Intermediate Multiplication

Lecture 7

The reason I like the factoring method is that it's easier on your memory, much easier than the addition or the subtraction method, because once you compute a number, … you immediately put it to use.

In this lecture, we'll extend our knowledge of 2-by-1 and 3-by-1 multiplication to learn five methods for 2-by-2 multiplication. First is the addition method, which can be applied to any multiplication problem, although it's best to use it when at least one of the numbers being multiplied ends in a small digit. With this method, we round that number down to the nearest easy number. For 41 × 17, we treat 41 as 40 + 1 and calculate (40 × 17) + (1 × 17) = 680 + 17 = 697.

A problem like 53 × 89 could be done by the addition method, but it's probably easier to use the **subtraction method**. With this method, we treat 89 as 90 – 1 and calculate (53 × 90) – (53 × 1) = 4770 – 53 = 4717. The subtraction method is especially handy when one of the numbers ends in a large digit, such as 7, 8, or 9. Here, we round up to the nearest easy number. For 97 × 22, we treat 97 as 100 – 3, then calculate (100 × 22) – (3 × 22) = 2200 – 66 = 2134.

A third strategy for 2-by-2 multiplication is the factoring method. Again, for the problem 97 × 22, instead of rounding 97 up or rounding 22 down, we factor 22 as 11 × 2. We now have 97 × 11 × 2, and we can use the 11s trick from Lecture 1. The result for 97 × 11 is 1067; we multiply that by 2 to get 2134.

When using the factoring method, you often have several choices for how to factor, and you may wonder in what order you should multiply the factors. If you're quick with 2-by-1 multiplications, you can practice the "**math of least resistance**"—look at the problem both ways and take the easier path. The factoring method can also be used with decimals, such as when converting temperatures from Celsius to Fahrenheit.

Another strategy for 2-digit multiplication is **squaring**. For a problem like 13^2, we can apply the close-together method. We replace one of the 13s with 10; then, since we've gone down 3, we need to go back up by adding 3 to the other 13 to get 16. The first part of the calculation is now 10 × 16. To that result, we add the square of the number that went up and down (3): $10 \times 16 = 160$ and $160 + 3^2 = 169$. Numbers that end in 5 are especially easy to square using this method, as are numbers near 100.

If you're quick with 2-by-1 multiplications, you can practice the "math of least resistance"—look at the problem both ways and take the easier path.

Finally, our fifth mental multiplication strategy is the close-together method, which we saw in the last lecture. For a problem like 26 × 23, we first find a round number that is close to both numbers in the problem; we'll use 20. Next, we note how far away each of the numbers is from 20: 26 is 6 away, and 23 is 3 away. Now, we multiply 20 × 29. We get the number 29 in several ways: It's either 26 + 3 or 23 + 6; it comes from adding the original numbers together (26 + 23 = 49), then splitting that sum into 20 + 29. After we multiply 29 × 20 (= 580), we add the product of the distances (6 × 3 = 18): 580 + 18 = 598.

After you've practiced these sorts of problems, you'll look for other opportunities to use the method. For example, for a problem like 17 × 76, you can make those numbers close together by doubling the first number and cutting the second number in half, which would leave you with the close-together problem 34 × 38.

The best method to use for mentally multiplying 2-digit numbers depends on the numbers you're given. If both numbers are the same, use the squaring method. If they're close to each other, use the close-together method. If one of the numbers can be factored into small numbers, use the factoring method. If one of the numbers is near 100 or it ends in 7, 8, or 9, try the subtraction method. If one of the numbers ends in a small digit, such as 1, 2, 3, or 4, or when all else fails, use the addition method. ■

Important Terms

math of least resistance: Choosing the easiest mental calculating strategy among several possibilities. For example, to do the problem 43 × 28, it is easier to do 43 × 7 × 4 = 301 × 4 = 1204 than to do 43 × 4 × 7 = 172 × 7.

squaring: Multiplying a number by itself. For example, the square of 5 is 25.

subtraction method: A method for multiplying numbers by turning the original problem into a subtraction problem. For example, 9 × 79 = (9 × 80) – (9 × 1) = 720 – 9 = 711, or 19 × 37 = (20 × 37) – (1 × 37) = 740 – 37 = 703.

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 3.

Kelly, *Short-Cut Math.*

Problems

Calculate the following 2-digit squares. Remember to begin by going up or down to the nearest multiple of 10.

1. 14^2

2. 18^2

3. 22^2

4. 23^2

5. 24^2

6. 25^2