# Calendar Calculating

## Lecture 10

**Sometimes people ask me the days of the week of ancient history, like what day of the week was January 1 in the year 0? The answer is "none of the above," since prior to the 3rd century, most places did not have seven days of the week. Instead, the situation was like what the Beatles once described as "Eight Days a Week."**

In this lecture, we'll learn how to figure out the day of the week of any date in history. Once you've mastered this skill, you'll be surprised how often you use it. Starting with the year 2000, every year gets a code number. The code for 2000 is 0. The codes for Monday through Saturday are 1 through 6; the code for Sunday is 7 or 0. There are also codes for every month of the year: 6 (Jan.), 2 (Feb.), 2 (March), 5 (April), 0 (May), 3 (June), 5 (July), 1 (Aug.), 4 (Sept.), 6 (Oct.), 2 (Nov.), 4 (Dec.). In a **leap year**, January is 5 and February is 1.

It's helpful to develop a set of mnemonic devices to establish a link in your mind between each month and its code. For example, January might be associated with the word WINTER, which has the same number of letters as its code; February is the second month, and its code is 2; and so on.

To determine the day of the week for any year, we use this formula: month code + date + year code. For the date January 1, 2000, we go through these steps: The year 2000 was a leap year, so the month code for January is 5; add 1 for the date and 0 for the year. Those numbers sum to 6, which means that January 1, 2000, was a Saturday. If the sum of the codes and date is 7 or greater, we subtract the largest possible multiple of 7 to reduce it.

For the year 2001, the year code changes from 0 to 1; for 2002, it's 2; for 2003, it's 3; for 2004, because that's a leap year, the code is 5; and for 2005, the code is 6. The year 2006 would have a code of 7, but because we subtract 7s in the process of figuring out dates, we can subtract 7 here and simplify this code to 0.

The formula for determining the code for any year from 2000 to 2099 is: years + leaps – multiples of 7. Let's try the year 2025. We first plug the last two digits in for years. To figure out the leaps, recall that 2000 has a year code of 0. After that, the calendar will shift once for each year and once more for each leap year. By 2025, the calendar will have shifted 25 times for each year, plus once more for each leap year, and there are six leap years from 2001 to 2025 (years ÷ 4, ignoring any remainder). Thus, we add 25 + 6 = 31, then subtract the largest possible multiple of 7: 31 – 28 = 3, which is the year code for 2025.

**Determining the year code is the hardest part of the calculation, so it helps to do that first.**

Determining the year code is the hardest part of the calculation, so it helps to do that first. There is also a shortcut that comes in handy when the year ends in a high number. Between 1901 and 2099, the calendar repeats every 28 years. Thus, if you have a year such as 1998, you can subtract any multiple of 28 to make that number smaller, and the calendar will be exactly the same.

The general rule for leap years is that they occur every 4 years, with the exception that years divisible by 100 are not leap years. An exception to this exception is that if the year is divisible by 400, then it is still a leap year.

The year 1900 has a code of 1, 1800 is 3, 1700 is 5, and 1600 is 0. To determine the code for a year in the 1900s, the formula is years + leaps + 1 – multiples of 7; for the 1800s, years + leaps + 3 – multiples of 7; for the 1700s, years + leaps + 5 – multiples of 7; and for the 1600s, years + leaps – multiples of 7.

The calculations we've done all use the **Gregorian calendar**, which was established by Pope Gregory XIII in 1582 but wasn't universally adopted until the 1920s. Before the Gregorian calendar, European countries used the Julian calendar, established by Julius Caesar in 46 B.C. Under the Julian calendar, leap years happened every four years with no exceptions, but this created problems because the Earth's orbit around the Sun is not exactly 365.25 days. For this reason, we can't give the days of the week for dates in ancient history. ■

## Important Terms

**Gregorian calendar**: Established by Pope Gregory XIII in 1582, it replaced the Julian calendar to more accurately reflect the length of the Earth's average orbit around the Sun; it did so by allowing three fewer leap years for every 400 years. Under the Julian calendar, every 4 years was a leap year, even when the year was divisible by 100.

**leap year**: A year with 366 days. According to our Gregorian calendar, a year is usually a leap year if it is divisible by 4. However, if the year is divisible by 100 and not by 400, then it is not a leap year. For example, 1700, 1800, and 1900 are not leap years, but 2000 is a leap year. In the 21st century, 2004, 2008, …, 2096 are leap years, but 2100 is not a leap year.

## Suggested Reading

Benjamin and Shermer, *Secrets of Mental Math: The Mathemagician's Guide to Lightning Calculation and Amazing Math Tricks*, chapter 9.

Duncan, *The Calendar: The 5000-Year Struggle to Align the Clock and the Heavens—and What Happened to the Missing Ten Days.*

Reingold and Dershowitz, *Calendrical Calculations: The Millennium Edition.*

## Problems

Here are the year codes for the years 2000 to 2040. The pattern repeats every 28 years (through 2099). For year codes in the 20th century, simply add 1 to the corresponding year code in the 21st century.

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|---|---|---|---|---|---|---|---|---|---|---|
| 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| 0 | 1 | 2 | 3 | 5 | 6 | 0 | 1 | 3 | 4 | 5 |
| | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| | 6 | 1 | 2 | 3 | 4 | 6 | 0 | 1 | 2 | 4 |
| | 2021 | 2022 | 2023 | 2024 | 2025 | 2026 | 2027 | 2028 | 2029 | 2030 |
| | 5 | 6 | 0 | 2 | 3 | 4 | 5 | 0 | 1 | 2 |
| | 2031 | 2032 | 2033 | 2034 | 2035 | 2036 | 2037 | 2038 | 2039 | 2040 |
| | 3 | 5 | 6 | 0 | 1 | 3 | 4 | 5 | 6 | 1 |