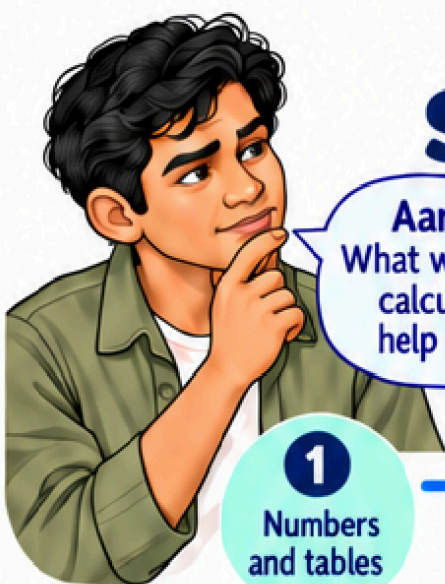


Linear algebra: see every calculation



Aarav:
What will these calculations help me do?

From lists of numbers to useful model calculations

SEE THE WHOLE CHAPTER



Maya:
Store measurements, combine them, and see what a model changes.

1
Numbers and tables

2
Vectors and geometry

3
Matrix transformations

4
Solve and undo changes

5
Find important directions

6
Use in machine learning

1 VECTORS AND MATRICES

A **vector** is an ordered list of numbers.

Delivery:
distance = 2 km,
parcels = 3



Vector:
[2, 3]

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Vector
(2 numbers, in order)

A **matrix** is a table of numbers (many vectors as rows).

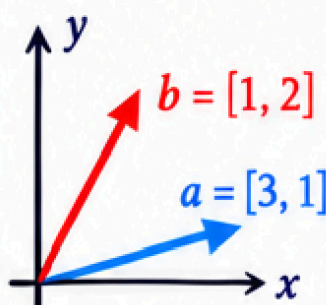
	Distance (km)	Parcels
Delivery 1	2	3
Delivery 2	4	1

$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

Matrix (2 deliveries, 2 numbers each)

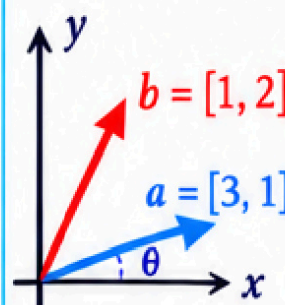
2 VECTORS: LENGTH, DIRECTION, AND DOT PRODUCT

Length (how long).



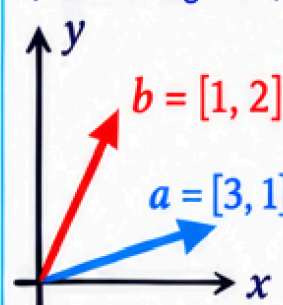
Length of a:
 $\|a\| = \sqrt{3^2 + 1^2} = \sqrt{10}$

Direction (which way).



Direction of a:
Angle of a:
about 18.4°
above horizontal.

Dot product
(measures alignment).



Dot product:
 $a \cdot b = 3 \cdot 1 + 1 \cdot 2 = 5$

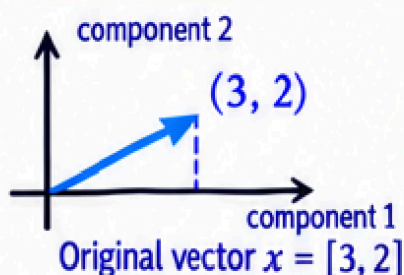
3 A MATRIX TRANSFORMS A VECTOR

A **matrix** can stretch, shrink, rotate, or shear a vector.

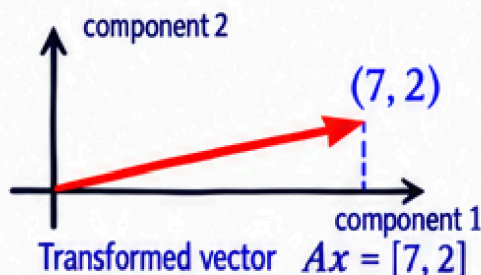
Matrix
(transformation)
 $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
(shear to the right)

Vector
(input)
 $x = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$$Ax = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$



Original vector $x = [3, 2]$



Transformed vector $Ax = [7, 2]$

The matrix changes the vector's coordinates, moving it to a new point.

4 SOLVE FOR UNKNOWN VALUES

Linear equations let us find unknowns.

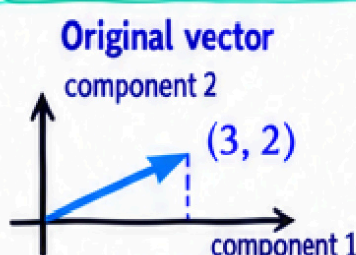
Example: solve for x, y

$$\begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases}$$

Solution:
 $x = 2$
 $y = 1$

$2x + y = 5$
 $x - y = 1$
Add:
 $3x = 6$
 $x = 2$
Then $y = 1$

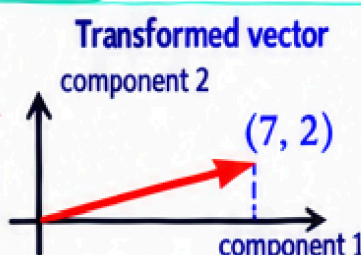
Undo reversible changes (back to the original).



Original vector
component 2
(3, 2)
component 1

Change
(matrix)

Solve
(inverse)

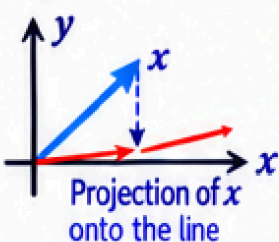


Transformed vector
component 2
(7, 2)
component 1

Solve $Ax = [7, 2]$ to recover $x = [3, 2]$.

5 FIND IMPORTANT DIRECTIONS

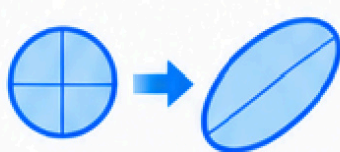
Projection: shadow onto a line (keep the part in an important direction).



Projection of x onto the line

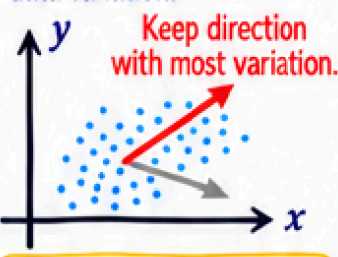
Drops the part perpendicular to the line.

Singular value decomposition (SVD): separate directions from stretching.



SVD shows the main directions and how much they are stretched.

Principal component analysis (PCA): keep directions with the most data variation.



PCA picks the direction where the data varies the most.

6 USE IN MACHINE LEARNING

From a house description to a price.



size
rooms

model
calculation

estimated
price


From a movie review to a prediction.



list of
numbers

model
calculation

positive or
negative
review
 