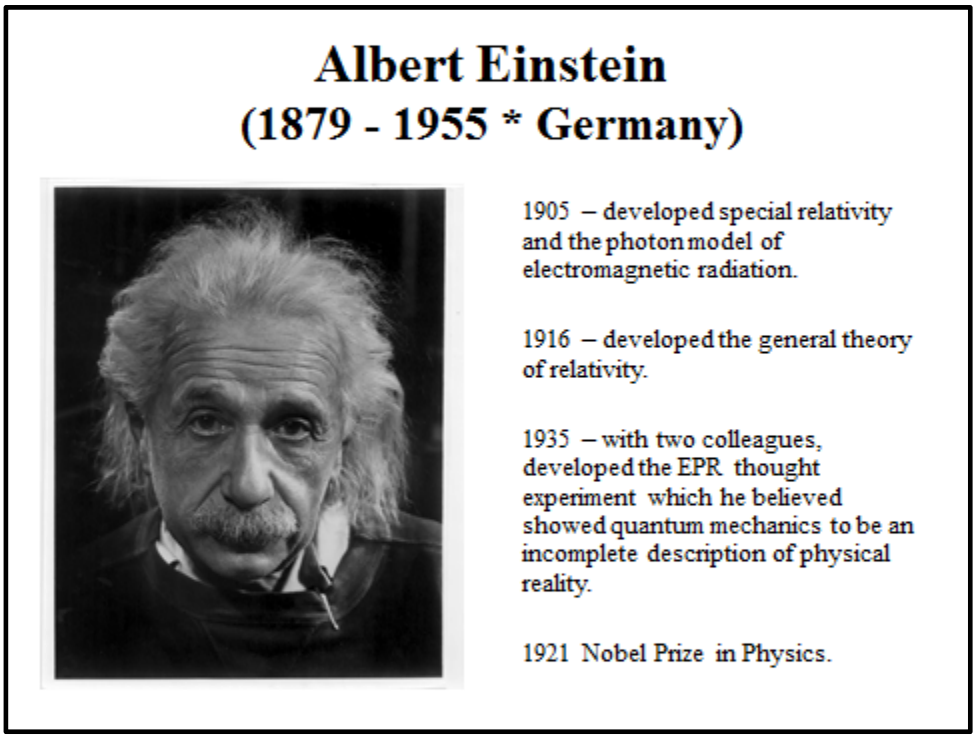
## Special Relativity

### Albert Einstein and the Miracle Year

The year 1905 is often referred to as the Annus Mirabilis (or year of miracles). In this year, Albert Einstein, a 23-year old with an undergraduate degree in physics—working as a Swiss patent clerk—published four papers in the *Annalen der Physik*, the leading German scientific journal. Each paper contributed significantly to modern physics, and one publication won him the Nobel Prize in Physics.

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The first of these papers proposed that the classical wave model was not a complete description of electromagnetic radiation. While there could be little doubt that electromagnetic radiation sometimes behaved as if it were a wave, Einstein proposed that the energy of the radiation could be absorbed or emitted only in discrete amounts. Borrowing Planck's term, he called these discrete amounts quanta. Although, this idea was rejected by most physicists for over 15 years, it is now a fundamental component of our worldview. In 1921, Einstein received the Nobel Prize in physics for this paper.

Although classical physicists generally accepted the reality of atoms, at the start of the twentieth century there was no empirical evidence for their existence. Some influential physicists and chemists at the time saw the atom as simply a useful model rather than an actual real entity.  In his second 1905 paper, Einstein provided a quantitative theory for the motion of suspended particles in a stationary fluid, a phenomenon described by Robert Brown in 1827 and known as Brownian motion.  The theory required the existence of real atoms of definite, finite size.  The experimental verification of Einstein's theory was made by the French physicists Jean Perrin.  Since the publication of Perrin's results in 1908, no one has seriously doubted the atomic theory of matter.  Perrin received the Nobel Prize in physics in 1926 for this work.

The third and fourth papers of 1905 introduced Einstein's most famous contribution to physics: the Theory of Relativity.  Because he generalized his theory in 1915, the 1905 theory is now known as the Special Theory of Relativity.

**Special Relativity and Space-time**

In his third 1905 paper, Einstein revolutionized our understanding of the nature of space and time and at the same time reconciled Maxwell's theory of electromagnetism with the laws of mechanics.

The classical laws of mechanics obey the principle of relativity.  That is, the laws of mechanics have the same mathematical form in all frames of reference that are in uniform motion with respect to one another.  In physics, a frame of reference is simply a coordinate system used for the mathematical description of physical phenomena.  The coordinate system is fixed to the state of motion of the observer of the phenomenon. How an object behaves when tossed in the air in a room on the surface of the earth (the earth frame of reference) is exactly the way it behaves on an airplane traveling with a constant velocity (the airplane frame of reference).  The laws of physics are the same in both frames of reference.  This principle of relativity for the laws of mechanics has been known since the time of Galileo.  (Rather than an airplane, Galileo's example was a ship sailing on a perfectly smooth sea with a constant speed.)

An inertial frame of reference is a frame of reference in which the laws of mechanics have their simplest physical and mathematical form.  All other frames traveling at a constant velocity (a constant speed in a straight line) with respect to an inertial frame of reference are also inertial frames of reference.  However, Maxwell's equations did not appear to satisfy the principle of relativity.  They have their simplest physical and mathematical form only in a single inertial frame of reference and have more complex forms in all other inertial frames.  At the time Maxwell proposed his theory, it was assumed that it was the inertial frame at rest with respect to the ether, the hypothetical medium through which electromagnetic radiation is propagated, that constituted this one special inertial frame.

For example, Maxwell's theory leads inescapably to the prediction that the speed of light in a vacuum is a constant, that is, it has a single unique value. This value is represented by the letter 'c,' and its approximate value is $3 \times 10^8$ m/s or 186,000 miles per second.[1] This is a law of physics in Maxwell's theory but is assumed to be true only in the inertial frame at rest with respect to the ether. In any other inertial frame, it would be expected to have a different value; two observers moving with respect to one another should obviously measure different values for the speed of a light beam. Because this law is not the same in all inertial frames of reference, Maxwell's theory does not satisfy the principle of relativity. Michelson and Morley attempted, unsuccessfully, to measure this predicted difference of the speed of light in different frames of reference in 1887.

Philosophy played an important role in physics for Einstein. The principle of relativity had tremendous philosophical appeal to him, "because it is so natural and simple." At the same time he had complete faith in Maxwell's theory of electromagnetism as it applied to the constancy of the speed of light in a vacuum. In his third paper, he wrote, "As a result of an analysis of the physical concepts of time and space, it became evident [evident to Einstein maybe] that in reality there is not the least incompatibility between the principle of relativity and the law of propagation of light, and that by systematically holding fast to both these laws, a logical rigid theory could be arrived at." This was exactly what Einstein did in the third paper.

He developed his theory, later called the Special Theory of Relativity, based solely on two postulates.

Postulate I: The laws of physics are the same, that is, have the same mathematical form, in all inertial frames of reference.
Postulate II: The speed of light in a vacuum is an absolute constant.

[1] In 1972 the speed of light in a vacuum was determined to be 299,792,456.2 ± 1.1 m/s. $3 \times 10^8$ m/s is within 0.067% of the correct value and is usually used. Light travels slightly slower in air than it does in a vacuum (about 0.37% under normal conditions) but $3 \times 10^8$ is also a very good approximation for the speed of light in air.

He accepted these postulates as true, and by "systematically holding fast" to them, he logically deduced the physical consequences, producing a host of very peculiar results.

For instance, this “logical rigid theory” of Einstein’s demands radical changes in our common sense, classical notions of time and space. Some of these very peculiar predictions will be treated in more detail later in this reading.

The second postulate automatically provides an explanation for the negative results of the Michelson-Morley experiment, where they failed to detect the predicted difference between the values of the speed of light in two frames of reference in motion with respect to one another. The speed of light is an absolute constant, the same in all inertial frames of reference. Although relativity explains the Michelson-Morley results, it seems fairly certain that Einstein’s thinking was neither motivated nor influenced by the Michelson-Morley experiments. It is entirely possible that Einstein was even unaware of them.

In addition to explaining the Michelson-Morley results, relativity eliminated the troublesome concept of the ether. In the classical wave model, some material substance must oscillate in order to produce a wave, much as air molecules oscillate to produce a sound wave and water molecules oscillate to produce a water wave. The ether was the proposed substance that oscillated to produce an electromagnetic wave. In the classical theory, the speed with which a wave propagates is determined by the rigidity of the oscillating material. Thus, sound travels much faster in a metal than it does in air. The extremely high speed of light would require the ether to be extremely rigid. However, in order for us to receive light from distant stars, this extremely rigid ether must fill all of the space of the universe, and the earth would have to move through the ether in its orbit around the sun. Because the earth does not lose energy as it orbits the sun, it would have to travel through this extremely rigid material experiencing no friction at all. The properties required of the ether are incompatible with one another, and physics is well rid of this hypothetical substance.

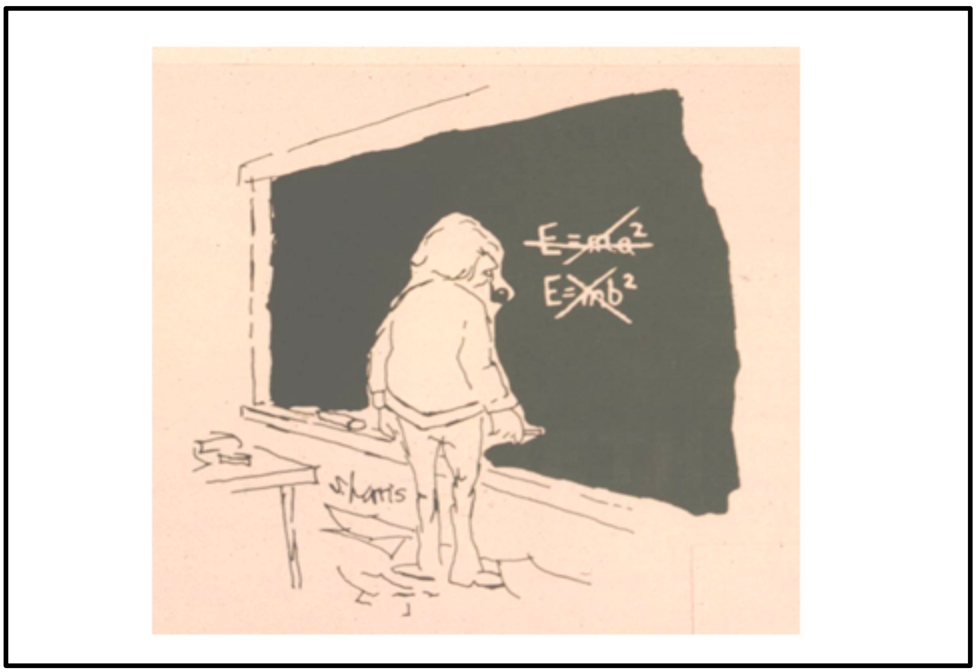
**The Equivalence of Matter and Energy**

In his fourth and final 1905 paper in the Annalen der Physik, Einstein developed an argument for mass as a form of energy. Prior to this paper, the conservation of mass and the conservation of energy were considered two distinct and independent laws. That is energy could not be created or destroyed, and neither could mass. Conservation of mass was consistent with all observations, and in the minds of physicists, mass was in no way related to energy. However, “systematically holding fast” to the postulates of the special theory of relativity and their implications for time and space required that mass could be created or destroyed as long as an equivalent amount of some other form of energy was destroyed or created.

Although at the time there was no evidence whatsoever for Einstein's ideas of time, space, and the equivalence of mass and energy, Einstein knew through physical intuition that the theoretical consequences of his theory must be true. When told shortly after the publication of his paper of an experiment that contradicted his theory, he said the theory is correct and redo the experiment. It turns out, he was right.

This paper produced what is arguably the most famous equation in the field of physics: $E = mc^2$, where c is the speed of light. The speed of light has nothing to do with how mass is transformed to other forms of energy; $c^2$ is simply a proportionally constant that converts the traditional units for mass to those for energy.

In chemical reactions, the amount of energy released or absorbed is not sufficient to produce a measurable change in the mass of the system, which is why the equivalence had not been noticed before. However, with the advent of nuclear physics in the early twentieth century, the much greater energies involved relative to the mass of the particles made the equivalence clear.




Cartoon by Sydney Harris. Used with permission from Sydney Harris.

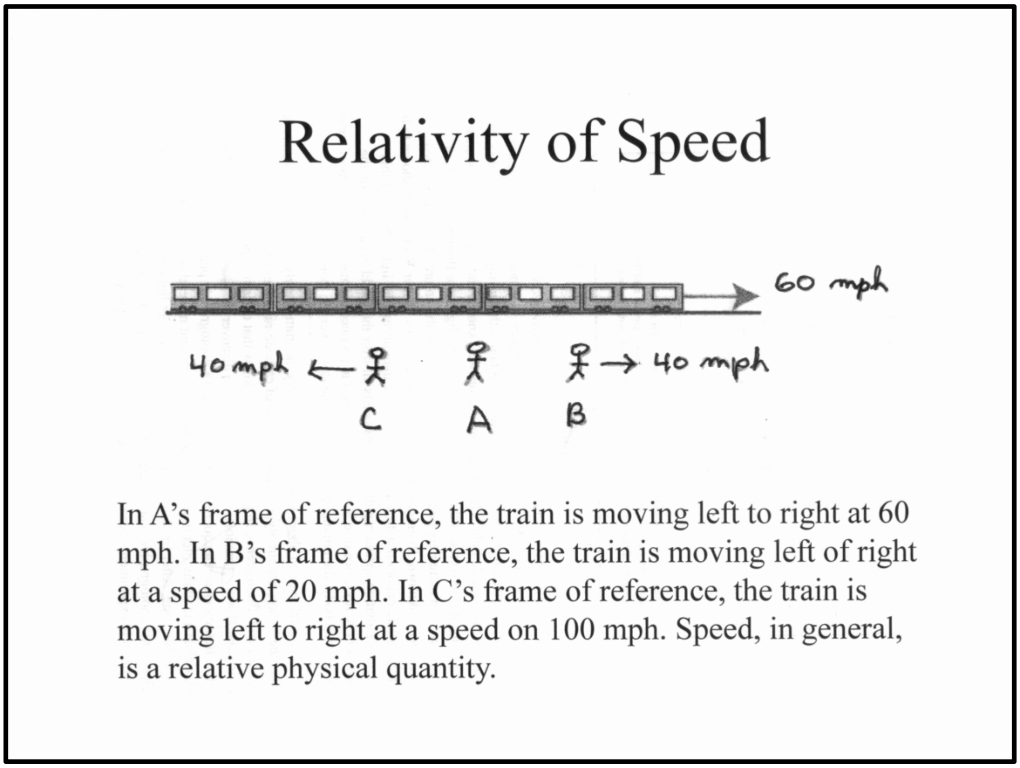
### Einstein's Fifth Paper

The four 1905 papers described above were Einstein's most significant contributions in the miracle year. But there was also a fifth paper written that year, a paper titled "A New Determination of Molecular Dimensions." He submitted it to the University of Zurich as a doctoral thesis. Although the title of the dissertation focused on the sizes of molecules, the technique Einstein described also gave a measurement of the number of molecules (or atoms) present in a solution. This paper was selected for the dissertation primarily because it was the least revolutionary of the five and less likely to stir up opposition from a thesis committee. The only official comment Einstein received on the dissertation was that it was too short. In response, he added a single sentence, and it was accepted. Einstein was awarded his doctorate in 1906, meaning that his four revolutionary papers published the year before were published by someone with an undergraduate degree in physics.

**Length Contraction and Time Dilation**

Einstein derived the laws of the Special Theory of Relativity from the logical consequences of his two postulates, first that the principle of relativity holds for all physical laws, both the laws of mechanics and the laws of electromagnetism, and that the speed of light is an absolute constant.

To say that something is an absolute constant means that its value is independent of the frame of reference in which it is measured. An example of an absolute constant is the charge on an electron. Speed is obviously a relative rather than an absolute, physical quantity. Consider a train moving at 60 mph with respect to an observer standing beside the tracks. Suppose a second observer is moving at 40 mph on a road parallel to the tracks in the same direction as the train. In the frame of reference of that observer, the train is moving forward with a speed of 20 mph. For an observer moving at 40 mph in the opposite direction, the train would move backwards through that frame of reference with a speed of 100 mph.



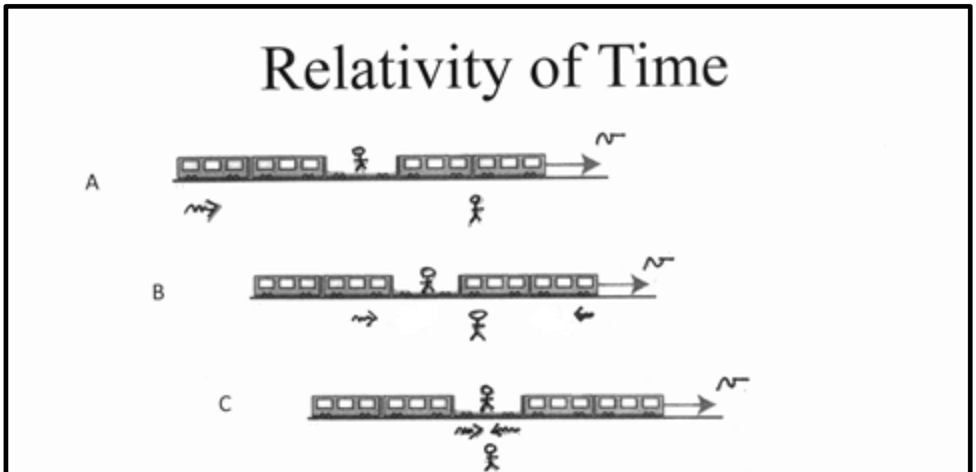

The relative nature of speed in general seems obvious. However, Einstein said that there was something special about the speed of light: it alone has a speed that is an absolute constant. In the diagram above, replace the train with a beam of light. Let observer B move at 80% of the speed of light, and observer C also moves at 80% of the speed of light. According to Einstein, all three of the observers would measure the speed of the light beam moving through their frame of reference as the same absolute value, $3 \times 10^8$ meters per second. Only a complete revamping of our concepts of space and time could allow for this to be true.

In classical physics space and time are absolute; space intervals and time intervals are completely independent of the frame of reference in which they are observed. However, a simple thought experiment can show that this is not true in relativity.

Imagine a train moving at close to the speed of light. An observer is positioned in the exact middle of the train. A second observer is at rest beside the tracks. At the exact instant the two observers pass one another, two flash bulb signals arrive simultaneously from the front and rear of the train.

Analyzing the situation from the frame of reference of the train, the observer on the train will conclude that the two flash bulbs went off simultaneously. Because each source of the light is equidistant from the train observer, and because the speed of light is an absolute constant, the two flashes of light must have traveled for the same amount of time. Thus, the two flash bulbs must have gone off at exactly the same time. That is they must have gone off simultaneously.

Now, analyze the situation from the frame of reference of the observer standing beside the tracks. The two flash bulb signals arrive simultaneously from the front and rear of the train. Both the observer on the train and the observer beside the tracks agree on this. They are instantaneously face-to-face and at that instant each sees the two flashes. However, the speed of light is finite. Therefore, it is clear that the light signals must have been emitted sometime in the past, sometime before the train reached its present position. The observer beside the tracks must have been closer to the front of the train and farther from the rear when the flashes were emitted. Thus, in the frame of reference of the observer beside the tracks, the flash from the rear had to travel a greater distance than the one from the front. Since the speed of light is an absolute constant, in order for the flashes to arrive simultaneously in the frame of reference of the observer beside the tracks, the flash from the rear must have been emitted before the flash from the front. In this frame, they did not go off simultaneously.




The above diagram is drawn as viewed in the frame of reference of the observer standing beside the train tracks. In that frame of reference, the flash from the rear of the train is emitted first (A). Later (B) the flash from the front is emitted. At a still later time (C), both flashes arrive simultaneously for both the train observer and the observer beside the tracks. Two events that are simultaneous in one frame of reference are not simultaneous in a different frame of reference. Time is relative.

The train observer concludes that the light signals were emitted at exactly the same time. The observer beside the tracks concludes that the light signal from the rear of the train was emitted before the one from the front. Who is right? Both are right; time is a relative physical quantity. Events separated in space that are simultaneous for one observer are not simultaneous for a second observer in motion relative to the first one.

Not only is the time interval between events relative, the order of certain events can also be relative. To an observer in a frame of reference in which the train is moving right to left (i.e. an observer moving in the same direction as the train, but with a greater speed), the signal from the front of the train will have been emitted before the signal from the rear, the opposite of what the observer beside the tracks sees.

The relative nature of the order of certain events does not conflict with cause and effect. If two events are causally related, that is if one event produced the other, the order of the events is not relative. Otherwise a paradox could occur, such as the ball striking the wall before it is thrown. For all observers regardless of their relative motion, analysis using the theory of relativity will always yield the correct order for causally related events. However, the time interval between the events will be different. In fact, if two events that are not causally related but are separated in space by a distance small enough for light to travel between them, all observers will agree on the relative order of the events.

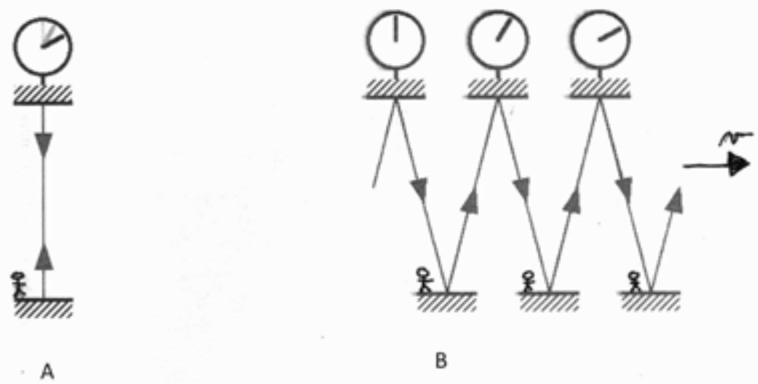
**Time Dilation**

As a further illustration of the relativity of time, consider the following situation. Observer A has a device for emitting and detecting light signals. It has a mirror located a certain distance above the device which will reflect the emitted light signal back to the device, which will then detect it.

Suppose that when the device receives a light signal, another light signal is immediately emitted. Suppose further that the distance is just right so that it takes the signal exactly one second for the round trip. That is, that a light signal is detected every second. (Actually this would require a device 93,000 miles high, but this is only a thought experiment.) The device is a clock—an instrument for measuring time—and the unit of time is one second, just as it is on ordinary clocks. The detection of the light signal is like the ticking of a clock.

An observer B has an identical clock. Imagine that observer B with his clock moves left to right with respect to observer A and his clock. The situation, diagramed from the viewpoint of observer A, looks like the following.

## Relativity of Time




Two observers with identical clocks are in different frames of reference. Observer A observes three ticks on B's clock to take longer than three ticks on his own clock, and concludes that B's clock runs slow.

How does the time interval between the ticks of the two clocks compare? The second postulate of relativity tells us that A observes that both light signals travel at exactly the same speed. It is clear that A observes the distance B's light signal travels between emission and detection to be greater than the distance for his own clock. Thus, A will determine that the time interval between the emission and detection of the light signal by B's clock is greater than the time interval between these two events on his own clock. B's light signal must travel a greater distance at the same speed, and this requires more time. While A observes his clock ticking 60 times, or one minute, A will observe B's clock to tick less than 60 times, or less than a minute. Thus, A concludes that B's clock runs slower than his own.

However, it should be clear that the situation just described from A's point of view will have exactly the opposite reasoning when described from B's point of view. In B's frame of reference, it is A's light signal that will travel the greater distance and therefore take more time to complete one tick. Each observer claims that the other's clock runs slow. There appears to be a contradiction. However, when stated differently, there is no contradiction. Both thought experiments yield the same result: the clock in motion with respect to the observer will measure time more slowly than an identical clock at rest with respect to the observer. This phenomenon is known as time dilation.

This result was derived using light-signal clocks. Is it the same for all clocks? The answer is yes. In a frame of reference in motion with respect to an observer, all processes occurring in time will occur more slowly than in the observer's frame. This includes biological as well as mechanical processes. Living beings age more slowly in moving frames of reference than will be the case in a frame of reference at rest with respect to the observer.

Going back to our thought experiment, it is clear that the added distance that the light signal must travel depends on the relative speed of the moving frame of reference. The faster the clock travels with respect to the observer, the greater the distance the signal will have to travel between emission and detection. Thus, time dilation must be a function of the relative speed.

The time-dilation equation is written as follows:

$$T=To1-v2/c2$$

where T is the time interval indicated on a clock at rest with respect to the observer, $T_o$ is the time interval indicated on a clock moving with a speed v relative to the observer, and c is the speed of light. Of course, if v = 0, the two times are the same.

In this equation, the relative speed must be on the order of the speed of light in order for the difference in the time intervals to be significant. This explains why this effect was not noticed empirically before 1905. Our ordinary experiences do not involve speeds anywhere near the speed of light and thus the consequence of time dilation produces no measurable effect. However, early in the twentieth century, physicists began studying the motion of subatomic particles, and time dilation was soon confirmed.

The speed at which the consequences of relativity become significant is usually taken to be about one-tenth the speed of light. The speed of light is 186,000 miles per second, so one-tenth would be 18,600 miles per second or 1,146,000 mph. This is a speed that will probably never be reached by ordinary macroscopic objects but is one easily obtained by submicroscopic particles such as electrons and protons. Even at this high speed, the difference in the rate at which time passes in the two frames of reference is only about 2%.

## Length Contraction

The Special Theory of Relativity united space and time, physical quantities that in classical physics were each absolute and independent. In relativistic physics, space-time intervals are absolute, rather than time intervals and space intervals separately. If space-time intervals are absolute and time intervals are relative, then space intervals must be relative as well.

For the space-time interval to be absolute, DT DL for the time and space intervals between two events must be a constant: it must have the same value for all observers. Let To be the time measured on a clock moving at a speed v with respect to the observer, and let Lo be the space interval (distance) measured for the moving frame. Let T and L be the time and distance intervals measured in the at rest frame. Thus, T L = To Lo. By substituting the time dilation equation into this equation, we get:

$$L = Lo 1- v2/c2.$$

This result is known as length contraction. It is important to realize that the phenomenon of length contraction involves only the space interval in the direction of the relative motion. Thus, the shape of an object in motion relative to the observer will be altered. For example, if a square moves relative to an observer, it will be a rectangle in the observer's frame of reference. The dimension in the direction of the motion will be contracted, while the perpendicular dimension will be unaffected. To say that the moving square will be a rectangle in the observer's frame of reference is not the same as saying the square will appear to be a rectangle. It will actually be a rectangle with all the properties of a rectangle in that frame of reference. The space and time intervals do not just appear to be different in different inertial frames; they must actually be different in order for the speed of light to be an absolute constant. This real difference has physical consequences that have been verified time and time again by experiments and observations.

Time dilation and length contraction can be summarized as:

1) Every clock goes at its fastest rate when it is at rest relative to the observer. If it moves relative to the observer with a speed v, its rate is slowed by a factor of $1- v2/c2$.

2) Every object is largest when it is at rest relative to the observer. If it moves relative to the observer with a speed v, it is contracted in the direction of motion by a factor of $1- v2/c2$.

As mentioned earlier, it is subatomic phenomena that are likely to show significant relativistic effects. One such phenomenon is the production of muons in the upper atmosphere of the earth. Muons are essentially overweight electrons. They are unstable, and after a very short time, they will decay to electrons. Muons have been studied extensively in the laboratory, and muons created with speeds small compared to the speed of light have a lifetime of about 2.2 x $10^{-6}$ seconds. That is after about 2.2 microseconds, they will spontaneously decay.

In addition to being created in the laboratory, they are also created when high speed cosmic rays strike the upper atmosphere of the earth. This occurs about 2 miles or 3000 meters above the earth's surface. Because of the high energies involved when cosmic rays strike the molecules of the upper atmosphere, the muons are created with extremely high speeds, speeds approaching the speed of light. Most of the muons created in this way pass through the atmosphere and bombard the surface of the earth.

This would be inexplicable to a pre-1905 physicist. Even if they could travel at the speed of light, $3 \times 10^8$ m/sec, with a lifetime of $2.2 \times 10^{-6}$ seconds, they could only travel a distance of 660 meters before decaying, well short of the 3000 m needed to reach the surface of the earth.

Because the muons are created with speeds approaching the speed of light, a relativistic analysis is required. In the earth frame of reference, the muon represents a moving clock; thus its clocks run slower, and time dilation needs to be taken into account. If the muon is traveling at a speed of 98% of the speed of light, (typical of the speeds involved), the factor $1- v2/c2$ will have the value:

$$1-(0.98c)2/c2 =.1- 0.96 = 0.04 = 0.20.$$

Thus, time on the muon clock will pass at only 20% of the rate of earth clocks. This means that the muon will not decay until earth clocks register 5 times the laboratory life time of the muon:

$$5 \times 2.2 \times 10^{-6} \text{ seconds or } 11 \times 10^{-6} \text{ seconds.}$$

Traveling at 98% of the speed of light, or $2.94 \times 10^8$ m/sec, the average distance traveled by the muon in $11 \times 10^{-6}$ seconds is 3230 meters. This analysis explains why most muons created in the upper atmosphere can make it to the earth's surface, 3000 meters below, before decaying, which they are, in fact, observed to do.

In the previous example, we analyzed the problem from the point of view of an earth observer. In that frame of reference, the muon is a moving clock, and time dilation accounts for the fact that muons are able to strike the surface. Now let's analyze the situation from the point of view of the muon. In the muon's frame of reference, it is not a moving clock; therefore, time dilation does not apply. The lifetime of the muon is not $11 \times 10^{-6}$ seconds but $2.2 \times 10^{-6}$ seconds. However, space intervals are relative, and the 3000 meters that the muon must travel to reach the earth will be contracted in the muon's frame of reference.

In the frame of reference of the muon, the muon is at rest, while the upper atmosphere recedes at 98% of the speed of light and the surface of the earth approaches at 98% of the speed of light. The atmosphere is the moving frame of reference, and the space interval or distance from the top of the atmosphere to the earth's surface is contracted. Using our value of $1- v2/c2$ = 0.20 from the previous calculation, and Lo, the distance in the moving frame of reference (3000 meters) gives L, the separation between the upper atmosphere and the surface in the muon's frame of reference, as 600 meters. In the muon's frame of reference, its $2.2 \times 10^{-6}$ second lifetime is enough time to travel the 600 meters of atmosphere, resulting in a collision with the surface.

**The Twin Paradox**

Because time passes at different rates in different frames of reference, the theoretical possibility exists for twins in different frames of reference being reunited with one twin older than the other. Suppose that one twin stays home and the other is in a rocket ship traveling at 98% of the speed of light. The time dilation factor is 0.20 as in the earlier calculations. From the point of view of the stay-at-home twin, the traveling twin will only age 6 years in the same time interval he ages 30 years. If the traveling twin turns around and immediately returns home, again at 98% of the speed of light, the return trip will take 30 years on the stay-at-home twin's clock, while again only 6 years pass on the rocket ship clocks. The twins will be reunited with the stay-at-home 60 years older than when the trip started, while the traveling twin will be only 12 years older.
This scenario is known as the Twin Paradox, although it is not a paradox at all. There is nothing paradoxical about the above calculation. The above situation is described from the point of view of the stay-at-home twin. This is because of the two, the stay-at-home twin's frame of reference is the inertial frame, while that of the rocket ship is not. It must turn around in order for the two to be reunited. That is, it must decelerate to a stop and then accelerate back up to 98% of the speed of light. During the time it takes to do this, the rocket ship frame is not an inertial frame, and the laws of the special theory do not apply.

The later General Theory of Relativity, as discussed in the reading for subunit 7.4, can be used in a non-inertial frame of reference. For the first half of the trip, both frames are inertial (neglecting the initial acceleration). The traveling twin agrees that he has aged 6 years. Because the stay-at-home twin is the moving frame, the rocket observer sees his twin age only 1.2 years (6 years times 0.20). On the return half of the trip, both are again inertial observers. The rocket traveler again ages 6 years and sees his twin age another 1.2 years. During the turn around, the equations of special relativity do not apply, and the equations of the general theory must be used. According to the general theory, during the turn around, the rocket observer will see the stay-at-home clocks suddenly speed up dramatically. According to precise calculations, the stay-at-home clock will speed forward and tick off 57.6 years during the turn-around time, even if it takes only a few minutes on the rocket ship clocks. Thus, each twin will be able to explain why the stay-at-home twin has aged 60 years, while the traveling twin has only aged 6 years.

## Some Consequences of Special Relativity

- 1. The time interval between two events is not absolutely defined. Time dilation.
- 2. The length of an object or the distance between two objects is not absolutely defined. Length contraction.
- 3. Mass is a form of energy and under appropriate circumstances can be converted to another form or vice versa. $E = mc^2$. Elementary particles can be created or destroyed.
- 4. The speed of light in a vacuum is an absolute physical constant. $c = 3 \times 10^8$ m/sec.
- 5. No object with rest mass can move relative to an observer at a speed equal to or greater than the speed of light in a vacuum. No information can be propagated at a speed exceeding the speed of light in a vacuum.

Although the twin paradox is strictly a thought experiment, the equivalent experiment has been done many times with atomic clocks. Atomic clocks can keep time with extreme precision. Two identical atomic clocks are prepared and set to exactly the same time. One is put on an earth-orbiting satellite. The other remains at rest in the laboratory. When they are reunited, the clocks show different times in exact accord with Einstein's theories. The speeds involved are very small compared to the speed of light, but the precision of the clocks is such that effects on the order of tiny fractions of a second are observable.