Rollercoaster!
Measuring Energy and Force on a Rollercoaster

PURPOSE
In this activity you will measure the potential energy at the top of a ramp, the kinetic energy at the bottom of the ramp, and the energy lost on the ramp. You will then use this value of kinetic energy to determine the height from which the car must roll in order to complete a loop and allow your car to successfully reach the end of the track.

MATERIALS
Each lab group will need the following:
- ball bearing, steel, 1-2 cm dia.
- calculator, TI® graphing
- clamp, pendulum
- clamp, single buret
- meter stick
- paper, carbon
- 10 paper, copy
- racetrack, extension set
- racetrack, sets
- ring stand
- ruler, clear metric
- tape, masking
- washer, 2 in.

PROCEDURE
PART I: TRACK CHARACTERISTICS
In Part I of the lab, you will measure the speed of a car as it rolls off the end of a curved track.

1. Set up the apparatus as shown in Figure 1 below. Line up the end of the molding track with the edge of the table top. If there is extra molding above the clamp at the top of the ring stand, have one of your lab partners hold it up, or find a way to prop it up so that it does not interfere with Part I of the lab.

![Figure 1](image-url)

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2. Measure the vertical distance $d_y$ from the floor to the end of the track, and the height $h$ at which you released the car above the bottom of the track, and record your values in the data table for Part I on your student answer page.

3. Use a tape measure to measure the length of the track $L$ in centimeters from the top of the track at height $h$ to the end of the track (from point A to point B), and record your value in the data table for Part I on your student answer page.

4. Roll the car down the track and allow it to roll off the end of the track and land on the floor. Place a white sheet of paper on the floor at the location where the car landed. Place a piece of carbon paper on top of the white paper with the carbon side down. The next time you roll the car down the ramp and allow it to land on the floor, the car will make a mark on the white paper. You may want to tape the white paper to the floor. Depending on the way the car hits, you may have to make a decision about the actual point of contact. If it lands on the wheels and makes 4 marks, use the center. If it hits on the front end use this mark. Other possibilities may require you to decide where the center of mass of the car hits.

5. Measure the horizontal distance $d_x$ from the point at which the hanging weight touches the floor to the point at which the car landed on the floor, and record your value in the data table for Part I on your student answer page.

6. Repeat step 4 two more times, record your values in the data table for Part I on your student answer page. Use these values to find the average horizontal distance traveled by the car.

7. Answer the Analysis questions for Part I before moving on to the procedure for Part II.

**PART II: LOOP HEIGHT**

In Part I of the lab, you determined the speed and kinetic energy of the car as it came off the end of the track, the initial potential energy of the car at the top of the track, the energy lost by the car on the track, and the energy lost per centimeter of track. In Part II, you will predict the height from which the car must be let go on the track in order to make it just go completely around a loop of specific height. You will also predict the speed of the car at the end of the track and confirm this speed by predicting and measuring the speed of the car as it goes off the table and strikes the floor.

1. Clamp one end of the track to the top of the ring stand or other structure used to elevate the ramp. You may use the same set up and same height $h$ as in Part I. If after making your calculations, you may need to increase the height, or you may need to start the car at some distance down the track. The exact distance $h$ from which you release the car will be calculated after other measurements are made.

2. Add the circular loop onto the end of the track and adjust your apparatus so that the end of the track on the outside of the loop ends at the exact edge of the table.
3. Use a tape measure or meter stick to measure the same length of the track \( L \) in centimeters as in Part I from the top of the track at height \( h \) to the beginning of the loop. Measure the diameter of the circle (\( H \)) and calculate the circumference of the circle. Also measure any distance from the outbound side of the loop.

![Diagram of rollercoaster track loop](image)

**Figure 2**

4. In the Analysis section for Part II of your student answer page, predict the minimum height \( h \) from which the car can be released in order to make it completely around the loop without ever losing contact with the track. Also predict the velocity of the car when it leaves the end of the track based on the amount of kinetic energy the car still posses as it reaches the edge of the track/table and becomes a projectile.

5. Roll the car down the ramp and test the accuracy of your prediction. Do not wind the spring in the car, let the car roll freely using only its gravitational potential energy.

6. If the car does not make it around the loop without losing contact with the track, or if you think the car could be moving slower, adjust the height \( h \) from which the car is released and roll the car again.
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DATA AND OBSERVATIONS

PART I: TRACK CHARACTERISTICS

Mass \( m \) of your car = _______________ kg

Length \( L \) of the track from the top of the ramp to the end of the track = _____________ cm

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Initial Height of Car above Table ( h ) (m)</th>
<th>Distance from Floor to Bottom of Ramp ( d_y ) (m)</th>
<th>Horizontal Distance ( d_x ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2</td>
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<td>3</td>
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</tr>
</tbody>
</table>

ANALYSIS

PART I: TRACK CHARACTERISTICS

In Part I of the lab, we want to find the speed of the car as it leaves the end of the ramp so that we can then find the kinetic energy at the end of the ramp. As the car leaves the end of the ramp, it is moving horizontally and vertically at the same time. In the horizontal direction, it does not accelerate, but moves with a constant speed. In the vertical direction, the car accelerates at 9.80 m/s² downward as it falls. We can combine the horizontal and vertical motions of the car to find its speed, \( v \), as it leaves the end of the track.
1. The time the car is in the air after it leaves the end of the ramp can be found using the motion equation

\[ d_y = \frac{1}{2} gt^2 \]

where \( d_y \) is the vertical distance from the floor to the end of the ramp, and \( g \) is the acceleration due to gravity. Rearrange this equation for the time of flight \( t \) in terms of \( d_y \) and \( g \). Show your steps in the space below.

2. Using your equation for time above, substitute your values and find the actual time the car is in the air after it leaves the ramp.

3. Find the average horizontal distance \( d_x \) the car travels before striking the floor. Show your work in the space below, and be sure to include the proper number of significant digits.

4. Knowing the average horizontal distance \( d_x \) your car traveled before striking the floor and the equation below, find the speed \( v \) of the car in m/s as it leaves the track.

\[ v = \frac{d_x}{t} \]

5. Using the equation for kinetic energy below, find the kinetic energy in Joules of the car as it leaves the end of the track.

\[ KE = \frac{1}{2} mv^2 \]
6. Using the equation for potential energy, find the potential energy in Joules of the car when it is at the top of the ramp at the height \( h \).

\[ PE = mgh \]

7. According to the law of conservation of energy, the total energy of a system is neither created nor destroyed. The total energy of the car at the top of the ramp (point A) is its potential energy, and the total energy of the car at the end of the track (point B) is its kinetic energy.

a. According to your calculations, is the potential energy of the car at point A greater than, less than, or equal to its kinetic energy at the point B? Calculate any loss or gain in energy between points A and B.

b. If the energy at point A is greater than at B, where did the energy go? If the energy at point B is greater than at A, where did it come from?

8. Using the equation below, find the percent of total energy lost or gained between points A and B.

\[ \% \text{Energy lost} = \frac{\text{Energy at A} - \text{Energy at B}}{\text{Energy at A}} \times 100 \]

9. Calculate the energy lost or gained per centimeter of track between points A and B.

\[ \text{Energy lost or gained per cm} = \frac{\text{Energy lost or gained}}{L} \]
PART II: HEIGHT TO COMPLETE A LOOP

Any time the car is in contact with the track, the car is losing energy. In question 9 in the Analysis section of Part I above, you calculated the energy lost per centimeter of track. You will use this value to help you predict the minimum height $h$ from which to release your car in order for it to complete your loop without losing contact with the track.

1. Should the height $H$ of the loop be greater than, less than, or equal to the initial height $h$ of the car at point A? Explain your answer.

2. Measure the maximum height $H$ in centimeters for your loop.

3. Calculate the potential energy PE of the car at the measured height $H$ of the loop. Be sure to use meters for the height so that the units for potential energy will be Joules.

4. Using your value for the energy lost per centimeter of track, calculate the energy lost on the track between the bottom of the loop and the top of the loop. (Hint: the amount of track between the bottom of the track and the top is half the circumference of the loop, and circumference $= \pi \times \text{diameter}$.)
5. Calculate the minimum velocity for the car when it reaches the top of the track in order for the car to complete the loop without losing contact with the track. Remember, the centripetal force must equal the gravitational force at this point.

\[ F_g = F_c \]
\[ mg = \frac{mv^2}{r} \]
\[ rg = v^2 \]
\[ v = \sqrt{rg} \]

6. In order for the car to make it around the loop at the top without losing contact with the track, it must have this minimum or critical velocity. Calculate the kinetic energy for this minimum velocity at the maximum height \( H \).

\[ \text{KE}_T = \underline{\text{__________}} \text{ J} \]

7. The kinetic energy at the bottom of the loop must be enough to convert to potential energy at the top of the loop, and also enough to overcome energy losses around half of the loop plus enough to supply the kinetic energy required to complete the loop. Add these values together to determine the required kinetic energy of the car just as it enters the loop. On the diagram below, label the values for kinetic energy at the top (\( \text{KE}_{\text{top}} \)), the energy lost on the track between the bottom and the top (\( \text{E}_{\text{lost}} \)), and the potential energy at the top (\( \text{PET} \)).

8. Add these values to obtain the required KE at the bottom or beginning of the loop.

\[ \text{Required KE}_{\text{bottom}} = \underline{\text{______}} \text{ J} \]
9. Either use the measured length of the track or calculate its total length. Assuming the length of track before the car reaches the loop is $\pi h/2$, a quarter circle with radius $h$, use this equation to calculate the minimum height from which the car must be released to complete the loop.

$$PE - E_{\text{lost on track}} = KE_{\text{beginning bottom of loop}}$$

$$mgh - (\frac{\pi h}{2} \times 8.7 \times 10^{-3} \text{ J / m}) = KE_{\text{beginning bottom of loop}}$$

PART III: EXTENSION BALL ON THE TRACK

After completing the lab and calculations with the car, replace the car with a steel ball of about 1.5 cm diameter. Before releasing the ball, predict whether the ball will complete the loop as the car did.

1. Prediction:

2. Release the ball from the same height as the car and describe your results.

CONCLUSION QUESTIONS

1. In terms of energy gains and losses, what are some things a rollercoaster design engineer must take into consideration when designing a rollercoaster?

2. If you chose to add a second hill in the track after the loop, what are some of the things you would need to consider to determine the height of the hill?
3. Would a passenger feel “heavier” as she passes the bottom of the loop or the top of the loop? Explain your answer, and draw and label vector arrows representing the forces acting on a passenger at the bottom of the loop and at the top of the loop. (Hint: There are two forces acting on the passenger at both the top of the loop and the bottom.)

![Diagram of forces at the bottom and top of the loop]

4. If we say that the passengers are experiencing an acceleration of one “g”, we mean that they are experiencing an acceleration of 9.80 m/s\(^2\). If a vertical rollercoaster loop has a radius of 10.0 meters and the speed of the rollercoaster at the bottom of the loop is 20.0 m/s and 5.0 m/s at the top, how many g’s of acceleration do the passengers experience at the following points on the loop? Recall that the equation for finding the centripetal acceleration is 
   \[ a_c = \frac{v^2}{r} \].
   a. at the bottom of the loop
   b. at the top of the loop?
5. Using the values for the energy of the car you obtained in Analysis questions 4 and 5 above, find the speed of the car at points 1, 2, and 3, as shown on the diagram below.

\[ v_1 = \_ \_ \_ \_ \_ m/s \]
\[ v_2 = \_ \_ \_ \_ \_ m/s \]
\[ v_3 = \_ \_ \_ \_ \_ m/s \]
6. On the diagram above, draw and label vector arrows representing the forces acting on a passenger at the bottom of the loop (1), halfway between the bottom and the top (2), and the top of the loop (3).
7. Calculate the centripetal force acting on the car at points 1, 2, and 3. Remember that the centripetal force acting on the car at any point on the circular track is the *net* force acting on the car.

8. Calculate the force $F_T$ the track exerts on the car at points 1, 2, and 3. Note that the centripetal force can include both the weight of the car and the force the track exerts on the car.
PART III: EXTENSION BALL ON THE TRACK

A.

1. Can you think of any reasons that the ball did or did not complete the loop. Discuss them here.

2. If the ball had the same mass as the car, would it complete the loop from the same height (H) on the track?

B.

1. Do people of different masses encounter different values of “g” when riding the same roller coaster?

2. Do all riders on the same roller coaster travel at the same speed at the same time?
3. Do all riders of the same roller coaster experience the same values of centripetal acceleration at the same time?

4. Do all riders of the same roller coaster experience the same values of centripetal acceleration at the same point on the track?