

Your route through calculus

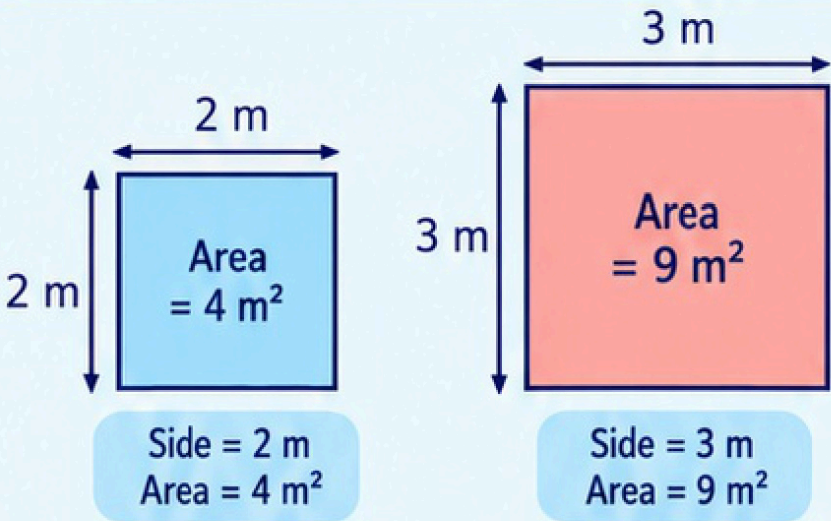


How a small change affects a result, and how small amounts add up.



1 One input changes

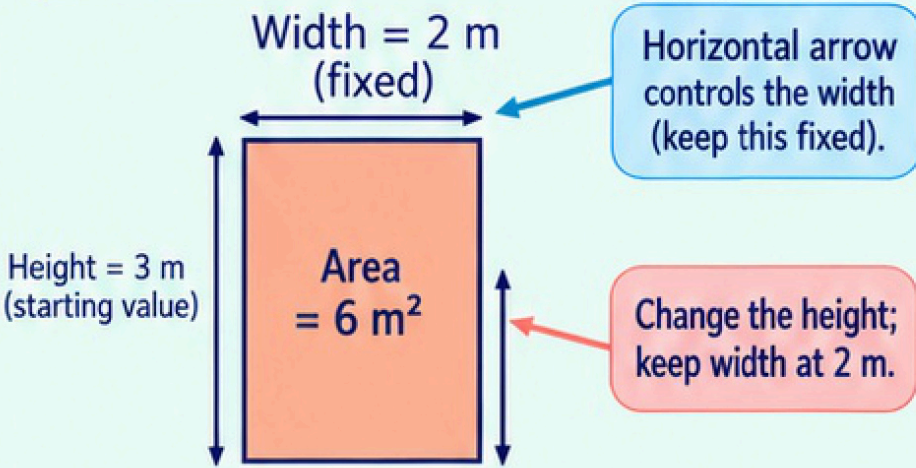
How much does area change when the side grows?



When the side increases from 2 m to 3 m, the area increases from 4 m² to 9 m².

2 Several inputs: change one at a time

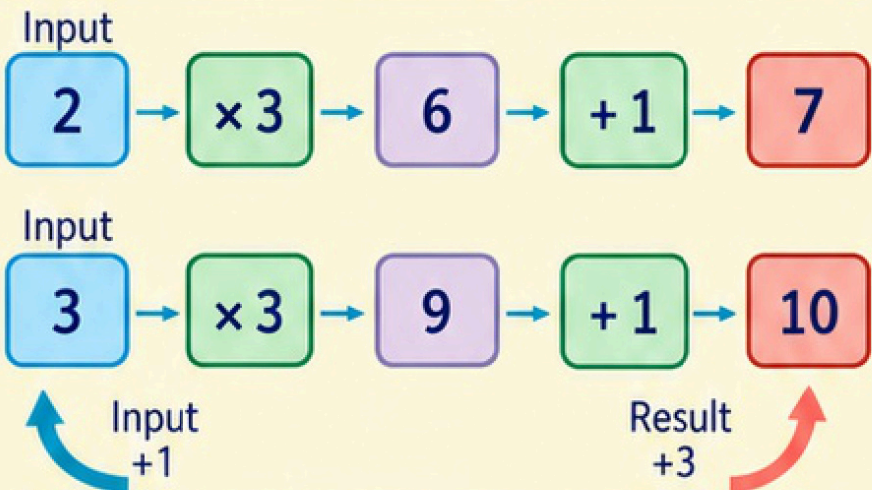
Change one side while keeping the other fixed.



If we change one side and keep the other fixed, the area changes.

3 Follow connected steps

Trace a change through each calculation.

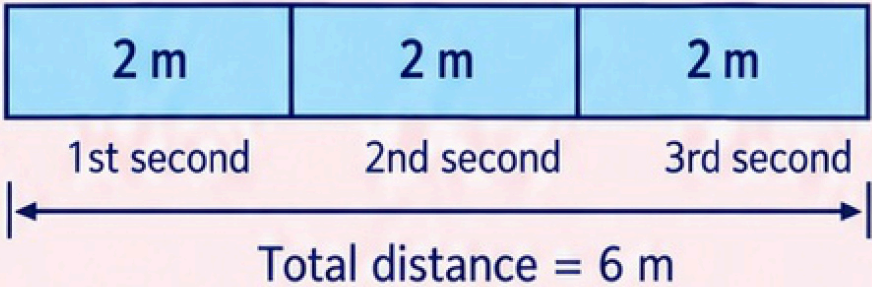


The input increased by 1 and the result increased by 3.

4 Add small amounts

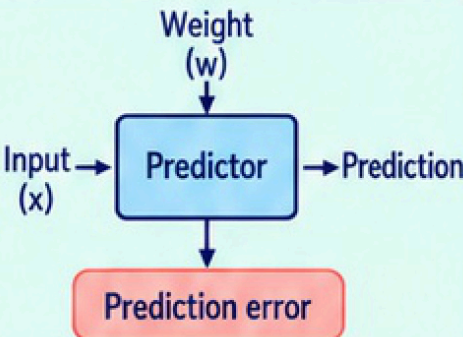
Add distance covered during each second.

Speed = 2 metres per second (constant)
Time = 3 seconds



How does this connect to machine learning?

We already met a predictor with an adjustable weight. Calculus measures how changing that weight would change prediction error. Training uses these measurements to choose updates.



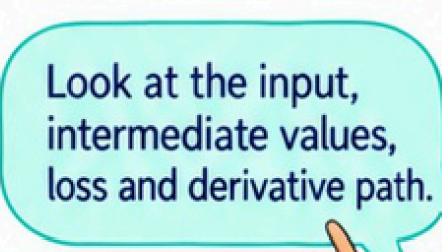
Measuring and changing are separate steps.



A reference map for calculus in machine learning



What should I revisit when a training formula is confusing?



Look at the input, intermediate values, loss and derivative path.

Keep in mind:

- These ideas build the local calculus needed for learning algorithms.
- More advanced topics (convergence proofs, integration theory) require additional study.

1 Single-variable foundations

Function

$$y = f(x) = x^2$$

Limit (difference quotient)

$$\frac{f(x+h) - f(x)}{h} = 2x + h$$

for $h \neq 0$
As h approaches 0,
 $\frac{f(x+h) - f(x)}{h} \rightarrow 2x$.

Derivative

$$\frac{d}{dx} x^2 = 2x$$

At $x = 3$:
 $\left. \frac{d}{dx} x^2 \right|_{x=3} = 2(3) = 6$

The vertical bar $|_{x=3}$ means evaluate at $x = 3$.

Integral

$$\int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

Focus on basics:

- Work through exercises to strengthen the basic operations.

2 Several variables

Partial derivatives

$$f(x, y) = x^2 + 3y$$
$$\frac{\partial f}{\partial x} = 2x$$
$$\frac{\partial f}{\partial y} = 3$$

Gradient

Components:

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

At $(1, 2)$:

$$\nabla f(1, 2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Directional derivative

$$D_{\vec{d}} f = \nabla f \cdot \vec{d}$$

At $(1, 2)$, $\vec{d} = [1, 0]$:
 $[2, 3] \cdot [1, 0] = 2 \times 1 + 3 \times 0 = 2$

$D_{\vec{d}} f$ is the rate of change in the unit direction $\vec{d}$.

Multivariable view

$$f(x, y) = x^2 + y^2$$

Draw and calculate small cases before generalizing.

3 Machine-learning connections

A simple model

Linear model: $\hat{y} = wx + b$
Loss (squared error): $L = \frac{1}{2} (\hat{y} - y)^2$

Input

x
(given)

Model

$\hat{y} = wx + b$
(intermediate value)

Loss

$L = \frac{1}{2} (\hat{y} - y)^2$
(scalar output)

Why the factor 1/2?
 $\frac{\partial}{\partial \hat{y}} \frac{1}{2} (\hat{y} - y)^2 = \frac{1}{2} \cdot 2(\hat{y} - y) = (\hat{y} - y)$.
The factor 1/2 cancels the 2 from the derivative, but it does **not** change the minimizer.

Derivative path

$$\frac{\partial L}{\partial w} = (\hat{y} - y)x$$

Example: $x = 2, w = 1, b = 0, y = 3$
 $\hat{y} = 1 \cdot 2 + 0 = 2$
 $\frac{\partial L}{\partial w} = (2 - 3)(2) = -2$

Chain rule view:
 $\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w} = (\hat{y} - y)x$

Learning step roadmap

Identify input
Start with the data example.

Calculate prediction
Use the model to get the output.

Compare with target
Compute the loss from the prediction.

Trace loss slope
Find how the loss changes with the parameter.

Choose update
Move the parameter in the direction that reduces the loss.

4 Reference table

When you see ...	Use ...	First question
A local change	Derivative or partial derivative	Which variable changes?
A chain of operations	Chain rule	Which routes connect input to loss?
Many outputs	Jacobian	Which output? Which input?
Curvature	Hessian and Taylor	Where are derivatives evaluated?
An update	Gradient plus optimizer	How large is the proposed step?

Keep learning:

- Practice these ideas often with your own examples.
- They are the foundation for more advanced machine learning topics.

5 Check your understanding

What comes next?

→

Revisit the worked examples. Then connect these ideas to your next machine-learning topic.

That's the next step!