Mental Addition and Subtraction

Lecture 2

The bad news is that most 3-digit subtraction problems require some sort of borrowing. But the good news is that they can be turned into easy addition problems.

When doing mental addition, we work one digit at a time. To add a 1-digit number, just add the 1s digits (52 + 4 → 2 + 4 = 6, so 52 + 4 = 56). With 2-digit numbers, first add the 10s digits, then the 1s digits (62 + 24 → 62 + 20 = 82 and 82 + 4 = 86).

With 3-digit numbers, addition is easy when one or both numbers are multiples of 100 (400 + 567 = 967) or when both numbers are multiples of 10 (450 + 320 → 450 + 300 = 750 and 750 + 20 = 770). Adding in this way is useful if you're counting calories.

To add 3-digit numbers, first add the 100s, then the 10s, then the 1s. For 314 + 159, first add 314 + 100 = 414. The problem is now simpler, 414 + 59; keep the 400 in mind and focus on 14 + 59. Add 14 + 50 = 64, then add 9 to get 73. The answer to the original problem is 473.

We could do 766 + 489 by adding the 100s, 10s, and 1s digits, but each step would involve a carry. Another way to do the problem is to notice that 489 = 500 – 11; we can add 766 + 500, then subtract 11 (answer: 1255). Addition problems that involve carrying can often be turned into easy subtraction problems.

With mental subtraction, we also work one digit at a time from left to right. With 74 – 29, first subtract 74 – 20 = 54. We know the answer to 54 – 9 will be 40-something, and 14 – 9 = 5, so the answer is 45.

A subtraction problem that would normally involve borrowing can usually be turned into an easy addition problem with no carrying. For 121 – 57, subtract 60, then add back 3: 121 – 60 = 61 and 61 + 3 = 64.

With 3-digit numbers, we again subtract the 100s, the 10s, then the 1s. For 846 – 225, first subtract 200: 846 – 200 = 646. Keep the 600 in mind, then do 46 – 25 by subtracting 20, then subtracting 5: 46 – 20 = 26 and 26 – 5 = 21. The answer is 621.

Three-digit subtraction problems can often be turned into easy addition problems. For 835 – 497, treat 497 as 500 – 3. Subtract 835 – 500, then add back 3: 835 – 500 = 335 and 335 + 3 = 338.

Understanding **complements** helps in doing difficult subtraction. The complement of 75 is 25 because 75 + 25 = 100. To find the complement of a 2-digit number, find the number that when added to the first digit will yield 9 and the number that when added to the second digit will yield 10. For 75, notice that 7 + 2 = 9 and 5 + 5 = 10. If the number ends in 0, such as 80, then the complement will also end in 0. In this case, find the number that when added to the first digit will yield 10 instead of 9; the complement of 80 is 20.

Understanding complements helps in doing difficult subtraction.

Knowing that, let's try 835 – 467. We first subtract 500 (835 – 500 = 335), but then we need to add back something. How far is 467 from 500, or how far is 67 from 100? Find the complement of 67 (33) and add it to 335: 335 + 33 = 368.

To find 3-digit complements, find the numbers that will yield 9, 9, 10 when added to each of the digits. For example, the complement of 234 is 766. Exception: If the original number ends in 0, so will its complement, and the 0 will be preceded by the 2-digit complement. For example, the complement of 670 will end in 0, preceded by the complement of 67, which is 33; the complement of 670 is 330.

Three-digit complements are used frequently in making change. If an item costs $6.75 and you pay with a $10 bill, the change you get will be the complement of 675, namely, 325, $3.25. The same strategy works with change from $100. What's the change for $23.58? For the complement of

2358, the digits must add to 9, 9, 9, and 10. The change would be $76.42. When you hear an amount like $23.58, think that the dollars add to 99 and the cents add to 100. With $23.58, 23 + 76 = 99 and 58 + 42 = 100. When making change from $20, the idea is essentially the same, but the dollars add to 19 and the cents add to 100.

As you practice mental addition and subtraction, remember to work one digit at a time and look for opportunities to use complements that turn hard addition problems into easy subtraction problems and vice versa. ■

Important Term

complement: The distance between a number and a convenient round number, typically, 100 or 1000. For example, the complement of 43 is 57 since 43 + 57 = 100.

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 1.

Julius, *More Rapid Math Tricks and Tips: 30 Days to Number Mastery.*

———, *Rapid Math Tricks and Tips: 30 Days to Number Power.*

Kelly, *Short-Cut Math.*

Problems

Because mental addition and subtraction are the building blocks to all mental calculations, plenty of practice exercises are provided. Solve the following mental addition problems by calculating from left to right. For an *added* challenge, look away from the numbers after reading the problem.

1. 52 + 7

2. 93 + 4