

PRIVATE COMPUTING

WITH

HOMOMORPHIC ENCRYPTION

FROM FOUNDATIONS TO PRODUCTION
FOR SOFTWARE ENGINEERS



MASTER THE
FOUNDATIONS



UNDERSTAND
SCHEMES AND
LIBRARIES



DESIGN HE-COMPATIBLE
ALGORITHMS AND
ARCHITECTURES



BUILD WORKING
APPLICATIONS



EVALUATE SECURITY
AND PERFORMANCE
TRADEOFFS

WARREN ASHFORD

Private Computing with Homomorphic Encryption

From Foundations to Production for Software Engineers

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Contents

From Foundations to Production for Software Engineers	1
Introduction	2
Chapter 1: The Promise and Limits of Private Computation	5
A Day in the Life of Sensitive Data	5
Trust Problems and Why Encryption Alone Is Not Enough	6
Threat Models That Matter	7
The Privacy-Preserving Computation Landscape	9
When Homomorphic Encryption Is the Right Tool	12
How This Book Is Organized	14
Chapter 2: Cryptographic Prerequisites for Software Engineers	17
What Modern Encryption Actually Guarantees	17
Symmetric and Asymmetric Cryptography in One Page Each	18
Semantic Security and Indistinguishability	19
Computational Hardness Assumptions	21
Randomness and Entropy in Cryptographic Systems	22
The Gap Between Textbook and Real-World Cryptography	23
Chapter 3: Number Theory Foundations – Modular and Polynomial	
Arithmetic	26
Integers Modulo N	26
Extended Euclidean Algorithm and Its Role in Cryptography	26
Polynomial Arithmetic Over Finite Rings	26
Cyclotomic Polynomials and Why They Matter for HE	26
Number Theoretic Transform – the Fast Way to Multiply Polynomials	26
Computational Complexity of Polynomial Operations	27
Chapter 4: Lattices, Learning With Errors, and the Hard Problems	
Behind HE	28

What Is a Lattice?	28
Hard Problems on Lattices	28
Learning With Errors – the Core Assumption	28
Ring-LWE – Polynomials Make It Efficient	28
Why Lattices Resist Quantum Attacks	28
Security Levels and Parameter Choices	28
Chapter 5: Homomorphic Encryption – Core Concepts and Mechanics	30
The Basic Idea	30
Partially, Somewhat, Fully – Levels of Homomorphism Defined	30
Noise as the Fundamental Resource	30
Ciphertext Expansion and Why Encrypted Data Is Big	30
Keys in HE	30
The Basic Operations: Encrypt, Decrypt, Add, Multiply	30
Chapter 6: Advanced HE Operations – Depth, Levels, and Management	
Techniques	32
Multiplicative Depth and Computational Levels	32
Relinearization – Keeping Ciphertexts Manageable After Multiplication	32
Rescaling – Making Room for More Computation	32
Modulus Switching – Reducing Noise Without Bootstrapping	32
Rotations and Permutations – Moving Data Within Encrypted Vectors	32
SIMD Batching and Vectorization – Doing Many Computations at Once	33
Chapter 7: Exact Arithmetic Schemes – BFV and BGV	34
The BFV Scheme End-to-End	34
The BGV Scheme and How It Differs from BFV	34
Parameter Selection for Exact Arithmetic	34
Encoding Integers and Vectors in BFV/BGV	34
Noise Budget Analysis for Exact Schemes	34
Practical Performance Characteristics	34
Chapter 8: Approximate Arithmetic – The CKKS Scheme for Real and	
Complex Numbers	36
Why Approximate? Motivation from ML and Scientific Computing	36
The CKKS Construction End-to-End	36
Encoding Real and Complex Numbers – Scaling and Precision	36
How Rescaling Works in CKKS – the Key to Multiplicative Depth	36
Managing Precision Loss Across Computation	36
Choosing Parameters for Target Accuracy	37

Chapter 9: Boolean and Circuit-Oriented HE – TFHE	38
Why Boolean Circuits? Comparison with Arithmetic Circuits	38
The TFHE Construction at a High Level	38
Gate Bootstrapping – How TFHE Refreshes Ciphertexts	38
NAND Gates, Multiplexers, and Building Complex Functions	38
Performance Characteristics of TFHE	38
Chapter 10: Scheme Comparison and Selection Framework	39
Decision Tree for Scheme Selection	39
Comprehensive Scheme Comparison Table	39
Library Landscape	39
Interoperability and Standardization Efforts	39
Migration Between Schemes and Libraries	39
Future-Proofing Your HE Choices	39
Chapter 11: Encoding, Key Management, and Core Operations	41
Plaintext Encoding Strategies	41
Key Generation in Practice	41
Encryption and Decryption Implementation Walkthrough	41
Basic Homomorphic Operations – Addition, Multiplication, Negation	41
Serialization and Storage of Keys and Ciphertexts	41
Debugging Your First HE Program	42
Chapter 12: Circuit Design and Algorithm Transformation for HE	43
Analyzing Algorithms for HE Compatibility	43
Arithmetic Circuits vs Boolean Circuits – Choosing Representation	43
Minimizing Multiplicative Depth – Algebraic Tricks and Approximations	43
Handling Conditionals and Branching in HE	43
Exploiting SIMD/Batching for Parallel Workloads	43
Approximating Non-Polynomial Functions – ReLU, Sigmoid, Division	44
Chapter 13: End-to-End Implementations – From Prototype to Working System	45
Implementation 1: Encrypted Arithmetic Calculator	45
Implementation 2: Private Aggregation and Analytics Pipeline	45
Implementation 3: Encrypted Database-Style Queries	45
Implementation 4: Private ML Inference Service	45
Testing, Validation, and Correctness Verification Strategies	45
Chapter 14: Architecture, Performance, and Operations in Production	47

Client/Server Architectures for HE Systems	47
Multi-User and Federated Scenarios	47
Performance Profiling and Optimization	47
Bandwidth Management for Large Ciphertexts	47
Observability, Logging, and Monitoring Without Leaking Secrets . . .	47
Key Rotation, Versioning, and Long-Term Maintenance	48
Chapter 15: Bootstrapping – Theory, Practice, and Trade-offs	49
Why Bootstrapping Is Necessary – the Noise Exhaustion Problem . .	49
How Bootstrapping Works Conceptually – Evaluate Decryption on Encrypted Data	49
Mathematical Details of Bootstrapping in Different Schemes	49
Performance Cost – Why It Is Expensive and When You Can Avoid It .	49
Leveled HE vs Fully Homomorphic HE in Practice	49
Emerging Optimizations and Hardware Acceleration	50
Chapter 16: Security Considerations – What HE Protects and What It Does Not	51
Honest-but-Curious vs Malicious Adversaries – What Each Scheme Assumes	51
Metadata Leakage – Sizes, Timing, Access Patterns	51
Side Channels in HE Implementations	51
Integrity and Authentication Limitations	51
Common Security Mistakes and Anti-Patterns	51
Defense-in-Depth with HE – Combining with TLS, MACs, and Other Mechanisms	52
Chapter 17: Comparing Technologies and Hybrid Architectures	53
Trusted Execution Environments – When They Beat HE	53
Secure Multi-Party Computation – Collaboration Without a Server . .	53
Zero-Knowledge Proofs – Verification Without Disclosure	53
Differential Privacy – Aggregate Privacy vs Individual Encryption . . .	53
Hybrid Architectures – HE + TEE, HE + MPC, HE + ZKP	53
Technology Selection Framework for Real Projects	54
Chapter 18: The Evolving Ecosystem and Future Directions	55
Standardization Landscape – NIST, IETF, ISO, Industry Consortia . . .	55
Hardware Acceleration – FPGAs, GPUs, ASICs for HE	55
Emerging Research Directions	55
Framework for Evaluating New HE Schemes and Libraries	55

Practical Readiness Assessment – Is HE Ready for Your Use Case? . . .	55
Conclusion	57
The Path from Understanding to Production	57
Final Checklist Before Deploying HE	57
Resources for Continued Learning	57
Glossary	58
Notation Reference	59
Scheme Comparison Reference	60
Parameter Selection Reference	61
Troubleshooting Guide	62
Architectural Decision Guide	63
References	64

From Foundations to Production for Software Engineers

This book takes software engineers from zero knowledge of homomorphic encryption to advanced production-ready competence. You will learn the mathematical foundations, master the major schemes and libraries, design HE-compatible algorithms and architectures, build working applications, and reason intelligently about security and performance. By the end, you can evaluate whether HE fits a real system, select an appropriate scheme and implementation, take prototypes toward production, and know when another technology is a better choice.

Introduction

Every modern software system faces the same uncomfortable tension: to be useful, data must be processed, and to be processed, it usually must be exposed. We encrypt data at rest in databases and in transit over networks, but the moment computation begins, encryption falls away. The server decrypts, the application runs, and for however long that operation takes, sensitive information sits in plaintext memory where operating systems can inspect it, where administrators might read it, where attackers could extract it if they breach the perimeter.

This is not a new problem, but it has become more urgent. Cloud computing moved computation away from organizations that own data, creating trust boundaries that did not exist before. Machine learning models trained on medical records, financial transactions, or personal behavior raise questions about who sees what during inference and training. Regulatory frameworks like GDPR, HIPAA, and sector-specific rules demand stronger guarantees than “we will try to keep your data private” can provide. And the threat landscape has only grown: supply chain attacks, insider threats, nation-state actors, and increasingly sophisticated malware all target the moment when data is most vulnerable, which is precisely when it is being computed upon.

Homomorphic encryption offers a radical answer to this problem: what if we could compute on encrypted data without ever decrypting it? What if the output of that computation, once decrypted, was identical to the result we would have gotten had we performed the same operations on plaintext? This is not science fiction or theoretical curiosity anymore. It is deployed technology, used in production systems by companies including Microsoft and Apple, and increasingly available through mature open-source libraries that engineers can integrate into real applications today.

This book exists because homomorphic encryption sits at a difficult intersection. To use it well, you need to understand cryptography deeply enough to choose correct parameters, reason about security guarantees, and avoid dangerous mistakes. You also need engineering pragmatism: HE is slow compared to plaintext computation, ciphertexts are large, and the operations you can perform are constrained. Choosing when HE is the right tool requires

understanding both its capabilities and its limits, as well as the alternatives available in the privacy-preserving computing landscape.

Most existing resources on homomorphic encryption fall into one of two categories: academic papers that assume mathematical sophistication but say little about practical deployment, or introductory blog posts that explain the concept at a high level but provide insufficient depth for real implementation work. This book bridges that gap. It is written for software engineers who want to understand HE well enough to build production systems with it, not just appreciate it conceptually. We will develop the necessary mathematical foundations from scratch, walk through the internals of major schemes, implement working code using current libraries, design architectures for real deployments, and critically examine where HE fits in the broader ecosystem of privacy technologies.

The journey ahead is substantial but structured deliberately. We begin with motivation and context, establishing what problems HE solves and how it compares to alternatives. We then build mathematical foundations progressively, introducing concepts only when needed and always connecting them back to practical implementation concerns. The core of the book examines major schemes in detail, explains the engineering techniques that make HE usable, and walks through complete implementations from prototype to production-ready systems. Throughout, we maintain a clear separation between educational examples using simplified parameters and configurations appropriate for real deployments.

You do not need prior knowledge of cryptography beyond basic programming and computer science concepts. If you understand what an algorithm is, have worked with data structures, and are comfortable reading mathematical notation when it is explained, you can follow this book. We introduce modular arithmetic, polynomial rings, lattices, and cryptographic security notions from first principles, always starting with intuition before formalism.

By the end of this book, you will be able to evaluate whether homomorphic encryption is appropriate for a given workload, select an appropriate scheme and library, design HE-compatible algorithms and system architectures, implement working applications with correct parameters, profile and optimize performance, conduct security reviews of HE systems, and make informed trade-offs between HE and alternative privacy-preserving technologies. More importantly, you will understand what HE does not protect, where its assumptions can fail, and how to avoid the common mistakes that undermine even

well-intentioned implementations.

Let us begin by understanding why this technology matters, what problems it is designed to solve, and how it fits into the broader landscape of privacy-preserving computation.

Chapter 1: The Promise and Limits of Private Computation

A Day in the Life of Sensitive Data

Consider a healthcare startup that has developed a machine learning model capable of predicting patient outcomes from medical records. Hospitals want to use this model, but they cannot send their patients' data to the startup due to privacy regulations and competitive concerns. The startup wants to provide accurate predictions without ever seeing raw patient data. Under conventional encryption, both sides are stuck: encrypt the data and it cannot be processed; decrypt it for processing and you have violated privacy requirements or trust boundaries.

Now consider a financial services company running fraud detection across multiple banks. Each bank has transaction data that would improve fraud models if combined with others' data, but regulatory restrictions and competitive dynamics prevent sharing raw transaction records. The goal is to compute aggregate statistics and run models on the union of all data without any single institution seeing another's transactions.

A third scenario: a cloud provider offers AI inference services. A client wants to use the provider's neural network on sensitive inputs, but does not want the provider to see either the input data or the intermediate activations during computation. The provider also wants to protect its model weights from extraction by curious clients.

These are not hypothetical edge cases. They represent concrete business requirements that organizations face today, and they all share a common structure: multiple parties need to perform computations on data that some of them cannot afford to expose in plaintext. Conventional encryption solves the problem of protecting data at rest and in transit, but it does nothing for data during computation. When you decrypt data to process it, you have left the protection that encryption provided.

Homomorphic encryption changes this equation. With HE, the healthcare startup can receive encrypted patient records from hospitals, run its prediction model on those encrypted records, and return encrypted predictions that only the hospital can decrypt. The financial consortium can aggregate encrypted transaction statistics without any bank seeing another's raw data. The cloud provider can perform inference on encrypted inputs without learning anything about them. In each case, computation happens on data that never exists in plaintext at the computing site.

This capability is powerful enough to warrant careful study, and powerful enough to be misused or misunderstood if studied superficially. Before we dive into how HE works, we need to understand what it promises, what it does not promise, and where it fits among other technologies designed for similar purposes.

Trust Problems and Why Encryption Alone Is Not Enough

Every modern distributed system is built on trust assumptions that are rarely examined explicitly. When you store data in a cloud database, you trust the cloud provider's security team to prevent unauthorized access. When you send data over HTTPS, you trust certificate authorities and TLS implementations. When you run code on a server, you trust that the operating system isolates your processes from others, that the administrator cannot read your memory, and that no attacker has compromised the machine.

These trust assumptions are often reasonable for many applications. But they break down in several important scenarios:

Multi-party data collaboration: When multiple organizations need to compute jointly on their combined data, there is usually no single entity that all parties trust enough to decrypt everyone's data. Even if such an entity existed, regulatory requirements may explicitly prohibit concentrating decrypted sensitive data in one place.

Cloud computing with untrusted providers: Organizations increasingly use cloud infrastructure for computation, but the cloud provider operates the physical hardware, manages the hypervisor, controls the operating system, and employs the personnel who maintain the system. Even if you trust

the provider's intentions, you may not be able to trust that every employee, contractor, or software component in the stack will behave correctly.

Insider threats: Within any organization, there are employees with legitimate access to systems who might misuse that access. Database administrators can read data they manage. DevOps engineers can inspect application memory. Security auditors have broad visibility. Encryption at rest protects against external attackers but not against authorized insiders who need to decrypt data to do their jobs.

Regulatory and compliance requirements: Frameworks like GDPR, HIPAA, PCI-DSS, and financial regulations impose strict requirements on how sensitive data may be accessed and processed. Even if technical controls are adequate, legal liability may require stronger guarantees than “we have good security practices” can provide.

Conventional encryption addresses only part of this trust problem. Data encrypted at rest in a database is protected from attackers who gain filesystem access but not from the application that queries the database. Data encrypted in transit with TLS is protected from network eavesdroppers but not from the server that receives it. The fundamental limitation is that conventional encryption is designed to protect data when it is stored or transmitted, not when it is being computed upon. To compute on encrypted data with traditional schemes, you must first decrypt it, which returns you to the original trust problem.

HE is designed specifically for this gap. It allows computation to proceed while data remains encrypted, so the computing party never sees plaintext inputs, intermediate values, or outputs. The only parties who see plaintext are those who hold the decryption key, and they see only the final results of computation, not the underlying data that produced them.

Threat Models That Matter

Understanding what HE protects requires understanding what it is protecting against. Different applications face different threats, and choosing whether to use HE depends critically on your threat model. Let us examine the most relevant ones.

Honest-but-curious adversary (semi-honest model): This is the most common assumption in HE literature and practice. An honest-but-curious

adversary follows the protocol correctly: they perform the computations they are supposed to perform, return results as specified, and do not deviate from the algorithm. However, they try to learn additional information from whatever they can observe during execution: ciphertexts they receive, timing of operations, memory usage patterns, or any side-channel information available to them.

Most modern HE schemes are designed to be secure against honest-but-curious adversaries. The security guarantee is that ciphertexts reveal no information about their underlying plaintexts beyond what is revealed by the computation's output. If a server computes $f(x)$ on encrypted input x and returns an encrypted result, everything the server observes during computation should be indistinguishable from what it would observe for any other input x' that produces the same output $f(x')$.

Malicious adversary: A malicious adversary may deviate arbitrarily from the protocol. They might modify ciphertexts to influence computation results, submit malformed inputs designed to extract information about keys or other data, tamper with intermediate values, or simply refuse to complete a computation. Security against malicious adversaries is significantly harder to achieve and often requires additional mechanisms beyond encryption alone, such as zero-knowledge proofs to verify correct execution or authentication codes to detect tampering.

Most production HE deployments today assume honest-but-curious adversaries, sometimes supplemented by other mechanisms for integrity verification. This is a practical choice: the performance overhead of full malicious security can be prohibitive, and many applications have operational controls that make the honest-but-curious assumption reasonable.

Network eavesdropping: An attacker who can observe all network traffic between parties but cannot modify it or inject messages. HE provides confidentiality against such attackers when combined with standard transport-layer encryption (like TLS) for key exchange and result delivery. HE itself does not replace TLS: keys must still be exchanged securely, and results that are decrypted at any endpoint are visible to whoever controls that endpoint.

Compromised endpoints: If the party that holds the decryption key is compromised, HE provides no additional protection. The attacker with the secret key can decrypt any ciphertext. This is why key management is critical in HE systems: the security of everything rests on protecting the secret key. HE

protects data during computation at untrusted servers, but it cannot protect data at endpoints where keys are present and decryption occurs.

Physical attackers: An attacker with physical access to devices performing encryption or decryption might attempt side-channel attacks: measuring power consumption, electromagnetic emissions, or timing variations to extract keys or plaintexts. Standard cryptographic implementations face this threat, and HE implementations are no exception. Some research has demonstrated side-channel attacks on HE libraries that can recover secret information from power traces or timing measurements. Production deployments may need additional countermeasures like constant-time implementations, masking techniques, or execution in hardware-isolated environments.

Understanding your actual threat model is essential before deciding to use HE. If you fully trust the server performing computation, conventional encryption with decryption at the server is simpler and faster. If you face a malicious adversary who might tamper with results, you need integrity verification mechanisms beyond what HE provides by itself. If your primary concern is protecting data from network eavesdropping, TLS is sufficient. HE is specifically valuable when you need to compute on data at a party that you cannot fully trust with plaintext access but that you expect to follow the computation protocol correctly.

The Privacy-Preserving Computation Landscape

HE is not the only technology for computing on sensitive data without exposing it. Several approaches exist, each with different trade-offs in security, performance, and applicability. Understanding these alternatives is crucial for choosing the right tool.

Trusted Execution Environments (TEEs): TEEs like Intel SGX/TDX, AMD SEV-SNP, and ARM TrustZone create hardware-enforced secure enclaves where code and data are protected from the rest of the system, including the operating system and hypervisor. Data is encrypted in memory outside the enclave and only decrypted inside it. The processor guarantees that no external entity can read or modify enclave memory.

TEEs offer near-native performance because computation happens on plaintext data inside the enclave. They are widely deployed in production for confidential computing workloads. However, they require trust in hardware

vendors, their firmware, and the attestation process. TEEs have also been subject to numerous side-channel attacks over the years, including Spectre variants that can leak data from enclaves through shared CPU caches. The trust boundary is small compared to trusting a cloud provider's full stack, but it is not zero.

Secure Multi-Party Computation (MPC): MPC protocols allow multiple parties to jointly compute a function on their combined inputs without revealing those inputs to each other. Each party holds a share of the data, and through a series of communication rounds, they collectively compute the result while learning nothing beyond what the output reveals.

MPC is mature and widely used in production, particularly in financial services for joint fraud detection and credit scoring. It provides strong security guarantees against honest-but-curious adversaries and can be extended to malicious security. The main limitations are communication overhead (multiple rounds of interaction between parties), complexity of implementation, and the requirement that all participating parties remain online and synchronized during computation.

Zero-Knowledge Proofs (ZKPs): ZKPs allow one party to prove to another that a statement is true without revealing anything beyond the truth of that statement. For example, you can prove you know a password without sending the password, or prove a computation was performed correctly on certain inputs without revealing those inputs.

ZKPs excel at verification scenarios: proving eligibility without revealing identity, proving correct computation without revealing data, proving compliance with rules without exposing underlying information. They are increasingly used in blockchain systems for privacy-preserving transactions and in authentication systems. However, ZKPs do not directly support general-purpose computation on encrypted data. You can prove that a computation was done correctly, but you cannot use ZKPs alone to compute on someone else's encrypted data without them decrypting it first.

Differential Privacy (DP): DP adds carefully calibrated noise to query results or model outputs to ensure that the presence or absence of any single individual's data does not significantly affect the output. It provides a rigorous mathematical guarantee about what an attacker can learn from observing outputs.

DP is widely used by major technology companies for analytics and increas-

ingly in machine learning training. It protects against inference attacks where an adversary tries to determine whether a specific person's data was included in a dataset or what that data contained. However, DP does not protect the raw data itself during computation: it only limits what can be learned from outputs. The computing party still sees plaintext inputs and can potentially learn information beyond what is protected by the DP guarantee.

Searchable Encryption: This allows searching over encrypted data without fully decrypting it. Various schemes exist with different trade-offs between search functionality, privacy guarantees, and efficiency. Searchable encryption is useful for specific patterns like keyword search but does not support general computation on encrypted data.

HE's position in this landscape: Homomorphic encryption occupies a unique niche. Unlike TEEs, it requires no trust in hardware vendors or firmware. Unlike MPC, it does not require interactive communication between multiple parties during computation: one party can encrypt data and send it to another party for non-interactive processing. Unlike ZKPs, it supports actual computation on encrypted data rather than just verification. Unlike DP, it protects the raw data itself during computation, not just the statistical properties of outputs.

The trade-off is performance. HE is significantly slower than plaintext computation, with overheads ranging from hundreds to millions of times depending on the operations and parameters. Ciphertexts are much larger than plaintexts, creating bandwidth costs. The set of supported operations is constrained: efficient addition and multiplication are available, but arbitrary functions require decomposition into these basic operations, which can be expensive.

HE is most appropriate when:

- You need to compute on data at an untrusted server without trusting hardware enclaves
- Interactive protocols like MPC are impractical due to latency or availability constraints
- The computation can be expressed efficiently as arithmetic or Boolean circuits
- Batch processing is possible, allowing the overhead to be amortized across many inputs
- The security requirements justify the performance cost

HE is generally not appropriate when:

- You can trust a TEE for your threat model and need near-native performance
- The computation requires many conditional branches or data-dependent memory access patterns
- Latency requirements are extremely tight (sub-millisecond response times)
- Only aggregate statistics are needed and differential privacy suffices

When Homomorphic Encryption Is the Right Tool

Let us be specific about scenarios where HE is currently a practical choice, based on real deployments and well-understood use cases.

Private set operations: Computing intersections, unions, or differences between encrypted sets is one of the most mature HE applications. Microsoft Edge's Password Checkup feature uses BFV-based HE to check whether a user's password appears in a database of known breached passwords without sending the actual password to Microsoft's servers. Apple's Live Caller ID and Visual Search features use similar techniques. These are private set intersection (PSI) problems that map well to HE because they can be expressed as polynomial evaluations over encrypted data.

Encrypted aggregation: Summing, averaging, or computing other aggregate statistics on encrypted values from multiple sources is efficient with additive homomorphic schemes and feasible with fully homomorphic schemes when more complex aggregations are needed. Healthcare organizations can compute aggregate statistics across patient populations without centralizing raw records. Financial institutions can compute combined risk metrics across institutions without sharing individual transaction data.

Encrypted machine learning inference: Running trained neural networks or other ML models on encrypted inputs is increasingly practical for certain model types and sizes. The CKKS scheme, designed for approximate arithmetic on real numbers, is particularly suited to this because neural network computations are naturally expressed as matrix multiplications and additions followed by activation functions that can be approximated with polynomials. Inference on small to medium models (like logistic regression, shallow networks, or

quantized models) can complete in seconds to minutes on modern hardware, which may be acceptable for batch processing or non-real-time applications.

Private analytics and data clean rooms: Organizations that need to perform joint analytics on combined datasets without sharing raw data can use HE-based “clean rooms” where queries are executed on encrypted data. This is particularly valuable when regulatory constraints prevent data sharing but business needs require cross-organization analysis. The computation overhead is significant, but for batch analytics run periodically rather than in real time, it can be acceptable.

Encrypted search and pattern matching: Searching over encrypted databases or matching patterns against encrypted text is possible with HE, though efficiency depends heavily on the specific search requirements. Exact string matching can be expressed as polynomial operations, while more complex searches may require careful algorithm design to minimize multiplicative depth.

Scenarios where you should probably not use HE right now:

Real-time interactive applications requiring sub-second latency: The overhead of HE operations is still too high for many real-time use cases. If your application must respond in milliseconds, HE will likely be a bottleneck unless the encrypted computation can be done entirely on-device with very small inputs.

General-purpose program execution: While FHE theoretically supports arbitrary computation, expressing a general program as an arithmetic or Boolean circuit and executing it homomorphically is typically orders of magnitude slower than running it in plaintext. HE is practical for specific computational patterns, not as a general replacement for unencrypted computation.

Workloads dominated by branching and data-dependent operations: HE schemes are efficient at addition and multiplication but struggle with conditional branches (if-then-else), comparisons, and data-dependent memory access. These operations require expensive circuit constructions that can make the overall computation prohibitively slow. If your algorithm relies heavily on control flow rather than arithmetic, consider MPC or TEEs instead.

Scenarios where a trusted party already exists: If all parties involved already trust a single entity to perform computation on their data, adding HE complexity provides no additional security benefit and only adds cost.

The key principle is this: HE should be used when the security benefit of computing on encrypted data justifies the performance, complexity, and operational costs it introduces. It is not a silver bullet for all privacy problems, but for the specific scenarios where it fits, it provides guarantees that no other technology can match.

How This Book Is Organized

This book is structured to take you progressively from foundational concepts to production-ready expertise. Each chapter builds on previous material, and while you can reference individual chapters as needed, reading sequentially will provide the most coherent learning path.

The book is organized into parts that correspond to increasing levels of depth and practicality:

Part I: Foundations establishes the motivation, threat models, and cryptographic background needed for everything that follows. You will learn the number-theoretic foundations (modular arithmetic, polynomial rings) that underlie modern HE schemes.

Part II: Core HE Concepts and Lattice Cryptography introduces lattice-based cryptography, the mathematical foundation of all modern practical HE schemes. You will understand the Learning With Errors problem, why it is believed to be hard, and how it enables encryption schemes with homomorphic properties. The core mechanics of HE are explained here: noise growth, multiplicative depth, relinearization, rescaling, and the techniques that make complex computation possible.

Part III: Major Schemes in Detail provides deep dives into the leading HE schemes used in production today. You will learn how BFV and BGV work for exact integer arithmetic, how CKKS handles approximate real and complex number computation, and how TFHE approaches the problem through Boolean circuits and fast bootstrapping. Each scheme is explained from first principles, with mathematical details and practical guidance on when to use it.

Part IV: Implementation Fundamentals covers the practical skills needed to work with HE libraries. You will learn encoding strategies, key management, core operations, serialization, and debugging techniques. Circuit design patterns are covered in depth, showing how to transform ordinary algorithms into HE-compatible forms that minimize expensive operations.

Part V: Production Systems and Advanced Topics walks through complete implementations from requirements to deployment. You will build progressively more sophisticated systems: encrypted calculators, private aggregation pipelines, encrypted database-style queries, and private ML inference services. Performance optimization, system architecture, bootstrapping, security considerations, and operational concerns are all covered with production-oriented guidance.

Part VI: Context, Security, and the Ecosystem places HE in its broader context. You will learn about security considerations specific to HE deployments, compare HE systematically with alternative technologies, understand the standardization landscape, and develop frameworks for evaluating new schemes and making technology choices.

The back matter includes a glossary of terms, notation reference, scheme comparison tables, parameter selection guidelines, troubleshooting guide, architectural decision frameworks, and a curated bibliography pointing to primary sources for further study.

Throughout the book, you will encounter several conventions:

- Mathematical notation is introduced when needed and always explained. Symbols are defined before use, and every formula includes an explanation of what each component represents.
- Code examples are written for current versions of major libraries (SEAL 4.4, OpenFHE 1.5.1, TFHE-rs latest) and are complete enough to be reproduced. Examples include comments explaining why each step is taken, not just what it does.
- Educational examples use simplified parameters that make computation fast for demonstration purposes. These are clearly marked as such and should never be used in production. Production parameter recommendations are provided separately with security level specifications.
- Cross-references to other chapters help you navigate the material and revisit concepts as their significance becomes clearer.

You do not need to master every detail on first reading. HE is a complex topic, and understanding deepens with repeated exposure. Focus on building intuition for the core concepts first: what homomorphism means, why noise matters, how schemes differ in their strengths, and what trade-offs are involved. The mathematical details will become clearer as you see them applied in concrete implementations.

Let us now begin the technical journey by establishing the cryptographic foundations that make modern HE possible.

Chapter 2: Cryptographic Prerequisites for Software Engineers

What Modern Encryption Actually Guarantees

Before diving into homomorphic encryption specifically, we need to establish what encryption means in modern cryptography and what guarantees it provides. Many software engineers use encryption libraries without fully understanding the security properties those libraries are designed to provide, which can lead to subtle vulnerabilities even when the encryption itself is mathematically sound.

At its core, encryption transforms readable data (plaintext) into apparently random data (ciphertext) using a key, with the property that someone possessing the correct key can reverse the transformation and recover the original plaintext. But this basic description hides important distinctions about what “secure” encryption actually means.

Confidentiality: The primary goal of encryption is confidentiality: ensuring that anyone who sees the ciphertext but does not possess the decryption key learns nothing meaningful about the plaintext. This seems obvious, but the precise meaning matters. A naive notion of security might say “an attacker cannot decrypt the message.” Modern cryptography demands more: an attacker should learn no partial information about the plaintext at all, not even whether two ciphertexts encrypt the same message or whether a particular word appears in the text.

Integrity: Encryption alone does not guarantee integrity. An attacker who can modify ciphertexts might cause the decrypted result to change in predictable ways, even without knowing the key. For example, flipping a bit in an AES-encrypted block will flip corresponding bits in the decrypted plaintext. If you need to detect tampering, encryption must be combined with a message authentication code (MAC) or use an authenticated encryption mode like AES-GCM.

Authenticity: Related to integrity, authenticity ensures that a message came from who it claims to come from. Encryption does not provide this by itself: if anyone with the key can encrypt messages, then anyone with the key could have produced any given ciphertext. Digital signatures or MACs are needed for authentication.

HE schemes primarily provide confidentiality during computation. Most do not provide integrity or authenticity guarantees by default, which means an honest-but-curious server cannot learn your plaintext data, but a malicious server could potentially modify ciphertexts to influence computation results. Understanding this limitation is essential when designing HE-based systems: you may need additional mechanisms like authenticated encryption for key exchange, MACs for verifying computation integrity, or zero-knowledge proofs for proving correct execution.

Symmetric and Asymmetric Cryptography in One Page Each

Modern cryptography divides into two main categories based on how keys are used: symmetric (same key for encryption and decryption) and asymmetric (different keys).

Symmetric encryption uses a single shared secret key for both encrypting and decrypting data. If Alice wants to send an encrypted message to Bob, they must first agree on a shared key through some secure channel. Once they share the key, Alice encrypts her message with it and sends the ciphertext to Bob, who decrypts it with the same key.

The most widely used symmetric encryption algorithm today is AES (Advanced Encryption Standard), which operates on 128-bit blocks of data using keys of 128, 192, or 256 bits. AES is extremely efficient in both software and hardware implementations, making it suitable for encrypting large amounts of data. Other symmetric algorithms include ChaCha20 (popular in TLS 1.3), Threefish, and various stream ciphers.

Symmetric encryption has two main advantages: it is fast, often processing gigabytes per second on modern CPUs, and keys are relatively short while providing strong security. A 256-bit AES key provides security that would require astronomical computational resources to break by brute force. The main disadvantage is key distribution: how do Alice and Bob agree on a shared

key without an attacker intercepting it? This problem led to the development of asymmetric cryptography.

Asymmetric encryption (also called public-key cryptography) uses a pair of mathematically related keys: a public key that anyone can use to encrypt messages, and a private key that only the owner can use to decrypt them. If Bob wants to receive encrypted messages, he generates a key pair, publishes his public key, and keeps his private key secret. Alice can then encrypt messages using Bob's public key, and only Bob can decrypt them with his private key.

The most well-known asymmetric algorithms are RSA (based on integer factorization), Diffie-Hellman key exchange (based on discrete logarithms), and elliptic curve variants like ECDH and ECDSA. These schemes enable secure communication between parties who have never met, because they can exchange public keys over insecure channels without compromising security.

Asymmetric encryption solves the key distribution problem but comes with significant costs: it is much slower than symmetric encryption (often thousands of times slower), keys are larger, and ciphertexts expand considerably compared to plaintext. For this reason, most practical systems use a hybrid approach: asymmetric encryption establishes a shared secret, and then symmetric encryption handles bulk data transfer. TLS, which secures web traffic, uses exactly this pattern.

HE schemes are fundamentally asymmetric: they use public keys (or evaluation keys) for encryption and homomorphic operations, while only the holder of the secret key can decrypt results. This structure is essential for HE's use case: a client encrypts data with a public key and sends it to an untrusted server, which performs computation using only public information and returns encrypted results that only the client can decrypt.

Semantic Security and Indistinguishability

The concept of “security” in cryptography has evolved considerably since encryption was first studied seriously. Early notions of security were informal: a cipher was considered secure if no one could break it with known techniques. Modern cryptography uses rigorous mathematical definitions that precisely specify what it means for an encryption scheme to be secure.

The central notion is **semantic security**, introduced by Goldwasser and Micali in 1982. An encryption scheme is semantically secure if, given a ciphertext,

an attacker cannot learn any partial information about the plaintext beyond its length. This is stronger than merely saying “the attacker cannot decrypt the message.” It means that even questions like “does this message contain the word ‘password’?” or “is this message longer than 100 characters?” should be unanswerable from the ciphertext alone (beyond what the ciphertext length already reveals).

Semantic security is typically formalized through **indistinguishability games**, which are thought experiments that define security precisely. The most common game is called IND-CPA (Indistinguishability under Chosen Plaintext Attack):

1. A challenger generates an encryption key pair and gives the public key to an attacker.
2. The attacker chooses two messages of equal length, m_0 and m_1 .
3. The challenger encrypts one of them (chosen at random) and sends the ciphertext to the attacker.
4. The attacker tries to determine which message was encrypted.

An encryption scheme achieves IND-CPA security if no efficient attacker can succeed at this game with probability significantly better than random guessing (50 percent). This definition captures the intuition that ciphertexts should reveal nothing about their plaintexts: if an attacker cannot distinguish between encryptions of two chosen messages, then the ciphertext gives away no useful information.

There are stronger notions as well:

- **IND-CCA1** (Indistinguishability under Non-Adaptive Chosen Ciphertext Attack): The attacker can request decryptions of chosen ciphertexts before receiving the challenge ciphertext.
- **IND-CCA2** (Indistinguishability under Adaptive Chosen Ciphertext Attack): The attacker can request decryptions even after seeing the challenge ciphertext, except for the challenge ciphertext itself. This is considered the gold standard for public-key encryption security.

Most modern HE schemes achieve IND-CPA security but not IND-CCA2. This is intentional: achieving CCA security typically requires mechanisms that break homomorphism. If an attacker could decrypt arbitrary ciphertexts,

they could potentially use that ability to learn information about the secret key by crafting special ciphertexts. The trade-off is acceptable because HE applications typically assume honest-but-curious adversaries who follow the protocol rather than actively trying to break it through adaptive attacks.

Understanding these security notions matters for practical deployment. If your application requires CCA security (for example, if an attacker might submit maliciously crafted ciphertexts for processing), you need to layer additional protections on top of HE, such as verifying ciphertext validity before processing or using authenticated encryption for result delivery.

Computational Hardness Assumptions

Cryptographic security ultimately rests on assumptions about the difficulty of certain mathematical problems. We do not prove that breaking an encryption scheme is impossible; we prove that breaking it would require solving a problem that appears computationally intractable with known algorithms. If someone discovers an efficient algorithm for that underlying problem, the cryptographic scheme becomes insecure.

Traditional public-key cryptography relies on two main hardness assumptions:

- **Integer factorization:** Given a large number n that is the product of two primes p and q , it is hard to find p and q . RSA's security depends on this.
- **Discrete logarithm:** Given a generator g and value $h = g^x$ in a finite group, it is hard to find x . Diffie-Hellman and elliptic curve cryptography depend on variants of this.

These problems are believed to be hard for classical computers but would be efficiently solvable by sufficiently large quantum computers using Shor's algorithm. This has motivated the development of post-quantum cryptography: schemes whose security relies on problems believed to be hard even for quantum computers.

Lattice-based cryptography, which underlies all modern practical HE schemes, is one of the leading post-quantum approaches. Its security rests on problems related to high-dimensional lattices (regular grids of points in many dimensions), particularly the Shortest Vector Problem and Learning With

Errors. These problems have strong worst-case to average-case reductions: if you can break the cryptographic scheme on typical inputs, you can solve hard lattice problems on worst-case inputs. This provides stronger confidence in security than factorization or discrete log, where no such reduction exists.

No known quantum algorithm solves these lattice problems efficiently. Grover's algorithm provides a quadratic speedup for search problems, which effectively halves the security level (a 256-bit classical security becomes 128-bit quantum security), but this is addressed by using larger parameters. Lattice-based schemes are considered among the strongest candidates for post-quantum security, which is why NIST's post-quantum standardization effort selected several lattice-based algorithms for standardization in 2024.

When selecting HE parameters, you will choose a “security level” that corresponds to the estimated computational cost of breaking the scheme. Common choices are 128-bit security (comparable to AES-128), 192-bit (AES-192), or 256-bit (AES-256). These numbers represent the estimated number of operations an attacker would need to perform, on a classical computer, to break the encryption with non-negligible probability. A scheme with 128-bit security requires approximately 2^{128} operations to break, which is computationally infeasible with current and foreseeable technology.

Randomness and Entropy in Cryptographic Systems

Cryptographic algorithms rely heavily on randomness. Keys must be generated randomly, encryption schemes use random values to ensure that encrypting the same message twice produces different ciphertexts, and many security proofs assume that certain values are drawn from uniform distributions. If the randomness is poor or predictable, even mathematically sound cryptographic schemes can be broken.

True randomness vs pseudo-randomness: True randomness comes from physical processes like thermal noise, radioactive decay, or atmospheric noise. Computers generate true random bits using hardware random number generators (HRNGs) that sample such physical phenomena. However, HRNGs are slow and may not produce enough entropy for all cryptographic needs.

Pseudo-random number generators (PRNGs) take a small amount of true randomness (called a seed) and expand it into a long sequence of bits that appear random but are actually deterministic: given the same seed, a PRNG

always produces the same output. For cryptographic use, we need cryptographically secure pseudo-random number generators (CSPRNGs), which have the property that even if an attacker sees some of the output bits, they cannot predict future outputs or reconstruct the seed.

Most modern operating systems provide CSPRNG interfaces: `/dev/urandom` on Linux and macOS, `CryptGenRandom` or `BCryptGenRandom` on Windows, `getrandom()` in modern POSIX systems. HE libraries use these system CSPRNGs to generate keys and random values during encryption. As a software engineer using HE, you should generally rely on your library's built-in randomness rather than implementing your own, unless you have specific requirements that the library does not meet.

Entropy requirements: The security of cryptographic keys depends on having sufficient entropy in their generation. A 256-bit key generated with only 32 bits of actual entropy is effectively a 32-bit key: an attacker can try all 2^{32} possible seeds rather than all 2^{256} possible keys. HE schemes use large keys (often megabytes in size), and all of that key material must be generated with adequate entropy.

In practice, modern operating systems maintain entropy pools that accumulate randomness from various sources (interrupt timing, hardware events, user input) and provide it to applications requesting random data. For most deployment scenarios, the system CSPRNG is sufficient. Special care may be needed in embedded systems, virtual machines, or other constrained environments where entropy sources are limited.

Reproducibility for testing: During development and testing, you may want to use fixed random seeds to make tests reproducible. This is acceptable for testing but must never be used in production. HE libraries typically provide ways to inject custom randomness sources for testing while using secure system randomness by default. Be explicit about when you are using test randomness versus production randomness, and ensure that test configurations cannot accidentally reach production.

The Gap Between Textbook and Real-World Cryptography

Textbook descriptions of cryptographic schemes often simplify or omit details that are essential for real-world security. Understanding these gaps is crucial

for deploying HE correctly.

Deterministic vs probabilistic encryption: Many textbook examples show deterministic encryption, where the same plaintext always encrypts to the same ciphertext. This is insecure: an attacker who sees two identical ciphertexts knows they encrypt the same plaintext, which leaks information. Real-world encryption schemes are probabilistic: they incorporate random values (called randomness or coins) during encryption so that each encryption of the same message produces a different ciphertext. HE schemes are probabilistic by design, and the randomness is essential for security.

Key size vs security level: Textbook cryptography often conflates key size with security level. In practice, the relationship depends on the best known attacks against the scheme. For AES, a 256-bit key provides approximately 256 bits of security because the best attack is essentially brute force. For lattice-based schemes, the relationship between parameters (like polynomial degree and modulus size) and security level is more complex and depends on sophisticated lattice reduction algorithms. HE libraries typically handle this mapping internally, but you should understand that choosing parameters is not simply about picking large numbers.

Implementation security: A mathematically secure scheme can be broken through implementation flaws. Timing attacks exploit variations in execution time that depend on secret data. Cache attacks use shared CPU caches to infer information about computations. Side-channel attacks measure power consumption, electromagnetic emissions, or other physical properties to extract secrets. HE implementations are vulnerable to these attacks just like any other cryptographic code. Production deployments should use implementations that have been reviewed for side-channel resistance and may need additional protections depending on the threat model.

Parameter selection: Textbook descriptions of cryptographic schemes often use small parameters for clarity, but those parameters would be trivially broken in practice. HE schemes are particularly sensitive to parameter choices: too small, and the scheme is insecure; too large, and it becomes impractically slow; poorly balanced, and noise grows too quickly during computation, causing decryption failures. Production parameter selection requires careful analysis of security requirements, computational constraints, and the specific operations to be performed. Modern HE libraries provide tools for selecting appropriate parameters, but understanding the trade-offs is essential.

Error handling: Cryptographic code must handle errors carefully. In many schemes, leaking information about why decryption failed (wrong key? corrupted ciphertext? noise too large?) can help an attacker refine their attack. HE schemes have a non-zero probability of decryption failure due to noise growth, and how this failure is reported can have security implications. Libraries typically abstract this away, but custom implementations must be careful.

Understanding these gaps between theory and practice will serve you well throughout the rest of this book. When we examine specific HE schemes and implement them using real libraries, we will pay attention to these practical concerns alongside the mathematical foundations. The goal is not just to understand how HE works in principle, but to use it correctly in systems that must be secure, reliable, and performant in production environments.

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