Math in Your Head!

Lecture 1

Just by watching this course, you will learn all the techniques that are required to become a fast mental calculator, but if you want to actually improve your calculating abilities, then just like with any skill, you need to practice.

In school, most of the math we learn is done with pencil and paper, yet in many situations, it makes more sense to do problems in your head. The ability to do rapid mental calculation can help students achieve higher scores on standardized tests and can keep the mind sharp as we age.

One of the first mental math tips you can practice is to calculate from **left to right**, rather than **right to left**. On paper, you might add 2300 + 45 from right to left, but in your head, it's more natural and faster to add from left to right.

These lectures assume that you know the multiplication table, but there are some tricks to memorizing it that may be of interest to parents and teachers. I teach students the multiples of 3, for example, by first having them practice counting by 3s, then giving them the multiplication problems in order (3 × 1, 3 × 2 …) so that they associate the problems with the counting sequence. Finally, I mix up the problems so that the students can practice them out of sequence.

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There's also a simple trick to multiplying by 9s: The multiples of 9 have the property that their digits add up to 9 (9 × 2 = 18 and 1 + 8 = 9). Also, the first digit of the answer when multiplying by 9 is 1 less than the multiplier (e.g., 9 × 3 = 27 begins with 2).

In many ways, mental calculation is a process of simplification. For example, the problem 432 × 3 sounds hard, but it's the sum of three easy problems: 3 × 400 = 1200, 3 × 30 = 90, and 3 × 2 = 6; 1200 + 90 + 6 = 1296. Notice that when adding the numbers, it's easier to add from largest to smallest, rather than smallest to largest.

Again, doing mental calculations from left to right is also generally easier because that's the way we read numbers. Consider 54 × 7. On paper, you might start by multiplying 7 × 4 to get 28, but when doing the problem mentally, it's better to start with 7 × 50 (350) to get an estimate of the answer. To get the exact answer, add the product of 7 × 50 and the product of 7 × 4: 350 + 28 = 378.

Below are some additional techniques that you can start using right away:

- The product of 11 and any 2-digit number begins and ends with the two digits of the multiplier; the number in the middle is the sum of the original two digits. Example: 23 × 11 → 2 + 3 = 5; answer: 253. For a multiplier whose digits sum to a number greater than 9, you have to carry. Example: 85 × 11 → 8 + 5 = 13; carry the 1 from 13 to the 8; answer: 935.

- The product of 11 and any 3-digit number also begins and ends with the first and last digits of the multiplier, although the first digit can change from carries. In the middle, insert the result of adding the first and second digits and the second and third digits. Example: 314 × 11 → 3 + 1 = 4 and 1 + 4 = 5; answer: 3454.

- To square a 2-digit number that ends in 5, multiply the first digit in the number by the next higher digit, then attach 25 at the end. Example: 35^2 → 3 × 4 = 12; answer: 1225. For 3-digit numbers, multiply the first two numbers together by the next higher number, then attach 25. Example: 305^2 → 30 × 31 = 930; answer: 93,025.

- To multiply two 2-digit numbers that have the same first digits and last digits that sum to 10, multiply the first digit by the next higher digit, then attach the product of the last digits in the original two numbers. Example: $84 \times 86 \rightarrow 8 \times 9 = 72$ and $4 \times 6 = 24$; answer: 7224.

- To multiply a number between 10 and 20 by a 1-digit number, multiply the 1-digit number by 10, then multiply it by the second digit in the 2-digit number, and add the products. Example: $13 \times 6 \rightarrow (6 \times 10) + (6 \times 3) = 60 + 18$; answer: 78.

- To multiply two numbers that are both between 10 and 20, add the first number and the last digit of the second number, multiply the result by 10, then add that result to the product of the last digits in both numbers of the original problem. Example: $13 \times 14 \rightarrow 13 + 4 = 17$, $17 \times 10 = 170$, $3 \times 4 = 12$, $170 + 12 = 182$; answer: 182. ■

Important Terms

left to right: The "right" way to do mental math.

right to left: The "wrong" way to do mental math.

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 0.

Hope, Reys, and Reys, *Mental Math in the Middle Grades.*

Julius, *Rapid Math Tricks and Tips: 30 Days to Number Power.*

Ryan, *Everyday Math for Everyday Life: A Handbook for When It Just Doesn't Add Up.*