Go Forth and Multiply

Lecture 3

You've now seen everything you need to know about doing 3-digit-by-1-digit multiplication. … [T]he basic idea is always the same. We calculate from left to right, and add numbers as we go.

Once you've mastered the multiplication table up through 10, you can multiply any two 1-digit numbers together. The next step is to multiply 2- and 3-digit numbers by 1-digit numbers. As we'll see, these 2-by-1s and 3-by-1s are the essential building blocks to all mental multiplication problems. Once you've mastered those skills, you will be able to multiply any 2-digit numbers.

We know how to multiply 1-digit numbers by numbers below 20, so let's warm up by doing a few simple 2-by-1 problems. For example, try 53 × 6. We start by multiplying 6 × 50 to get 300, then keep that 300 in mind. We know the answer will not change to 400 because the next step is to add the result of a 1-by-1 problem: 6 × 3. A 1-by-1 problem can't get any larger than 9 × 9, which is less than 100. Since 6 × 3 = 18, the answer to our original problem, 53 × 6, is 318.

Here's an area problem: Find the area of a triangle with a height of 14 inches and a base of 59 inches. The formula here is 1/2(*bh*), so we have to calculate 1/2 × (59 × 14). The commutative law allows us to multiply numbers in any order, so we rearrange the problem to (1/2 × 14) × 59. Half of 14 is 7, leaving us with the simplified problem 7 × 59. We multiply 7 × 50 to get 350, then 7 × 9 to get 63; we then add 350 + 63 = to get 413 square inches in the triangle. Another way to do the same calculation is to treat 59 × 7 as (7 × 60) – (7 × 1): 7 × 60 = 420 and 7 × 1 = 7; 420 – 7 = 413. This approach turns a hard addition problem into an easy subtraction problem. When you're first practicing mental math, it's helpful to do such problems both ways; if you get the same answer both times, you can be pretty sure it's right.

The goal of mental math is to solve the problem without writing anything down. At first, it's helpful to be able to see the problem, but as you gain skill, allow yourself to see only half of the problem. Enter the problem on a calculator, but don't hit the equals button until you have an answer. This allows you to see one number but not the other.

The **distributive law** tells us that 3 × 87 is the same as (3 × 80) + (3 × 7), but here's a more intuitive way to think about this concept: Imagine we have three bags containing 87 marbles each. Obviously, we have 3 × 87 marbles. But suppose we know that in each bag, 80 of the marbles are blue and 7 are crimson. The total number of marbles is still 3 × 87, but we can also think of the total as 3 × 80 (the number of blue marbles) and 3 × 7 (the number of crimson marbles). Drawing a picture can also help in understanding the distributive law.

Most 2-digit numbers can be factored into smaller numbers, and we can often take advantage of this.

We now turn to multiplying 3-digit numbers by 1-digit numbers. Again, we begin with a few warm-up problems. For 324 × 7, we start with 7 × 300 to get 2100. Then we do 7 × 20, which is 140. We add the first two results to get 2240; then we do 7 × 4 to get 28 and add that to 2240. The answer is 2268. One of the virtues of working from left to right is that this method gives us an idea of the overall answer; working from right to left tells us only what the last number in the answer will be. Another good reason to work from left to right is that you can often say part of the answer while you're still calculating, which helps to boost your memory.

Once you've mastered 2-by-1 and 3-by-1 multiplication, you can actually do most 2-by-2 multiplication problems, using the **factoring method**. Most 2-digit numbers can be factored into smaller numbers, and we can often take advantage of this. Consider the problem 23 × 16. When you see 16, think of it as 8 × 2, which makes the problem 23 × (8 × 2). First, multiply by 8 (8 × 20 = 160 and 8 × 3 = 24; 160 + 24 = 184), then multiply 184 × 2 to get the answer to the original problem, 368. We could also do this problem by thinking of 16 as 2 × 8 or as 4 × 4.

For most 2-by-1 and 3-by-1 multiplication problems, we use the **addition method**, but sometimes it may be faster to use subtraction. By practicing these skills, you will be able to move on to multiplying most 2-digit numbers together. ■

Important Terms

addition method: A method for multiplying numbers by breaking the problem into sums of numbers. For example, 4 × 17 = (4 × 10) + (4 × 7) = 40 + 28 = 68, or 41 × 17 = (40 × 17) + (1 × 17) = 680 + 17 = 697.

distributive law: The rule of arithmetic that combines addition with multiplication, specifically $a \times (b + c) = (a \times b) + (a \times c)$.

factoring method: A method for multiplying numbers by factoring one of the numbers into smaller parts. For example, 35 × 14 = 35 × 2 × 7 = 70 × 7 = 490.

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 2.

Julius, *More Rapid Math Tricks and Tips: 30 Days to Number Mastery.*

———, *Rapid Math Tricks and Tips: 30 Days to Number Power.*

Kelly, *Short-Cut Math.*

Problems

Because 2-by-1 and 3-by-1 multiplication problems are so important, an ample number of practice problems are provided. Calculate the following 2-by-1 multiplication problems in your head using the addition method.

1. 40 × 8

2. 42 × 8