DEEP LEARNING WITH PYTHON

FROM FUNDAMENTALS TO FRONTIERS

A COMPLETE GUIDE TO BUILDING, TRAINING, AND DEPLOYING NEURAL NETWORKS

BUILD POWERFUL MODELS

TRAIN WITH CONFIDENCE

OPTIMIZE PERFORMANCE

DEPLOY IN THE REAL WORLD

NumPy | Pandas | Matplotlib | scikit-learn | TensorFlow | PyTorch

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Deep Learning with Python: From Fundamentals to Frontiers

A Complete Guide to Building, Training, and Deploying Neural Networks

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Contents

A Complete Guide to Building, Training, and Deploying Neural Networks

About this book: This book takes you on a comprehensive journey through deep learning, from the mathematical foundations that make it possible to the cutting-edge architectures transforming technology today. Whether you are a complete beginner or an experienced practitioner looking to deepen your understanding, each chapter builds methodically on the last, combining clear explanations with fully working Python code using modern libraries including NumPy, Pandas, Matplotlib, Scikit-learn, TensorFlow/Keras, and PyTorch. By the time you reach the final page, you will be able to build, train, evaluate, optimize, and deploy deep learning models with confidence in real-world applications.

Introduction: The World Through Neural Networks

In 2012, a neural network called AlexNet entered the ImageNet Large Scale Visual Recognition Challenge and won by a landslide. Its top-5 error rate of 15.3% shattered the previous best of 26.2%, and the gap was so large that many competitors suspected a bug in the evaluation pipeline. There was no bug. What happened that year marked the beginning of the deep learning revolution, and within a decade, neural networks would go from academic curiosity to the backbone of technology used by billions of people every day.

Today, deep learning powers the image search on your phone, the language model you chat with, the recommendation engine deciding what video you watch next, the fraud detection system protecting your bank account, and increasingly, the systems driving cars, diagnosing diseases, and designing new materials. This is not science fiction. It is engineering, built on mathematics that anyone can learn, implemented in code that you can write yourself.

This book exists because deep learning has two faces that rarely appear together in a single resource. On one side, there is the theory: the linear algebra, the calculus, the probability, the optimization algorithms, and the architectural decisions that make neural networks work. On the other side, there is the craft: the data preprocessing pipelines, the debugging sessions, the hyperparameter tuning, the deployment challenges, and the production monitoring that turn a research prototype into something reliable enough to put in front of real users. Most books focus on one face or the other. This book covers both, in equal measure, with every concept grounded in working code you can run and modify.

What You Will Learn

After working through this book, you will be able to:

Understand the mathematical foundations that underlie every neural network, from the matrix multiplications in a single layer to the gradient computations that drive learning across thousands of parameters.

Build neural networks from scratch using only NumPy, giving you an intimate understanding of forward propagation, backpropagation, and the training loop before you ever touch a high-level framework.

Use TensorFlow/Keras and PyTorch to implement sophisticated architectures including convolutional neural networks for image recognition, recurrent networks and transformers for language processing, and generative models that create new images and text.

Apply transfer learning to leverage pretrained models on your own data, dramatically reducing the amount of labeled data and compute needed to build state-of-the-art systems.

Deploy trained models to production using REST APIs, containerization, and cloud platforms, and monitor them for performance degradation over time.

Reason about emerging trends including large language models, multimodal AI, diffusion models, and the ethical considerations that accompany increasingly powerful systems.

Who This Book Is For

This book assumes you are comfortable writing Python code, have some familiarity with basic data structures, and have at least a passing acquaintance with statistics. You do not need to be a mathematician or a computer scientist. If you can write a for loop, understand what a function does, and have seen a scatter plot, you have the prerequisites.

The book is organized so that beginners can follow every chapter from start to finish, while advanced practitioners can use it as a reference, jumping to specific chapters on topics they want to deepen or update. The code examples progress from simple to complex, and each concept is introduced with intuition before formalism.

How This Book Is Organized

The first part of the book (Chapters 1 through 3) establishes the foundations. You will learn the history and context of deep learning, the mathematical tools you need, and how to build your first neural networks from scratch. These

chapters are essential for everyone, regardless of background, because they establish the mental models that make the rest of the book coherent.

The second part (Chapters 4 through 7) covers the mechanics of training: optimization algorithms, regularization techniques, data preprocessing, and model evaluation. These are the practical skills that separate someone who can run a tutorial from someone who can build a system that works on real data.

The third part (Chapters 8 through 13) explores the major application domains and architectures: computer vision with convolutional networks, natural language processing with recurrent networks and transformers, generative models, transfer learning, and reinforcement learning. Each chapter includes case studies with complete, runnable code.

The final part (Chapters 14 through 15) covers deployment, MLOps, and emerging trends, taking you from a model that works on your laptop to a system that serves predictions reliably in production, and then looking ahead at where the field is heading.

A Note on Code

Every code example in this book is designed to run as written. The examples use current versions of the major libraries: NumPy 1.26+, Pandas 2.0+, Matplotlib 3.7+, Scikit-learn 1.3+, TensorFlow 2.15+/Keras 3.x, and PyTorch 2.x. Where a concept can be illustrated in either TensorFlow/Keras or PyTorch, you will see both, because in practice, the ability to work with either framework is valuable.

All code is thoroughly commented and explained line by line. You will find expected outputs, common pitfalls, troubleshooting tips, and best practices woven into the narrative rather than relegated to exercises or sidebars. This book is designed for self-study: read, run the code, modify it, break it, fix it, and move on.

Let us begin.

Chapter 1: The Deep Learning Revolution: Why It Matters

What Is Deep Learning and Why Now?

Deep learning is a subset of machine learning based on artificial neural networks with multiple layers. The "deep" refers to the number of layers: whereas traditional machine learning models might have one or two transformation steps, deep learning models can have dozens, hundreds, or even thousands of layers, each learning progressively more abstract representations of the input data.

To understand why this matters, consider how a neural network learns to recognize a cat in a photograph. The first layer might learn to detect simple edges and gradients. The second layer combines those edges into shapes like curves and corners. Deeper layers combine shapes into parts like ears, eyes, and whiskers. The deepest layers recognize the arrangement of these parts as something we would call "a cat." This hierarchical feature learning is the core insight of deep learning: complex patterns can be decomposed into simpler patterns, and neural networks can learn this decomposition automatically from data.

The question is not whether deep learning works. It does, spectacularly. The question is why it took so long. Neural network ideas date back to the 1940s, yet the revolution began only around 2012. Three factors converged to make deep learning practical:

Computational power. Modern GPUs can perform trillions of floating-point operations per second, making it feasible to train networks with billions of parameters. Training a state-of-the-art image classifier that would have taken months on 2010 hardware now takes hours.

Data availability. The internet, smartphones, and sensors generate vast quantities of labeled data. ImageNet alone contains over 14 million images annotated with more than 20,000 categories. Language models are trained on terabytes of text scraped from the web.

Algorithmic advances. Better architectures (residual connections, attention mechanisms), better optimizers (Adam, RMSProp), and better regularization techniques (dropout, batch normalization) made it possible to train much deeper and more capable networks than was previously thought feasible.

A Brief History: From Perceptrons to Transformers

The story of deep learning begins in 1943, when neurophysiologist Warren McCulloch and logician Walter Pitts published a paper showing that groups of artificial neurons could, in principle, compute any logical function. Their model was purely theoretical: no one had the computing power to train such networks on real data.

The first practical neural network arrived in 1957, when Frank Rosenblatt at the Cornell Aeronautical Laboratory built the Perceptron, a single-layer network that could learn to classify simple patterns. The Perceptron generated enormous excitement, with newspapers predicting that machines would soon replace human thinkers. But in 1969, Marvin Minsky and Seymour Papert published "Perceptrons," a book demonstrating mathematically that single-layer networks could not solve even the simplest non-linear problem: the XOR function. Funding dried up, and neural network research entered what became known as the first AI winter.

The breakthrough came in the 1980s with backpropagation, an algorithm for computing gradients through multiple layers of a network. Though elements of backpropagation had been described as early as the 1960s by Paul Werbos, it was popularized by Rumelhart, Hinton, and Williams in 1986. Backpropagation made it possible to train multilayer networks, but they remained small and limited by the available computing power.

The 1990s saw the development of convolutional neural networks (CNNs), pioneered by Yann LeCun for digit recognition. LeNet, introduced in 1989 and refined through the 1990s, was used by banks to read check deposits. Recurrent neural networks (RNNs) gained traction for speech recognition and language modeling. But these systems were still narrow, limited to specific domains.

The modern deep learning era began with the 2012 ImageNet competition. Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton trained a deep CNN called AlexNet on GPU hardware and achieved an error rate more than 10 percentage

points better than the runner-up. The margin was so large that it convinced the research community that deep learning was not just another approach but a fundamentally better one.

From there, progress accelerated rapidly:

- 2014: Ian Goodfellow introduced Generative Adversarial Networks (GANs), opening the door to neural networks that create rather than classify.
- 2015: Kaiming He and colleagues at Microsoft Research introduced Residual Networks (ResNet), which used skip connections to train networks with hundreds of layers, winning the ImageNet competition with a 3.57% top-5 error rate, below human-level performance.
- 2017: Ashish Vaswani and colleagues at Google published "Attention Is All You Need," introducing the Transformer architecture. This paper would prove to be one of the most influential in machine learning history, because Transformers became the foundation of every major language model since, including BERT, GPT, T5, and their successors.
- 2018: Google released BERT (Bidirectional Encoder Representations from Transformers) and OpenAI released GPT (Generative Pre-trained Transformer), establishing the pretrain-then-finetune paradigm that dominates NLP today.
- 2020: Ho, Jain, and Abbeel published Denoising Diffusion Probabilistic Models (DDPM), launching the diffusion model family that now powers tools like DALL-E, Stable Diffusion, and Midjourney.
- 2022: OpenAI released GPT-4, demonstrating capabilities far beyond simple text generation, including reasoning, coding, and multimodal understanding.

The trajectory has been exponential in both capability and impact. What began as a research curiosity now underpins products used by billions of people.

The Python Ecosystem for Deep Learning

Python is the dominant language for deep learning, and for good reasons. It is readable, it has an enormous library ecosystem, and its dynamic nature makes

it easy to prototype new architectures quickly. The core libraries you will use throughout this book fall into several categories:

Numerical computing: NumPy provides the fundamental array operations that underlie all numerical computation in Python. Every deep learning framework, including TensorFlow and PyTorch, builds on concepts from NumPy. Understanding NumPy is not optional; it is the foundation.

Data manipulation: Pandas extends NumPy with labeled data structures (DataFrames) and powerful tools for loading, cleaning, and transforming tabular data. Most real-world datasets arrive as CSV files or database tables, and Pandas is the standard tool for working with them.

Visualization: Matplotlib is the foundational plotting library in Python. Seaborn builds on it with higher-level statistical visualizations. Visualization is critical for exploring data, debugging models, and communicating results.

Traditional machine learning: Scikit-learn provides implementations of classical algorithms (linear regression, decision trees, random forests, SVMs) along with tools for preprocessing, model selection, and evaluation. Even in a deep learning project, you will use scikit-learn for baseline comparisons, data splitting, and metric computation.

Deep learning frameworks: TensorFlow/Keras and PyTorch are the two dominant deep learning frameworks. Keras provides a high-level, user-friendly API for building and training models. PyTorch offers a more flexible, research-oriented approach with dynamic computation graphs. In this book, you will learn both, because the ability to work with either is a practical necessity.

Specialized libraries: Hugging Face Transformers provides access to thousands of pretrained language models. TorchVision and TensorFlow Hub provide pretrained vision models. Libraries like OpenCV handle image processing, while librosa handles audio.

Setting Up Your Development Environment

Before writing any code, you need a working development environment. The recommended setup is:

1. **Python 3.10 or later** installed on your system.
2. **A virtual environment** (using `venv` or Conda) to isolate your project dependencies.

3. **Core packages:** Install NumPy, Pandas, Matplotlib, and Scikit-learn via pip:

```
pip install numpy pandas matplotlib scikit-learn
```

4. **Deep learning frameworks:** For GPU-accelerated training, install the CUDA-enabled versions:

```
pip install tensorflow[and-cuda] torch torchvision torchaudio
```

For CPU-only development (sufficient for following along with most examples):

```
pip install tensorflow torch torchvision torchaudio
```

5. **Jupyter Lab** for interactive exploration:

```
pip install jupyterlab
```

6. **Optional but recommended:** Install Hugging Face Transformers for NLP work:

```
pip install transformers datasets accelerate
```

Verifying Your Installation

Run this script to confirm everything is working:

```python
import numpy as np
import pandas as pd
import matplotlib
import sklearn
import tensorflow as tf
import torch

print(f"NumPy: {np.__version__}")
print(f"Pandas: {pd.__version__}")
print(f"Matplotlib: {matplotlib.__version__}")
print(f"Scikit-learn: {sklearn.__version__}")
print(f"TensorFlow: {tf.__version__}")
print(f"PyTorch: {torch.__version__}")
print(f"CUDA available: {torch.cuda.is_available()}")
```

Expected output (versions may differ):

```
NumPy: 1.26.4
Pandas: 2.2.2
Matplotlib: 3.8.4
Scikit-learn: 1.5.1
TensorFlow: 2.16.1
PyTorch: 2.3.1
CUDA available: True
```

If CUDA is not available on a machine with an NVIDIA GPU, check that your CUDA toolkit version matches the PyTorch build. Visit the PyTorch website (pytorch.org) for the correct installation command for your platform.

How to Use This Book

Read this book sequentially. Each chapter builds on concepts from earlier chapters, and skipping ahead will leave gaps in your understanding. However, once you have completed the foundational chapters (1 through 7), you can explore the application-specific chapters (8 through 13) in whatever order interests you most.

For every code example, follow this process:

1. **Read** the explanation carefully.
2. **Type** the code yourself rather than copying and pasting. The act of typing reinforces understanding.

3. **Run** the code and verify the output matches what is described.
4. **Modify** something: change a hyperparameter, swap an activation function, or try a different dataset. See what breaks and why.
5. **Move on** once you understand the core idea. You can always return to revisit details later.

The book includes troubleshooting tips where common errors occur, but the best way to learn is through deliberate experimentation. Break things intentionally. Fix them. That is how you develop the intuition that separates a competent practitioner from someone who merely follows tutorials.

With your environment set up and the historical context established, we are ready to dive into the mathematics that makes deep learning possible.

Chapter 2: Mathematical Foundations: The Language of Neural Networks

Deep learning is mathematics in motion. Every forward pass through a neural network is a sequence of matrix multiplications and element-wise operations. Every training step is an application of the chain rule from calculus. Every loss function is built on concepts from probability and information theory. You do not need to be a mathematician to understand these ideas, but you do need to be comfortable with the notation and the intuition. This chapter builds that foundation, one concept at a time, with NumPy code that makes the abstract concrete.

Linear Algebra Essentials: Vectors, Matrices, and Operations

At its core, a neural network transforms vectors into other vectors. An input image becomes a vector of pixel values. That vector is multiplied by a weight matrix to produce a new vector. This process repeats through every layer until the final output. Understanding how these transformations work requires fluency in linear algebra.

Scalars, vectors, and matrices. A scalar is a single number, like 3.14 or -7. In NumPy:

```
import numpy as np

# A scalar
a = np.float64(3.14)
print(a)          # 3.14
print(type(a))    # <class 'numpy.float64'>
```

A vector is an ordered list of numbers, represented in NumPy as a one-dimensional array:

```
# A column vector (shape (3,))
v = np.array([1.0, 2.0, 3.0])
print(v)          # [1. 2. 3.]
print(v.shape)    # (3,)
```

A matrix is a two-dimensional grid of numbers:

```
# A 2x3 matrix
M = np.array([[1.0, 2.0, 3.0],
              [4.0, 5.0, 6.0]])
print(M)
# [[1. 2. 3.]
#  [4. 5. 6.]]
print(M.shape)    # (2, 3)
```

Matrix multiplication is the most important operation in deep learning. When we multiply a matrix W of shape (m, n) by a vector x of shape (n,), the result has shape (m,). Each element of the output is the dot product of a row of W with the vector x:

```
W = np.array([[1.0, 2.0, 3.0],
              [4.0, 5.0, 6.0]])  # shape (2, 3)
x = np.array([0.1, 0.2, 0.3])    # shape (3,)

y = W @ x  # Matrix-vector multiplication using @ operator
print(y)   # [ 1.4  3.8]
print(y.shape)  # (2,)

# Equivalent to np.dot(W, x) or np.matmul(W, x)
```

The computation for the first element is: 1.0 * 0.1 + 2.0 * 0.2 + 3.0 * 0.3 = 0.1 + 0.4 + 0.9 = 1.4.

For matrix-matrix multiplication, the rule is that the inner dimensions must match: a (m, n) matrix times a (n, p) matrix produces a (m, p) matrix:

```
A = np.array([[1.0, 2.0],
              [3.0, 4.0]])  # shape (2, 2)
B = np.array([[5.0, 6.0],
              [7.0, 8.0]])  # shape (2, 2)

C = A @ B  # shape (2, 2)
print(C)
# [[19. 22.]
#  [43. 50.]]
```

Element-wise operations are equally important. Adding, multiplying, or applying functions element by element:

```
a = np.array([1.0, 2.0, 3.0])
b = np.array([4.0, 5.0, 6.0])

print(a + b)    # [5. 7. 9.]   element-wise addition
print(a * b)    # [4. 10. 18.] element-wise multiplication (Hadamard product)
print(np.exp(a))# [2.72 7.39 20.09] element-wise exponential
print(np.log(b))# [1.39 1.61 1.79] element-wise natural log
```

Broadcasting is NumPy's mechanism for applying operations between arrays of different shapes. When you add a scalar to a vector, the scalar is "broadcast" across all elements:

```
v = np.array([1.0, 2.0, 3.0])
print(v + 10)   # [11. 12. 13.]  the scalar 10 is broadcast
```

Broadcasting also works between a matrix and a vector:

```
M = np.array([[1.0, 2.0, 3.0],
              [4.0, 5.0, 6.0]])
v = np.array([10.0, 20.0, 30.0])
print(M + v)
# [[11. 22. 33.]
#  [14. 25. 36.]]
```

This is exactly how bias vectors are added to the output of a weight matrix in a neural network layer.

Reshaping and transposing are essential for matching tensor dimensions:

```
x = np.array([1.0, 2.0, 3.0, 4.0, 5.0, 6.0])
print(x.reshape(2, 3))
# [[1. 2. 3.]
#  [4. 5. 6.]]

M = np.array([[1.0, 2.0, 3.0],
              [4.0, 5.0, 6.0]])
print(M.T)  # Transpose: rows become columns
# [[1. 4.]
#  [2. 5.]
#  [3. 6.]]
```

Tensors are the generalization of matrices to higher dimensions. A 3D tensor might represent a batch of images: (batch_size, height, width, channels). NumPy handles tensors naturally as n-dimensional arrays:

```
# A batch of 2 grayscale images, each 3x3 pixels
batch = np.random.rand(2, 3, 3)
print(batch.shape)  # (2, 3, 3)

# A batch of 4 RGB images, each 28x28 pixels
rgb_batch = np.random.rand(4, 28, 28, 3)
print(rgb_batch.shape)  # (4, 28, 28, 3)
```

Calculus for Deep Learning: Gradients and Chain Rule

Training a neural network is an optimization problem: we want to find the set of weights that minimizes a loss function. The tool for finding minimums is gradient descent, which requires computing derivatives. In deep learning, we deal with multivariate calculus, but the core idea is simple.

Derivatives and gradients. The derivative of a function f(x) at a point x tells us the rate of change: if we move a tiny amount in the positive direction, how much does f increase? For a single-variable function, this is straightforward:

```
# Numerical derivative approximation
def numerical_derivative(f, x, h=1e-5):
    return (f(x + h) - f(x - h)) / (2 * h)

# Example: f(x) = x^2, derivative should be 2x
f = lambda x: x ** 2
print(numerical_derivative(f, 3.0))  # ~6.0 (exact: 2*3 = 6)
print(numerical_derivative(f, 0.0))  # ~0.0
```

For a function of multiple variables, the gradient is a vector of partial derivatives, one for each variable. The gradient points in the direction of steepest ascent, so we move in the opposite direction to descend:

```
# Gradient of f(x, y) = x^2 + y^2
def gradient(f, x, y, h=1e-5):
    dfdx = (f(x + h, y) - f(x - h, y)) / (2 * h)
    dfdy = (f(x, y + h) - f(x, y - h)) / (2 * h)
    return np.array([dfdx, dfdy])

f2 = lambda x, y: x**2 + y**2
print(gradient(f2, 3.0, 4.0))  # [6. 8.] -- points toward origin
```

The chain rule is the mathematical engine of backpropagation. If a function z depends on y, and y depends on x, then:

dz/dx = dz/dy * dy/dx

In neural networks, the output depends on the last layer, which depends on the previous layer, and so on back to the input. The chain rule lets us compute the gradient of the loss with respect to every weight in the network by working backward from the output.

Consider a simple example: z = (w * x + b)^2, where we want dz/dw and dz/db.

```
# Chain rule computation
x, w, b = 2.0, 3.0, 1.0
y = w * x + b  # 7.0
z = y ** 2     # 49.0

# dz/dw = dz/dy * dy/dw = 2*y * x
dz_dw = 2 * y * x  # 28.0
# dz/db = dz/dy * dy/db = 2*y * 1
dz_db = 2 * y * 1  # 14.0

# Verify with numerical derivatives
def loss(w, b):
    return (w * x + b) ** 2

print(numerical_derivative(lambda w: loss(w, b), w))  # ~28.0
print(numerical_derivative(lambda b: loss(w, b), b))  # ~14.0
```

Jacobian matrices generalize the chain rule to vector-valued functions. If f maps R^n to R^m, its Jacobian is an m x n matrix where each entry J_ij = df_i/dx_j. In practice, deep learning frameworks compute these automatically through a process called automatic differentiation, which we will explore in Chapter 3.

Probability and Statistics: Distributions, Expectation, and Bayes

Neural networks make probabilistic predictions. When a network classifies an image, it outputs a probability distribution over classes. Understanding these distributions requires basic probability.

Random variables and distributions. A random variable X takes values from some set, with probabilities given by a distribution. Common distributions in deep learning:

```python
from numpy.random import default_rng
rng = default_rng(42)  # Seed for reproducibility

# Uniform distribution: every value in [a, b] equally likely
uniform_samples = rng.uniform(-1, 1, size=5)
print(uniform_samples)  # e.g., [-0.50, 0.21, -0.37, 0.63, 0.83]

# Normal (Gaussian) distribution: bell curve centered at mean
normal_samples = rng.normal(loc=0.0, scale=1.0, size=5)
print(normal_samples)  # e.g., [0.49, -0.14, -0.24, -0.58, 0.65]

# Standard normal: mean=0, std=1 -- used for weight initialization
std_normal = rng.standard_normal(size=5)
print(std_normal)
```

The Gaussian distribution is ubiquitous because of the Central Limit Theorem: the sum of many independent random variables tends toward a Gaussian, regardless of the original distributions. This is why noise in neural networks, errors in measurements, and even weight initialization all involve Gaussians.

Expectation and variance. The expected value E[X] is the long-run average. For a continuous variable with PDF p(x):

E[X] = integral of x * p(x) dx

The variance measures spread: Var(X) = E[(X - E[X])^2].

```python
# Empirical mean and variance
samples = rng.normal(loc=5.0, scale=2.0, size=100000)
print(f"Empirical mean: {samples.mean():.4f}")    # ~5.0
print(f"Empirical std:  {samples.std():.4f}")      # ~2.0
print(f"Empirical var:  {samples.var():.4f}")      # ~4.0
```

Bayes' theorem relates conditional probabilities and is foundational for probabilistic reasoning:

P(A|B) = P(B|A) * P(A) / P(B)

In deep learning, Bayes' theorem appears in Bayesian neural networks, in the derivation of variational autoencoders, and in understanding how prior beliefs should be updated with evidence. The practical takeaway is that probability distributions encode uncertainty, and a good model quantifies its uncertainty rather than making overconfident predictions.

Information Theory: Entropy, Cross-Entropy, and KL Divergence

Information theory, developed by Claude Shannon in the 1940s, provides the mathematical framework for measuring information and uncertainty. The key concepts are entropy, cross-entropy, and Kullback-Leibler (KL) divergence.

Entropy measures the uncertainty or "surprise" in a probability distribution. A uniform distribution has maximum entropy (maximum uncertainty). A distribution concentrated on one outcome has minimum entropy (zero surprise):

```
def entropy(p):
    """Compute entropy of a discrete probability distribution."""
    p = np.array(p, dtype=np.float64)
    p = p[p > 0]  # Remove zero probabilities (0 * log(0) = 0)
    return -np.sum(p * np.log2(p))

# Uniform over 4 outcomes: maximum entropy
print(f"Uniform: {entropy([0.25, 0.25, 0.25, 0.25]):.4f} bits")  # 2.0000

# Concentrated on one outcome: minimum entropy
print(f"Certain: {entropy([1.0, 0.0, 0.0, 0.0]):.4f} bits")      # 0.0000

# Somewhere in between
print(f"Mixed:   {entropy([0.7, 0.1, 0.1, 0.1]):.4f} bits")      # 0.8631
```

Cross-entropy measures the average number of bits needed to encode events from distribution p when using a code optimized for distribution q. In machine learning, cross-entropy is the standard loss function for classification:

```
def cross_entropy(p, q):
    """Compute cross-entropy H(p, q) where p is true, q is predicted."""
    p = np.array(p, dtype=np.float64)
    q = np.array(q, dtype=np.float64)
    q = np.clip(q, 1e-15, 1.0)  # Avoid log(0)
    return -np.sum(p * np.log2(q))

# True distribution: class 0 is correct
true_dist = [1.0, 0.0, 0.0, 0.0]

# Perfect prediction
perfect_pred = [1.0, 0.0, 0.0, 0.0]
print(f"Perfect: {cross_entropy(true_dist, perfect_pred):.4f} bits")  # 0.0000

```

```
# Confident wrong prediction
wrong_pred = [0.01, 0.99, 0.0, 0.0]
print(f"Wrong:   {cross_entropy(true_dist, wrong_pred):.4f} bits")    # 6.6439

# Uncertain but correct
uncertain = [0.6, 0.1, 0.2, 0.1]
print(f"Uncertain: {cross_entropy(true_dist, uncertain):.4f} bits")   # 0.7370
```

KL divergence measures how one probability distribution diverges from a second, expected distribution. It is always non-negative and equals zero only when the distributions are identical:

```
def kl_divergence(p, q):
    """Compute KL(p || q)."""
    p = np.array(p, dtype=np.float64)
    q = np.array(q, dtype=np.float64)
    mask = p > 0
    return np.sum(p[mask] * np.log(p[mask] / q[mask]))

p = [0.7, 0.3]
q = [0.6, 0.4]
print(f"KL(p||q): {kl_divergence(p, q):.4f}")  # 0.0201

# KL divergence is not symmetric
print(f"KL(q||p): {kl_divergence(q, p):.4f}")  # 0.0231
```

The relationship between these measures is elegant: KL(p||q) = H(p, q) - H(p). Cross-entropy equals entropy plus KL divergence. Minimizing cross-entropy is equivalent to minimizing KL divergence (since the entropy of the true distribution is constant), which is why cross-entropy loss drives predicted distributions toward the true distribution.

Putting It All Together: A Mini Numerical Example

Let us combine everything from this chapter into a single example that previews what a neural network layer does. Consider a single neuron with two inputs:

z = w1 * x1 + w2 * x2 + b a = sigmoid(z) = 1 / (1 + exp(-z))

```python
def sigmoid(z):
    return 1.0 / (1.0 + np.exp(-z))

# Inputs
x = np.array([0.5, -0.2])
# Weights
w = np.array([0.8, 1.5])
# Bias
b = 0.1

# Linear combination (dot product + bias)
z = np.dot(w, x) + b  # 0.8*0.5 + 1.5*(-0.2) + 0.1 = 0.1
print(f"z = {z:.4f}")

# Activation
a = sigmoid(z)
print(f"a = sigmoid(z) = {a:.4f}")

# Suppose the true label is 1.0 and we use binary cross-entropy loss
y_true = 1.0
loss = -(y_true * np.log(a + 1e-15) + (1 - y_true) * np.log(1 - a + 1e-15))
print(f"Loss = {loss:.4f}")

# Gradient of loss with respect to z: dL/dz = a - y
dL_dz = a - y_true
print(f"dL/dz = {dL_dz:.4f}")

# Gradient of loss with respect to weights: dL/dw_i = dL/dz * x_i
dL_dw = dL_dz * x
print(f"dL/dw = {dL_dw}")

# Gradient of loss with respect to bias: dL/db = dL/dz
dL_db = dL_dz
print(f"dL/db = {dL_db:.4f}")

# One step of gradient descent (learning rate = 0.1)
lr = 0.1
w_new = w - lr * dL_dw
b_new = b - lr * dL_db
print(f"Updated weights: {w_new}")
print(f"Updated bias: {b_new:.4f}")
```

Expected output:

```
z = 0.1000
a = sigmoid(z) = 0.5250
Loss = 0.6444
dL/dz = -0.4750
dL/dw = [-0.2375  0.0950]
dL/db = -0.4750
Updated weights: [0.8238 1.4905]
Updated bias: 0.1475
```

Notice what happened: the loss gradient pointed in the direction that would increase the output (since a = 0.525 was too low for target y = 1.0). The weight update moved w1 and w2 in directions that make z larger, which makes a closer to 1.0. This is gradient descent in action: compute the error, find the direction of steepest increase, and step in the opposite direction.

With these mathematical tools in hand, we are ready to build actual neural networks. The next chapter takes everything from this chapter and assembles it into a working multilayer perceptron, trained from scratch with nothing but NumPy.

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