The Art of Guesstimation

Lecture 5

Your body is like a walking yardstick, and it's worth knowing things like the width of your hand from pinkie to thumb, or the size of your footsteps, or parts of your hand that measure to almost exactly one or two inches or one or two centimeters.

Mental estimation techniques give us quick answers to everyday questions when we don't need to know the answer to the last penny or decimal point. We estimate the answers to addition and subtraction problems by rounding, which can be useful when estimating the grocery bill. As each item is rung up, round it up or down to the nearest 50 cents.

To estimate answers to multiplication or division problems, it's important to first determine the order of magnitude of the answer. The general rules are as follows:

- For a multiplication problem, if the first number has x digits and the second number has y digits, then their product will have $x + y$ digits or, perhaps, $x + y - 1$ digits. Example: A 5-digit number times a 3-digit number creates a 7- or 8-digit number.

- To find out if the answer to $a \times b$ will have the larger or smaller number of digits, multiply the first digit of each number. If that product is 10 or more, then the answer will be the larger number. If that product is between 5 and 9, then the answer could go either way. If the product is 4 or less, then the answer will be the smaller number.

- For a division problem, the length of the answer is the difference of the lengths of the numbers being divided or 1 more. (Example: With an 8-digit number divided by a 3-digit number, the answer will have $8 - 3 = 5$ or 6 digits before the decimal point.)

- To find out how many digits come before the decimal point in the answer to $a \div b$, if the first digit of a is the same as the first digit of b, then compare the second digits of each number. If the first digit of a is larger than the first digit of b, then the answer will be the longer choice. If the first digit of a is less than the first digit of b, then the answer will be the shorter choice.

In estimating sales tax, if the tax is a whole number, such as 4%, then estimating it is just a straight multiplication problem. For instance, if you're purchasing a car for $23,456, then to estimate 4% tax, simply multiply 23,000 × 0.04 (= $920; exact answer: $938). If the tax is not a whole number, such as 4.5%, you can calculate it using 4%, but then divide that amount by 8 to get the additional 0.5%.

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Suppose a bank offers an interest rate of 3% per year on its savings accounts. You can find out how long it will take to double your money using the "Rule of 70"; this calculation is 70 divided by the interest rate.

Suppose you borrow $200,000 to buy a house, and the bank charges an interest rate of 6% per year, compounded monthly. What that means is that the bank is charging you 6/12%, or 1/2%, interest for every month of your loan. If you have 30 years to repay your loan, how much will you need to pay each month? To estimate the answer, follow these steps:

- Find the total number of payments to be made: 30 × 12 = 360.

- Determine the monthly payment without interest: $200,000 ÷ 360. Simplify the problem by dividing everything by 10 (= 20,000 ÷ 36), then by dividing everything by 4 (= 5000 ÷ 9, or 1000 × 5/9). The fraction 5/9 is about 0.555, which means the monthly payment *without interest* would be about 1000 × 0.555, or $555.

- Determine the amount of interest owed in the first month: $200,000 × 0.5% = $1000.

A quick estimate of your monthly payment, then, would be $1000 to cover the interest plus $555 to go toward the principal, or $1555. This estimate will always be on the high side, because after each payment, you'll owe the bank slightly less than the original amount.

Square roots arise in many physical and statistical calculations, and we can estimate square roots using the divide-and-average method. To find the square root of a number, such as 40, start by taking any reasonable guess. We'll choose $6^2 = 36$. Next, divide 40 by 6, which is 6 with a remainder of 4, or 6 2/3. In other words, 6 × 6 2/3 = 40. The square root must lie between 6 and 6 2/3. If we average 6 and 6 2/3, we get 6 1/3, or about 6.33; the exact answer begins 6.32! ■

Important Term

square root: A number that, when multiplied by itself, produces a given number. For example, the square root of 9 is 3 and the square root of 2 begins 1.414…. Incidentally, the square root is defined to be greater than or equal to zero, so the square root of 9 is *not* –3, even though –3 multiplied by itself is also 9.

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 6.

Doerfler, *Dead Reckoning: Calculating Without Instruments.*

Hope, Reys, and Reys, *Mental Math in the Middle Grades.*

Kelly, *Short-Cut Math.*

Ryan, *Everyday Math for Everyday Life: A Handbook for When It Just Doesn't Add Up.*

Weinstein and Adam, *Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin*.