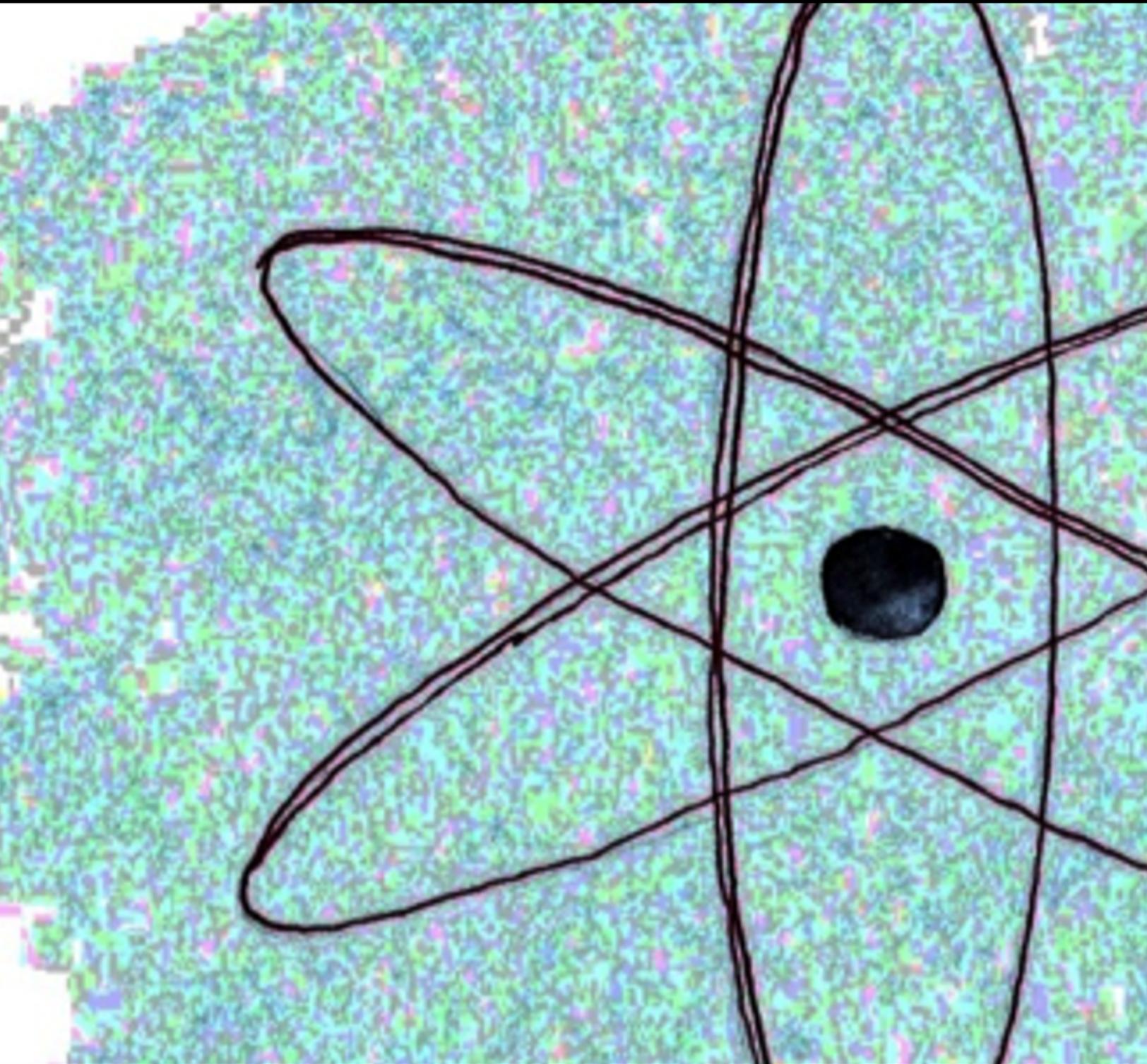


The Eclectic Watercolourists's Guide to:
Special Relativity



Walther Johns

The Eclectic Watercolourist's Guide to: SPECIAL RELATIVITY

[with a bit of General thrown in]

Walther Johns and Walther Johns

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*To my wife, **Colleen***

For her undying love and support and for whom this book was written

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There's something wrong with speed

As every schoolchild knows if a bow can fire an arrow at 60kph and a man galloping on a horse at 30kph fires an arrow forwards, then that arrow will be going at 90kph.

Well, the schoolchild is wrong, as every physicist in the world will now agree. The true answer is – well, my calculator does not have enough decimal places on it to do the calculation, but it is something like 89.99...kph with about 15 nines in total. This is very, very close to 90kph and would need extremely sophisticated measuring equipment to detect the difference.

Before 1905, when Einstein told us the correct answer, all schoolchildren and most physicists knew perfectly well that the true answer was 90kph, exactly. Not almost 90kph, not so close to 90kph that it hardly makes any difference, but exactly 90kph. It stood to reason and was obvious and never, ever needed to be discussed, except to teach schoolchildren.



The speed of sound was first measured in the time of Galileo. Galileo (1564-1642), the man who invented telescopes (no, he didn't actually- a spectacle-making Dutchman (well, he was German by birth but was living in the Netherlands) called Hans Lipperhey did- or it might have been his kids, nobody's sure, but Hans applied for the patent – Galileo improved the telescope and used it to study the night sky) also put forward a method for measuring the speed of light, but it was too fast for him. In fact, it was so fast that Galileo thought that it might be infinitely fast. But it isn't (or is it? - see later).

Light goes along at about 1 billion kph (bkph). A cracking pace, to be sure, and very, very fast indeed, but a long, long way short of infinitely fast.

Once the speed of light could be measured accurately, the trouble started.

If a man galloping on a horse at 30kph shone a torch forward then how fast would the light be traveling? 1 billion and 30kph said all the schoolchildren. Some physicists were not so sure and thought they should measure it. When they did they found that light was moving at 1 bkph and not 1 billion and 30kph.

Of course, they didn't use a horse and a torch, they were a little more sophisticated than that, but the result was the same. The speed of light was independent of the speed of the source of the light. Thus, if the source (the torch, let us say) was stationary or moving backwards or moving forwards at whatever speed (or sideways, come to that), the speed of light was the same.

This did not really surprise the physicists – they had seen this sort of thing before, with sound. Sound, as every schoolchild should know, is a wave that moves through the air at a fixed speed that is dependent on the density, temperature and other characteristics of the air through which it is moving.

If somebody shouts at you, the sound travels through the air at whatever the fixed speed applicable to the air between her and you happens to be. If she is running towards you, or away from you, the sound still travels at that fixed rate and you measure the speed of the sound she made to be the same. Just like the light.

Of course, with the sound, if there is a breeze blowing towards you this will increase the speed of the sound that you measure by the speed of the breeze and if it is blowing away from you this will reduce your measurement. Light is quite independent of the speed of any breeze but physicists were quite happy with this. More or less, anyway.

Also, with sound, if you run towards the person shouting at you then you are moving with respect to the air. It is like having a breeze. And when you run towards the sound you will find that the speed of the sound coming past you is increased by the speed of your running, and if you run away then the speed of the sound passing you reduces by the speed of your running.

But when they measured the speed of a light beam going past a train moving towards the light at 60kph, say, the speed of light stayed at 1 bkph.

Think of it, there is a light shining down the railway track. You are on the platform measuring the speed of that light and a train comes thundering past as fast as it can go. Your sister is on the train measuring the speed of that same light. And you both come up with exactly the same answer: - 1 bkph. Even if your sister was on a jet plane going at top speed, or zooming along on the fastest imaginable rocket she would still measure the speed of that light beam to be exactly the same as you do. 1 bkph.

The physicists did not know what to make of this. It turned out that however fast and in whatever direction you were traveling and however fast and in whatever direction the source of the light was traveling, the speed of light measured by you (and everyone else) was the same. 1 bkph. If this does not surprise you then either (i) you have heard of this before or (ii) you have not understood what I am saying. Let's put this into a schoolchild example: you are walking down the road on the left with the railway track and the sea to your right - see the picture.



You are strolling along the road at about 4kph, lost in the beauty of the scene and wondering idly about the speed of light.

There is a train coming down the track towards you- you can't see it yet but it will be there in a moment. The train driver, looking at his speedometer sees that he is going at 80kph. Coming up behind you, not yet in the picture, is a car. For your sake we must hope that the driver is paying more attention to what is happening around her than you are, for her speedometer says that she is going at 140kph.

The driver of the car would say, if anyone asked her, that she was going at 140kph with respect to the road, at 136kph with respect to that pedestrian who seems to be wandering all over the place and at 220kph with respect to the train coming towards her. [We know, since Einstein told us, that this is not exactly accurate but it is accurate enough for my purposes].

Similarly, you would say that you were going at 4kph with respect to the road, at 136kph (or, if you were being really fancy, maybe you would say at -136kph) with respect to the motorist who is going to give you a hell of a fright in a couple of seconds because you don't yet know she is there and at 84kph with respect to the train.

And it is easy to see what the train driver would say. Let us now imagine that there are two beams of light- one coming from in front and one from behind. Everybody – you, the train driver and the motorist– measure the speed of both beams of light to be 1bkph. And it does not matter if the source of the beam is moving forwards, backwards, sideways or whatever. And if the car and you and the train were jet planes or interstellar rockets travelling at hundreds of millions of kilometers per hour

you would all still measure the speed of the light beams to be 1bkph.

If you have understood this and you have never heard of it before and if you are not surprised then you are truly something else.

The physicists did not know what to make of this. Light, they thought, must be a very funny thing. And they set about constructing wonderful theories to explain just what light was and how this all worked. But they got their knickers in a huge knot. Because they were looking at the wrong thing. Light wasn't funny at all- something else entirely different was funny.

It was the genius of Einstein that solved the problem. Effectively he said "Light travels at a fixed speed that is the same for everyone no matter how they or the light are moving. All our measurements show this to be the case. So let's stop trying to explain it. If light travels at a fixed speed for everybody then there is something odd about our understanding of speed. And what is speed? Speed is distance divided by time. If you travel 360 kilometers in 4 hours your speed is 90kph. So, if our understanding of speed is odd, then it is probably because we don't understand what distance or time really is".

Space and distance are closely related, so another way of saying all this is to say that light only seems odd because our understanding of space and time is wrong.

Proper velocity, momentum, conservation

Oh dear, back to the French. You might want a cup of coffee.

Your sister lives in a town 500 km away from you. You invite her to come visit for a week. She accepts, hops into her car and 5 hours later she's on your doorstep. Average speed? 100 kph. Is it really that simple?

OK, your sister lives in the next solar system 44 trillion km away from you. You invite her to come for a visit. She accepts, hops into her space cruiser and 6 years and 3 months later she is on your doorstep. Average speed? You say, my sister did 44 trillion kilometers in 6 years and 3 months which is an average speed of 800 million kph or $.8c$.

Your sister says, I traveled 26.4 trillion km and it took me 3 years and 9 months which is an average speed of 800 million kph or $.8c$.

Remember- the proper length is the length measured by someone at rest wrt the distance being measured. In the case of the distance between the two solar systems this is you. Anyone moving wrt that distance, such as your sister, will measure the distance to be shorter.

Also remember- the proper time is the time measured by someone present at both events. In this case the two events are your sister leaving her home and then arriving at yours. The person present at both is your sister and anyone moving wrt her, such as you, will measure a longer time.

But you both agree that you are both moving at $.8c$ wrt each other. So, is it really that simple? It seems logical to take the speed as distance divided by time and to use the distances and times as recorded by each observer.

But, in being logical in your calculation (44 trillion divided by $6\frac{1}{4}$ years) you have divided the proper length (or distance) by an improper time. And your sister, in being equally logical in her calculation (26.4 trillion divided by $3\frac{3}{4}$ years) has divided an improper length by a proper time.

There is nothing wrong with this but it does raise the obvious question- why not calculate velocity to be the proper length divided by the proper time? Why not, indeed? This is called the proper velocity- again an unfortunate word.

In this case the proper length is 44 trillion kms and the proper time is $3\frac{3}{4}$ years, which gives the proper velocity to be 1340 million kph or $1.34c$. Faster than the speed of light. Well no, not really. What is the proper velocity of light?

Well, in your frame of reference (i.e. the frame of reference in which you are stationary) the speed of light is 1 bkph. So in 1 hour light travels 1 billion km. The proper distance is 1 billion km but the

time of 1 hour is not the proper time. The proper time is the time measured by a photon itself to do the trip- i.e. the time measured by a clock traveling with the photon.

In Chapter 3 we saw the relationship between improper and proper time. Calling T the improper time and T_0 the proper time we had

$$T = \gamma T_0$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$

Now, if $T = \gamma T_0$ then we have

$$\begin{aligned} T_0 &= T/\gamma \\ &= T\sqrt{(1 - v^2/c^2)} \end{aligned}$$

if $\gamma = 1/\sqrt{1 - v^2/c^2}$ then $1/\gamma = \sqrt{1 - v^2/c^2}$

We have $T=1$ hour which is the improper time that you measure as the photon of light travels 1 billion km. In our formula v is the speed of the traveling object- in this case the photon that, like all photons, goes at c . Hence $v=c$.

Thus $1 - v^2/c^2 = 1 - c^2/c^2 = 1 - 1 = 0$. And the square root of 0 is also 0.

Thus $T_0 = 1$ times 0 =0.

Thus the proper time, the time as measured by the photon itself to go 1 billion kilometers is nothing. Niente, nada. In fact, the photon could go 2 billion kilometers in no time at all. 10 billion kilometers in no time at all.

A trillion, a quadrillion, umpteen gazillions in no time at all. And if you do umpteen gazillion kilometers in no time at all you are going infinitely fast. So Galileo was right! Clever boy. The proper velocity of light is infinite.



We use η to denote the proper velocity of something. η is the lower-case Greek letter called eta. You can pronounce this as either 'etta' (as in letter) or as 'eater' - for reasons which may become clear a bit later it is slightly more fun to pronounce it as 'eater'.

So v is the velocity calculated by someone using the distance and time that they measure in their reference frame. η is the velocity calculated by dividing the proper length by the proper time. And don't fall into the trap of thinking that proper is better- it's not, it's just different.

Why have we brought in proper velocity? No, it's not just to confuse you. Wait a bit, and all will become clear(ish).

There is something called momentum. It is denoted by p . Prior to Einstein physicists said that momentum equals mass times velocity

$$p = mv$$

Momentum was also said to be conserved. Physicists love things that are conserved because it makes calculations and everything easier to understand and do.

If you have 2 snooker balls on a snooker table and they are moving you can calculate the momentum of each ball and add them up to get the total momentum. The balls then hit each other and bounce off at different speeds and directions. You re-calculate the momentum of each ball and add them up. Although the momentum of each ball has changed the total has not- it is the same as before. The total momentum of the system consisting of 2 balls has been conserved. Ok, you have to assume that the balls glide over the table top without any friction and that the snooker table is perfectly flat and so on, but you get the idea.

Scientists talk about a closed, isolated system. System means all the balls or whatever it is that you are looking at and closed and isolated means that there is no external force acting on the system and the system does not act on anything outside itself- but don't worry too much about this.

The important point was that when they did all the experiments they found that their theoretical idea of the conservation of momentum was actually true. They were, naturally, very pleased. What they didn't realize was that it was only almost true. It was only (almost) true at slow speeds.

Einstein said (assumption A) that physics is the same for everybody traveling at a constant velocity. Thus if you are not accelerating (i.e. your velocity is constant) and you carry out a snooker ball experiment and you find that momentum is conserved then your sister coming past at high speed should find the same (provided that she also is not accelerating). But she doesn't. Not only does your sister measure distances in the x-direction to be smaller, but she also has to use the velocity addition equation and, furthermore, mass changes. None of this was known to those who first thought up the idea of conservation of momentum.

But, nonetheless, they had got a firm grip on the tail of a real truth. Momentum *is* conserved but $p = mv$ is not true, while $p = m\eta$ is.

At slow speeds the difference between v and η is minute. You would never notice. At high speeds it can be massive. At the speed of light it is the difference between 1 bkph and infinity, and you can't get anything bigger than that.



Before Einstein mass was always the same. The mass of an object at rest wrt you- called the rest mass, but it could have been called the proper mass- was exactly the same as the mass when the object was moving wrt you. Therefore you did not have to say *which* mass you meant when you said $p = mv$. After Einstein we know that mass is relative- the mass of a moving object is greater than when it is stationary. Thus when you say $p = m\eta$ you need to say which mass you mean. You mean the rest mass. That is the correct mass to use. To make it clear you could write $p = m_0\eta$ where m_0 denotes the rest mass, but I have not done this. Laziness, probably, but a little later on I will have to use m_0 to stop us getting even more muddled

Now, there are 3 things you need to understand.

Firstly: Conservation of Momentum is not an inevitable and logical consequence of Einstein's two assumptions- you can't derive it from the maths. It is an idea that stands or falls on its own.

Secondly: Conservation of Momentum is only a theory. We cannot be sure that it is true. It is an idea, a very nice idea- some say that it is a beautiful idea- but it is only an idea. Is it true? Yes, we think so, we hope so, but all we can do is dream up experiments to test the idea. Having dreamt up an experiment we do it and see if the idea passes or fails. So far it has always passed- or, at least, we think so.

If you had the satisfying and beautiful idea that a car that was 4 meters long was damned well always 4 meters long and you carried out an experiment to measure it as it came past you at 100 kph would you think your result was a pass or a fail?

You will recall that the car coming past at 100 kph is not 4m long but only 3 point then 13 9's and an 8 meters long. If you only measured to 10 decimal places (and this is *very, very* accurate) you would get an answer of 4. If you *could* only measure to 10 places (that's all your technology was capable of) would your idea pass or fail?

It would pass, but it would still be wrong. And if, 3 years later, technology allowed you to be 1000 times more accurate you could measure to 13 decimal places and your wrong idea would still pass.

You can never be sure, when you are doing experiments, if just a little more accuracy (or maybe a lot more accuracy) wouldn't change your conclusions.

But all the experiments we have ever done have shown Conservation of Momentum (using η) to be true at the level of accuracy we are capable of. But if we had more capability and more accuracy...?

But the idea is too beautiful- it has to be true. Isn't truth supposed to be beautiful? So we will take it as true. For now.

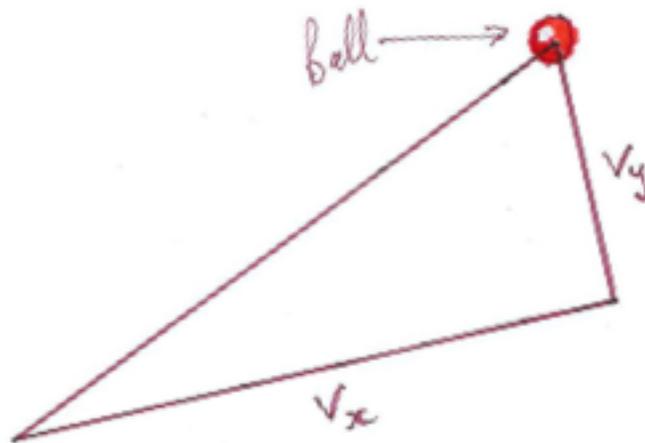
Thirdly: So far we have always arranged for the moving object to be moving along our x-axis. This got rid of y's and z's. But once we start talking about balls on a snooker table we are, well, a bit snookered. It is just too limiting to only allow the balls to move on the x-axis, even if there are only two. We have to bring in y as well.

Here's a bird's-eye view of a snooker table with a ball on it. The path of the ball is also shown.



Let us, arbitrarily, take the x and y directions as shown. The ball is clearly moving in both the x and the y directions. The ball has both a velocity in the x -direction (called v_x) and in the y -direction (v_y).

Here is the ball again



Another right-angled triangle. Pythagoras rides to the rescue again.

The long side (opposite the right angle) represents the actual velocity of the ball (which we call v without any subscript), while the v_x and v_y represents the velocity along the x and y directions

respectively.

So

$$v^2 = v_x^2 + v_y^2$$

and if we had balls flying through the air in all directions we would have

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

The same is true for η

$$\eta^2 = \eta_x^2 + \eta_y^2 + \eta_z^2$$

And when we talk about momentum ($p = m\eta$) we can have momentum in the x-direction, momentum in the y-direction and momentum in the z-direction.

The momentum in the x-direction is called p_x and

$$p_x = m\eta_x$$

Also $p_y = m\eta_y$ and $p_z = m\eta_z$

And the exciting thing is that momentum is conserved in each direction. You have a whole bunch of balls flying through the air- you are in a spaceship, not accelerating and away from gravity. It is a closed, isolated system.

You calculate the *proper* velocity of each ball in its x, y and z- directions. You calculate the p_x , p_y and p_z for each ball. You then add up all the p_x 's, all the p_y 's and all the p_z 's getting three totals.

The balls collide with each other- smash, bang, crash- and fly off at new speeds in new directions. You once again calculate all the p_x 's, p_y 's and p_z 's and add them up getting another three totals. *No you don't!* You get the *same* three totals. Momentum is conserved in each direction. And this is also what your sister coming past, lickety-split, will measure because you and she have both used η and not v in your calculations.

Another thing that used to be conserved until Einstein came along was energy. Remember, the old-style momentum ($p = mv$) was also conserved until he pointed out that you had to redefine momentum ($p = m\eta$).

In the good old bad old days when men were men it was thought (known) that although energy came in different forms and could change from one form to another the total energy was always conserved.

A rifle bullet zipping through the air has energy because it is moving- this is called kinetic energy. If the bullet slams into a brick wall it comes to a dead stop and no longer has any kinetic energy left. But energy is conserved so the kinetic energy has to go somewhere.

Some of it goes into the phwat that you hear when the bullet hits the wall. Some brick chips may well fly off taking some kinetic energy with them. The brick wall may well vibrate a bit, taking another bit of the energy. But the bulk of the kinetic energy is translated into thermal energy (fancy term for heat energy) and the bullet gets hot and may even melt. Ah, you say, but the bullet cools down and therefore its heat energy is destroyed. No, well, yes, the bullet cools down but its heat energy is not destroyed. It is passed on to the things near the bullet which get a bit warmer themselves and which then in turn pass on their extra heat to the stuff near them. Eventually the whole world is just a little bit warmer than it was. Of course, the rifle bullet got its energy from the explosion of the cartridge in the rifle, so this all started with some chemical energy.

So, in the good old bad old days you could add up all the energy in a closed and isolated system and know that whatever happened in this system the total energy would remain unchanged. If one part of the system got more energetic then some other part would have to become less energetic, and by exactly the same amount.

Nowadays, of course, we know (think) that energy and mass are linked and so what is conserved in a closed and isolated system is the total of mass and energy combined and the unit of conversion between them is c^2 .

And every experiment we have ever carried out has shown this to be the case- at the level of accuracy we are capable of...

Is Conservation of Mass plus Energy true? We hope so and we think so and even if it is not true it is certainly almost true.