} \]

29. **Predict** If Sam rides at the same rate, how long will it take him to ride 80 miles?
**Warm Up**

1. **Vocabulary** A _________ is the set of all possible outcomes of an event.

A number cube is labeled 1–6. Suppose the number cube is rolled once.

2. List all the possible outcomes.

3. What is the probability of rolling a prime number?

Simplify.

4. \( \frac{4}{5} \cdot \frac{15}{22} \)

5. \( \frac{18}{55} \left( \frac{33}{54} \right) \)

**New Concepts**

Events where the outcome of one does not affect the probability of the other are called **independent events**. To find the probability of two independent events, multiply the probabilities of the two events.

Spinning a spinner and flipping a coin are independent events. The result of one does not affect the result of the other. What is the probability of spinning a 3 and landing on heads?

\[
P(3 \text{ and heads}) = P(3) \cdot P(\text{heads})
\]

\[
= \frac{1}{5} \cdot \frac{1}{2}
\]

\[
= \frac{1}{10}
\]

Spinning the spinner twice also creates two independent events. The first spin does not affect the second spin. What is the probability of spinning a 5 and then a 1?

\[
P(5 \text{ and 1}) = P(5) \cdot P(1)
\]

\[
= \frac{1}{5} \cdot \frac{1}{5}
\]

\[
= \frac{1}{25}
\]

With **dependent events**, the outcome of one event does affect the probability of the other event. To find the probability of two dependent events, you multiply the probability of the first event by the probability of the second event, given the results of the first event.
Lesson 33

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Type of Events | Definition | Calculating the Probability
---|---|---
Independent Events | The outcome of the first event does not affect the second event. | \( P(A \text{ and } B) = P(A) \cdot P(B) \)
Dependent Events | The outcome of the first event does affect the second event. | \( P(A \text{ and } B) = P(A) \cdot P(B) \), where \( P(B) \) is calculated under the new conditions.

**Example 1** Identifying Situations Involving Independent and Dependent Events

Identify each set of events as independent or dependent.

a. rolling a 6 on one number cube and a 4 on another number cube

**SOLUTION** These events are independent. Rolling one number cube does not affect the outcome of rolling the other number cube.

b. rolling a 6 on a number cube and then a 4 on the same number cube

**SOLUTION** These events are independent. Both rolls of this number cube have the same possible outcomes, and the result of the first roll does not affect the second roll.

c. drawing a red marble from a bag, keeping it out of the bag, and then drawing a blue marble

**SOLUTION** These events are dependent. By not replacing the first marble, the outcome of the second draw is affected. There are fewer marbles to choose from.

d. drawing a red marble from a bag, putting it back in the bag, and then drawing a blue marble

**SOLUTION** These events are independent. Because the first marble is replaced, the second draw is not affected. It has the same choices as the first.

A tree diagram can help demonstrate the sample space for events.

**Example 2** Using a Tree Diagram

A coin is flipped twice. Make a tree diagram showing all possible outcomes. What is the probability of the coin landing on heads both times?

**SOLUTION**

\[
P(H, H) = \frac{1}{4}
\]

```
<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>HH</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>HT</td>
</tr>
<tr>
<td>T</td>
<td>H</td>
<td>TH</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>TT</td>
</tr>
</tbody>
</table>
```
Example 3  Calculating the Probability of Dependent Events

Natalia has two squares and three circles in a bag.

a. Find the probability of drawing a circle, keeping it, and then drawing another circle without the use of a tree diagram.

**SOLUTION**

For the first draw, the bag has 5 shapes and 3 are circles.

\[ P(\text{1st circle}) = \frac{3}{5} \]

For the second draw, a circle has been removed. There is one less circle and one less shape.

\[ P(\text{2nd circle}) = \frac{2}{4} \]

To find the probability of these two events, multiply their probabilities.

\[ P(\text{1st circle}) \cdot P(\text{2nd circle}) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10} \]

b. Find the probability of drawing a square, keeping it, and then drawing a circle.

**SOLUTION**

For the first draw, the bag has 5 shapes and 2 are squares.

\[ P(\text{square}) = \frac{2}{5} \]

For the second draw, a square has been removed. There is one less shape, but the number of circles is the same.

\[ P(\text{circle}) = \frac{3}{4} \]

To find the probability of these two events, multiply their probabilities.

\[ P(\text{square}) \cdot P(\text{circle}) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} = \frac{3}{10} \]

**Odds** are another way of describing the likelihood of an event. Odds are expressed as a ratio, usually written with a colon. Odds can be calculated for something or against something happening.

<table>
<thead>
<tr>
<th>Definition of Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Odds of an event</strong></td>
</tr>
<tr>
<td>Assume that all outcomes are equally likely, and that there are ( m ) favorable and ( n ) unfavorable outcomes.</td>
</tr>
<tr>
<td>The odds for the event are ( m:n ).</td>
</tr>
</tbody>
</table>
Example 4 Calculating Odds

A bag contains 6 red marbles, 2 yellow marbles, and 1 blue marble.

a. What are the odds of drawing a red marble?

**SOLUTION**

Look at the favorable outcomes and the unfavorable outcomes.

There are 6 red marbles (favorable outcomes).

There are 3 marbles that are not red (unfavorable outcomes).

The odds of drawing a red marble are 6:3 or 2:1.

b. What are the odds against drawing a blue marble?

**SOLUTION**

Look at the favorable and the unfavorable outcomes.

There are 8 marbles that are not blue (unfavorable outcome).

There is 1 blue marble (favorable outcome).

The odds against drawing a blue marble are 8:1.

Example 5 Solving Multi-Step Problems Involving Probability

Isaac has 6 blue and 4 white shirts in his closet. There are also 2 pairs of navy pants and 3 pairs of khaki pants in his closet.

a. What is the probability Isaac will choose khaki pants and a white shirt from his closet?

**SOLUTION**

\[ P(\text{khaki pants}) = \frac{3}{5} \]

\[ P(\text{white shirt}) = \frac{4}{10} = \frac{2}{5} \]

\[ P(\text{khaki pants and white shirt}) = \frac{3}{5} \cdot \frac{2}{5} = \frac{6}{25} \]

b. Assume that after the pants and shirt are worn, they are put in the laundry hamper. What is the probability that he will choose khaki pants and a white shirt from the closet to wear the next day?

**SOLUTION**

\[ P(\text{khaki pants}) = \frac{2}{4} = \frac{1}{2} \]

\[ P(\text{white shirt}) = \frac{3}{9} = \frac{1}{3} \]

\[ P(\text{khaki pants and white shirt}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \]
Lesson Practice

Identify each set of events as independent or dependent.

a. A card is chosen from a deck of cards, replaced, and then a second card is chosen.

b. A marble is drawn from a bag, kept, and then a second marble is drawn.

c. A coin is flipped, and a number cube is rolled.

d. A spinner is spun and the result is recorded. Then the spinner is spun a second time.

e. A coin is flipped and a six-sided number cube is tossed. Make a tree diagram showing all possible outcomes. What is the probability of landing on tails and on an even number?

A bag contains 4 red blocks and 3 blue blocks.

f. Find the probability of drawing a red block, keeping it, and then drawing another red block.

g. Find the probability of drawing a blue block, keeping it, and then drawing a red block.

Use the spinner to answer the problems.

h. What are the odds of spinning black?

i. What are the odds against spinning gray?

Campers select one inside activity and one outside activity daily. There are 5 inside activities and 8 outside activities.

j. What is the probability of choosing pottery and horseback riding on the first day?

k. Inside activities can be repeated, but outside activities cannot be repeated. What is the probability of choosing pottery and swimming the second day?

Practice Distributed and Integrated

Solve.

1. \(-5v = 6v + 5 - v\)  \(\text{(28)}\)

2. \(-3(b + 9) = -6\)  \(\text{(26)}\)

3. \(-22 = -p - 12\)  \(\text{(28)}\)

4. \(-\frac{2}{5} = -\frac{1}{3}m + \frac{3}{5}\)  \(\text{(26)}\)

5. \(\frac{2}{x} = \frac{30}{-6}\)  \(\text{(34)}\)

6. \(\frac{x - 4}{6} = \frac{x + 2}{12}\)  \(\text{(34)}\)

Simplify.

7. \(\frac{y^6z^5}{y^5x^7}\)  \(\text{(32)}\)

8. \(\frac{w^{-2}z^{-3}}{w^{-3}z^2}\)  \(\text{(32)}\)

9. \(\frac{4x^2z^0}{2x^3z}\)  \(\text{(32)}\)
*10. Model  There are 10 little marbles and 4 big marbles in a bag. A big marble is drawn and not replaced. Draw a picture that represents how the contents of the bag change between the first draw and the second draw.

*11. Write  Explain the difference between probability and odds.

*12. True or False: Two rolls of a number cube are independent events.

*13. Is the set of whole numbers closed under subtraction? Explain.

*14. Multiple Choice  A bag contains 3 blue stones, 5 red stones, and 2 white stones. What is the probability of picking a blue stone, keeping it, and then picking a white stone?

A \( \frac{3}{50} \)  B \( \frac{1}{15} \)  C \( \frac{3}{28} \)  D \( \frac{1}{2} \)

15. Stock Market  The value of an investor’s stock changed by \( -1\frac{3}{4} \) points last week. This week the value changed by 3 times as much. How much did the value of the investor’s stock change this week?

*16. Predict  What is the probability of rolling a 3 twice in a row on a six-sided number cube?

17. Write  Give an example of a situation in which someone may want to use large intervals on a graph to persuade people to come to a certain conclusion.

*18. Analyze  Simplify \( x^3 \cdot x^{-3} \). What is the mathematical relationship between \( x^a \) and \( x^{-a} \)?

19. Time  A nanosecond is \( 10^{-9} \) times as fast as 1 second and a microsecond is \( 10^{-6} \) times as fast as 1 second. How much faster is the nanosecond than the microsecond?

20. Convert 30 quarts per mile to gallons per mile.

*21. Error Analysis  Two students solved the proportion \( \frac{5}{9} = \frac{c}{45} \). Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{9} = \frac{c}{45} ) 9c = 225  ( c = 25 )</td>
<td>( \frac{5}{9} = \frac{c}{45} ) 9c = 225  ( c = 2025 )</td>
</tr>
</tbody>
</table>

*22. Vehicle Rental  One moving company charges $19.85 plus $0.20 per mile to rent a van. The company also rents trucks for $24.95 plus $0.17 per mile. At how many miles is the price the same for renting the vehicles?

23. If a set of ordered pairs is not a relation, can the set still be a function? Explain.
24. **Write** Explain how to find the solution of \(0.09n + 0.2 = 2.9\).

25. **Keeping Cool** The British thermal unit (BTU) is a unit of energy used globally in air conditioning industries. The number of BTUs needed to cool a room depends on the area of the room. To find the number of BTUs recommended for any size room, use the formula \(B = 377lw\), where \(B\) is the number of BTUs, \(l\) is the length of the room, and \(w\) is the width of the room. The room you want to cool uses the recommended number of 12,252.5 BTUs and is 5 meters wide. Find the area of the room.

26. **Multi-Step** A quarterback throws the ball approximately 30 times per game. He has already thrown the ball 125 times this season. The equation \(y = 30x + 125\) predicts how many times he will have thrown the ball after \(x\) more games.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

**a.** Copy and complete the table using a graphing calculator and then graph the solutions.

**b.** When will he have thrown the ball more than 300 times?

27. **Marathons** A marathon is 26.2 miles long. In order to qualify for the Boston Marathon, Jill must first complete a different marathon within \(3\frac{2}{3}\) hours. Her average speed in the last marathon she completed was 7.8 miles per hour. Did she qualify for the Boston Marathon? Explain.

28. **Geometry** The rectangles shown are similar.

![Similar Rectangles Diagram]

**a.** Find the ratio of the side lengths of the smaller rectangle to the larger rectangle.

**b.** Find the longer side length of the larger rectangle using proportions.

29. Jack is building a square pen for his dog. If he wants the area of the pen to be 144 square feet, how long should he make each side of the pen?

30. **True or False:** Whole numbers include negative numbers.
Recognizing and Extending Arithmetic Sequences

Warm Up

1. **Vocabulary** Any quantity whose value does not change is called a ________.

Simplify.

2. \(7.2 - 5.8 - (-15)\)
3. \(-0.12 - (-43.7) - 73.5\)
4. \(6(-2.5)\)
5. \((-15)(-4.2)\)

New Concepts

Sequences of numbers can be formed using a variety of patterns and operations. A **sequence** is a list of numbers that follow a rule, and each number in the sequence is called a **term of the sequence**. Here are a few examples of sequences:

1, 3, 5, 7, …
7, 4, 1, -2, …
2, 6, 18, 54, …
1, 4, 9, 16, …

In the above examples, the first two sequences are a special type of sequence called an arithmetic sequence. An **arithmetic sequence** is a sequence that has a constant difference between two consecutive terms called the **common difference**.

To find the common difference, choose any term and subtract the previous term. In the first sequence above, the common difference is 2, while in the second sequence, the common difference is -3.

If the sequence does not have a common difference, then it is not arithmetic.

**Example 1**  Recognizing Arithmetic Sequences

Determine if each sequence is an arithmetic sequence. If yes, find the common difference and the next two terms.

a. 7, 12, 17, 22, …

**SOLUTION** Since \(12 - 7 = 5\), \(17 - 12 = 5\), and \(22 - 17 = 5\), the sequence is arithmetic with a common difference of 5. The next two terms are 22 + 5 = 27 and 27 + 5 = 32.

b. 3, 6, 12, 24, …

**SOLUTION** Since \(6 - 3 = 3\) and \(12 - 6 = 6\), there is no common difference and the sequence is not arithmetic.
The first term of a sequence is denoted as \(a_1\), the second term as \(a_2\), the third term \(a_3\), and so on. The \(n\)th term of an arithmetic sequence is denoted \(a_n\). The term preceding \(a_n\) is denoted \(a_{n-1}\). For example, if \(n = 6\), then the term preceding \(a_6\) is \(a_5\) or \(a_{6-1}\) or \(a_5\).

<table>
<thead>
<tr>
<th>Term Number ((n))</th>
<th>Term</th>
<th>Sequence Pattern</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1(^{st}) or (a_1)</td>
<td>7</td>
<td>(a_1)</td>
</tr>
<tr>
<td>2</td>
<td>2(^{nd}) or (a_2)</td>
<td>((7) + 4)</td>
<td>(a_1 + d)</td>
</tr>
<tr>
<td>3</td>
<td>3(^{rd}) or (a_3)</td>
<td>((7 + 4) + 4)</td>
<td>(a_2 + d)</td>
</tr>
<tr>
<td>4</td>
<td>4(^{th}) or (a_4)</td>
<td>((7 + 4 + 4) + 4)</td>
<td>(a_3 + d)</td>
</tr>
<tr>
<td>5</td>
<td>5(^{th}) or (a_5)</td>
<td>((7 + 4 + 4 + 4) + 4)</td>
<td>(a_4 + d)</td>
</tr>
<tr>
<td>(n)</td>
<td>(n^{th}) or (a_n)</td>
<td>(a_{n-1} + 4)</td>
<td>(a_{n-1} + d)</td>
</tr>
</tbody>
</table>

Arithmetic sequences can be represented using a formula.

### Arithmetic Sequence Formula

Use the formula below to find the next term in a sequence.

\[
a_n = a_{n-1} + d
\]

- \(a_1\) = first term
- \(d\) = common difference
- \(n\) = term number

In the arithmetic sequence 7, 11, 15, 19, \ldots, \(a_1 = 7\), \(a_2 = 11\), \(a_3 = 15\), and \(a_4 = 19\). The common difference is 4.

### Example 2 Using a Recursive Formula

Use a recursive formula to find the first four terms of an arithmetic sequence where \(a_1 = -2\) and the common difference \(d = 7\).

**SOLUTION**

\[
a_n = a_{n-1} + d
\]

Write the formula.

\[
a_n = a_{n-1} + 7
\]

Substitute 7 for \(d\).

\(a_1 = -2\) Write the first term.

\(a_2 = -2 + 7 = 5\) Find the second term.

\(a_3 = 5 + 7 = 12\) Find the third term.

\(a_4 = 12 + 7 = 19\) Find the fourth term.

The first four terms of the sequence are \(-2, 5, 12,\) and 19.

A rule for finding any term in an arithmetic sequence can be developed by looking at a different pattern in the sequence 7, 11, 15, 19, \ldots.
Math Reasoning

Write Explain the difference between \(a^1\) and \(a^n\).

<table>
<thead>
<tr>
<th>Term</th>
<th>Term Number ((n))</th>
<th>Sequence Pattern</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} or (a_1)</td>
<td>1</td>
<td>7</td>
<td>(a_1)</td>
</tr>
<tr>
<td>2\textsuperscript{nd} or (a_2)</td>
<td>2</td>
<td>(7 + 4 = 7 + (1)4)</td>
<td>(a_1 + (1)d)</td>
</tr>
<tr>
<td>3\textsuperscript{rd} or (a_3)</td>
<td>3</td>
<td>(7 + 4 + 4 = 7 + (2)4)</td>
<td>(a_1 + (2)d)</td>
</tr>
<tr>
<td>4\textsuperscript{th} or (a_4)</td>
<td>4</td>
<td>(7 + 4 + 4 + 4 = 7 + (3)4)</td>
<td>(a_1 + (3)d)</td>
</tr>
<tr>
<td>5\textsuperscript{th} or (a_5)</td>
<td>5</td>
<td>(7 + 4 + 4 + 4 + 4 = 7 + 4(4))</td>
<td>(a_1 + (4)d)</td>
</tr>
<tr>
<td>(n\textsuperscript{th}) or (a_n)</td>
<td>(n)</td>
<td>(7 + (n - 1)4)</td>
<td>(a_1 + (n - 1)d)</td>
</tr>
</tbody>
</table>

**Finding the \(n\textsuperscript{th}\) Term of an Arithmetic Sequence**

\[a_n = a_1 + (n - 1)d\]

\(a_1 = \text{first term}\) \hspace{1em} \(d = \text{common difference}\)

**Example 3** Finding the \(n\textsuperscript{th}\) Term in Arithmetic Sequences

\(a\). Use the rule \(a_n = 6 + (n - 1)2\) to find the 4\textsuperscript{th} and 11\textsuperscript{th} terms of the sequence.

**SOLUTION**

4\textsuperscript{th} term: 
\[a_4 = 6 + (4 - 1)2 = 6 + (3)2 = 6 + 6 = 12\]

11\textsuperscript{th} term: 
\[a_{11} = 6 + (11 - 1)2 = 6 + (10)2 = 6 + 20 = 26\]

\(b\). Find the 10\textsuperscript{th} term of the sequence 3, 11, 19, 27, ... .

**SOLUTION**

\(a_1 = 3\) and the common difference \(d = 11 - 3 = 8\)
\[a_n = 3 + (n - 1)8\] Write the rule, substituting for \(a\), and \(d\).
\[a_{10} = 3 + (10 - 1)8\] Substitute the value for \(n\).
\[a_{10} = 75\] Simplify using the order of operations.

\(c\). Find the 10\textsuperscript{th} term of the sequence \(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots\) .

**SOLUTION**

\(a_1 = \frac{1}{4}\) and \(d = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}\)
\[a_n = \frac{1}{4} + (n - 1)\frac{1}{2}\] Write the rule, substituting for \(a\), and \(d\).
\[a_{10} = \frac{1}{4} + (10 - 1)\frac{1}{2}\] Substitute the value for \(n\).
\[a_{10} = \frac{19}{4}\] Simplify using the order of operations.
Solve each proportion

1. \( \frac{2}{10} = \frac{x}{-20} \)

2. \( \frac{32}{4} = \frac{x + 4}{3} \)

*3. Construction* An amphitheater with tiered rows is being constructed. The first row will have 24 seats and each row after that will have an additional 2 seats. If there will be a total of 15 rows, how many seats will be in the last row?
4. Use a graphing calculator to complete the table of values for the function \( f(x) = 2x^2 - 5 \). Graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

5. Solve \( y = x + \frac{2}{3} \) for \( z \).

Solve each equation. Check your answer.

6. \( 4x + 2 = 5(x + 10) \)

7. \( 2\left(n + \frac{1}{3}\right) = \frac{3}{2}n + 1 + \frac{1}{2}n - \frac{1}{3} \)

8. A bead is drawn from a bag, kept, and then a second bead is drawn. Identify these events as independent or dependent.

9. Justify Is the sequence 0.3, -0.5, -1.3, -2.1, … an arithmetic sequence? Justify your answer.

10. Write Explain why the sequence 2, 4, 8, 12, 16, … is not an arithmetic sequence.

11. Multiple Choice In the rule for the \( n^{th} \) term of an arithmetic sequence \( a_n = a_1 + (n - 1)d \), what does \( d \) represent?
   
   A the number of terms  
   B the first term  
   C the \( n^{th} \) term  
   D the common difference

12. Is the sequence 7, 14, 21, 28, … an arithmetic sequence? If it is, then find the common difference and the next two terms. If it is not, then find the next two terms.

13. Statistics A poll is taken and each person is asked two questions.
The results are shown in the table. What is the probability that someone answered “yes” to both questions?

<table>
<thead>
<tr>
<th>Question 1</th>
<th>Question 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>55</td>
</tr>
<tr>
<td>No</td>
<td>45</td>
</tr>
</tbody>
</table>

14. Predict A number cube labeled 1–6 is rolled two times. What is the probability of rolling a 2 and then a 3?

15. Geometry A rectangle with perimeter 10 units has a length of 3 units and a width of 2 units. Additional rectangles are added as shown below.

![Rectangles](image)

a. Write a rule for the perimeter of \( n \) rectangles.
b. Use the rule to find the perimeter of 12 rectangles.

16. Work uniforms include pants or a skirt, a shirt, and a tie or a vest. There are 3 pairs of pants, 5 skirts, 10 shirts, 2 ties, and 1 vest in a wardrobe.
a. What is the probability of choosing a pair of pants, a shirt, and a tie?
b. The pants and shirt from the previous day must be washed, but the tie returns to the wardrobe. What is the probability of choosing pants, a shirt, and a tie the next day?
17. Evaluate the expression \( d = 6 \cdot \frac{1}{c^2} \) for \( c = 2 \).

18. **Physical Science** The wavelengths of microwaves can range from \( 10^{-3} \) m to \( 10^{-1} \) m. Express the range of wavelengths using positive exponents.

19. **Multiple Choice** Ms. Markelsden baked 36 cookies in 45 minutes. How many cookies can she bake in 3 hours?
   A  45 cookies  
   B  81 cookies  
   C  64 cookies  
   D  144 cookies

20. Does \( y = x^2 + 2 \) represent a function? Explain how you know.

21. **Architecture** A model of a building is 15 inches tall. In the scale drawing, 1 inch represents 20 feet. How tall is the building?

22. Generalize Given \( \frac{2}{b} = \frac{1}{a} \), where \( a \) and \( b \) are positive numbers, write an equation that shows how to find \( a \).

23. The line graph at right shows the costs of tuition at a university over the past 5 years. How might this graph be misleading?

24. Write The equation \( x + 5 = x - 5 \) has no solution. Explain why it has no solution.

25. Verify Solve \( 16x + 4(2x - 6) = 60 \) for \( x \). Check your answer.

26. True or False: Any irrational number divided by an irrational number will be an irrational number. Explain your answer.

27. **Savings** Hector has $400 in his savings account. Each week he deposits his $612.50 paycheck and takes out $250 to live on for the week. If he wants to buy a car for $5500, about how many months will it take him to save up for the car?

28. **Multi-Step** Jamal is riding his bike at a rate of about 8 miles per hour. How many hours will it take Jamal to ride 50 miles?

29. **Car Rental** A family rented a car that cost $45 per day plus $0.23 per mile. If the family rented the car for 7 days and paid $395.50 altogether, how many miles did they drive?

30. **Error Analysis** Two students determine whether the ordered pairs in the table represent a function. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

**Student A**
(7, 12) and (7, 10)
The \( x \)-values are the same, so it is not a function.

**Student B**
All the \( y \)-values are different, so it is not a function.
Warm Up

1. **Vocabulary** A pair of numbers that can be used to locate a point on a coordinate plane is called a(n) __________.

Evaluate.

2. $3x + 14; x = -9$  
3. $7.5w - 84.3; w = 15$

Solve.

4. $7x - 18 = -74$  
5. $57 + 19y = -76$

New Concepts

Linear equations can be graphed by making a table of ordered pairs that satisfy the equation and then graphing the corresponding points $(x, y)$. An ordered pair or set of ordered pairs that satisfy an equation is called the **solution of a linear equation in two variables**. When an equation is in standard form, the linear equation can be graphed another way.

### Standard Form of a Linear Equation

The **standard form of a linear equation** is $Ax + By = C$, where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both zero.

The $x$-coordinate of the point where the graph of an equation intersects the $x$-axis is called the **$x$-intercept**. The $y$-coordinate of the point where the graph of an equation intersects the $y$-axis is called the **$y$-intercept**. The coordinate pairs $(x, 0)$ and $(0, y)$ that satisfy a linear equation are two solutions of the linear equation in two variables.

#### Example 1  Finding $x$- and $y$-Intercepts

Find the $x$- and $y$-intercepts for $3x + 4y = 24$.

**SOLUTION**

To find the intercepts, make a table. Substitute 0 for $y$ and solve for $x$. Substitute 0 for $x$ and solve for $y$.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 4y = 24$</td>
<td>$3x + 4y = 24$</td>
</tr>
<tr>
<td>$3x + 4(0) = 24$</td>
<td>$3(0) + 4y = 24$</td>
</tr>
<tr>
<td>$3x = 24$</td>
<td>$4y = 24$</td>
</tr>
<tr>
<td>$\frac{3x}{3} = \frac{24}{3}$</td>
<td>$\frac{4y}{4} = \frac{24}{4}$</td>
</tr>
<tr>
<td>$x = 8$</td>
<td>$y = 6$</td>
</tr>
</tbody>
</table>

The $x$-intercept is 8. The $y$-intercept is 6.
An efficient method for graphing a linear equation in two variables is to plot the \( x \)- and \( y \)-intercepts and then to draw a line through them.

**Example 2  Graphing Using the \( x \)- and \( y \)-Intercepts**

Graph \( 5x - 6y = 30 \) using the \( x \)- and \( y \)-intercepts.

**SOLUTION**

To find the intercepts, make a table. Substitute 0 for \( y \) and solve for \( x \).

\[
\begin{array}{c|c}
\text{ } & \text{ } \\
5x - 6y = 30 & 5x - 6y = 30 \\
5x - 6(0) = 30 & 5(0) - 6y = 30 \\
5x = 30 & -6y = 30 \\
\frac{5x}{5} = \frac{30}{5} & -\frac{6y}{-6} = \frac{30}{-6} \\
x = 6 & y = -5 \\
\end{array}
\]

The \( x \)-intercept is 6. The \( y \)-intercept is -5.

To graph the equation, plot the points (6, 0) and (0, -5). Then draw a line through them.

**Example 3  Locating \( x \)- and \( y \)-Intercepts on a Graph**

Find the \( x \)- and \( y \)-intercepts on the graph.

**SOLUTION**

The \( x \)-intercept is the \( x \)-coordinate of the point where the line crosses the \( x \)-axis. The point where the line crosses the \( x \)-axis is (–4, 0). The \( x \)-intercept is –4.

The \( y \)-intercept is the \( y \)-coordinate of the point where the line crosses the \( y \)-axis. The point where the line crosses the \( y \)-axis is (0, 7). The \( y \)-intercept is 7.
Any linear equation can be rearranged into standard form. Put both variables and their coefficients on one side of the equal sign and the constants on the other side.

**Example 4  Using Standard Form to Graph**

Write the equation $y = -\frac{2}{3}x + 5$ in standard form. Then graph the equation using the $x$- and $y$-intercepts.

**SOLUTION**

Write the equation in standard form.

\[
y = -\frac{2}{3}x + 5
\]

\[
+\frac{2}{3}x = +\frac{2}{3}x
\]

\[
\frac{2}{3}x + y = 5
\]

To find the $x$-intercept, substitute 0 for $y$ and solve for $x$.

\[
\frac{2}{3}x + 0 = 5
\]

\[
\frac{2}{3}x = 5
\]

\[
\left(\frac{3}{2}\right)\frac{2}{3}x = 5\left(\frac{3}{2}\right)
\]

\[
x = 7\frac{1}{2}
\]

The $x$-intercept is $7\frac{1}{2}$.

To find the $y$-intercept, substitute 0 for $x$ and solve for $y$.

\[
\frac{2}{3}x + y = 5
\]

\[
\frac{2}{3}(0) + y = 5
\]

\[
y = 5
\]

The $y$-intercept is 5.

Graph the $x$- and $y$-intercepts and draw a line through them.
Example 5  Application: Play Tickets

At a school play, student tickets are $5 and adult tickets are $8. Let \( x \) be the number of student tickets sold. Let \( y \) be the number of adult tickets sold. The equation \( 5x + 8y = 400 \) shows that one drama club member raised $400 from ticket sales. Find the intercepts and explain what each means.

**SOLUTION**

Substitute 0 for \( y \) and solve for \( x \). Then substitute 0 for \( x \) and solve for \( y \).

\[
\begin{align*}
5x + 8y &= 400 \\
5x + 8(0) &= 400 \\
5x &= 400 \\
8(0) + 8y &= 400 \\
8y &= 400 \\
\frac{5x}{5} &= \frac{400}{5} \\
\frac{8y}{8} &= \frac{400}{8} \\
x &= 80 \\
y &= 50
\end{align*}
\]

The \( x \)-intercept is 80. The \( y \)-intercept is 50.

The \( x \)-intercept shows that if the drama club member sold no adult tickets, 80 student tickets were sold. The \( y \)-intercept shows that if no student tickets were sold, 50 adult tickets were sold.

Lesson Practice

a. Find the \( x \)- and \( y \)-intercepts for \(-6x + 9y = 36\).

b. Graph \( 4x + 7y = 28 \) using the \( x \)- and \( y \)-intercepts.

c. Find the \( x \)- and \( y \)-intercepts on the graph.

d. Write \( 4y = 12x - 12 \) in standard form. Then graph the equation using the \( x \)- and \( y \)-intercepts.

e. **Athletics** Hirva jogs 6 miles per hour and bikes 12 miles per hour. The equation \( 6x + 12y = 24 \) shows that she has gone a total of 24 miles. Find the intercepts and explain what each means.
Solve.

1. \( \frac{-2.25}{x} = \frac{9}{6} \)  
2. \( \frac{y + 2}{y + 7} = \frac{11}{31} \)

3. \( 2(f + 3) + 4f = 6 + 6f \)  
4. \( 3x + 7 - 2x = 4x + 10 \)

Evaluate each expression for the given value of the variable.

5. \( (m + 6) ÷ (2 - 5) \) for \( m = 9 \)  
6. \( -3(x + 12 ÷ 2) \) for \( x = -8 \)

Simplify by combining like terms.

7. \( 10y^3 + 5y - 4y^3 \)  
8. \( 10xy^2 - 5x^2y + 3y^3x \)

9. Identify the subsets of real numbers to which the number \( \sqrt{7} \) belongs.

*10. Find the \( x \)- and \( y \)-intercepts for \( 5x + 10y = -20 \).

*11. Find the \( x \)- and \( y \)-intercepts for \( -8x + 20y = 40 \).

*12. Write Explain how knowing the \( x \)- and \( y \)-intercepts is helpful in graphing a linear equation.

*13. Multiple Choice What is the \( x \)-intercept for the equation \( 15x + 9y = 45 \)?

A (0, 3)  
B (3, 0)  
C (5, 0)  
D (0, 5)

*14. Fishery A Pacific salmon can swim at a maximum speed of 8 miles per hour. The function \( y = 8x \) describes how many miles \( y \) the fish swims in \( x \) hours. Graph the function. Use the graph to estimate the number of miles the fish swims in 2.5 hours.

15. Determine if the sequence 34, 29, 24, 19, … is an arithmetic sequence or not. If yes, find the common difference and the next two terms. If no, find the next two terms.

16. Error Analysis Two students are finding the common difference for the arithmetic sequence 18, 15, 12, 9, …. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 − 15 = 3</td>
<td>15 − 18 = −3</td>
</tr>
<tr>
<td>15 − 12 = 3</td>
<td>12 − 15 = −3</td>
</tr>
<tr>
<td>12 − 9 = 3</td>
<td>9 − 12 = −3</td>
</tr>
</tbody>
</table>

*17. Geometry A right triangle is formed by the origin and the \( x \)- and \( y \)-intercepts of \( 14x + 7y = 56 \). Find the area of the triangle.
18. **Data Analysis**  The table shows the weights of a newborn baby who was 7.5 lb at birth.
   a. Write a recursive formula for the baby’s weight gain.
   b. If the pattern continues, how much will the baby weigh after 7 weeks?

<table>
<thead>
<tr>
<th>Week Number</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>10.5</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>13.5</td>
</tr>
</tbody>
</table>

*19. **Multi-Step**  Use the arithmetic sequence $-65, -72, -79, -86, \ldots$.
   a. What is the value of $a_i$?
   b. What is the common difference $d$?
   c. Write a rule for the $n^{th}$ term of the sequence.

20. **Write**  A coin is flipped and lands on heads. It is flipped again and lands on tails. Identify these events as independent or dependent.

21. **Economics**  You have agreed to a babysitting job that will last 14 days. On the first day, you earn $25, but on each day after that you will earn $15. How much will you earn if you babysit for 7 days?

22. **Probability**  A bag holds 5 red marbles, 3 white marbles, and 2 green marbles. A marble is drawn, kept out, and then another marble is drawn. What is the probability of drawing two white marbles?

*23. **Multiple Choice**  Which of the following expressions is the simplified solution of $\frac{m^3n^{-10}p^5}{mn^p^{2-1}}$?
   A $\frac{m^3p^3}{n^{10}}$
   B $\frac{m^3p^7}{n^{10}}$
   C $\frac{m^2p^3}{n^9}$
   D $\frac{m^2p^7}{n^{10}}$

*24. **Verify**  Is the statement $4^{-2} = -16$ correct? Explain your reasoning.

25. **Convert**  Convert 45 miles per hour to miles per minute.

26. **Rewrite**  Rewrite the following question so it is not biased: Would you rather buy a brand new luxury SUV or a cheap used car?

27. **Identify**  Identify the independent variable and the dependent variable: money earned, hours worked.

28. **Temperature**  Use the formula $F = \frac{9}{5}C + 32$ to find an equivalent Fahrenheit temperature when the temperature is $-12^\circ C$.

29. **Homework**  A student has to write a book report on a book that contains 1440 pages. Suppose she plans to read 32 pages per day. Using function notation, express how many pages remain after reading for $d$ days.

30. **Soccer**  For every hour a player practices soccer, he must drink 8 fluid ounces of liquid to stay hydrated. Write an equation describing this relation and determine whether it is a function.
Lesson 36

Writing and Solving Proportions

Warm Up

1. **Vocabulary** A _________ is a comparison of two quantities using division.

Solve.

2. \( \frac{13}{52} = \frac{x}{36} \)

3. \( \frac{42}{56} = \frac{63}{w} \)

4. \( 15x - 37 = 143 \)

5. \( 78 + 22y = -230 \)

New Concepts

Proportions are frequently used to solve problems in mathematics. Proportional reasoning can be applied in many situations, including reading and drawing maps, architecture, and construction. Solving problems in these situations requires knowledge of similar figures.

If two geometric objects or figures are similar, they have the same shape but are not necessarily the same size. The triangles below are similar.

\[ \triangle ABC \sim \triangle DEF \]

When two figures are similar, they have sides and angles that correspond. Corresponding sides and angles are found using the order of the letters in the similarity statement. In the triangles above, \( \angle A \) and \( \angle D \) correspond, \( \angle B \) and \( \angle E \) correspond, and \( \angle C \) and \( \angle F \) correspond. Corresponding angles of similar figures are congruent, or have the same measure.

\[ \angle A \cong \angle D \quad \angle B \cong \angle E \quad \angle C \cong \angle F \]

Sides of similar figures also correspond. In the example above, \( \overline{AB} \) and \( \overline{DE} \) correspond, \( \overline{BC} \) and \( \overline{EF} \) correspond, and \( \overline{AC} \) and \( \overline{DF} \) correspond. Corresponding sides of similar figures do not have to be congruent. However, they do have to be in proportion. The ratio of the all pairs of corresponding sides must be the same.

\[ \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \]

In the example above, the ratio of the sides of \( \triangle ABC \) to \( \triangle DEF \) is 2 to 1. This ratio, which can also be written as \( \frac{2}{1} \) or 2:1, is called the scale factor of \( \triangle ABC \) to \( \triangle DEF \).

\[ \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{1} \]

A scale factor is the ratio of a side length of a figure to the side length of a similar figure. The scale factor of \( \triangle DEF \) to \( \triangle ABC \) is 1 to 2.
Example 1  Finding Measures in Similar Figures

\( PQRS \sim WXYZ \)

a. Find \( m\angle Q \) and \( m\angle W \).

**SOLUTION**

\( \angle Q \) and \( \angle X \) correspond, so they are equal, and \( m\angle Q = 80^\circ \). \( \angle W \) and \( \angle P \) correspond, so they are equal and \( m\angle W = 120^\circ \).

b. Find the scale factor of \( PQRS \) to \( WXYZ \).

**SOLUTION**

\( PS \) and \( WZ \) correspond, so the scale factor of \( PQRS \) to \( WXYZ \) is \( \frac{PS}{WZ} = \frac{3}{2} \).

c. Use the scale factor to find \( QR \) and \( ZY \).

**SOLUTION**

All corresponding side lengths must be in a ratio of 3 to 2. \( QR \) corresponds with \( XY \) and \( ZY \) corresponds with \( SR \).

\[
\frac{PS}{WZ} = \frac{QR}{XY} \quad \frac{PS}{WZ} = \frac{SR}{ZY}
\]

\[
\frac{3}{2} = \frac{a}{5} \quad \frac{3}{2} = \frac{6}{b}
\]

\[
2a = 15 \quad 3b = 12
\]

\[
a = 7.5 \quad b = 4
\]

So, \( QR = 7.5 \) and \( ZY = 4 \).

Another application of proportional reasoning is indirect measurement. Indirect measurement involves using similar figures to find unknown lengths.

Example 2  Using Indirect Measurement

A radio tower casts a shadow 10 feet long. A woman who is 5.5 feet tall casts a shadow 4 feet long. The triangle drawn with the tower and its shadow is similar to the triangle drawn with the woman and her shadow. How tall is the radio tower?

**SOLUTION**

Set up a proportion to solve the problem.

\[
\frac{10}{4} = \frac{x}{5.5}
\]

\[
10(5.5) = 4 \cdot x
\]

\[
55 = 4x
\]

\[
x = 13.75
\]

The radio tower is 13.75 feet tall.
A **scale drawing** is a drawing that reduces or enlarges the dimensions of an object by a constant factor. Maps and blueprints are examples of scale drawings. The **scale** is a ratio showing the relationship between a scale drawing or model and the actual object.

### Example 3 Application: Scale Drawings

A desk is designed to have three drawers on the right side. In the scaled design drawing, the width of the drawers is 4 centimeters. If the scale of the drawing is 1 cm:7 cm, how wide will the actual desk drawers be?

**SOLUTION** Set up a proportion to solve the problem.

\[
\frac{\text{drawing length}}{\text{actual length}} = \frac{1}{7} = \frac{4}{x}
\]

\[
1 \cdot x = 4 \cdot 7
\]

\[
x = 28
\]

The width of the actual desk drawers will be 28 centimeters.

Scale factors of side lengths can also be used to determine the ratios of perimeters, areas, and volumes of figures and solids.

### Exploration Changing Dimensions

Students may work in groups of two or three.

**a.** Begin with 1 cube and let the length of an edge equal 1 unit. Then the perimeter of the base of the cube is \(4 \cdot 1 = 4\) units, the area of the base of the cube is \(1 \cdot 1 = 1\) square unit, and the volume of the cube is \(1 \cdot 1 \cdot 1 = 1\) cubic unit. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Length of Edge (units)</th>
<th>Perimeter of the Base (units)</th>
<th>Area of the Base (square units)</th>
<th>Volume of the Base (cubic units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**b.** Use the cubes to build another cube with edge lengths of 2 units. Record your answers in the table.

**c.** Repeat for a cube with a side length of 3 units. Record your answers in the table.

**Use the rows with edge lengths of 2 and 3 from the table.**

**d.** What is the scale factor of the edge lengths?

**e.** Find the ratio of the perimeters. How does this ratio compare to the scale factor?

**f.** Find the ratio of the areas. How does this ratio compare to the scale factor?
Find the ratio of the volumes. How does this ratio compare to the scale factor?

### Ratios of the Perimeter, Area, and Volume of Similar Figures

If two similar figures have a scale factor of \( \frac{a}{b} \), then the ratio of their perimeters is \( \frac{a}{b} \), the ratio of their areas is \( \frac{a^2}{b^2} \), and the ratio of their volumes is \( \frac{a^3}{b^3} \).

#### Example 4 Application: Changing Dimensions

A dartboard is composed of concentric circles. The radius of the smallest inner circle is 1 inch, and with each consecutive circle, the radius increases by 1 inch.

**a.** What is the ratio of the circumference of the two inner circles to the circumference of the outermost circle?

**SOLUTION**

\[
\text{circumference of two inner circles} \div \text{circumference of outermost circle} = \frac{2\pi(2)}{2\pi(5)} = \frac{2}{5}
\]

**b.** What is the ratio of the area of the two inner circles to the area of the outermost circle?

**SOLUTION**

\[
\text{area of two inner circles} \div \text{area of outermost circle} = \frac{\pi(2)^2}{\pi(5)^2} = \frac{4\pi}{25\pi} = \frac{4}{25}
\]

### Lesson Practice

**a.** \( \triangle ABC \sim \triangle LKM \). Find \( m\angle K \) and \( m\angle C \).

**b.** The figures are similar. Find the scale factor. Then use the scale factor to find \( x \).
c. The side of a building casts a shadow 21 meters long. A statue that is 5 meters tall casts a shadow 4 meters long. The triangles are similar. How tall is the building?

\[ \text{Ex 2} \]
\[ \begin{align*}
\text{5 m} & \quad 4 \text{ m} \\
\text{21 m} & \quad x
\end{align*} \]

\[ \text{Ex 3} \]

\[ \text{Ex 4} \]

\[ \text{Ex 5} \]

\[ \text{Ex 6} \]

\[ \text{Ex 7} \]

\[ \text{Ex 8} \]

\[ \text{Ex 9} \]

\[ \text{Ex 10} \]

\[ \text{Ex 11} \]

**Practice**

**Distributed and Integrated**

Solve each proportion.

\[ \begin{align*}
\text{1.} & \quad \frac{3}{4} = \frac{x}{100} && \text{2.} & \quad \frac{5.5}{x} = \frac{1.375}{11}
\end{align*} \]

Simplify each expression.

\[ \begin{align*}
\text{3.} & \quad 2^2 + 6(8 - 5) + 2 && \text{4.} & \quad \frac{(3 + 2)(4 + 3) + 5^2}{6 - 2^2} && \text{5.} & \quad \frac{14 - 8}{-2^2 + 1}
\end{align*} \]

6. The point (3, 5) is graphed in which quadrant of a coordinate plane?

7. True or False: The set of ordered pairs below defines a function.
\[ \{(1, 3), (2, 3), (3, 3), (4, 3)\} \]

8. The triangles at right are similar. Find the missing length.

\[ \text{Ex 2} \]
\[ \begin{align*}
9 & \quad 16 & \quad x & \quad 12
\end{align*} \]

9. **Multiple Choice** One triangle has side lengths 3, 5, and 6. A similar triangle has side lengths 18, 15, and 9. Which of the following ratios is the scale factor of the triangles?

\[ \begin{align*}
\text{A} & \quad \frac{1}{6} && \text{B} & \quad \frac{1}{3} && \text{C} & \quad \frac{1}{5} && \text{D} & \quad \frac{2}{3}
\end{align*} \]

10. **Landscaping** A landscaping company needs to measure the height of a tree. The tree casts a shadow that is 6 feet long. A person who is 5 feet tall casts a shadow that is 2 feet long.

\[ \begin{align*}
a. & \quad \text{Draw a picture to represent the problem.} \\
b. & \quad \text{Use your picture to find the height of the tree.}
\end{align*} \]
11. A real estate company sells small models of houses. The scale factor of the models to the actual houses is 0.5 ft:10 ft. Find the ratio of the areas of the model to the actual house.

12. (Entrepreneurship) A small child decided to sell his artwork. He sold black-and-white drawings for $2 and colored drawings for $3. The equation $2x + 3y = 24$ shows that he earned $24. Find the $x$- and $y$-intercepts.

13. Verify A tree casts a shadow 14 feet long. A flagpole that is 20 feet tall casts a shadow 8 feet long. The triangle formed by the tree and its shadow is similar to the triangle formed by the flagpole and its shadow. Verify that the tree is 35 feet tall.

14. Find the $x$-intercept for $11x - 33y = 99$.

15. Find the $y$-intercept for $-7x - 8y = 56$.

16. Geometry A right triangle is formed by the origin and the $x$- and $y$-intercepts of the line $11x - 4y = 22$. Find the area of the triangle.

17. Multi-Step A car wash is held as a school fundraiser to earn $280 for a field trip. The charge is $7 for a car and $10 for an SUV. Let $x$ be the number of cars and $y$ be the number of SUVs washed. The profits are calculated using the equation $7x = -10y + 280$.
   a. Rewrite the profit equation in standard form.
   b. Calculate the $y$-intercept and explain its real-world meaning.
   c. Calculate the $x$-intercept and explain its real-world meaning.

18. Determine if the sequence $0.4, 0.1, -0.2, -0.5, \ldots$ is an arithmetic sequence or not. If yes, find the common difference and the next two terms. If no, find the next two terms.

19. Error Analysis Two students are writing an example of an arithmetic sequence. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th></th>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 1, -3, -7, ...</td>
<td>1, 4, 16, 44, ...</td>
<td></td>
</tr>
</tbody>
</table>

20. Verify At a raffle, 5 students’ names are in a hat. There are 3 prizes in a bag: 2 books and a free lunch. Once the name and a prize are drawn, they are not replaced. After giving out a book in the first drawing, a remaining student quickly calculates her probability of winning a book in the second drawing as $\frac{1}{5}$. Show that she is correct.

21. (Astronomy) The force of gravity on the moon is about one-sixth of that on Earth. If an object on Earth weighs 200 pounds, about how much does it weigh on the moon?
22. **Multiple Choice**  What are the odds against spinning a B on the spinner?

   - A 1:2
   - B 1:3
   - C 2:1
   - D 3:1

23. **Multi-Step**  Steve has $300 in the bank. Each week he spends $10. Mario has $100 in the bank and deposits $5 each week.

   a. Write expressions to represent how many dollars each person has in the bank after $w$ weeks.
   
   b. Write an expression that represents how many dollars they have altogether.
   
   c. After 6 weeks, how much money do they have?

24. Identify the independent variable and the dependent variable in the following statement: The fire was very large, so many firefighters were there.

25. **Multi-Step**  The measure of angle $B$ is three times the measure of angle $A$. The sum of the angle measures is 128°. Find the value of $x$.

26. **Piano Lessons**  A piano student has a $250 scholarship and an additional $422 saved for piano lessons. If each lesson costs $42, how many lessons will he have?

27. Simplify $\frac{4x^2z^0}{2x^3z}$.

28. **Meteorology**  Meteorologists sometimes use a measure known as virtual temperature ($T_v$) in kelvins (K) to compare dry and moist air. It can be calculated as $T_v = T(1 + 0.61r)$, where $T$ is the temperature of the air and $r$ is the mixing ratio of water vapor and dry air. For temperatures of $T = 282.5 K$ and $T_v = 285 K$, find the mixing ratio to the nearest thousandth.

29. **Measurement**  For a science project, Joe must measure out 5 samples of a liquid. The graph shows the size of his samples. Why might the data require smaller intervals on the graph?

30. **Justify**  Solve the equation $34 - 2(x + 17) = 23x - 15 - 3x$. Write out and justify each step.
**Warm Up**

1. **Vocabulary** The ________ tells how many times the base of a power is used as a factor.

Simplify.

2. \(7^4\)

3. \((8.34)(-4)(100)\)

4. \(5^{-5}\)

5. \(\frac{3x^{-3}}{5y^{-5}}\)

**New Concepts**

Very large or very small numbers are often written in **scientific notation**, a method of writing a number as the product of a number greater than or equal to 1 but less than 10 and a power of ten.

**Exploration**

**Applying Scientific Notation**

a. Copy the table. Multiply to find each number in standard form.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.08 \times 10^2)</td>
<td></td>
</tr>
<tr>
<td>(1.08 \times 10^3)</td>
<td></td>
</tr>
<tr>
<td>(1.08 \times 10^4)</td>
<td></td>
</tr>
<tr>
<td>(1.08 \times 10^5)</td>
<td></td>
</tr>
</tbody>
</table>

b. What pattern do you see in the table?

c. Copy the table. Multiply to find each number in standard form.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.08 \times 10^{-1})</td>
<td></td>
</tr>
<tr>
<td>(1.08 \times 10^{-2})</td>
<td></td>
</tr>
<tr>
<td>(1.08 \times 10^{-3})</td>
<td></td>
</tr>
<tr>
<td>(1.08 \times 10^{-4})</td>
<td></td>
</tr>
</tbody>
</table>

d. What pattern do you see in the table?

e. Which direction does the decimal move when the exponent is positive?

f. Which direction does the decimal move when the exponent is negative?
Scientific Notation

A number written as the product of two factors in the form \( a \times 10^n \), where \( 1 \leq a < 10 \) and \( n \) is an integer.

**Example 1**  Writing Numbers in Scientific Notation

Write each number in scientific notation.

**a.** 856,000

**SOLUTION**

Because this is a number greater than 1, the exponent will be positive. The decimal point moves to be after the 8 so that there is one digit to the left of the decimal.

856,000.

\[ \begin{array}{c}
\hline
5 & 4 & 3 & 2 & 1 \\
\hline
\end{array} \]

Move the decimal five places, and write the number as \( 8.56000 \times 10^5 \).

So, \( 856,000 = 8.56 \times 10^5 \).

**b.** 0.0005

**SOLUTION**

Because this is a number between 0 and 1, the exponent will be negative. The decimal point moves to be after the 5, so that there is one digit to its left.

\[ \begin{array}{c}
\hline
1 & 2 & 3 & 4 \\
\hline
\end{array} \]

Move the decimal four places, and write the number as \( 5 \times 10^{-4} \).

To multiply numbers in scientific notation, multiply the coefficients and then multiply the powers. If the result is not in scientific notation, adjust it so that it is.

**Example 2**  Multiplying Numbers in Scientific Notation

Find the product. Write the answer in scientific notation.

\[ (5.7 \times 10^3)(1.8 \times 10^5) \]

**SOLUTION**

\[ (5.7 \times 10^5)(1.8 \times 10^3) \]

Use the Commutative and Associative Properties of Multiplication to group the numbers and the powers.

\[ = (5.7 \cdot 1.8)(10^5 \cdot 10^3) \]

\[ = 10.26 \times 10^8 \]

Simplify.

Notice that the result is not in scientific notation. There is more than one digit before the decimal point. Move the decimal to the left one place and add one to the exponent.

\[ 10.26 \times 10^8 = 1.026 \times 10^9 \]
To divide numbers in scientific notation, divide the coefficients, and then divide the powers. If the result is not in scientific notation, adjust it so that it is.

**Example 3** Dividing Numbers in Scientific Notation

Find the quotient. Write the answer in scientific notation.

\[
\frac{1.2 \times 10^3}{9.6 \times 10^6}
\]

**SOLUTION**

\[
\frac{1.2}{9.6} \times \frac{10^3}{10^6} = \frac{0.125 \times 10^{-3}}{10^6}
\]

Divide the coefficients and divide the powers.

\[
= 0.125 \times 10^{-3}
\]

Simplify.

Notice that this number is not in scientific notation. There is not one nonzero digit before the decimal point. Move the decimal to the right one place and subtract one from the exponent.

\[
0.125 \times 10^{-3} = 1.25 \times 10^{-4}
\]

**Example 4** Comparing Expressions with Scientific Notation

Compare. Use <, >, or =.

\[
\frac{7.2 \times 10^6}{3.6 \times 10^4} \bigg\rangle \frac{1.05 \times 10^7}{3.5 \times 10^5}
\]

**SOLUTION**

\[
\frac{7.2 \times 10^6}{3.6 \times 10^4} = \frac{2 \times 10^2 = 200}{3.6 \times 10^4}
\]

\[
\frac{1.05 \times 10^7}{3.5 \times 10^5} = 0.3 \times 10^2 = 30
\]

Since 200 > 30, then

\[
\frac{7.2 \times 10^6}{3.6 \times 10^4} > \frac{1.05 \times 10^7}{3.5 \times 10^5}
\]

**Example 5** Application: Speed of Light

The speed of light is \(3 \times 10^8\) meters per second. If Earth is \(1.47 \times 10^{11}\) meters from the sun, how many seconds does it take light to reach Earth from the sun? Write the answer in scientific notation.

**SOLUTION** Divide the earth’s distance from the sun by the speed of light.

\[
\frac{1.47 \times 10^{11}}{3 \times 10^8} = \frac{0.49 \times 10^3}{3 \times 10^8}
\]

\[
= 4.9 \times 10^2
\]

Write the answer in scientific notation.

It takes light about \(4.9 \times 10^2\) seconds to reach the earth from the sun.
Lesson Practice

Write each number in scientific notation.

\( a. 1,234,000. \)  \( b. 0.0306. \)

\( c. \) Find the product. Write the answer in scientific notation. 
\[ (5.82 \times 10^3)(6.13 \times 10^{11}) \]

\( d. \) Find the quotient. Write the answer in scientific notation. 
\[ \frac{7.29 \times 10^{-2}}{8.1 \times 10^{-6}} \]

\( e. \) Compare. Use \(<, >, \text{ or } =.\) 
\[ \frac{4.56 \times 10^9}{3 \times 10^5} \bigcirc \frac{5.2 \times 10^8}{1.3 \times 10^3}. \]

\( f. \) \textbf{Astronomy} The speed of light is \( 3 \times 10^8 \text{ meters per second}. \) If Mars is \( 2.25 \times 10^{11} \text{ meters from the Sun}, \) how many seconds does it take light to reach Mars from the Sun? Write the answer in scientific notation.

Practice Distributed and Integrated

Simplify each expression.

\( 1. \) \( 18 \div 3^2 - 5 + 2 \)
\( 2. \) \( 7^2 + 4^2 + 3 \)
\( 3. \) \( 3[-2(8 - 13)] \)

Simplify each expression by combining like terms.

\( 4. \) \( 13h^2 + 5b - b^2 \)
\( 5. \) \( -3(8x + 4) + \frac{1}{2}(6x - 24) \)

\( *6. \) Write \( 7.4 \times 10^{-9} \) in standard notation.

\( *7. \) Write Explain how to recognize if a number is in scientific notation.

\( *8. \) Write Explain why anyone would want to use scientific notation.

\( *9. \) \textbf{Multiple Choice} What is \( (3.4 \times 10^{10})(4.8 \times 10^5) \) in scientific notation?
\( A \) \( 1.632 \times 10^{15} \)
\( B \) \( 1.632 \times 10^{16} \)
\( C \) \( 16.32 \times 10^{15} \)
\( D \) \( 16.32 \times 10^{16} \)

\( *10. \) \textbf{Physiology} The diameter of a red blood cell is about \( 4 \times 10^{-5} \text{ inches}. \) Write this number in standard notation.

\( 11. \) The triangles shown are similar. Find the missing length.

\[ \text{20} \]
\[ \text{18} \]
\[ \text{x} \]
\[ \text{12} \]

\( 12. \) A student’s final grade is determined by adding four test grades and dividing by 4. The student’s first three test grades are 79, 88, and 94. What must the student make on the last test to get a final grade of 90?
13. Graph $50x - 100y = 300$ using the $x$- and $y$-intercepts.

14. **Geometry** A square has side lengths of 3 centimeters. Another square has side lengths of 6 centimeters.
   a. What is the scale factor of the sides of the smaller square to the larger square?
   b. What is the perimeter of each square?
   c. What is the ratio of the perimeter of the smaller square to the perimeter of the larger square?
   d. What is the area of each square?
   e. What is the ratio of the area of the smaller square to the area of the larger square?

15. **Error Analysis** In the figures at right, $\angle A$ and $\angle F$ correspond. Two students are finding the measure of $\angle F$. Which student is correct? Explain the error.

   **Student A**
   \[
   \frac{5}{4} \cdot 80 = x \\
   5x = 320 \\
   x = 64 \\
   m\angle F = 64^\circ
   \]

   **Student B**
   \[
   m\angle A = m\angle F \\
   80^\circ = m\angle F
   \]

16. **Architecture** A room is 10 feet by 12 feet. If the scale of the blueprints to the room is 1 inch to 2 feet, find the dimensions of the room on the blueprints.

17. **Measurement** What is the ratio of the area of the smaller circle to the area of the larger circle?

18. **Verify** Verify that the sequence $4, 2\frac{2}{3}, 1\frac{1}{3}, 0, \ldots$ is an arithmetic sequence.

19. A piece of fruit is chosen from a box, eaten, and then a second piece of fruit is chosen. Identify these events as independent or dependent.

20. **Predict** An estimate of the number of tagged foxes to the total number of foxes in a forest is 3 to 13. A forest warden noted 21 tagged foxes during a trip in the forest. Write a proportion to indicate the total number of foxes that might be in the forest.

21. **Multiple Choice** In the rule for the $n^{th}$ term of arithmetic sequence $a_n = a_1 + (n - 1)d$, what does $a_1$ represent?
   - A the number of terms
   - B the first term
   - C the $n$th term
   - D the common difference

22. **Physical Science** The wavelengths of ultraviolet light can range from $10^{-9}$ meter to $10^{-7}$ meter. Express the range of wavelengths using positive exponents.
23. **Estimate** Between which two whole numbers is the solution to \( \frac{13}{14} = \frac{x}{10} \)?

24. **Dog Breeds** The table shows the number of dogs of the top five breeds registered with the American Kennel Club in 2006. Describe how the data could be displayed in a potentially misleading way.

<table>
<thead>
<tr>
<th>Breed</th>
<th>Labrador Retriever</th>
<th>Yorkshire Terrier</th>
<th>German Shepherd</th>
<th>Golden Retriever</th>
<th>Beagle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>123,760</td>
<td>48,346</td>
<td>43,575</td>
<td>42,962</td>
<td>39,484</td>
</tr>
</tbody>
</table>

25. Choose an appropriate graph to display a survey showing what type of sport most people like. Explain your answer.

26. **Exercising** A weightlifter averages 2 minutes on each exercise. Each workout includes a 20-minute swim. Write a rule in function notation to describe the time it takes to complete \( w \) exercises and the swim.

27. **Estimate** Using the order of magnitude, estimate the value of 89,678 multiplied by 11,004,734.

28. **Justify** Solve for \( x \): \( 7x + 9 = 2(4x + 2) \). Justify each step.

29. An oceanographer wants to convert measurements that are above and below sea level from yards to feet. He takes measurements of depths and heights in yards and feet.

<table>
<thead>
<tr>
<th>yards</th>
<th>-679</th>
<th>-125</th>
<th>32</th>
<th>79</th>
</tr>
</thead>
<tbody>
<tr>
<td>feet</td>
<td>-2037</td>
<td>-375</td>
<td>96</td>
<td>237</td>
</tr>
</tbody>
</table>

a. **Formulate** Use the table to write a formula to convert from yards to feet.

b. **Predict** Use the formula to convert 27.5 yards to feet.

c. Write a formula to convert yards to inches.

30. **Multi-Step** A rectangle has a perimeter of \( 38 + x \) centimeters. The rectangle has a length of \( 3x - 2 \) centimeters and a width of \( x \) centimeters. What is the length of the rectangle?

a. Substitute the dimensions of the rectangle into the perimeter formula \( P = 2w + 2l \).

b. Solve for \( x \).

c. Find the length of the rectangle.
Warm Up

1. **Vocabulary** When two or more quantities are multiplied, each is a ________ (term, factor) of the product.

Simplify.

2. \(2x(3x - 5)\)

3. \(-3x^2y(4x^2 - 7xy)\)

4. \(\frac{x^5}{x^{-3}}\)

5. \(\frac{1}{(-4)^3}\)

New Concepts

Simplifying expressions that contain numbers often requires knowledge of prime numbers and factors. Recall that a prime number is a whole number that is only divisible by itself and 1.

\[
\begin{align*}
2 & = 1 \cdot 2 \\
5 & = 1 \cdot 5 \\
13 & = 1 \cdot 13 \\
19 & = 1 \cdot 19 \\
\end{align*}
\]

All whole numbers other than 1 that are not prime are composite numbers. Composite numbers have whole-number factors other than 1 and the number itself. They can be written as a product of prime numbers, which is called the prime factorization of a number.

\[
\begin{align*}
4 & = 2 \cdot 2 \\
6 & = 2 \cdot 3 \\
8 & = 2 \cdot 2 \cdot 2 \\
\end{align*}
\]

Several methods can be used to find the prime factorization of a number. The process requires breaking down the composite numbers until all the factors are prime numbers.

The prime factorization for the number 24 can be found in at least three ways.

\[
\begin{align*}
24 & = 2 \cdot 12 \\
& = 2 \cdot 2 \cdot 6 \\
& = 2 \cdot 2 \cdot 2 \cdot 3 \\
\end{align*}
\]

It does not matter which method is used to find a prime factorization. The final product, however, must consist of only prime numbers. The factors are usually written in ascending order.
Example 1  Finding the Prime Factorization of a Number

Find the prime factorization of each number.

a. 120

SOLUTION

Method 1: List the factors and then the prime factors.

\[
120 = 2 \cdot 60 = 2 \cdot 2 \cdot 30 = 2 \cdot 2 \cdot 2 \cdot 15 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5
\]

Method 2: Use a factor tree.

\[
\begin{array}{c}
120 \\
10 \\
5 \\
2
\end{array}
\begin{array}{c}
12 \\
3 \\
2
\end{array}
\begin{array}{c}
5 \\
3
\end{array}
\begin{array}{c}
4 \\
2
\end{array}
\]

The prime factors are 2, 2, 2, 3, and 5.

Method 3: Use division by primes.

<table>
<thead>
<tr>
<th>2</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

The prime factors are 2, 2, 2, 3, and 5.

The prime factorization of 120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5.

b. 924

SOLUTION

\[
924 = 2 \cdot 462 = 2 \cdot 2 \cdot 231 = 2 \cdot 2 \cdot 3 \cdot 77 = 2 \cdot 2 \cdot 3 \cdot 7 \cdot 11
\]

The prime factorization of 924 = 2 \cdot 2 \cdot 3 \cdot 7 \cdot 11.

Prime factorization can be used when determining the greatest common factor (GCF) of monomials, which is the product of the greatest integer that divides without a remainder into the coefficients and the greatest power of each variable that divides without a remainder into each term.

Finding the GCF means finding the largest monomial that divides without a remainder into each term of a polynomial.
Example 2  Determining the GCF of Algebraic Expressions

Find the GCF of each expression.

a. $6a^2b^3 + 8a^2b^2c$

**SOLUTION**

Write the prime factorization for both terms.

$6a^2b^3 = 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b$

$8a^2b^2c = 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b \cdot b \cdot c$

Find all factors that are common to both terms.

$6a^2b^3 = 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b$

$8a^2b^2c = 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b \cdot b \cdot c$

Each term has one factor of 2, two factors of $a$ and two factors of $b$, so the GCF of $6a^2b^3$ and $8a^2b^2c$ is $2 \cdot a \cdot a \cdot b \cdot b = 2a^2b^2$.

b. $8c^3d^2e - 12c^3d^2e^2$

**SOLUTION**

$8c^3d^2e = 2 \cdot 2 \cdot 2 \cdot c \cdot c \cdot c \cdot d \cdot d \cdot e$

$12c^3d^2e^2 = 2 \cdot 2 \cdot 3 \cdot c \cdot c \cdot c \cdot d \cdot d \cdot d \cdot e \cdot e$

The GCF is $4c^3d^2e$.

Finding the GCF of a polynomial allows you to factor it and to write the polynomial as a product of factors instead of the sum or difference of monomials.

Factoring a polynomial is the inverse of the Distributive Property. Using the Distributive Property will “undo” the factoring of the GCF.

Example 3  Factoring a Polynomial

Factor each polynomial completely.

a. $6x^3 + 8x^2 - 2x$

**SOLUTION**

Find the GCF of the terms. The GCF is $2x$.

Write each term of the polynomial with the GCF as a factor.

$6x^3 + 8x^2 - 2x = 2x \cdot 3x^2 + 2x \cdot 4x - 2x \cdot 1$

$2x(3x^2 + 4x - 1)$

**Check**

$2x \cdot (3x^2 + 4x - 1)$

$2x \cdot (3x^2) + 2x \cdot (4x) - 2x \cdot (1)$ Use the Distributive Property.

$6x^3 + 8x^2 - 2x$ Multiply each term by the GCF.

The factored polynomial is the same as the original polynomial.
Lesson 38

b. \(9x^4y^2 - 9x^6y\)

**SOLUTION**

The GCF of the polynomial is \(9x^4y\).

\[9x^4y^2 - 9x^6y = 9x^4y \cdot y - 9x^4y \cdot x^2\]

The factored polynomial is \(9x^4y (y - x^2)\).

Fractions can be simplified if the numerator and denominator contain common factors. This is because the operations of multiplication and division undo each other.

**numeric fractions:** \(\frac{4}{10} = \frac{2}{5}\) and \(\frac{8}{4} = \frac{2}{1}\) or 2

**algebraic fractions:** \(\frac{4x}{10} = \frac{2x}{5}\) and \(\frac{8x}{4} = 2x\)

Notice that there is no addition or subtraction involved in the fractions above. Simplifying fractions with addition or subtraction in the numerator follows similar rules to adding or subtracting numeric fractions. A fraction can only be simplified if the numerator and the denominator have common factors.

**Example 4 Simplifying Algebraic Fractions**

Simplify each expression.

**a.** \(\frac{3p + 3}{3}\)

**SOLUTION**

\[
\frac{3p + 3}{3} = \frac{3(p + 1)}{3} \quad \text{Factor out the GCF.}
\]

\[
= p + 1 \quad \text{Simplify.}
\]

**b.** \(\frac{5x - 25x^2}{5xy}\)

**SOLUTION**

\[
\frac{5x - 25x^2}{5xy} = \frac{5x(1 - 5x)}{5xy} \quad \text{Factor out the GCF.}
\]

\[
= \frac{1 - 5x}{y} \quad \text{Simplify.}
\]
**Example 5**  Application: Finding the Height of an Object

The formula \( h = -16t^2 + 72t + 12 \) can be used to represent the height of an object that is launched into the air from 12 feet off the ground with an initial velocity of 72 feet/second. Rewrite the formula by factoring the right side using the GCF and making the \( t^2 \)-term positive.

**SOLUTION**

The GCF of the monomials is 4. To keep the \( t^2 \)-term positive, factor out \(-4\). So \( h = -4(4t^2 - 18t - 3)\).

**Lesson Practice**

**Find the prime factorization of each number.**

- a. 100
- b. 51

**Find the GCF of each expression.**

- c. \( 24mn^4 + 32m^5p \)
- d. \( 5p^2q^3r^2 - 10pq^2r^3 \)

**Factor each polynomial completely.**

- e. \( 8d^3e^3 + 12d^3e^2 \)
- f. \( 12x^4y^2z - 42x^3y^3z^2 \)

**Factor each expression completely.**

- g. \( \frac{6x + 18}{6} \)
- h. \( \frac{18x + 45x^3}{9x} \)

**i.** The formula \( h = -16t^2 + 60t + 4 \) can be used to find the height of an object that is launched into the air from 4 feet off the ground with an initial velocity of 60 feet/second. Rewrite the formula by factoring the right side of the equation using the GCF and making the \( t^2 \)-term positive.

**Practice**  Distributed and Integrated

Solve each equation for the variable indicated.

1. \( 6 = hj + k \) for \( j \)

2. \( \frac{a + 3}{b} = c \) for \( a \)

**Draw a graph that represents each situation.**

- 3. A tomato plant grows taller at a steady pace.
- 4. A tomato plant grows at a slow pace, and then grows rapidly with more sun and water.
- 5. A tomato plant grows slowly at first, remains a constant height during a dry spell, and then grows rapidly with more sun and water.

**Find each unit rate.**

- 6. Thirty textbooks weigh 144 pounds.
- 7. Doug makes $43.45 in 5.5 hours.
8. Write $2 \times 10^6$ in standard notation.

9. Solve $\frac{p}{3} = \frac{18}{21}$.

10. Find the prime factorization of 140.

11. **Multiple Choice** Which of the following expressions is the correct simplification of $\frac{10x + 5}{5}$?
   - A $2x + 5$
   - B $2x + 1$
   - C $10x + 1$
   - D $5x$

12. **Free Fall** The function $h = 40 - 16t^2$ can be used to find the height of an object as it falls to the ground after being dropped from 40 feet in the air. Rewrite the equation by factoring the right side.

13. **Write** Explain how the Distributive Property and factoring a polynomial are related.

14. **Generalize** Explain why the algebraic fraction $\frac{6(x - 1)}{6}$ can be reduced, and why the fraction $\frac{6x - 1}{6}$ cannot be reduced.

15. **Biology** The approximate diameter of a DNA helix is $0.000000002$ meters. Write this number in scientific notation.

16. **Measurement** A nanosecond is one-billionth of a second. Write this number in scientific notation.

17. Write $78,000,000$ in scientific notation.

18. **Geometry** A square has side length $6.04 \times 10^{-5}$ meters. What is its area?

19. The triangles are similar. Find the missing length.

   ![Diagram of similar triangles with side lengths 12, x, 9, and 12, 9, 12.]

20. **Multi-Step** An adult brain weighs about 3 pounds.
   - a. There are about 100 billion brain cells in the brain. Write this number in scientific notation.
   - b. Divide the weight of an adult brain by the number of cells and find how many pounds one brain cell weighs. Write the answer in scientific notation.

21. **Analyze** Find the $x$- and $y$-intercepts for $y = 12x$ and explain how they relate to the graph of the equation.

22. **Fundraising** The math club has a carwash to raise money. Out of the first 40 vehicles, 22 are SUVs and 18 are cars. What are the odds against the next one being a car?

23. **Justify** Explain why the statement $3^{-2} = -6$ is false.
24. **Analysis** (Inv 3) A bookstore wants to show the number of different types of books that were sold on a given day. Why is this graph misleading?

![Graph showing Number of Books Sold by Type]

*25. Determine if the sequence $\frac{5}{4}, 2, \frac{11}{4}, \frac{7}{2}, \ldots$ is an arithmetic sequence. If yes, find the common difference and the next two terms.

26. **Multiple Choice** (Inv 3) Which equation is in standard form?
   
   A $y - 6 = 3(x + 4)$  
   B $y = -6x + 13$  
   C $10y = 12y + 25$  
   D $9x + 11y = 65$

27. How is the value 30 represented in a stem-and-leaf plot?

28. **Pool Charges** (Inv 3) Barton Springs Pool charges $2 a visit plus a membership fee of $20.90 a month. Blue Danube Pool charges $2.95 a visit, with no membership fee. At what number of visits per month will the total fees for each pool be the same?

29. **Stock Market** (Inv 3) On a day of heavy trading, one share of ABC Industries’ stock originally decreased by $5 only to increase later in the day by twice the original value. The stock ended the day at $43 a share. What was the starting price of one share?

30. **Multi-Step** (Inv 3) The table shows the total number of shrubs a gardener planted after each half hour.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Shrubs</strong></td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

a. Plot this data on a coordinate grid.

b. Determine if the graph is a function. Explain.

c. **Predict** Can you predict the number of shrubs the gardener will plant in 3 hours? Why or why not?
Warm Up

1. **Vocabulary** The set of \( \underline{\text{real}} \) numbers includes all rational and irrational numbers.

Simplify.

2. \(-3x^2y (4x^2y^{-1} - xy)\)

3. \(mn(2x - 3my + 5ny)\)

4. \(\frac{5x - 25x^2}{5x}\)

5. Factor. \(3a^2b^3 - 6a^4b + 12ab\)

New Concepts

A **rational expression** is an expression with a variable in the denominator. Rational expressions can be treated just like fractions. As with fractions, the denominator cannot equal zero. Therefore, any value of the variable that makes the denominator equal to zero is not permitted.

Variables stand for unknown real numbers. So, all properties that apply to real numbers also apply to rational expressions. The Distributive Property can be used to simplify rational expressions.

**Example 1** Distributing Over Addition

Simplify \(\frac{x^2}{y^2}\left(\frac{x^2}{y} + \frac{3y^3}{m}\right)\).

**SOLUTION**

\[
\frac{x^2}{y^2}\left(\frac{x^2}{y} + \frac{3y^3}{m}\right) = \left(\frac{x^2}{y^2} \cdot \frac{x^2}{y}\right) + \left(\frac{x^2}{y^2} \cdot \frac{3y^3}{m}\right) \\
= \frac{x^4}{y^3} + \frac{3x^2y^3}{ym} \\
= \frac{x^4}{y^3} + \frac{3x^2y^2}{ym}; \quad y \neq 0, m \neq 0
\]

Note that \(y \neq 0\) and \(m \neq 0\) because either value would make the denominator equal to zero.
**Example 2** Distributing Over Subtraction

Simplify \( \frac{m}{z} \left( \frac{apx}{mk} - 2m^4p^4 \right) \).

**SOLUTION**

\[
\frac{m}{z} \left( \frac{apx}{mk} - 2m^4p^4 \right) = \frac{mapx}{zk} - \frac{2m^4p^4}{z} \quad \text{Distribute } \frac{m}{z}.
\]

Note that \( z \neq 0, k \neq 0, m \neq 0 \) because any of those values would make a denominator equal to zero. Although there is not an \( m \) in the denominator of the final expression, there is one in the denominator of the original expression; that is why \( m \neq 0 \).

When simplifying an expression with negative exponents, the final expression should not have negative exponents.

**Example 3** Simplifying with Negative Exponents

Simplify each expression.

a. \( \frac{b^3}{d^3} \left( \frac{2b^2}{d} - \frac{f^{-3}d}{b} \right) \)

**SOLUTION**

\[
\frac{b^3}{d^3} \left( \frac{2b^2}{d} - \frac{f^{-3}d}{b} \right) = \frac{b^3 \cdot 2b^2}{d^3 \cdot d} - \frac{b^3 \cdot f^{-3}d}{d^3b} \quad \text{Distribute } \frac{b^3}{d^3}.
\]

\[
= \frac{2b^5}{d^2} - \frac{b^4f^{-3}d}{d^3b} \quad \text{Product Property of Exponents}
\]

\[
= 2b^5d^{-2} - \frac{b^4d^4}{f^3} \quad d \neq 0, b \neq 0, f \neq 0 \quad \text{Simplify.}
\]

b. \( \frac{n^{-1}}{m} \left( \frac{mx}{cn^{-3}p^{-5}} + 5n^{-4}p^{-5} \right) \)

**SOLUTION**

\[
\frac{n^{-1}}{m} \left( \frac{mx}{cn^{-3}p^{-5}} + 5n^{-4}p^{-5} \right) = \frac{n^{-1}mx}{mcn^{-3}p^{-5}} + \frac{n^{-1} \cdot 5n^{-4}p^{-5}}{m} \quad \text{Distribute } \frac{n^{-1}}{m}.
\]

\[
= \frac{n^{-1}x}{cn^{-3}p^{-5}} + \frac{5n^{-5}p^{-5}}{m} \quad \text{Simplify.}
\]

\[
= \frac{n^2xp^5}{c} + \frac{5}{mm^2p^5}; \quad c \neq 0, m \neq 0, n \neq 0, p \neq 0 \quad \text{Simplify.}
\]
Example 4 Distributing Over Multiple Operations

Simplify each expression.

a. \( \frac{ab}{c^2} \left( \frac{axb}{c} + 2bx - \frac{4}{c^2} \right) \)

**SOLUTION**

\[ \frac{ab}{c^2} \left( \frac{axb}{c} + 2bx - \frac{4}{c^2} \right) \]

\[ = \frac{ab \cdot axb}{c^2} + \frac{ab \cdot 2bx}{c^2} - \frac{ab \cdot 4}{c^2} \]

Distribute \( \frac{ab}{c^2} \).

\[ = \frac{a^2b^2x}{c^2} + \frac{2ab^2x}{c^2} - \frac{4ab}{c^2}; c \neq 0 \]

Simplify.

b. \( \frac{g^2h}{d^2} \left( \frac{g^{-2}xh}{d} - 2h^4x^{-1} + \frac{9}{d^{-3}} \right) \)

**SOLUTION**

\[ \frac{g^2h}{d^2} \left( \frac{g^{-2}xh}{d} - 2h^4x^{-1} + \frac{9}{d^{-3}} \right) \]

\[ = \frac{g^0xh^2}{d} - \frac{2h^4x^{-1}g^2}{d^2} + \frac{9g^2h}{d^{-1}} \]

Distribute \( \frac{g^2h}{d^2} \).

\[ = \frac{xh^2}{d} - \frac{2h^4g^2}{xd^2} + \frac{9g^2h}{d}; d \neq 0 \text{ and } x \neq 0 \]

Simplify.

Example 5 Application: Furniture

A tabletop that is in the shape of a trapezoid has height \( \frac{a^2c}{b} \), and bases \( \frac{b^3}{c} \) and \( \frac{da}{c^2} \). The area of the tabletop is represented by the expression

\[ \frac{a^2c}{2b} \left( \frac{b^3}{c} + \frac{da}{c^2} \right) \].

Simplify the expression.

**SOLUTION**

\[ \frac{a^2c}{2b} \left( \frac{b^3}{c} + \frac{da}{c^2} \right) \]

\[ = \frac{a^2cb^3}{2bc} + \frac{a^2c \cdot da}{2bc^2} \]

Distribute.

\[ = \frac{a^2b^3c}{2bc} + \frac{a^3cd}{2bc^2} \]

Multiply.

\[ = \frac{a^2b^3}{2} + \frac{a^3d}{2bc} \]

Simplify.

The area of the tabletop can be represented by the simplified expression

\[ \frac{a^2b^3}{2} + \frac{a^3d}{2bc} \].
Lesson Practice

Simplify each expression.

\[ \frac{r^2}{q^3} \left( \frac{r^2}{q^3} + \frac{7q^3}{w} \right) \]
\[ \frac{t^{uay}}{tq} - 2t^3y^2 \]
\[ \frac{j^{-2}}{m} \left( \frac{j^{-3}}{m^{-2}} + \frac{9m^3}{k} \right) \]
\[ \frac{n^{-2}}{z} \left( \frac{v^{-2}cb}{nv^{-1}} - 4n^2b^{-3} \right) \]
\[ \frac{fs}{d^3} \left( fhs + 2sk - \frac{7}{d^5} \right) \]
\[ \frac{z^2x}{w^2} \left( \frac{zd^2x}{w} + 5tz - \frac{2}{w^{-2}} \right) \]

\[ \frac{t^2y}{z} \left( \frac{t^3}{y^2} + \frac{z^4}{y^t} \right) \]

Math Reasoning

Generalize For problem (f), which variable in the numerator cannot equal zero? Explain.

Practice Distributed and Integrated

Solve each equation. Check your answer.

1. \( \frac{4y + \frac{3}{2}}{2} = -18 \)
2. \( x - 4 + 2x = 14 \)
3. True or False: The set of integers is closed under division. If false, give a counterexample.

Translate words into algebraic expressions.

4. the sum of \( a \) and 3
5. 2.5 more than \( k \)
6. 3 less than \( x \)
7. 2 more than the product of 3 and \( y \)

*8. Simplify \( \frac{d^2}{s^3} \left( \frac{d^2}{s} + \frac{9s^3}{h} \right) \).

*9. Write Why isn’t division by zero allowed?

*10. Justify Simplify \( \frac{x^{-2}}{n^{-3}} \left( 2x^{-4} + n^{-3} \right) \) and explain each step.

*11. Multiple Choice Simplify \( \frac{g^{-2}s}{b^3} \left( \frac{g^{-3}s^{-1}}{b^{-1}} + \frac{4}{b^3} \right) \).

A \( \frac{4g^{-5}s^{-1}}{b^4} \)
B \( \frac{g^{-5}s^{-1}}{b} + \frac{4g^{-2}s}{b^5} \)
C \( \frac{g^{-5}}{b} + \frac{4g^{-2}s}{b^5} \)
D \( \frac{1}{bg^5} + \frac{4s}{b^5g^2} \)
*12. Simplify the expression \( \frac{w^2 p}{t} \left( \frac{4}{w^4} - \frac{t^3}{p^5} \right) \).

*13. Find the prime factorization of 918.

*14. **Error Analysis** Two students factor the polynomial \( 16x^4y^2z + 28x^3y^4z^2 + 4x^3y^2z^3 \) as shown below. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4x^3y^2z(4x + 7y^2z) )</td>
<td>( 4x^3y^2z(4x + 7y^2z + 1) )</td>
</tr>
</tbody>
</table>

15. **Geometry** The area of a rectangle is represented by the polynomial \( 6a^2b + 15ab \). Find two factors that could be used to represent the length and width of the rectangle.

16. **Multiple Choice** Complete the following statement: The side lengths of similar figures ________.
   A. must be congruent
   B. cannot be congruent
   C. are in proportion
   D. must be whole numbers

17. **Multi-Step** Use the expression \( 24x^2y^3 + 18xy^2 + 6xy \).
   a. What is the GCF of the polynomial?
   b. Use the GCF to factor the polynomial completely.

*18. **Probability** The probability that a point selected at random is in the shaded region of the figure is represented by the fraction \( \frac{\text{area of shaded rectangle}}{\text{area of entire rectangle}} \). Find the probability. Write your answer in simplest form.

*19. **Analyze** In order to double the volume of water in a fish tank, is it necessary to double the length, width, and height of the tank? If yes, explain why. If no, explain how to double the volume of water.

*20. Graph \( 27x + 9y = 54 \) using the \( x \)- and \( y \)-intercepts.

*21. **Fundraising** For a fundraiser, the science club sold posters for $5 and mugs for $8. The equation \( 5x + 8y = 480 \) shows that they made $480. Find the \( x \)- and \( y \)-intercepts.

22. **Entertainment** A contestant is in the bonus round of a game show where she can win $1500 for answering the first question correctly and then an additional $500 for each correct response to each of the next five questions. If she answers all of the questions correctly, how much money will she receive when she answers the sixth question?
23. Write 0.00608 in scientific notation.

24. **Error Analysis** Two students write $1.32 \times 10^{-5}$ in standard form. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000132</td>
<td>0.0000132</td>
</tr>
</tbody>
</table>

25. **Verify** Show that $4\frac{3}{4}$ is the solution to $\frac{1}{n-1} = \frac{4}{15}$.

26. Evaluate the expression $\frac{x^3y^2}{z^2}$ if $x = 3$, $y = 4$, and $z = -2$.

27. **Analyze** The odds of winning a CD in a raffle are 3:7. Explain how to find the probability of not winning a CD.

28. **Stamp Collecting** The table shows some collectible stamps with their estimated values. Explain whether the ordered pairs, such as (2, $2) and (2, $3), will be a function.

<table>
<thead>
<tr>
<th>Number</th>
<th>Stamp</th>
<th>Value (low)</th>
<th>Value (high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11¢ President Hayes (1931)</td>
<td>$2</td>
<td>$4</td>
</tr>
<tr>
<td>2</td>
<td>14¢ American Indian (1931)</td>
<td>$2</td>
<td>$3</td>
</tr>
<tr>
<td>3</td>
<td>4¢ President Taft (1930)</td>
<td>$1</td>
<td>$3</td>
</tr>
<tr>
<td>4</td>
<td>1¢ Benjamin Franklin (1911)</td>
<td>$5</td>
<td>$50</td>
</tr>
</tbody>
</table>

29. **Salaries** In an interview with a potential employee, an employer shows a line graph displaying the average salary of employees over several years. Explain why the graph is potentially misleading and why the employer might have shown this graph.

30. **Formulate** Write a rule for the table in function notation.

<table>
<thead>
<tr>
<th>g</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(g)</td>
<td>1.5</td>
<td>2.5</td>
<td>3.5</td>
<td>4.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>
**Simplifying and Evaluating Expressions Using the Power Property of Exponents**

### Warm Up

1. **Vocabulary** The _________ is the number that tells how many times the base of a power is used as a factor.

Simplify.

2. \((4x^2y^3)(5x^4y^4)\)

3. \(\frac{24x^3y^6}{36x^2y^3}\)

4. \((-3)^2 - 3^2\)

5. Compare: \(4^2 + \sqrt{36} \bigcirc -(\sqrt{3})^2 + \sqrt{25}\). Use >, <, or =.

### New Concepts

Previous lessons have explored expressions involving exponents. Several rules and definitions have been developed.

\[ x^0 = 1 \]

\[ x^m \cdot x^n = x^{m+n} \]

\[ x^1 = x \]

\[ x^{-n} = \frac{1}{x^n} \]

\[ \frac{x^m}{x^n} = x^{m-n} \]

There is another property of exponents that involves raising a power to a power.

#### Exploration: Raising a Power to a Power

This Exploration shows how to raise a power to a power.

The expression \((2^4)^3\) means to use \(2^4\) as a factor three times.

\[(2^4)^3 = 2^4 \cdot 2^4 \cdot 2^4 = 2^{12}\]

Simplify.

\[ a. \quad (3^1)^3 \]

\[ b. \quad (4^2)^2 \]

\[ c. \quad (7^3)^4 \]

\[ d. \quad Are \ there \ any \ patterns? \ What \ conclusions \ can \ you \ draw \ from \ the \ patterns? \]

The expression \((a^2)^3\) means to use \(a^2\) as a factor three times.

\[(a^2)^3 = a^2 \cdot a^2 \cdot a^2 = a^6\]

Simplify.

\[ e. \quad (a^3)^5 \]

\[ f. \quad (b^5)^2 \]

\[ g. \quad (d^4)^4 \]

\[ h. \quad Using \ the \ results \ from \ a \ through \ g \ above, \ write \ a \ rule \ for \ raising \ a \ power \ to \ a \ power. \]
The pattern in the Exploration leads to the Power of a Power Property.

### Power of a Power Property

If \( m \) and \( n \) are real numbers and \( x \neq 0 \), then

\[
(x^m)^n = x^{mn}.
\]

#### Example 1 Simplifying a Power of a Power

Simplify each expression.

**a.** \((2^3)^2\)

**SOLUTION**

\[
(2^3)^2 = 2^{3 \cdot 2} = 2^6
\]

**b.** \((a^6)^3\)

**SOLUTION**

\[
(a^6)^3 = a^{6 \cdot 3} = a^{18}
\]

The Power of a Power Property and the Product Property of Exponents can be used together to formulate a rule for the power of a product. Look at the expression \((5x^2)^3\). The outer exponent of 3 means to use everything inside the parentheses as a factor three times, or to multiply \(5x^2\) three times.

\[
(5x^2)^3 = 5x^2 \cdot 5x^2 \cdot 5x^2
\]

\[
= (5 \cdot 5 \cdot 5) \cdot (x^2 \cdot x^2 \cdot x^2) = 5^3 \cdot x^6 = 125x^6
\]

This pattern is summarized in the Power of a Product Property.

### Power of a Product Property

If \( m \) is a real number with \( x \neq 0 \) and \( y \neq 0 \), then

\[
(xy)^m = x^my^m.
\]

#### Example 2 Simplifying a Power of a Product

Simplify each expression.

**a.** \((7a^3b^5)^3\)

**SOLUTION**

\[
(7a^3b^5)^3 = 7^3 \cdot a^{3 \cdot 3} \cdot b^{5 \cdot 3} = 343a^9b^{15}
\]

**b.** \((-2y^4)^3\)

**SOLUTION**

\[
(-2y^4)^3 = (-2)^3 \cdot y^{4 \cdot 3} = -8y^{12}
\]

#### Example 3 Application: Interior Design

A square family room is being measured for carpeting. If the length of one side of the room is \(2x\) feet, what is the area of the room?

**SOLUTION**

The area is \((2x)^2 = 4x^2\) square feet.
Power of a Quotient Property

If \( x \) and \( y \) are any nonzero real numbers and \( m \) is an integer, then
\[
\left( \frac{x}{y} \right)^m = \frac{x^m}{y^m}.
\]

**Example 4**  Simplifying a Power of a Quotient

Simplify each expression.

**a.** \( \left( \frac{2x}{5} \right)^2 \)

**SOLUTION**
\[
\left( \frac{2x}{5} \right)^2 = \frac{4x^2}{25}
\]

**b.** \( \left( \frac{-x^2}{3y^3} \right)^4 \)

**SOLUTION**
\[
\left( \frac{-x^2}{3y^3} \right)^4 = \frac{(-1)^4(x^2)^4}{(3y^3)^4} = \frac{x^8}{81y^{12}}
\]

The rules for exponents apply to many expressions with powers. When simplifying expressions, be sure to follow the order of operations.

**Example 5**  Simplifying Expressions with Powers

Simplify each expression.

**a.** \( (4xy^2)^2(2x^3y)^2 \)

**SOLUTION**
\[
(4xy^2)^2(2x^3y)^2 = (16x^4y^4)(4x^6y^2) = 64x^{10}y^6
\]

**b.** \( (-5x^{-2})^2(3xy^2)^4 \)

**SOLUTION**
\[
(-5x^{-2})^2(3xy^2)^4 = (-25x^{-4})(81x^4y^8) = 2025x^0y^8 = 2025y^8
\]

**Lesson Practice**

Simplify each expression.

**a.** \( (5^2)^2 \)

**b.** \( (b^2)^7 \)

**c.** \( (-3n^4)^2 \)

**d.** \( (9ab^{-2})^2(2a^2b^4) \)

**e.** \( \left( \frac{3y^4}{4} \right)^3 \)

**f.** \( \left( \frac{-x}{7y^5} \right)^2 \)

**g.** A shipping container is in the shape of a cube with a side length of \( 3x \) inches. What is the volume of the container?
Solve each proportion.

1. \( \frac{3}{12} = \frac{-24}{m} \)
2. \( \frac{-4}{0.8} = \frac{2}{x - 1} \)
3. \( \frac{5}{12} = \frac{1.25}{k} \)

4. True or False: All whole numbers are integers. If false, give a counterexample.

5. Simplify \((4^4)^5\) using exponents.

6. **Multiple Choice** Which expression simplifies to \(-24x^4y^3z^3\)?
   - A \(-(2x^2y)^3(6y)\)
   - B \(-2(x^2y)^3(6y)\)
   - C \((2x^2y)^3(6y)\)
   - D \((2xy)^3(3)\)

7. Simplify \(\frac{e^3}{r^3}\left(\frac{e^2}{4r} + \frac{r^3}{k}\right)\).

8. **Cooking** Use the formula \(A = \pi r^2\) for the area of a circle. A 6-inch pizza covers an area of \(\pi(6)^2 = 36\pi\) square inches. What happens to the area of the pizza if you double the radius and make a 12-inch pizza?

9. **Verify** Is the statement \((a + b)^n = a^n + b^n\) true? Verify your answer with a numeric example.

10. **Generalize** When do you know to add exponents and when to multiply exponents?

11. **Painting** A rectangular top on a bench is to be painted.
    Its area is \(\frac{wd^3}{c}\left(\frac{d}{w^4} + \frac{c^3}{wd}\right)\). Simplify.

12. Simplify \(\frac{d^2}{d^3}\left(\frac{a - x}{d} - \frac{2x}{d^3}\right)\).

13. **Geometry** The equation of an ellipse is \(\frac{w^2y^2}{g^2} + \frac{w^2y^2}{w^2} = 1\). To enlarge the ellipse, the left side is multiplied by \(\frac{g^5}{w^2}\). This expression is \(\frac{g^5}{w^2}\left(\frac{w^2y^2}{g^2} + \frac{w^2y^2}{w^2}\right)\). Simplify.

14. The trim around a window has a total length of \(\frac{rt}{w^3}\left(\frac{rt}{w} + 2ty - \frac{8}{w^2}\right)\).
   a. Simplify the expression.
   b. Identify the variables that cannot equal zero.

15. Find the GCF of \(4xy^2z^4 - 2x^2y^3z^2 + 6x^3y^4z\).

16. **Error Analysis** Two students are simplifying the fraction \(\frac{3x - 6}{9}\) as shown below. Which student is correct? Explain the error.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3x - 6}{9}) = (\frac{x - 2}{3})</td>
<td>(\frac{3x - 6}{9}) = (\frac{x - 2}{3})</td>
</tr>
</tbody>
</table>
17. **Shipping** A shipping container is in the shape of a rectangular box that has a length of 10x + 15 units, a width of 5x units, and a height of 2 units.
   a. Write an expression that can be used to find the volume of the box.
   b. Factor the expression completely.

18. 0.78 of 250 is what number?

19. Give the domain and range of \{(4, 9); (4, 7); (2, 4); (5, 12); (9, 4)\}.

20. The heights of 8 trees were 250, 190, 225, 205, 180, 240, 210, and 220 feet. How could a misleading graph make you think the trees are all very similar in size?
   a. Make a bar graph of the data using a broken axis.
   b. Make a bar graph of the data using large increments.
   c. Compare the two graphs.

21. **Justify** Without changing the number to standard form, explain how you can tell that -10 < 1 \times 10^{-4}.

22. **Multiple Choice** What is \(\frac{1.6 \times 10^7}{6.4 \times 10^5}\) in scientific notation?
   A. \(2.5 \times 10^4\)  
   B. \(0.25 \times 10^5\)  
   C. \(2.5 \times 10^6\)  
   D. \(4 \times 10^5\)

23. **Astronomy** The diameter of the moon is approximately 3,480,000 meters. Write this distance in scientific notation.

24. The rectangles below are similar. Find the missing length.

25. **Drama** The cost of presenting a play was $110. Each ticket was sold for $5.50. The equation \(11x - 2y = 110\) shows how much money was made after ticket sales. Graph this equation using the intercepts.

26. **Justify** Is the sequence 0.2, 2, 20, 200, … an arithmetic sequence? Justify your answer.

27. There are 2 yellow stickers and 4 purple stickers. Make a tree diagram showing all possible outcomes of drawing two stickers. How many possible ways are there to draw a purple sticker, keep it, and then draw another purple sticker?

28. In a stem-and-leaf plot, which digit of the number 65 would be a leaf?

29. **Measurement** How many inches are there in 18 yards?

30. **Analyze** The rule for negative exponents states that for every nonzero number \(x\), \(x^{-n} = \frac{1}{x^n}\). Explain why the base, \(x\), cannot be zero.
There are two basic kinds of reasoning: deductive and inductive. Deductive reasoning bases a conclusion on laws or rules. Inductive reasoning bases a conclusion on an observed pattern. Both types of reasoning can be used to support or justify conclusions.

All fruit have seeds. An apple is a fruit.

The two statements form an argument. The first statement is the premise, and the second statement is the conclusion. In deductive reasoning, if the argument is solid, the conclusion is guaranteed. In inductive reasoning, if the argument is solid, the conclusion is supported but not guaranteed. Consider the following examples:

<table>
<thead>
<tr>
<th>Daryl</th>
<th>Aliya</th>
</tr>
</thead>
<tbody>
<tr>
<td>According to Newton’s First Law, every object will remain in uniform motion in a straight line unless compelled to change its state by the action of an external force. So, if I kick a ball, it will travel forward at a constant speed until it hits the wall.</td>
<td>In the past, I’ve noticed that every time I kick a soccer ball, it travels forward at a constant speed until it hits the wall. The next time I kick a ball, it will keep going until it hits the wall.</td>
</tr>
</tbody>
</table>

Daryl’s reasoning is deductive because it is based on his knowledge of Newton’s First Law of Motion. Aliya reasons inductively, basing her conclusions on her observations.

Identify the type of reasoning used. Explain your answer.

1. Premise: A student has earned a score of 100 on the last five math tests. Conclusion: The student will earn a score of 100 on the next math test.
2. Premise: The measures of three angles of a rectangle are all 90°. Conclusion: The measure of the fourth angle is 90°.
3. Premise: A number pattern begins with 3, 5, 7, 9, 11, ... Conclusion: The next number in the pattern will be 13.

Each premise and conclusion above can be written as one sentence. For instance, the second set could be restated as, “If the measures of three angles of a rectangle are all 90°, then the measure of the fourth angle is 90°.” This is called a conditional statement. A conditional statement is a logical statement that can be written in “if-then” form.

A conditional statement is made up of two parts: a hypothesis and a conclusion. The hypothesis is the condition. It follows the word “if.” The conclusion is the judgment. It follows the word “then.” A conditional statement can either be true or false.
Write Use the given hypothesis to write a true or false conditional statement.
4. Write a true conditional statement: If you stay in the sun too long, ....
5. Write a true conditional statement: If a student has a temperature higher than 101 degrees, ....
6. Write a false conditional statement: If a number is divisible by 5, ....
The statement “If a figure has four sides, then it is a square” is false. It is not true because a rectangle has four sides, but a rectangle is not a square. An example that contradicts a statement is called a counterexample. One counterexample is sufficient to show that a statement is false.
Provide a counterexample for each statement.
7. The sum of a positive number and a negative number is negative.
8. If a student is a teenager, then she is 14 years old.

Investigation Practice

Identify the type of reasoning used. Explain your answer.
a. My friend has an allergic reaction when he eats peanuts.
b. If a driver sees a red light, she should stop.
Use the given hypothesis to write a true or false conditional statement.
c. Write a true conditional statement: If it rains today, ....
d. Write a false conditional statement: If \(x = 2\), ....
Use the diagram to write a counterexample for each statement.

![Diagram showing the relationships between Animals, Cats, Female, and Siamese]
e. If an animal is a cat, then it is Siamese.
f. All female cats are Siamese.