Masters of Mental Math

Lecture 12

You'll notice that cube rooting of 2-digit cubes doesn't really require much in the way of calculation. It's more like observation—looking at the number and taking advantage of a beautiful pattern.

We started this course using little more than the multiplication table, and we've since learned how to add, subtract, multiply, and divide enormous numbers. In this lecture, we'll review some of the larger lessons we've learned.

One of these lessons is that it pays to look at the numbers in a problem to see if they can somehow help to make the job of finding a solution easier. Can one of the numbers be broken into small factors; are the numbers close together; or can one of the numbers be rounded to give a good approximation of the answer?

We've also learned that difficult addition problems can often be made into easy subtraction problems and vice versa. In fact, if you want to become a "mental mathlete," it's useful to try to do problems in more than one way. We can approach a problem like 21 × 29, for example, using the addition, subtraction, factoring, or close-together methods.

We know that if we multiply a 5-digit number by a 3-digit number, the answer will have 8 (5 + 3) digits or maybe 7. If we pick the first digit of each number at random, then we would assume, just from knowing the multiplication table, that there's a good chance the product of those digits will be greater than 10, which would give us an 8-digit answer. According to **Benford's law**, however, it's far more likely that the original 5-digit and 3-digit numbers will start with a small number, such as 1, 2, or 3, which means that there's about a 50-50 chance of getting an answer with 8 digits or an answer with 7 digits. For most collections of numbers in the real world, such as street addresses or numbers on a tax return, there are considerably more numbers that start with 1 than start with 9.

Also in this course, we've learned how to apply the phonetic code to numbers that we have to remember and to use a set of codes to determine the day of the week for any date in the year. If anything, this course should have taught you to look at numbers differently, even when they don't involve a math problem.

As we've said, it usually pays to try to find features of problems that you can exploit. As an example, let's look at how to find the **cube root** of a number when the answer is a 2-digit number. Let's try 54,872; to find its cube root, all we need to know are the cubes of the numbers from 1 through 10. Notice that the last digits of these cubes are all different, and the last digit either matches the original number or is the 10s complement of the original number.

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To find the cube root of 54,872, we look at how the cube begins and ends. The number 54 falls between 3^3 and 4^3. Thus, we know that 54,000 falls between 30^3 (= 27,000) and 40^3 (= 64,000); its cube root must be in the 30s. The last digit of the cube is 2, and there's only one number from 1 to 10 whose cube ends in 2, namely, 8^3 (= 512); thus, the cube root of 54,872 is 38. Note that this method works only with perfect cubes.

Finally, we've learned that mental calculation is a process of constant simplification. Even very large problems can be broken down into simple steps. The problem $47{,}893^2$, for example, can be broken down into $47{,}000^2 + 893^2 + 47{,}000 \times 893 \times 2$. As we go through this problem, we make use of the criss-cross method, squaring smaller numbers, complements, and phonetic code—essentially, this is the math of least resistance.

To get into the *Guinness Book of World Records* for mental calculation, it used to be that contestants had to quickly determine the 13th root of a 100-digit number. To break the record now, contestants have to find the 13th root of a 200-digit number. Every two years, mathletes can also enter the Mental Calculation World Cup, which tests computation skills similar to what we've discussed in this course. Most of you watching this course are probably not aiming for these world championships, but the material we've covered should be useful to you throughout your life.

All mathematics begins with arithmetic, but it certainly doesn't end there. I encourage you to explore the joy that more advanced mathematics can bring in light of the experiences you've had with mental math. Some people lose confidence in their math skills at an early age, but I hope this course has given you the belief that you can do it. It's never too late to start looking at numbers in a new way. ■

Important Terms

Benford's law: The phenomenon that most of the numbers we encounter begin with smaller digits rather than larger digits. Specifically, for many real-world problems (from home addresses, to tax returns, to distances to galaxies), the first digit is N with probability $\log(N+1) - \log(N)$, where $\log(N)$ is the base 10 logarithm of N satisfying $10^{\log(N)} = N$.

cube root: A number that, when cubed, produces a given number. For example, the cube root of 8 is 2 since $2 \times 2 \times 2 = 8$.

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapters 8 and 9.

Doerfler, *Dead Reckoning: Calculating Without Instruments.*

Julius, *Rapid Math Tricks and Tips: 30 Days to Number Power*.

Lane, *Mind Games: Amazing Mental Arithmetic Tricks Made Easy.*

Rusczyk, *Introduction to Algebra.*

Smith, *The Great Mental Calculators: The Psychology, Methods and Lives of Calculating Prodigies Past and Present.*