Divide and Conquer

Lecture 4

When I was a kid, I remember doing lots of 1-digit division problems on a bowling league. If I had a score of 45 after three frames, I would divide 45 by 3 to get 15, and would think, "At this rate, I'm on pace to get a score of 150."

We begin by reviewing some tricks for determining when one number divides evenly into another, then move on to 1-digit division. Let's first try 79 ÷ 3. On paper, you might write 3 goes into 7 twice, subtract 6, then bring down the 9, and so on. But instead of subtracting 6 from 7, think of subtracting 60 from 79. The number of times 3 goes into 7 is 2, so the number of times it goes into 79 is 20. We keep the 20 in mind as part of the answer. Now our problem is 19 ÷ 3, which gives us 6 and a remainder of 1. The answer, then, is 26 with a remainder of 1.

We can do the problem 1234 ÷ 5 with the process used above or an easier method. Keep in mind that if we double both numbers in a division problem, the answer will stay the same. Thus, the problem 1234 ÷ 5 is the same as 2468 ÷ 10, and dividing by 10 is easy. The answer is 246.8.

With 2-digit division, our rapid 2-by-1 multiplication skills pay off. Let's determine the gas mileage if your car travels 353 miles on 14 gallons of gas. The problem is 353 ÷ 14; 14 goes into 35 twice, and 14 × 20 = 280. We keep the 20 in mind and subtract 280 from 353, which is 73. We now have a simpler division problem: 73 ÷ 14; the number of times 14 goes into 73 is 5 (14 × 5 = 70). The answer, then, is 25 with a remainder of 3.

Let's try 500 ÷ 73. How many times does 73 go into 500? It's natural to guess 7, but 7 × 73 = 511, which is a little too big. We now know that the quotient is 6, so we keep that in mind. We then multiply 6 × 73 to get 438, and using complements, we know that 500 – 438 = 62. The answer is 6 with a remainder of 62.

We can also do this problem another way. We originally found that 73 × 7 was too big, but we can take advantage of that calculation. We can think of the answer as 7 with a remainder of –11. That sounds a bit ridiculous, but it's the same as an answer of 6 with a remainder of 73 – 11 (= 62), and that agrees with our previous answer. This technique is called overshooting. With the problem 770 ÷ 79, we know that 79 × 10 = 790, which is too big by 20. Our first answer is 10 with a remainder of –20, but the final answer is 9 with a remainder of 79 – 20, which is 59.

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A 4-digit number divided by a 2-digit number is about as large a mental division problem as most people can handle. Consider the problem 2001 ÷ 23. We start with a 2-by-1 multiplication problem: 23 × 8 = 184; thus, 23 × 80 = 1840. We know that 80 will be part of the answer; now we subtract 2001 – 1840. Using complements, we find that 1840 is 160 away from 2000. Finally, we do 161 ÷ 23, and 23 × 7 = 161 exactly, which gives us 87 as the answer.

The problem 2012 ÷ 24 is easier. Both numbers here are divisible by 4; specifically, 2012 = 503 × 4, and 24 = 6 × 4. We simplify the problem to 503 ÷ 6, which reduces the 2-digit problem to a 1-digit division problem. The simplified problem gives us an answer of 83 5/6; as long as this answer and the one for 2012 ÷ 24 are expressed in fractions, they're the same.

To convert fractions to decimals, most of us know the decimal expansions when the denominator is 2, 3, 4, 5 or 10. The fractions with a denominator of 7 are the trickiest, but if you memorize the fraction for 1/7 (0.142857…), then you know the expansions for all the other sevenths fractions. The trick here is to think of drawing these numbers in a circle; you can then go around the circle to find the expansions for 2/7, 3/7, and so on. For example, 2/7 = 0.285714…, and 3/7 = 0.428571….

When dealing with fractions with larger denominators, we treat the fraction as a normal division problem, but we can occasionally take shortcuts, especially when the denominator is even. With odd denominators, you may not be able to find a shortcut unless the denominator is a multiple of 5, in which case you can double the numerator and denominator to make the problem easier.

Keep practicing the division techniques we've learned in this lecture, and you'll be dividing and conquering numbers mentally in no time. ■

Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 5.

Julius, *More Rapid Math Tricks and Tips: 30 Days to Number Mastery.*

Kelly, *Short-Cut Math.*

Problems

Determine which numbers between 2 and 12 divide into each of the numbers below.

1. 4410

2. 7062

3. 2744

4. 33,957

Use the create-a-zero, kill-a-zero method to test the following.

5. Is 4913 divisible by 17?

6. Is 3141 divisible by 59?