

Lecture Series in Porous Media

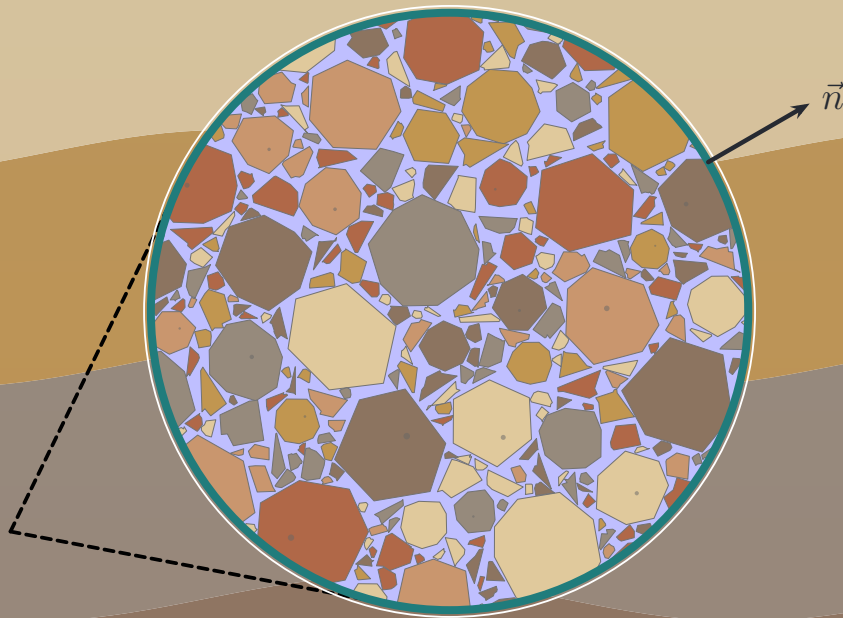
Concepts, Methods and Applications
Focused on Geological Systems Engineering

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Professor of Geo-Energy Solid and Fluid Mechanics • TU Delft • Summer 2026

Release Note: This book is a work in progress. It currently spans 8 chapters.

Further chapters (e.g. multiphase flow, numerical methods, well testing) are planned.



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Concepts, Methods and Applications Focused on Geological Systems
Engineering

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Materials are relevant to the courses:

Dynamics of Solids and Fluids, Multiphase Flow in Porous Media,
Grand Challenges in Earth Sciences, Subsurface Storage, Reservoir Simulation,
and Fluid Dynamics.

Based on the Video Lectures available on [the course YouTube channel](#).

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